Lecture No. # 21
**Burn Rate of Solid Propellants and Equilibrium Pressure in Solid Propellant Rockets**

Good afternoon. We will quickly recap what we were doing earlier, and then go to the burn rate of composite propellants, and also address whether there are some particular values of n, the exponent in burn rate law r is equal to a p^n ,which is necessary, when we design the solid propellant rockets.
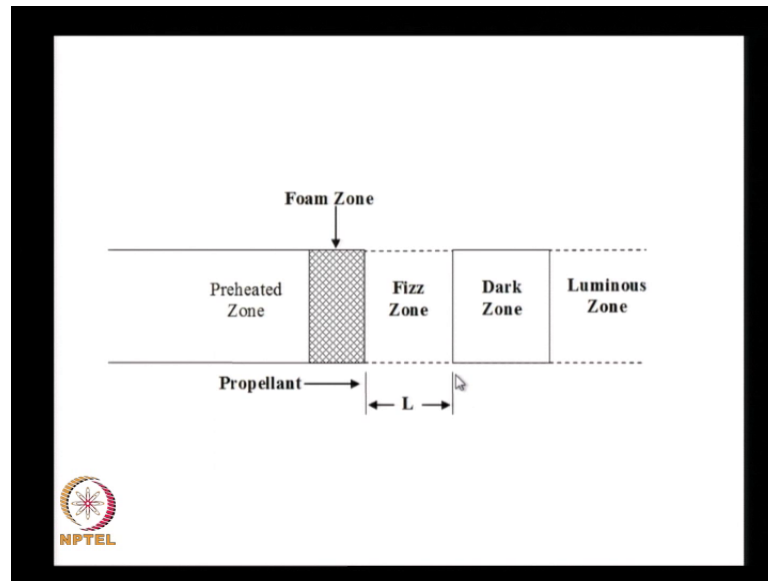
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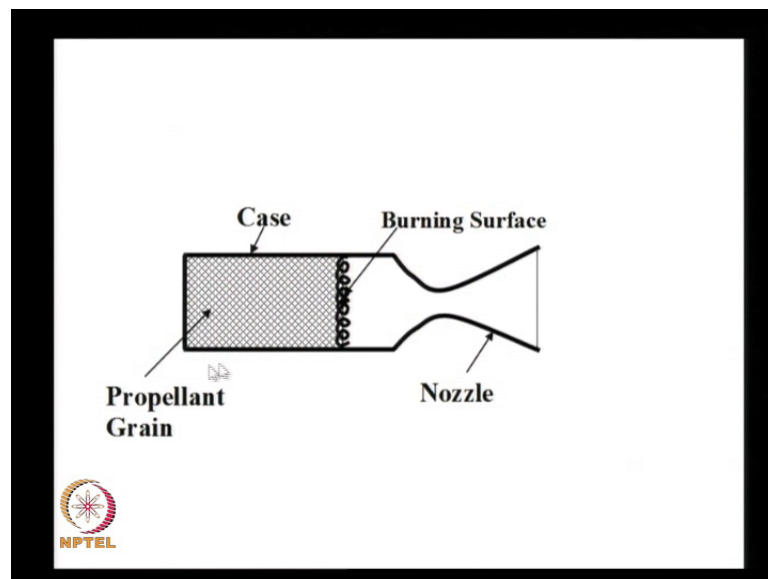
What we did in the last class was that we considered the propellant grain which is inside the case and the nozzle, as shown above, which constitutes the solid propellant rocket. We ignite the surface of the propellant, the surface burns or regresses at a particular rate which we call as the burn rate.
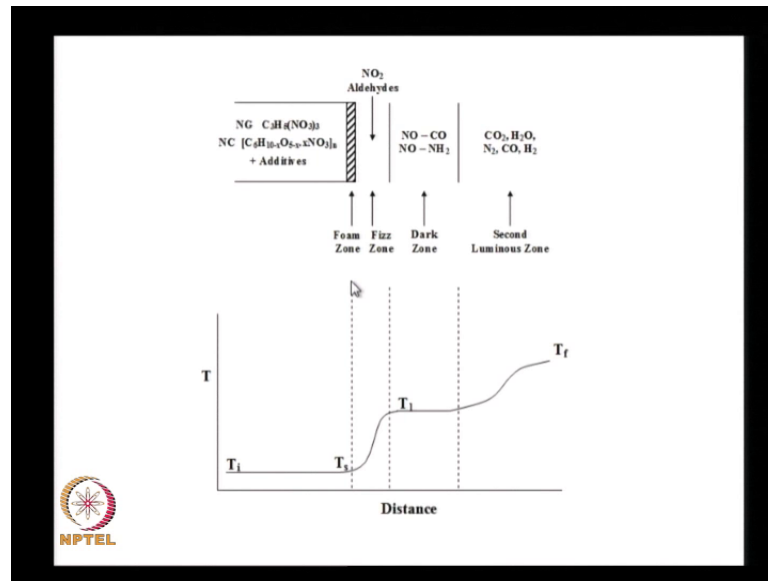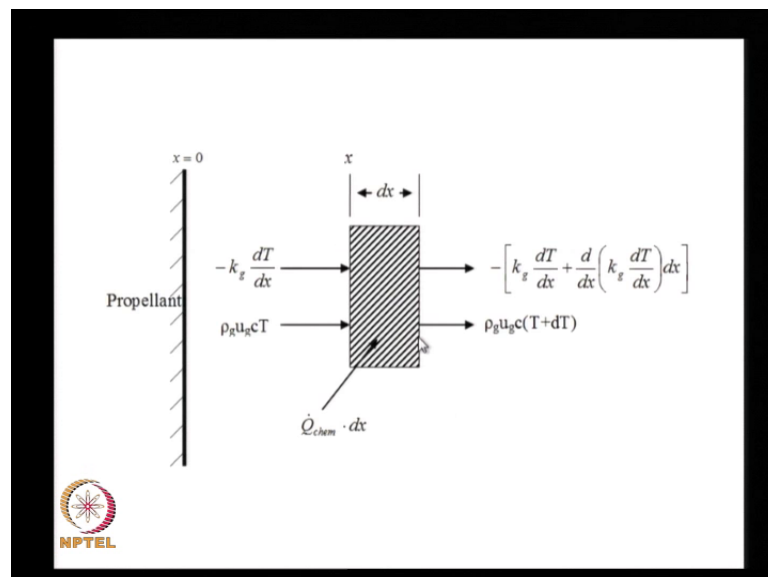
And what did we do for the case of double base propellants? We told that we have nitrocellulose and nitroglycerin in the double base propellant, we had NO_2, aldehydes and also some other constituents, which are formed during the combustion. The temperature increased from the surface temperature to the temperature at the edge of the fizz zone. This was the foam zone, which is preheated solid surface of the propellant from which the gases got generated. And then you had the fizz zone where the temperature went to T_1; thereafter it remained constant at this value in this dark zone where essentially the reaction between NO and NH_2, NO and CO do not generate much heat. But thereafter the reactions generate heat and you get CO_2, CO, H_2O and in this zone the temperature went to the final value T_f. This zone, this particular zone of dark zone is there only for pressures less than around 10 MPa or 100 atmospheres. And at higher values of pressures, what we got was that the temperature directly goes to the value of T_f.

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Foam Zone

Preheated Zone | Fizz Zone | Dark Zone | Luminous Zone

Propellant

← L →

NPTEL

Having said that we wanted to translate this information into the burn rate and therefore we again looked at the preheated zone, a foam zone, a fizz zone, a dark zone and a luminous zone as shown in the slide above.
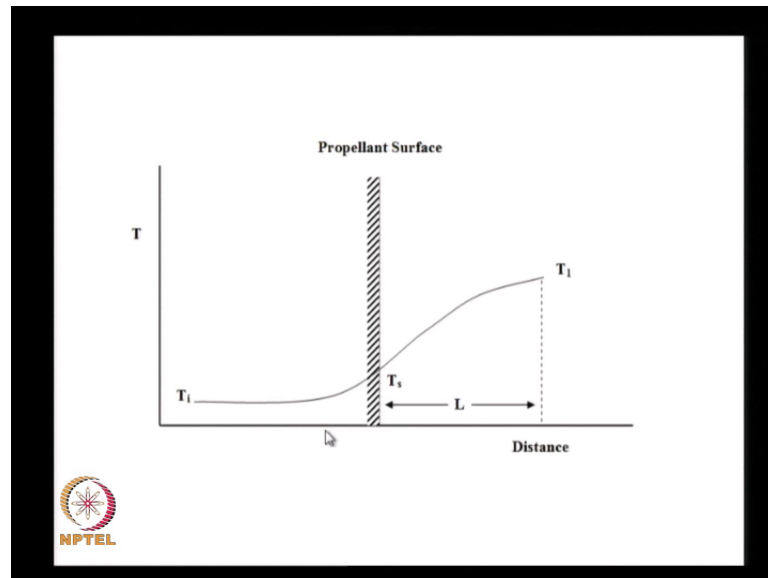
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$x = 0$

x

← dx →

Propellant

$-k_g \dfrac{dT}{dx}$

$-\left[k_g \dfrac{dT}{dx} + \dfrac{d}{dx}\left(k_g \dfrac{dT}{dx}\right)dx\right]$

$\rho_g u_g c T$

$\rho_g u_g c(T + dT)$

$\dot{Q}_{chem} \cdot dx$

NPTEL

We wrote the equation in this particular zone, the fizz zone, and what we wrote was for a small element in the fizz zone. This was at x from the surface of the propellant though the, fizz zone extended from x = 0 upto x = L. The length of the fizz zone is a fraction of a mm. We just show it exaggerated to be able to visualize the phenomenon. We had this

small element here with unit surface area and we said heat enters and heat leaves it. The heat leaving is in excess by some amount. The enthalpy, per unit volume is equal to ρ_g into u_g into specific heat into temperature. The gases leave the element at a temperature T plus dT and therefore, we equated the excess heat transfer and the heat generated in the fizz zone and we were able to write the equation for heat balance.
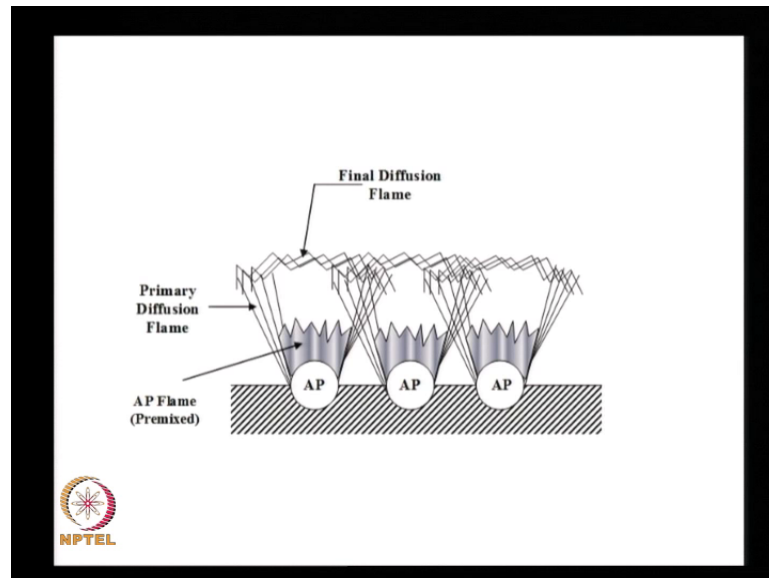
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And based on this equation, we said we could get the temperature profiles for given values of u_g, p_g and ρ_g. We related the burn rate to the pressure.

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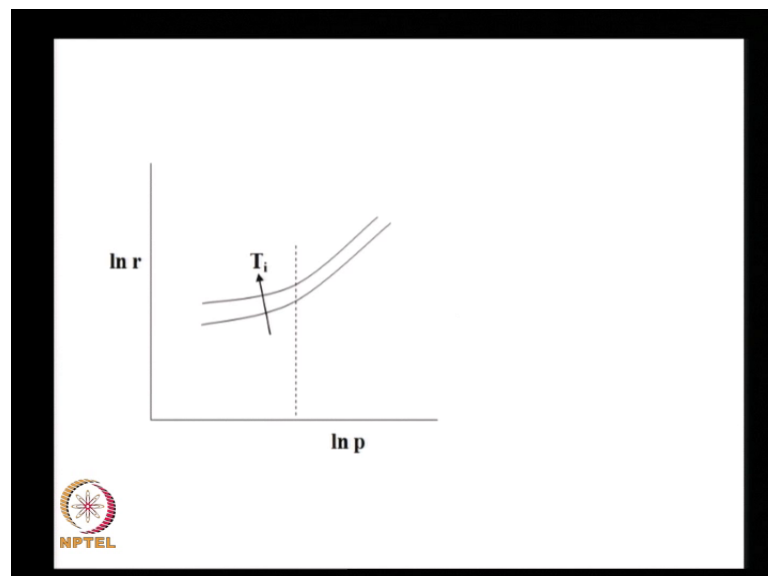
We also found that in the region of pressure for which we have the dark zone, the variation of the logarithm of the burn rate versus logarithm of pressure was slightly less, because the temperature T_1 which supplies heat to the surfaces is lower than the final temperature T_f supplying heat in the fizz zone. And therefore, we have low exponent here at low pressures a higher exponent at higher pressures as shown.

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We would like to do a similar analysis for a composite propellant. But a composite propellant is distinctly different from a double base propellant. What is the distinction between the two?

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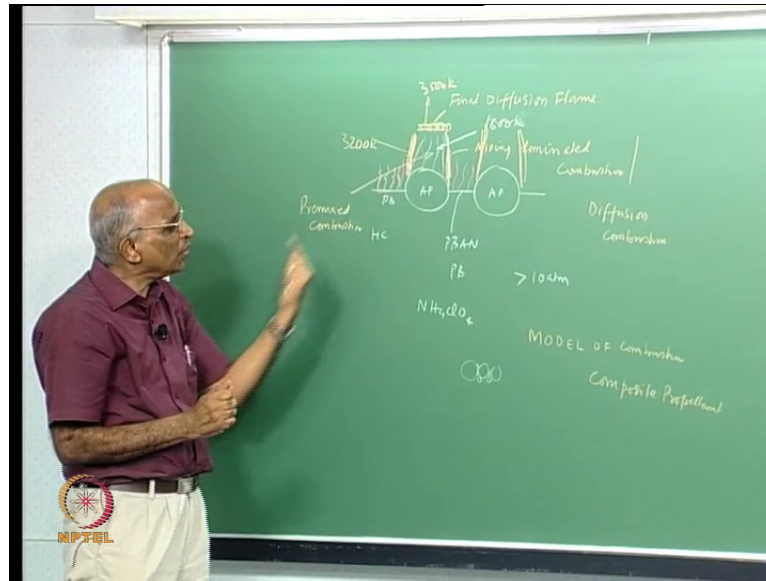
Let us put it on the board. We have in composite propelalnts, AP in the form of crystals. And this AP is contained in the polybutadiene. We could also have aluminum just like we saw in this sparkler when there is metal in it and the temperatures are much higher. It is much hotter and that is why metals like aluminum are introduced in it. Therefore, you have something like solid crystals of AP and in between may be it is bounded by the polybutadiene, let say PBAN. Instead of PBAN we could have HTPB or some other polybutadiene. It could be polybutadiene acrylic acid acrylonitrile or it could be HTPB, it could be CTPB and these are the fuels or the binders.

And when we supply heat and start making the propellant to burn, AP is NH_4ClO_4. It contains oxygen, it contains hydrogen, therefore, we could get a flame something like a monopropellant flame; this AP itself will burn. But the hydrocarbon which is over adjacent to it may be the polybutadiene cannot burn because it is just hydrogen and carbon, it does not contain oxygen. Therefore, the volatiles of hydrogen and carbon from the polybutadiene are generated above it.

Therefore, let me use a slightly different color chalk over here to show this; may be this is the fuel vapor which is coming out; this is a monopropellant flame and this flame is typically at a temperature of around 1600 Kelvin. Maybe whenever we are talking of the combustion taking place we presume that combustion takes place at a pressure of the order of greater than around 10 atmospheres. Because rockets do not generally operate at

chamber pressure less than this value. Therefore, we are talking of something like 1 MPa pressure. And at this pressure may be we have the flame of AP which is coming over here, we have the vapor of fuel coming over here and this is the picture that we can visualize in our mind of what takes place.

Mind you, in practice when I look at the propellant, I will have AP all over the place. I have small particles of AP and I could have aluminum in between. I have all these particles and we are just magnifying this zone and expanding this distance and putting this scheme to be able to formulate a model. We are interested in having a model, which we can express as an equation and solve the equation for burn rate. That is we have fuel vapor coming over here. We have oxidizer rich vapor burning and when the fuel vapor the burning oxidizer rich gases meet a little later on, we could have something like a zone of burning again. We have a zone over here wherein again burning will take place between the oxidizer rich flame which is coming from AP. and here mixing will be taking place between the oxidizer rich gases and the fuel vapor or a mixing dominated combustion will take place. Whereas in the AP flame above the AP crystal, it is just premixed.

Whatever AP decomposes, its vapor comes out as decomposed products of combustion. i.e.,as premixed combustion; there is no mixing involved. At the edges of the AP crystal, the fuel vapor is formed from the binder and the oxidizer rich vapor are transported from AP flame. They meet, mix and burn and therefore we call this mixing dominated combustion as diffusion flame. Essentially the flame is controlled by diffusion process. Therefore, above the vapor we have premixed combustion, at the edges of AP wherein the fuel vapor comes and meets these hot gases we have mixing dominated combustion. And the temperatures here are typically at these pressures - greater than about 10 atmospheres - around something like 3200 Kelvin. Well, the gases are still reacting, these reacting gases meet the oxidizer rich gases again and products of combustion are again formed. Products from this lower diffusion flame come over here again meet the oxidizer rich gases and therefore we have another mixing combustion here on top. This is the final diffusion flame. And the temperature here is typically around 3500 Kelvin.

Therefore what is the picture that we are trying to create? We are trying to draw a picture wherein we picture in our mind viz., , the mind model. We have AP giving the oxidizer rich products, which are essentially decomposition products and are premixed. They

form a diffusion flame at the edges. The first diffusion flame is formed at the edges and we again get a diffusion flame over the AP flame, a final flame. And typically, the temperature of the first diffusion flame is about 3200 Kelvin, while for the AP flame the temperature is around 1000 Kelvin. The highest temperature is reached in the final diffusion flame. This could be my mental picture or the model of combustion or burning taking place in a composite propellant.

Well, this is distinctly different from what we had for the double base propellant, wherein we had aldehydes, NH_2, NO reacting to give a fizz zone, you had a dark zone, you had the second luminous zone. In the case of composite propellants, we have a premixed zone, a first diffusion flame and a second diffusion flame. Now, how do we solve this? It becomes a little bit tricky.

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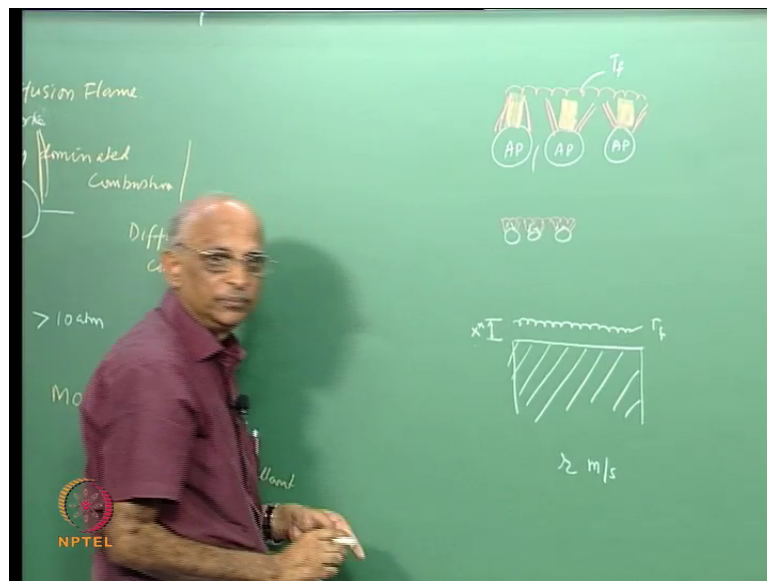
For the composite propellant, it is becoming a little complicated. Because we have a series of oxidizer particles, we have something like premixed combustion taking place just above. It is associated with low temperature. We have something like a diffusion flame over at the several edges above AP crystals over here, may be something like this as shown. And then we have a final diffusion flame on top.

Now, what is it that we are discussing? We have AP here, the polybutadiene over here and this is what I expect. If we have AP of smaller particle sizes, the zones will be something like this near to the surface and the final diffusion flame and heat release zone

will be nearer to the surface. In other wards it tells us that the height of the final diffusion flame will be proportional to the size of AP. If AP size is less, then the final diffusion term will be nearer. We still have these diffusion flames over here, the premixed flame over here. Therefore, AP size will decide the distance at which the final diffusion flame or heat release will take place.

Now, let us go through the assumptions. Let us assume that the final temperature of the combustion products is T_f. And now I want make an assumption involving the three flames and how do we do it? We cannot have all these premixed and diffusion flames and solve the equations. Why not make an assumption? I say this my propellant surface, it is a composite propellant; I know the size of the AP crystals are around 300 microns, the fine AP is around 30 microns. We cannot see that closely at the micro level. Now, we tell that the final diffusion flame formed is at a temperature T_f and it is formed at some distance away something like let say X*, a standoff distance. It forms after some distance from the propellant surface. Why does it form after some distance? Why does it not form at the surface?

Well, we first have the A P flame first which forms over AP crystals, then the mixing takes place then the secondary mixing takes place and then the final diffusion flames forms at some distance away. But this distance again is a fraction of an mm or so in practice, just like in the sparkler. It looks as if the burning is at the surface, but if we take a magnifying glass and see well there is a distance above the surface at which the burning takes place.

Therefore, now we are in a position to write an equation for this simplified scheme. The final temperature is T_f. This is the propellant surface. We are interested in writing an equation for the burn rate so many meters per second.

We can readily do this. Let us let us look at the slide again. This is where we have three AP crystals over here. We have the shaded portion which is the monopropellant flame or the premixed flame. And then I have the vapor coming from this hydrocarbon, this is pure vapor when it mixes with the oxidizer rich gases, I have a zone of diffusion flame over here. And the products from this diffusion flame and the AP monopropellant give me a final diffusion flame here. And the distance between this final diffusion flame and the surface is what we call as the standoff distance or the height X*.

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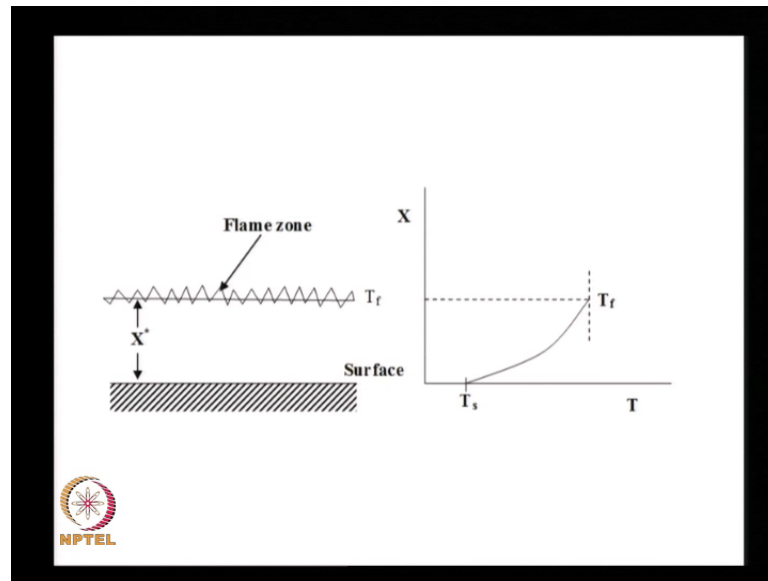
If this part is clear, let us go to the next slide. We say that this my final zone of combustion at a standoff distance X star and this is my surface and the surface temperature is Ts. Mind you the propellant get heated here from initial temperature to the surface temperature and in this gas zone, I say that the temperature varies between Ts to the final value T_f. This is a mental picture that we use or model for the combustion or burning of a composite propellant. At this point I thought I should illustrate what happens, many of you are working in combustion. These are the experiments which we were do in the lab; I have something like butane gas coming over here.

And when we ignite butane gas issuing from an orifice, at the orifice itself we seem to have a flame like this. We increase the velocity of butane gas flows and combustion takes off after a certain distance. I have mixing taking place in this zone and thereafter I have combustion. At still at higher velocity, we have this zone of standoff wherein mixing taking place is at high velocity. The heat is insufficient to propagate the flame into the stand off distance. We have something like a standoff distance here. Well, the standoff distance is something on these lines, but not precisely because of velocity in the case of the composite propellants. But it takes a certain distance for the final diffusion flame to form.
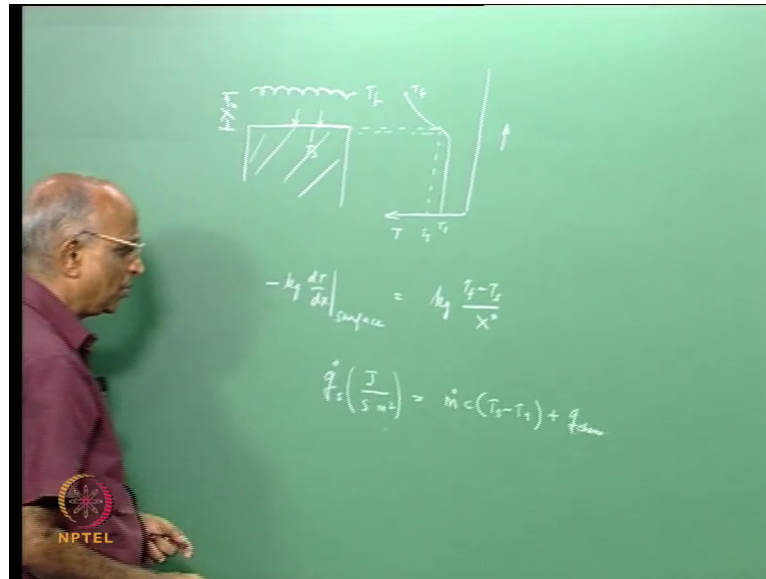
Therefore, I would like to write an equation for this configuration. We have the propellant surface here. We have the final diffusion flame over here at a distance X^* away. The temperature of the final diffusion flame is T_f and the surface temperature is T_s. If we were to plot the temperature distribution along this stand off distance; this is the propellant surface at temperature T_s; this as the distance over here; in the depth of the propellant the temperature is the initial value that is the initial temperature. And then what happens near the surface, the temperature increases to T_s at the surface and then in the flame zone, it further increases to something like T_f. Mind you this is my increasing direction of T and this is the distance from the in-depth of the propellant wherein at the depth of the propellant is still at the initial value of the temperature. We have an increasing temperature to T_s and then goes to the gas value T_f.
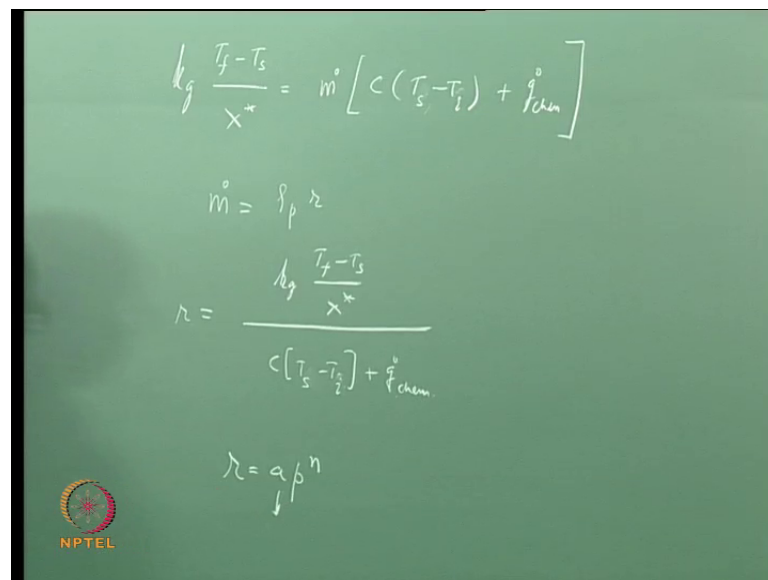
Therefore if we want to know what is the heat, which is coming on the surface, we need the value of thermal conductivity of the gas above the surface kg × dT/dX at the surface of the propellant. And the gradient in temperature can be approximated by the temperature increase to T_f from the surface value T_s over a distance of X star to be linear. We use this X^* as the flame standoff distance and we shall see what it represents subsequently. The heat transfer to the surface per unit area is equal to kg ×(T_f – T_s)/X^*. This is a very simple way of estimation where we assume the gradient to be linear. There are various flame models which are used for describing the combustion of composite

propellants. But this particular one was formulated by Professor Hermance at the University of Waterloo in Canada. It is very simple and is very illustrative.

There are various models; we have the granular diffusion model and other models, but this simple model gives a reasonable picture of the combustion behavior. This is the rate of heat which is coming to the surface equal to $q°_s$. We consider unit surface area and therefore it is so much Joules per second per meter square. This is the unit of heat flux. Mind you, we consider unit surface area. Where does this heat go? The heat goes to increase the surface temperature from the initial value Ti to the Ts. If we say that the rate at which the propellant regresses or burns is equal to m° then what happens? The heat corresponding to temperature increase into the specific heat into it has to be accounted for.

Suppose, at the surface I have some endothermic reactions taking place because I have the binder, which we said is polybutadiene. It has to get heated. It also requires heat to vaporize it. We need to supply this rate of heat $q°_{chem}$ to it. So, some endothermic heat of reaction q chemical is supplies to the surface.

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Therefore, we can write the heat balance equation as thermal conductivity of the gas kg × the flame temperature − the surface temperature of the propellant ÷ the flame standoff distance X* = mass which is getting released at the surface × the specific heat of the propellant × the surface temperature changes to Ts from the initial value Ti plus the heat

which gets released at the surface might be an endothermic reaction at the surface q°_{chem}. And therefore, this gives us the energy balance equation. Namely, this is the heat, which gets transmitted from the flame to the surface. And this helps to increase the surface temperature to Ts from the initial value and also supplies the energy required to vaporize this surface or to convert the surface from solid to vapor through a set of endothermic reactions.

Therefore, now, using this particular equation, if we have write the value m°: , $m^\circ = kg(T_f{-}Ts)/X^* \div \{Cp_s(Ts{-}Ti) + q_{chem}\}$ .. Actually, in this we need not even put q° here because it is just the magnitude of heat release and therefore, maybe we should do away with the dot here. And just write $kg(T_f{-}Ts)/X^* \div$ the sensible heat required plus the endothermic reactions at the surface.

Now, we would like to discuss this particular value of mass release rate. The mass release rate at the surface m° per unit area comes from regression of the propellant and is equal to ρ_p into the burn rate r. Or rather the burn rate r can now be written as $r = [kg(T_f {-}Ts)/X^*]/ \{\rho_p [C_s(Ts{-}Ti) + q_{chem}]\}$. The value of r is in meters per second, ρ_p in kilograms per meter cube.

Now, let us examine this equation under different conditions. We find that there is a flame, which is standing off at a distance X^* from the propellant surface. Let us again sketch it. We have seen this sketch several times during this class. We have the flame at a temperature Tf standing at a distance X^* away from the propellant surface. If the size of the ammonium perchlorate particles are small, then the mixing will take place immediately near to the surface and the flame will be near. X star would then be small. This is point one. If the chamber pressure, on the other hand, or if the pressure at which the burning rate is evaluated is high, we have more number of molecules which can react and therefore, the X^* will come down.

Therefore, what is the inference which can we can draw? Well, if we have fine AP particles, fine ammonium perchlorate particles in the composite propellant, then it is quite possible that X^* will be smaller and the burn rate will be higher. If the value of pressure at which the burning takes place is somewhat higher or large let us say, then chemical reactions get finished in a very short time and therefore, X^* will be small. And

if X* is small well, the burn rate will be higher. Therefore, a higher value of chamber pressure also leads to a higher value of the burn rate r.

If the pressure is higher, the thermal conductivity of the gas will be higher and therefore, again burn rate will be higher. ρ_p is the density of propellant and does not change. The surface temperature effects will come in, may be the endothermic reaction activation energy will come in, but we find that pressure is a major factor because pressure decides the value of X*.

When we did double base propellants, we found that the burn rate law could be expressed as in terms of Saint Robert's law as r is equal to a constant 'a' × pressure^n. We find here also as pressure increases the flame comes nearer and therefore, a similar law viz., $r = ap^n$ can be used to determine the burn rate of composite propellants.

Now, depending on the value of n the effects of pressure are modeled, but what is the value of 'a'? a will depend on the size of ammonium perchlorate particle size. The activation energy of the endothermic reactions and may be some of the compositional aspects like initial temperature of the propellant. As the initial temperature increases, we find that the denominator decreases and r increases. Therefore, the effect of the initial temperature and the activation energy would also influence the burn rate. The ammonium perchlorate particle size and the other parameters are embedded in the value of 'a'. Therefore, we can conclude that the same law $r = ap^n$ which describe the burn rate of double base propellants could also used can be used for composite propellants.

The burn rate of composite propellants can therefore be expressed in this particular form $r = ap^n$. Very simple;but a very illustrative derivation is used. We must be able to write such expressions for any system such as wood smoldering, may be carbon or may be charcoal burning.

Let us let us summarize it again. We said we had a flame zone at a distance X* away and can be assumed though we had two zones like premixed zone and the diffusion zones. And we got the expression for r and let us now go back and examine the value of r, how should it change with pressure? Could I rub this off? Is it is it clear?
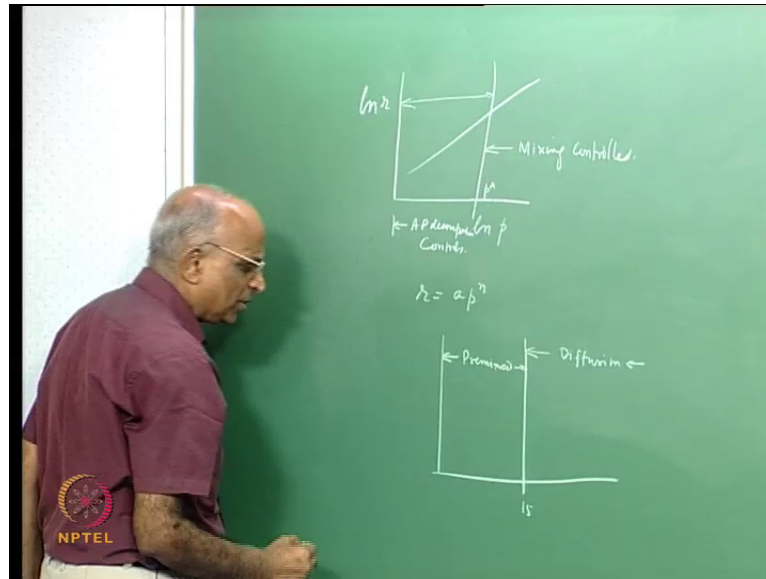
Now, we would like to use the equation and plot the value of the burn rate, logarithm of burn rate versus logarithm of pressure. Because we said that for composite propellant also the burn rate can be described by Vielle or the Saint Robert's law which is $r = ap^n$. Well logarithm of r as a function of logarithm of pressure should be a straight line as per this equation. But if we burn the propellant at low pressures, i.e., operate in the low pressure region of pressures less than some threshold value, p^*. We are yet to define this value of pressure; however, we say at low pressures. What happens to the burn rate at low values of pressure? May be AP will decompose and the hydrocarbon vapors are being generated.

And what happens to the rate of decomposition at low pressures? It is limited by the kinetics; why kinetics? We write $m°_{chem} = Ap^m \exp(-E/R_0T)$ and therefore, at lower pressure, we have lesser amount the decomposition and less of energy getting liberated. Or this becomes the controlling parameter because the decomposition generates vapor, but the vapors are in short supply. Therefore at low pressures, we presume that chemical kinetics and AP decomposition controls the rate of burning. Sufficient molecules are not available for the reaction.

When we talk of higher pressures; the diffusion process is independent of pressure and at higher pressures we get copious amount of fuel and oxidizer vapor coming out because the reactivity has increased. But the mixing of gases is independent of pressure and

therefore, mixing now is an impediment and controls the burning. And therefore, in this zone of high pressures, we have something like mixing controlling the final diffusion flame.

All what we now state is we may not get the same value of n over the entire region of pressure as was done earlier. At low pressures, we get premixing or a premixed flame dominating and this is the limiting factor, which controls the burn rate. Whereas, at high pressures, diffusion is limited and therefore it becomes the controlling factor limiting the burn rate. The diffusion process does not allow the availability of fuel and oxidizer vapors to mix and burn. Therefore we say that the burn rate at high pressures is diffusion or mixing controlled.

In other words, if we were to again re-plot burn rat edependence on pressure, we have a region in which we have premixed combustion controlling. And at higher pressures, we have something like diffusion mechanism controlling. And this pressure is of the order of 15 atmospheres or so. And what happens is whenever it is premixed control, we have a higher value of n because it goes the rate of a reaction goes as p to the power of an exponent And if it is diffusion controlled we have something like n which is very much lower.

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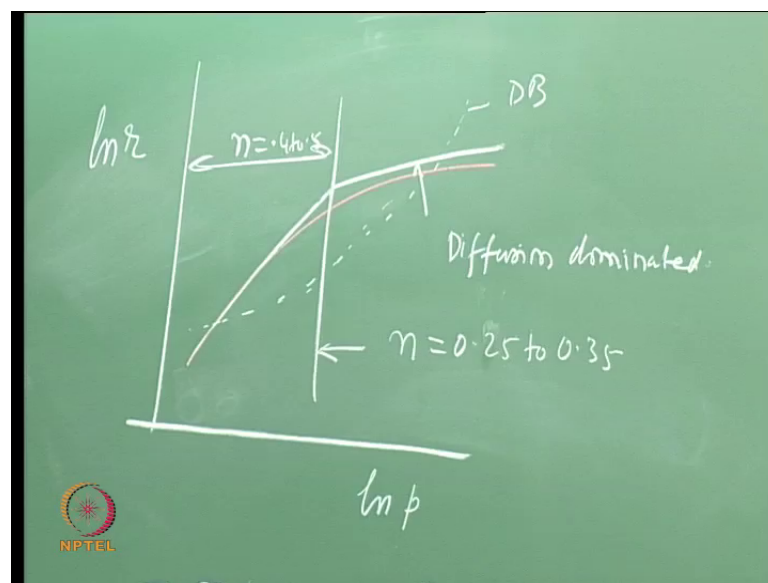
Therefore, for composite propellants, what happens? It is different from double base propellants in that if I were to plot the value of logarithm of burn rate versus logarithm of

pressure, I have something like high value of 'n' at the lower pressures and smaller value of 'n' at the higher pressures. Or rather the burn rate goes something like convex upwards. What did we have in the case of double base propellant? It was just the opposite; we had at higher values of 'n' at higher pressures due to the absence of the dark zone. In the case composite propellants, it is the reverse and we have at the lower pressures a higher value of n while at higher pressures, we have a lower value of n.

This distinction must be clear. But normally most of the rockets are operated at pressures excess of this threshold value of pressure. And therefore, we will say that n typically is around 0.25 to 0.35 for the composite propellants. In the lower pressure region, wherein n is nearer to something like 0.4 to 0.5, we have premixed combustion dominating. I show this in this particular slide wherein at a pressure less than a threshold value, we have premixed combustion dominating with a higher value of n followed by diffusion combustion dominating with a lower value of n.

With this background, let us determine the choice of 'n' necessary in the choice of a propellant.

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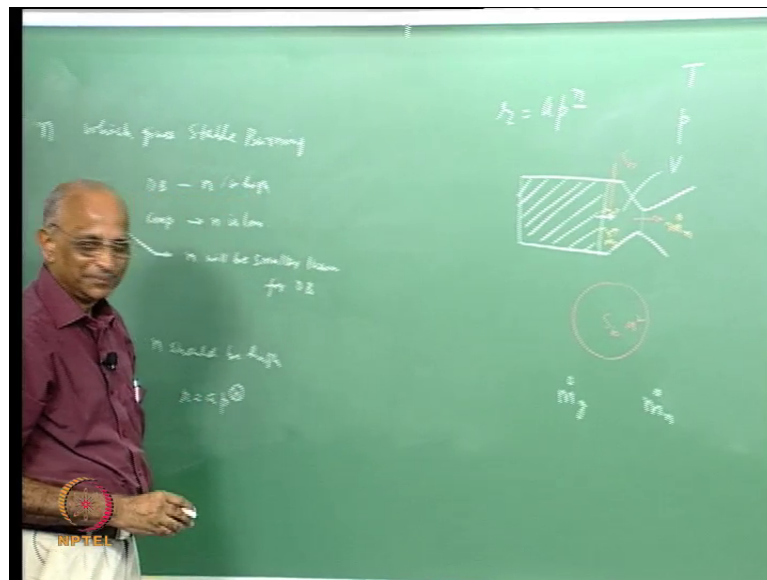
What must be the value of n which gives stable burning? Based on our understanding of the burn rates, can we recommend certain values on n? Before answering this question, what should be the burn rate law for a composite modified double base? I know for a double base propellant, the burn rate law. I know the law for a composite propellant. We

can say composite modified double base should incorporate both these feature and give something like average of this two. In other words, composite modified double base will give a value of n which is slightly lower than for double base, for double base you know it is all premixed combustion.

The value of 'n' is high for double base; for composite at the regions of interest 'n' is less. For the composite modified double base propelalnt "n' will be smaller than for double base propellant. If we were to consider something like nitramine propellants well, in nitramine propellants we had the explosive HMX, we had binder which were mixed together both of them are premixed and therefore the value of 'n' should be high. That means, in the burn rate law, $r = ap^n$, n is higher for nitramine propellants, for CMDB it is less than for double base propellants, but higher than for composite. In this way one could address the values of the burning rate index 'n'.

We should work out some numerical problems for burn rate and put things together. But what we have done is that for double base propellants, we looked at the evolving layers like fizz zone, dark zone and also the second luminal zone. For composite propellants, we used the simple Hermance model wherein we talk of a flame standing away from the surface by a standoff distance X*. We also know what this standoff means. When a gas burns, it take some time for the constituents to chemically react and form a flame.

If the burn rate r is equal to a p to the power n, can we now assess whether n should be small n or should be large? Should we have n equal to infinity or should we have n is equal to 0, what is the value, which will give proper burning in a rocket?

Let us consider a solid propellant rocket. Well! this is the propellant we have in a rocket as shown. This is the burning surface area Sb. In other words I have if we take a section over here, this is my surface, this is my burning surface area Sb meter square. And this is where the burning takes place and gas is getting released from the surface. And the gas gets pushed out through the particular nozzle.

Let us put some numbers down. We will call rate at which mass is getting generated from this surface area as equal to m° generated m°g. The rate at which gases are leaving the nozzle, we will call it as m°n.

Can we write an equation for the dependence on m°g and m°n? Let us consider the case wherein we have a certain volume. Let say this particular volume of the port be V. Let the pressure in this volume be p and the temperature of the gases be T. How do we write an equation for mass balance? Mass gets generated over here m°g it leaves the control volume or this volume at a rate m°n. What is the equation that we get?

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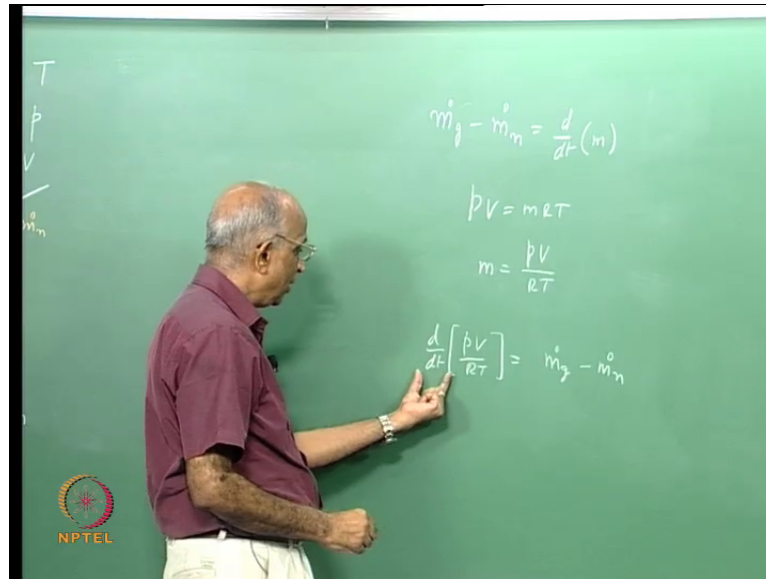
You immediately tell me that m°g −m°n = the rate of accumulation of the mass of gas in the particular volume or the chamber. What is the value of mass from gas equation? We have p V = m R T. The pressure is p, the volume is V and the temperature is T. Therefore, m = pV/RT. Therefore, we can now write d/dt [pV/ R T] = rate of accumulation of mass in the volume.. Is it alright?

Let us simplify this term before I come to the other terms? How will I simplify this particular term? Temperature is a constant viz., the flame temperature; pressure is a variable. As the propellant regresses the volume varies. Therefore, we have two variables pressure and volume while the temperature is constant.
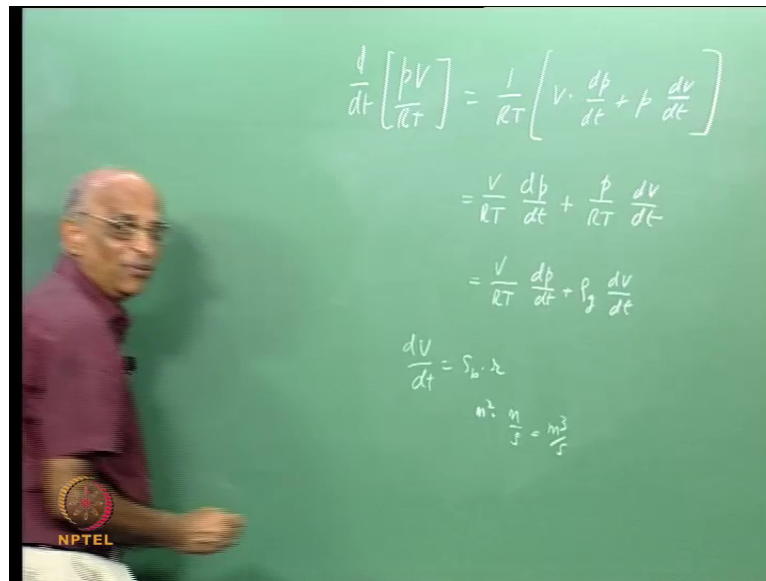
Therefore, for temperature being a constant or a specific gas constant R, we can write this equation $\frac{d}{dt}(pV/RT) = 1/RT \times \{V \times dp/dt + p \times dV/dt\}$. Again we simplify this and write it as $V/RT \times dp/dt + p/RT \times dV/dt$. What is p/RT equal to? Let us take a look at the gas equation or equation of state $pV = mRT$, giving p by RT as equal to mass/volume which is the gas density. We can write this as equal to $V/RT \times dp/dt$ + density of the gas $\rho_g \times dV/dt$.

What is dV by dt? Rate at which the volume is increasing and what is the rate at which the volume is increasing? The burning surface area is S_b, regression rate is r and therefore, dV/dt = burning surface area × the propellant burn rate so many meter^2 × meter/second, i.e., meter^3 by second. This is dV/dt. In one second, the distance would have been moved by r. Therefore, the increase in volume $dV/dt = r \times S_b$. Therefore, we write the left side of the equation as $V/RT \{ dp/dt$ plus the term $\rho_g \times S_b \times r\}$.
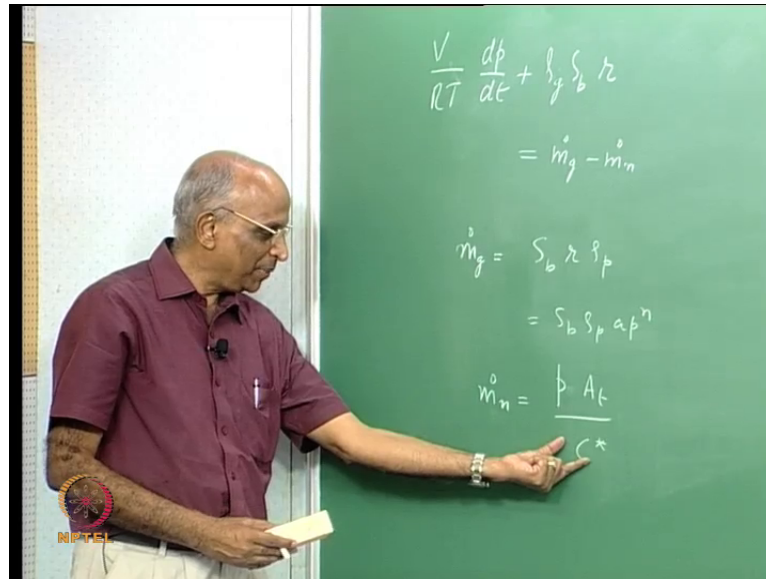
This is equal to what m°generated minus m°nozzle which is leaving.

The value m°g can be expressed in terms of pressure or in terms of burn rate? The burning surface area $S_b \times r \times \rho_p$ because $S_b \times r$ tells the volumetric rate at which the propellant is getting consumed and if it is multiplied by the propellant density ρ_p, it represents the mass at which is getting generated. This is equal to $S_b \times \rho_p \times a\,p^n$. Here the burn rate law $r = a\,p^n$ is used.

What is the rate at which mass is leaving the nozzle? We have been doing it all along. We had defined C* as solid propellant property that is $m°n = p_C\,A_t / C^*$. . C* star is the transfer function between chamber pressure and mass flow rate per unit throat area. And therefore, let us just substitute it and get the equation as:

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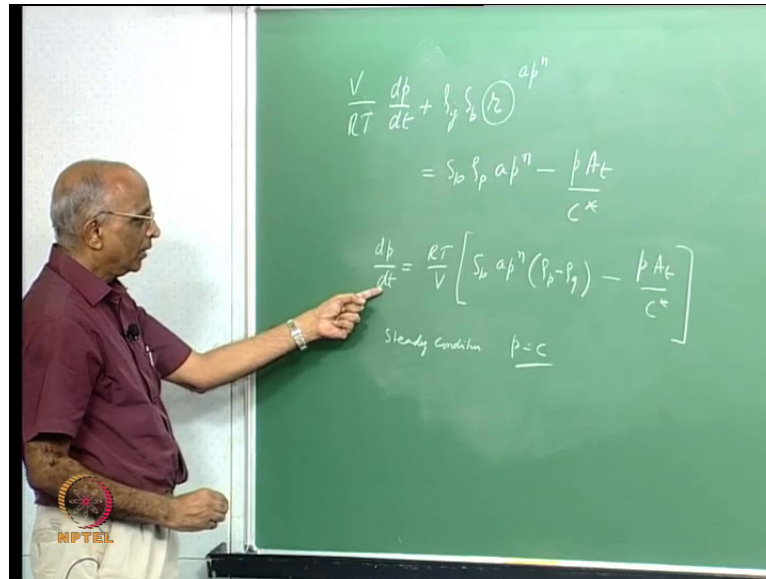
We substitute this again to get V/RT × [dp/dt] = Sb×ρ_p × a p^n − p ×At/C*. Here we wrote r = ap^n from the burn rate law. What is it we get? We get dp/dt = RT/V{Sb×a p^n ×(ρ_p − ρg) − p At /C*}. All what we have done is that we brought ρg on the right side, because it is with the minus sign now.

Now, we want to know what happens under steady conditions. By steady conditions, we mean the pressure with respective time is a constant, pressure does not change and therefore, dp/dt = 0. We therefore get:

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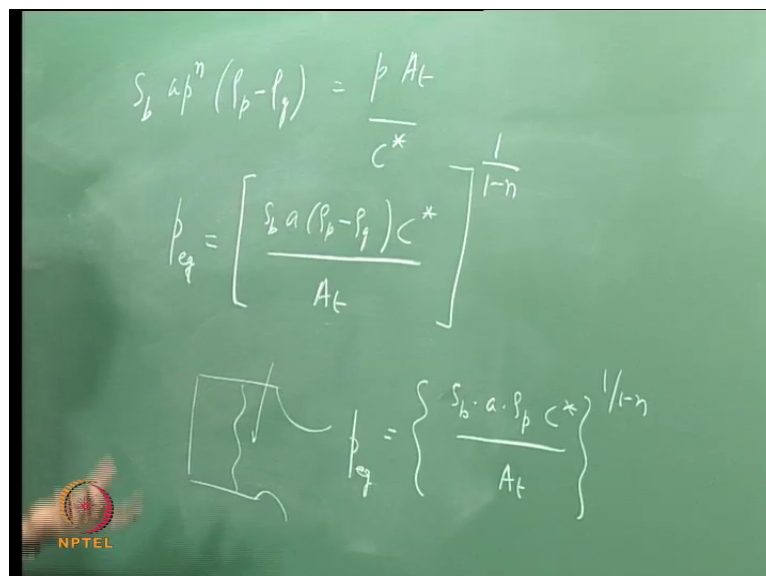
The value of $Sb \times a^n \times (\rho_p - \rho g) = p \, At/C^*$. If we were to call this steady value of chamber pressure which, is actually the steady equilibrium value in the chamber, the value of equilibrium pressure p equilibrium is determined as $\{Sb \times a \times (\rho_p - \rho g) \times C \, star \, /At\}^{1/(1-n)}$. We take here 1 minus n and this becomes 1 over 1 minus n. Please check whether it is right. We take p on this left side, it becomes p into 1 minus n and I still retain Sb a $\rho_p - \rho g$ and I take C star upstairs over here and At downstairs and this is equilibrium pressure.

When we burn a propellant in a rocket chamber, whatever be the configuration of the grain, the equilibrium pressure or steady state pressure is given in terms of the burning surface area, the pre-exponent a, the difference between propellant density and the gas density and the C star of the propellant divided by At raised to the power 1/(1-n). This gives the value of the equilibrium pressure.

Normally the gas density will be very much lower than propellant density. In fact, had we neglected the gas density, we would have got this expression by equating the mass generation with the mass leaving through the nozzle. We would have got this to be equal to $(Sb \times a \times \rho p \times C^* \, /At)^{1/(1-n)}$. This should have been the value of the equilibrium pressure. The gas density would be around one thousandth of the solid propellant density. Therefore, what are the conclusions that could be drawn?

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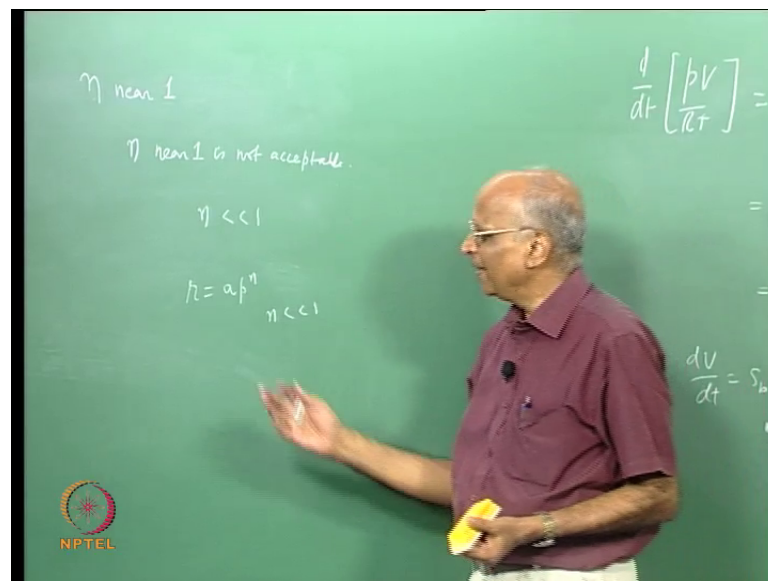
If n has a value near 1, what will happen to this equation? If n has a value of 1 well, we have infinite pressure it is just not possible. If n is around 0.995 very near to 1, then we

have a really large exponent and small perturbations or changes in this will give me a high value. Therefore, a value n near 1 is not acceptable. That means, n must be as small as possible. And any changes in burning surface area while the propellant is burning, some changes in the gas density while it is burning, some changes in C* or throat area should not lead to an explosion by giving an abnormally high value of pressure.

Therefore, we tell ourselves the one of the quality is required in the burning rate of a propellant r = a p^n, is thatn must be a small number. A number around 1 is just not acceptable. And since for composite propellants the value of 'n' is around 0.25 to 0.4 while for double base it is around near to 0.4 to 0.5, therefore composites are better off than double base propellants.

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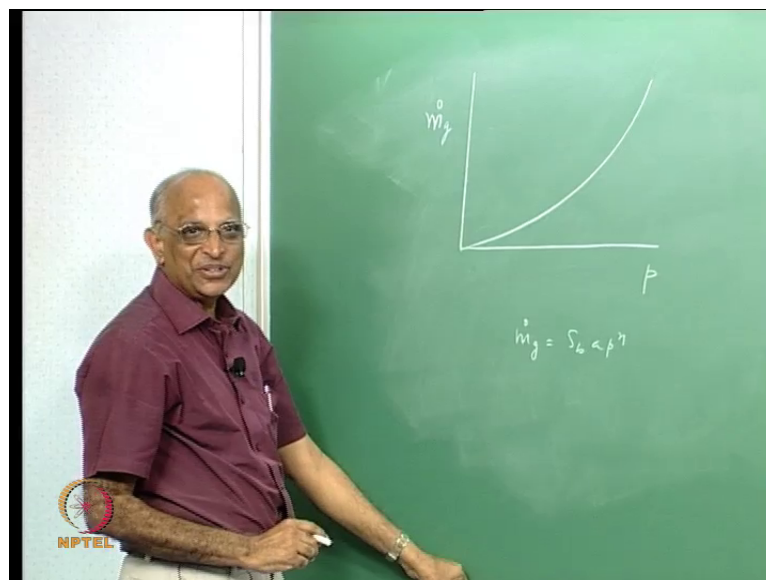
Let us also plot the rate of mass generation and mass leaving the nozzle. We have the mass generation rate of propellant as a function of pressure. $m°g = ap^n \times Sb \times \rho_p$. Therefore, it will go up progressively as shown. If we have n which is greater than 1, this will be my shape. If I have n less than 1, what will be the shape of the curve?
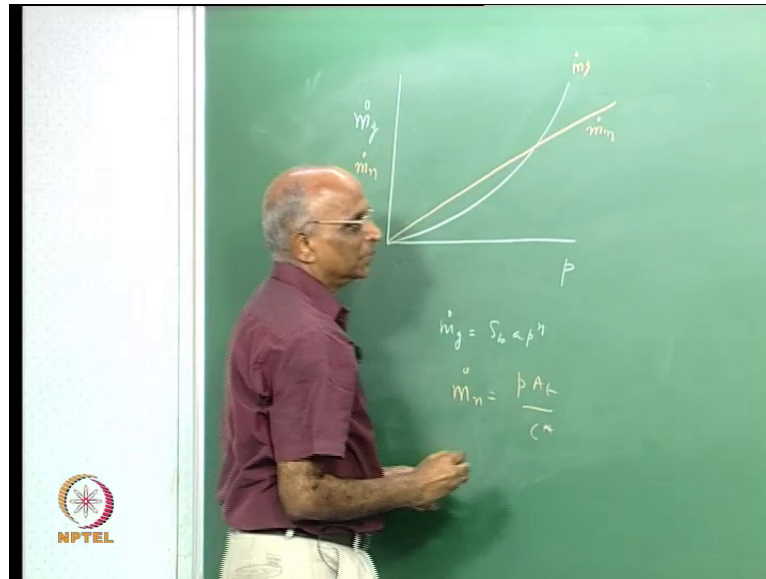
We must be able to do these things that is the basic aim of the course. The curve begins to droop as the pressure increases to higher values.

Now, what is the value of $m°_n$? $m°_n$ is linear function of p viz., $p \times A_t / C^*$. This is a straight line in the plot versus p. Now, we put both $m°_g$ and $m°_n$ on the same plot and let the two intersect at a point. Let us take a look at this point of intersection. What is this point wherein both are equal; steady state i.e., p equilibrium. And we know the expression for the p equilibrium, we have just derived it as saying $Sb \times a \times C^*/A_t$ to the power $1/(1-n)$.

Now, let us examine the characteristics around this particular point for $n > 1$. Suppose, if by chance as the motor is functioning, there is a small dispersion in pressure and the pressure slightly increases. If pressure slightly increases the mass generation rate increases. The pressure therefore further increases, it goes like this leading to very high pressures and the rocket explodes. If pressure slightly falls, the mass flow through the nozzle is higher compared to mass generation rate and therefore the pressure comes down. If the pressure decreases, the mass generation rate further decreases and the process continues till the rocket ceases to function. Therefore, when we have $n > 1$, we cannot get equilibrium, the motor either quenches or it explodes. Therefore values of n greater than one cannot be recommended.

Let us get back to the curves for n < 1 and examine it again. If there is a slight increase in pressure, what is going to happen? The pressure has increased the nozzle flow rate has increased compared to the mass generation rate and therefore, the pressure again comes to the point. If the pressure falls slightly, the mass generation rate is higher, it gets back to this point. Therefore, the point of intersection becomes a stable operating condition. Therefore, now, we are very clear that 'n' must be very much less than 1 and the reason for it. We learnt how to do this using these two plots. We also learnt how to do this using the equations wherein we had the expression raised to the power $1/(1 - n)$. And this is what how we decide the choice of n.

I will continue with this in the next class. But to summarize, we derived an expression for the burn rate of composite propellants using the standoff distance X^*. We found that the burn rate can still be modeled using the Saint Robert's law namely $r = a\, p^n$. We related it to a rocket and we said well n must be very much less than 1. In the next class, we will look at the temperature sensitivity of r, we will also look at some other parameters, which are important for the burn rate r, and then go to designing a solid propellant rocket. And what do we do in a design? We have to have some particular burning surface area; it becomes a simple geometric problem, and this what we will do in the next class.