Rocket Propulsion

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Lecture No. # 22
Design Aspects of Solid Propellant Rockets

We will continue with solid propellant rockets. What is it we have done so far? Let us take a quick review; we know that the propellant could be composite, it could be double base or it could be nitramine or it could be composite modified double base propellant.

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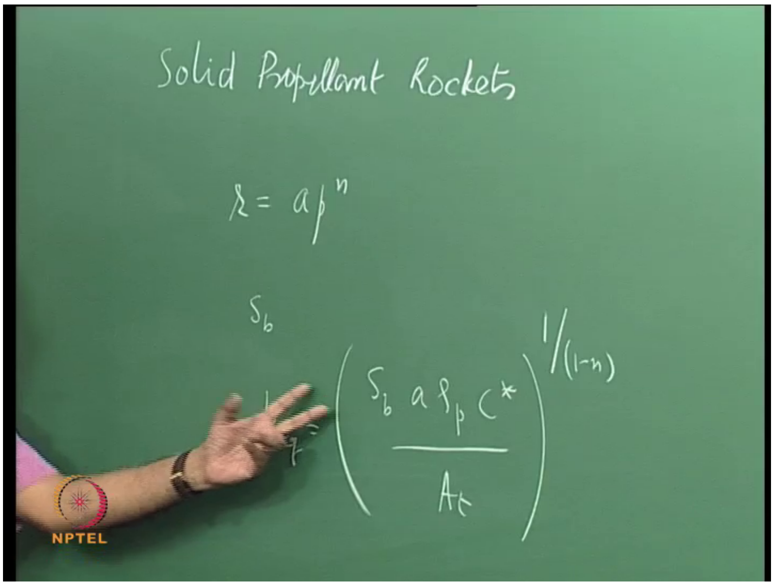

We can write the burn rate as a linear regression rate, $r = a\,p^n$. We also said that if we have a rocket in which the burning surface area is Sb, the equilibrium value of pressure can be derived as the burning surface area into this particular constant a in the burn rate law into ρ_p into C* divided At to the power $1/(1 - n)$. How did this come?

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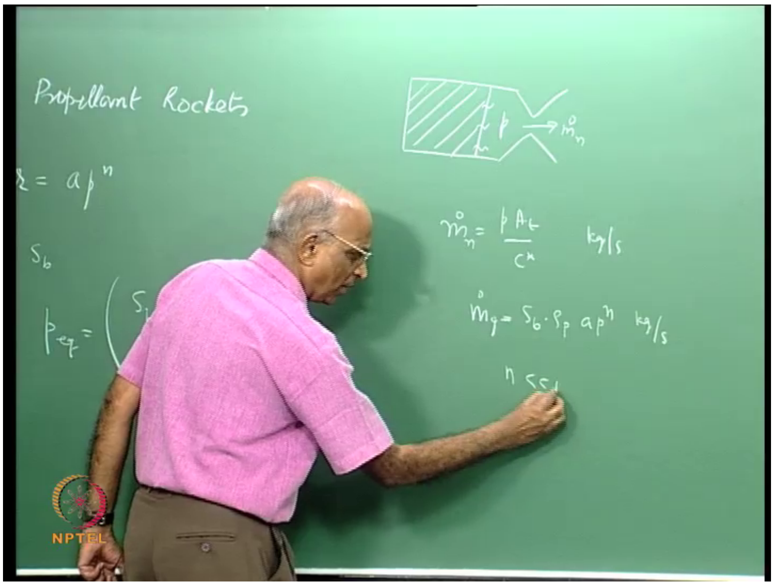

We said if we had a rocket and we considered a simple scheme where in we had propellant, which was enclosed in a case, we have something like a nozzle attached to it, we said the rate at which the mass leaves through the nozzle $m°_n$ can be written as p At/C*, where p is the pressure.

And we said the nozzle is always choked at the throat. Therefore, this is the mass being generated and this is the mass, which is leaving through the nozzle. And the rate at which the mass is getting generated from the burning of propellants, we got from the regression rate is r and that was equal to Sb × the propellant density × the burn rate r, which is equal to a p^n so much kilograms per second. We equated the two viz., mass generation and mass leaving rates and got the value of equilibrium pressure p^{1-n} = Sb × a × ρ_p × C* / At.

What does this tell us? Let us take a relook at this equation. We looked at it from the point of view of n and found that n cannot be anywhere near 1, because then what happens for any small change in the parameters, there is a large magnification here. And therefore, n should very much less than 1, because if n is near 1, I get a very large exponent and a small change can magnify into a large value o pressure. Therefore, we say from stable considerations, n must be very much less than 1, but we also learnt to look at it graphically.

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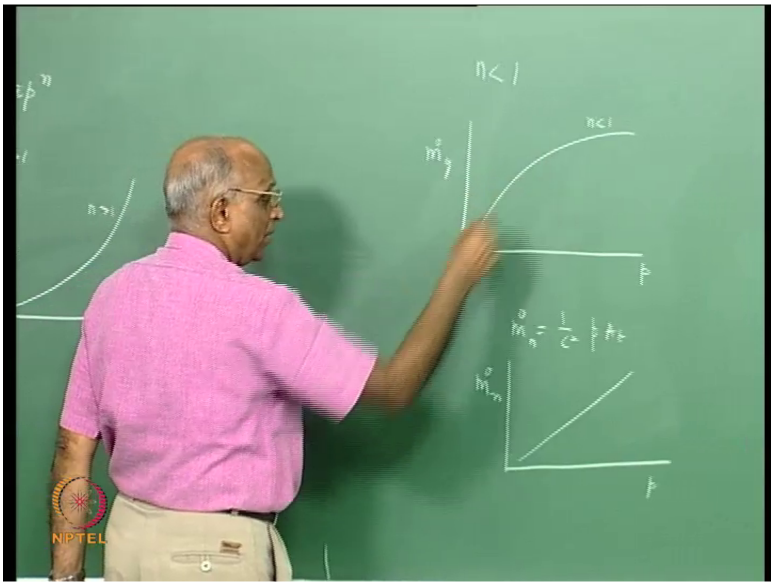

When we have n in the burn rate equation $r = a p^n$, if n > 1, how does the burn rate law change as pressure changes. Let us make a plot: as burn rate increases, if you have a given burning surface area and a given density of propellant we can plot the rate at which mass is generated due to burning. In other words, the rate at which mass flow gets generated depends on the burning surface area into the density of the propellant into the burn rate law; we want to plot it as a function of pressure.

Then if n > 1; well it keeps increasing higher than a linear value i.e., concave upwards.and the mass generation rate rapidly increases rather exponentially. If however, n < 1, then the mass generation rate will have something like a drooping characteristic i.e., convex upwards. This is for n less than 1 with variation of pressure; this is for n greater than 1. How does the mass flow rate, which leaves the nozzle change with change in pressure? We have been writing this on and off as m° nozzle is equal to (1/C*) p×At. The mass flow rate through the nozzle with respect to pressure will increase as a straight line.

Therefore, let us now plot the mass which leaves a nozzle and the mass generation rate for ‘n’ is greater than 1 and ‘n’ is less than 1 and see whether we can conclude on the type of exponent ‘n’ which we require.

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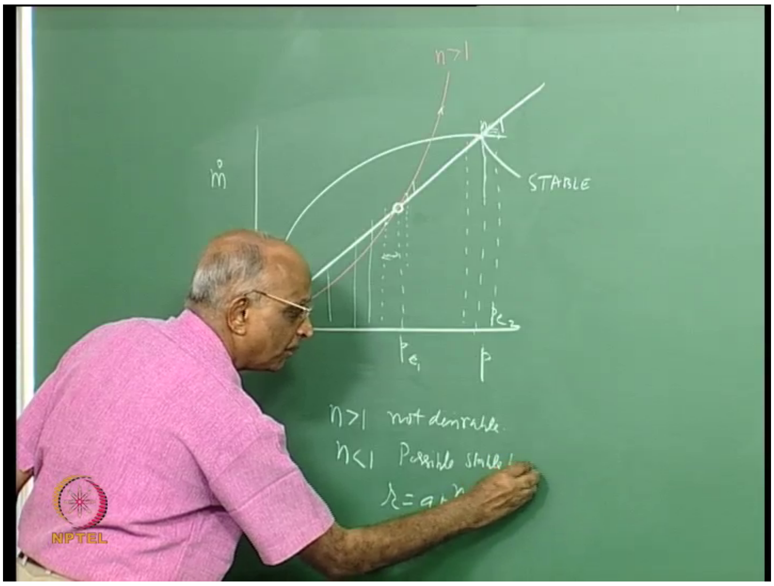

Let me now plot all the three together in a single figure. Y axis which is mass generation or let us say mass which is leaving the nozzle while pressure on the X axis. Let me put the value first for n less than 1 we get a curve like this drooping one. If we have n greater than 1, the curve exponentially increases. And what is the rate at which mass leaving the nozzle? I show it by a white line - a straight line like this with respect to pressure. Now we find the point at which the mass rate of generation for $n > 1$, and the mass, which is leaving through the nozzle are the same at this point of intersection. Therefore this will correspond to let us say equilibrium pressure for the case of $n > 1$. The red line is for $n > 1$ therefore p equilibrium corresponding to let us say case 1.

Let me also say tell that may be for $n < 1$ this is the p equilibrium value. We would like to examine whether are these two equilibrium pressures are possible? Well theoretically, this is the rate at which mass is leaving the nozzle, the rate at which mass is getting produced in the chamber and therefore this is equilibrium pressure. Similarly, for $n < 1$ this is the pressure for mass balance. Let us try to get some idea whether these two points are possible and if so are there some problems with the equilibrium pressures?

When $n > 1$, let us say we have a small perturbation in pressure. A small perturbation can always come and let say that the pressure reaches this higher value. That means the pressure is slightly higher than the equilibrium value. Since the pressure is slightly higher, what we find at this point the mass generation rate is higher than the mass which is leaving the nozzle. Therefore, the pressure will increase further; when pressure further increases, the mass generation made is further increased. Therefore, this point cannot be a stable equilibrium pressure as any small perturbation will make the pressure increase further and further till the rocket explodes.

If we considered points to the left of the equilibrium point by saying that by some chance there is a small pressure perturbation and the pressure in the motor falls to a value less than the equilibrium pressure pe_1, then what happens? The mass generation rate is lower than mass, which is flowing out through the nozzle. In other words mass flowing out through the nozzle is more than what is generated. Therefore, the pressure further falls, the pressure continues to fall till the rocket is extinguished.

Therefore, we tell that in case $n > 1$, we cannot really get an equilibrium pressure since with small changes in it due to perturbations, the chamber will either explode or pressure will become zero. Therefore, we tell that n greater than 1 is not desirable or not possible.

Let us examine the case, when n in the burn rate law, what was the burn rate law? $r = a p^n$. We would like to know if the equilibrium pressure obtained when $n < 1$ is possible. Let us have the same set of arguments again. If the pressure falls slightly less than the equilibrium value; we now find that the mass generation rate is higher than the mass which is leaving the nozzle. Therefore the pressure will go back to the equilibrium value.

If by chance the pressure exceeds the equilibrium value i.e., moves to the right of the equilibrium point. Here again we find the mass leaving the nozzle is higher than the mass generation rate and pushes the back to the equilibrium value. Therefore this point becomes a stable point. The value of $n < 1$ is therefore possible and it gives to rise to what we say is a stable situation for equilibrium pressure. Therefore, in the burn rate law $r = a p^n$; ‘n’ must be less than 1. This is what we have found.

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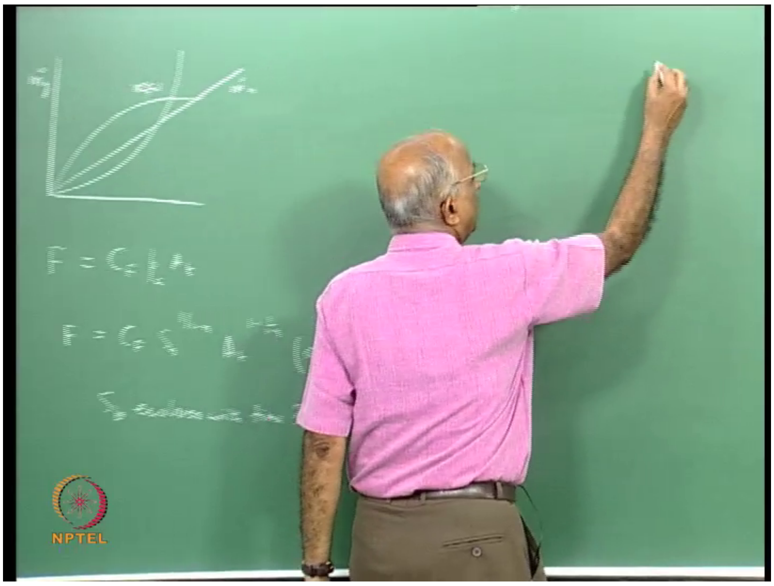

We need to put everything together and design a solid propellant rocket. What is meant by design of a rocket? We must be able to generate a given thrust from the rocket, and what is the thrust? We said it is equal to $C_F \times p \times At$. We drop the subscript c in p_C. We can express in terms of the nozzle effectiveness into chamber pressure and At and this is the thrust which is developed. Therefore, if we want a rocket to develop a particular thrust, we know that p equilibrium goes as Sb × the constant a × the propellant density × C* / throat area to the power $1/(1 - n)$. All what we need to do is to configure the burning surface area Sb such that we obtain the desired value of thrust. But it is not that easy as we shall see in the in the subsequent class. We can write the thrust as C_F × the value of p equilibrium from here × throat area; so we get the thrust as as $Sb^{1/(1-n)}$.

We take p equilibrium as $Sb^{1/(1-n)}$. Then we write the other terms together namely and get At. Now At is in the denominator here to the power $1 /(1 - n)$. And then we solve the other parameters namely a value of ρp into C* to the power $1 / (1 - n)$. Therefore, we find for a given constant throat area, if we know the evolution of burning surface area as the surface regresses, we can find out the value of thrust varying with time. This is how a solid rocket is designed. It is a simple geometric problem of evolution of the burning surface area Sb. And how Sb, the burning the surface area in meter squared, evolves with time would be dealt with in the class today.

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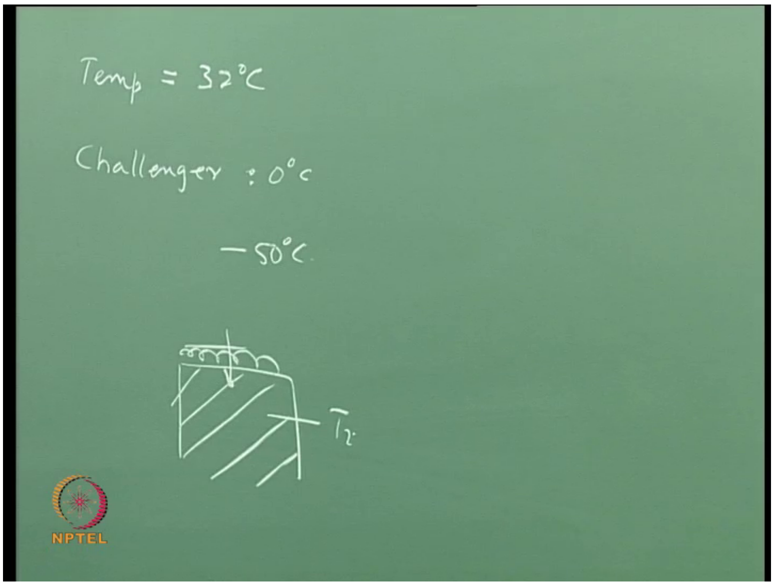

But before we do that, let us recall that the burn rate law r = a constant ‘a’ into p to the power ‘n’. We considered explicitly the effect of pressure alone. But we said ‘a’ includes the effect of the initial temperature of the propellant. Let us take an example. We can consider a temperature of the propellant to be ambient that is a rocket motor is tested today. The temperature is quite hot today may be 32^{o} C. Well, you all would read about a solid propellant rocket, which misbehaved in one of the space shuttle flight. It was a challenger rocket, which was launched on a very cold day, when the temperature was around 0^{o}C. And we will look at the failure after completing the portion on solid propellant rockets. Well, we could have a missile, which is operated from mountains in the Himalayan ranges where the temperature could be as low as minus 50^{o}C.

What is the effect of burn rate on temperature that is the initial temperature of the propellant itself? We are not looking at the flame temperature all. What we say is that we have a propellant block, the initial temperature of this block before it burns or just begins to burn is what we call as initial temperature of the propellant. We would like to know the effect of initial temperature of the propellant on the regression rate of the particular propellant.

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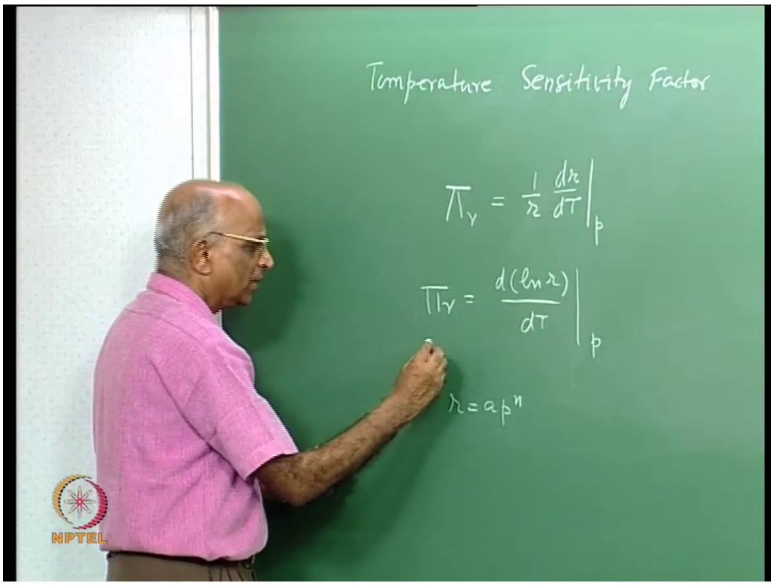

We define a term known as temperature sensitivity factor for the propellant. Let us see, what it is. We would like to know how sensitive the burn rate is to the propellant temperature changes. Therefore, we were interested in finding out the change in burn rate with temperature dr/dT. And we are considering the effect of temperature alone. This implies that the pressure is fixed; we are considering at constant ambient pressure or otherwise. But then instead of just saying burn rate variations with temperature or the variations in burn rate due to unit change of temperature, we now say fractional variation in burn rate. This is known as temperature sensitivity factor for burn factor r. In other words, we define a term π_r as equal to this dr/r i.e., d ln r divided by dT at constant pressure. This is defined as the temperature sensitivity factor for a solid propellant.

And just like how we determined 'n' by conducting two experiments on burn rates at pressures p_1 and p_2 and measured the burn rate r_1 and r_2 and $n = (\ln r_1 - \ln r_2) / (\ln p_1 - \ln p_2)$, in the same way the temperature sensitivity of burn rate is measured at different temperatures at the specified pressure. The factor π_r, which defines the sensitivity to temperature is determined. The value is around 3×10^{-3}. What should be units?

Well ln r has no units; dr by r; the units cancelled and it is only dT. Therefore ^oC inverse are the units. Typically for most composite problems the value is around 3×10^{-3} $^oC^{-1}$ and about 5×10^{-3} $^oC^{-1}$ for double base propellants. And for HMX based propellants, it is even lower; it is 2×10^{-3} $^oC^{-1}$. This is one of the reasons for the choice of HMX propellants for missiles.

We can integrate this equation for temperature sensitivity and find out explicitly, how the burn rate changes with temperature. Let us do it. We take the expression for π_r. We write d ln r = $\pi_r \times$ dT at constant pressure. We have taken the change in the logarithmic burn rate is equal to $\pi_r \times$ dT. Let us solve this equation. If we have at temperature T_1 the burn rate as r_1 and at temperature T_2 the burn rate is r_2 and we are interested in finding out the burn rate at a temperature T_2; and therefore, we just integrate out this to get ln r_2 − ln r_1 = π_r ($T_2 - T_1$).

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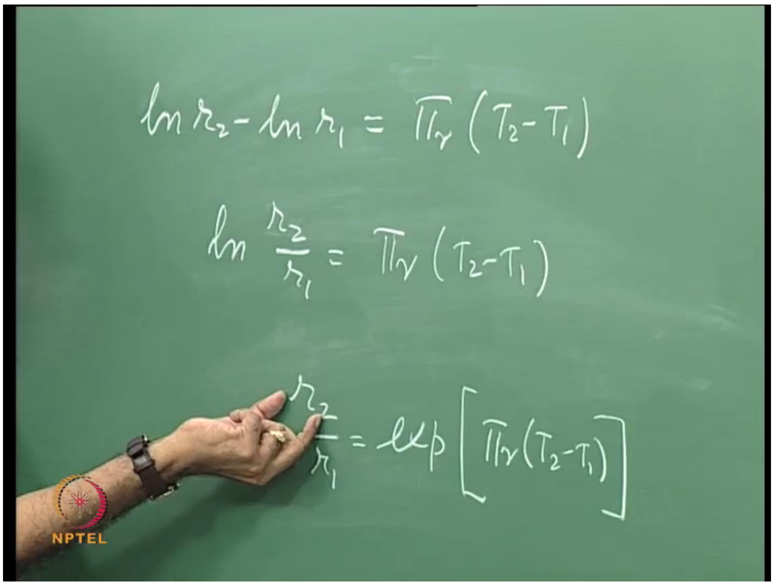

Therefore ln (r_2 / r_1) = π_r ($T_2 - T_1$) or rather we get r_2 / r_1 = exponential of π_r ($T_2 - T_1$). Therefore, if I know the burn rate at temperature T_1, using the value of the temperature sensitivity factor we can find out the burn rate at a temperature T_2 and this is how the effect of the variations of temperature are taken into account.

We are now in a position to design solid rockets. This means essentially we find out how much burning surface area is required and how it should evolve with time. However, let us ensure whether there any concerns about the burn rate equation which is needed for the design.

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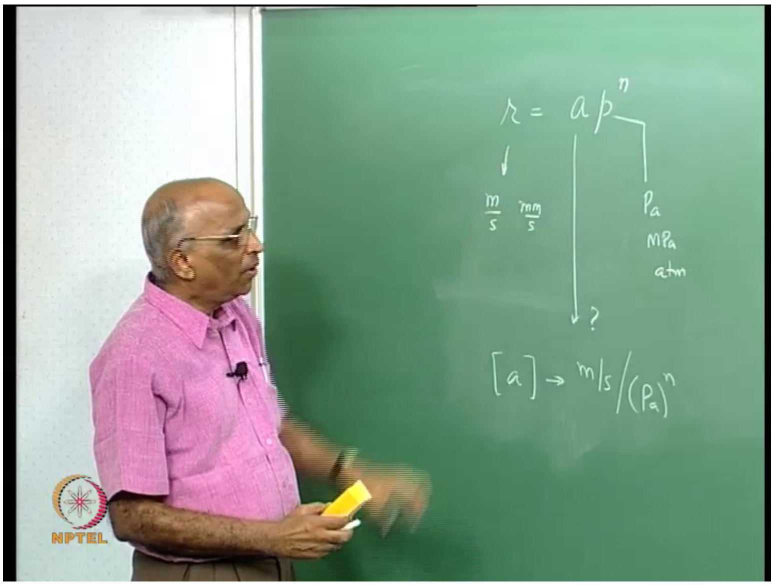

The burn rate is expressed in millimeters per second or meters per second and is given by $a \times p^n$. What is the unit for r? We say meters per second, millimeter per second or centimeters per second. The burn rate is the rate of regression of the propellant surface. What is the unit of pressure? Could be Pascal. It may be mega Pascal, could be atmosphere also. Then what is the unit for a? The constant 'a'; it may be noted, depends on temperature and on composition of the propellant including the AP particle size.

The unit of a is clumsy; it becomes meter per second divided by let us say Pascal raised to the power n. This is not a correct way of expressing a constant. We have a constant, which is a function of the parameters and the units of pressure. We cannot say that the constant is so much meter per second to the power of pressure to the exponent 'n'. How do we get over this problem? The form of equation is, however, correct. If we can find the burn rate let us say at pressure, which we call as reference pressure, and we evaluate the burn rate r with reference to this particular value p reference to the power n, the problem can be overcome.

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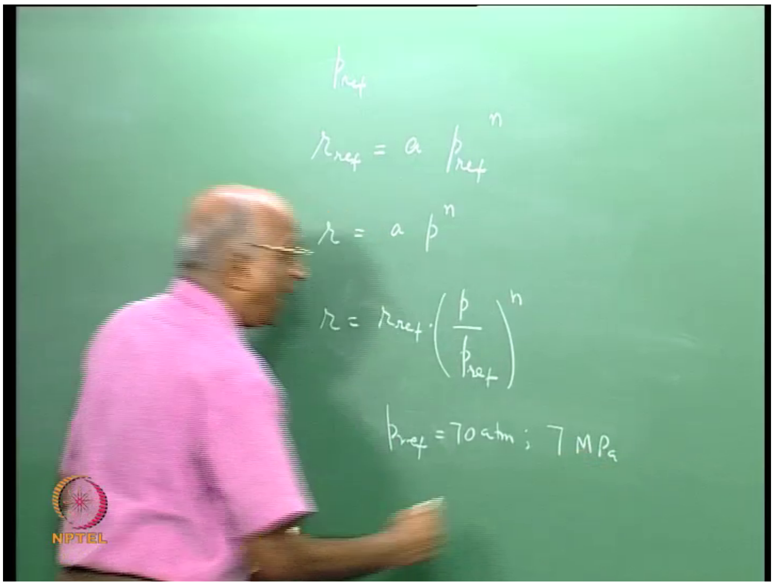

But we are interested in burn rate r at any pressure p as a into pressure the power n. We can write the value of r as equal r reference × p by p reference to the power n and this is one way we get over the units of pressure in the constant 'a'. In other words, the constant 'a' is the burn rate at the given reference pressure provided the reference pressure is used for non dimensionalising the value of pressure. This is how the burn rate is expressed through a non-dimensional pressure, which is based on the reference pressure. The reference pressure is normally taken as 70 atmospheres, which is about 1000 psi.

You may recall that when we studied nozzle we said under sea level conditions and vacuum conditions for evaluation of the specific impulse; we took the chamber pressure as 70 atmosphere. This is about the pressure at which a solid rocket or a good performing solid rocket works. This is equal to 7 MPa. And therefore, the burn rate law can now be written as r at a reference pressure 70 atmospheres or 7 MPa into pressure divided by 70 or 7 provided p is in atmosphere or MPa to a power n. This is at 7 Mega Pascal pressure. Some books write the value of 'a' as r at 7 MPa or 70 atmospheres into p divided by 7 or 70 to the power n.

The constant 'a' is therefore denoted by a_7 or a_{70}, which is the burn rate at the reference pressure of 7 MPa or 70 atmospheres. This is all about burn rates, effect of temperature on burn rates, etc., but there are many more problems which would be considered later like for instance in a rocket chamber; there could be velocity at the propellant surface, there could be thermal radiation in the chamber, there could be external heating.

We now return to the design of a solid propellant rocket for which, we wanted to find out the burning surface area and the evolution of the burning surface area. Let us do a simple problem, and then go to the evolution of the burning surface area to give a certain thrust and thrust profile. Let us consider a propellant block, which is contained in the rocket case and we put insulation and connect a nozzle here. Well I have a solid propellant rocket.

If this propellant block is ignited over here towards the nozzle, and it burns from the exposed end, that means the end of the propellant is ignited. We call it as end burning, because it burns from one end to the other; it does not burn from the sides here, because it is prevented from burning from the sides. The burning or the flame can go normally in this particular direction. Now supposing the throat area of the nozzle is At. What is the value of pressure in the cavity? We have already done it as equilibrium pressure. It is equal to let us write it down: $(\rho_p \times a \times C^* \times Sb / At)^{1/(1-n)}$. The burning surface area is known, and we determine the pressure in the cavity. The value of burn rate r is equal to a p^n and knowing the pressure, we can find the burn rate. And therefore, if the propellant grain has a length L, the time taken for the propellant to be consumed can be determined as L / burn rate. The burning surface area Sb is a constant and therefore the pressure p remains the same.

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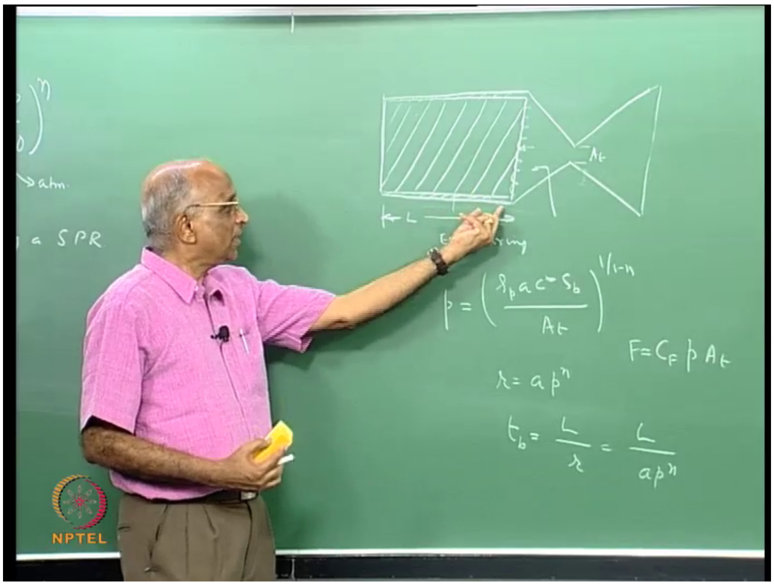

And therefore, the burning time t_b is equal to the value of L, the length of the grain divided by the burn rate r and is equal to L divided by a p^n. The thrust developed by this particular end burning is C_F into pressure into At. We know the value of pressure, the throat area and the burning surface area and the thrust developed can be determined.

We said that solid rockets are generally used when we want large thrust as in booster stage. Suppose we want a thrust of several 1000 tones. Then in that case, the burning surface area and hence the diameter of the solid propellant rocket is going to be extremely large.

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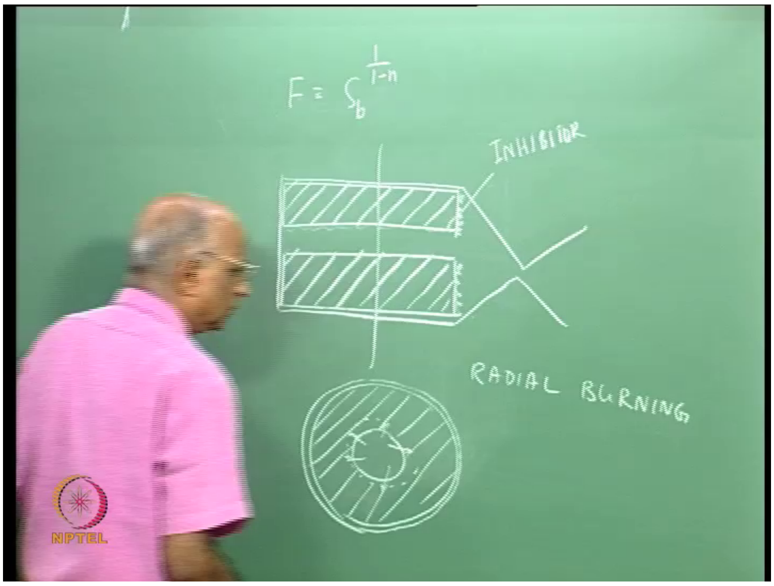

Therefore to be able to get some meaningful values of large burning surface area, we need some innovations. We have the propellant block; the same propellant block as in the end burning solid propellant rocket. But instead of burning it from the end, we burn it radially. We make a cylindrical hole along the axis in the propellant and again put it in a motor case with the nozzle. The propellant block now looks as shown with a central hole and we coat the propellant block at the nozzle edge with some material, which is an insulator. This will prevent it from burning on this side. We call this insulator, which prevents or inhibits the burning viz., an inhibitor. We coat it with the inhibitor, which will prevent the propellant from burning on the nozzle face of the propellant. We ignite this inner cylindrical surface, which now becomes the burning surface area.

If we take a cross section, what is it we get? We get this outer surface, we have the case over here, and then we have the inner diameter over here, and this is my propellant in between. We ignite this internal surface of the propellant, and the propellant burns normal to the surface; it therefore burns radially outward, and this type of burning is known as radial burning. The propellant block is in the form of a cylindrical annulus between the outer and inner diameters, and now we ignite the inner surface area of this particular annulus of propellant. And then what happens is the burning will progress, let us say normal the inner cylindrical surface towards the outer cylindrical surface.

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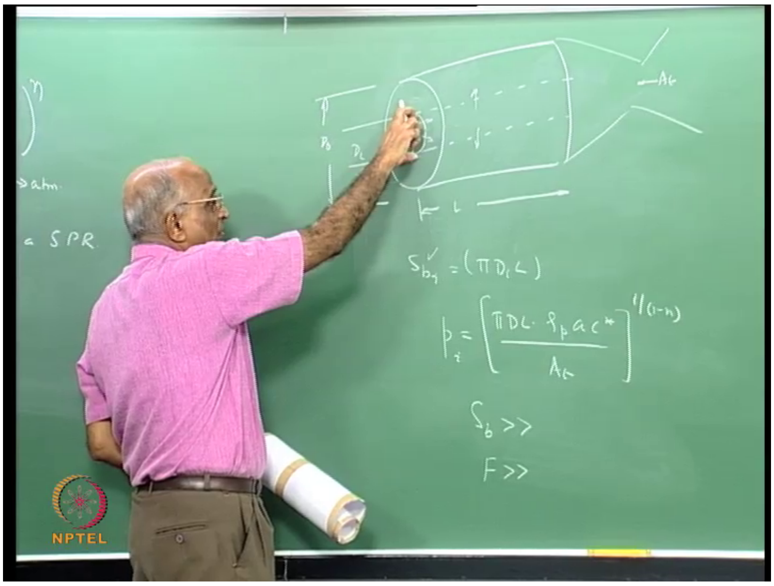

And therefore, now we have the initial burning surface area. If we were to write an expression for this burning surface area; this is the length L of the grain; the annulus is between the inner and the outer diameters. We take inner diameter as Di, outer diameter is Do, and the length of the propellant grain is L. So, how does the burning take place? It takes place radial, normal in other words to the surface as burning proceeds the burning surface area changes. We need to determine the pressure.

The initial burning surface area is equal to the perimeter into length. It is equal to πDi which is the perimeter × L. And what is the final burning surface area? It is equal to π Do the perimeter × length L. And therefore the equilibrium pressure to begin with corresponds to $(\pi Di\ L \times a \times C^* / At)^{1/(1-n)}$.

The throat area of the nozzle is equal to At. What is the change we are made as compared to the end burning grain? We now have the entire length of the inner perimeter πDi as the burning surface area to begin with. Therefore, we could have a longer grain to give a much greater burning surface area. And in radial burning grain, which burns in the radial direction, we can get Sb to be much larger than in an end burning grain of the same diameter. And therefore, we can get a large value of thrust. This is the modification that could be done; but you know there is some limit to the burning surface area. We still need to explore if for a particular diameter and length it is possible to increase this surface area even further. In other words all what we are asking is whether for a radial burning grain is it possible to increase this surface area by some means?

How can we do it? If we can wrinkle the surface; we can wrinkle it in some form, and how do I wrinkle it? I show in this small model, this was the original circular perimeter over here. We wrinkle this surface i.e., we make stars or some other shapes in the inner surface. In other words instead of having something like a cylinder over here, I make the inner surface in the form of a few star. Now I find that this surface has something like 5 vertices; 5 vertex star. And therefore, now I find my surface area has increased enormously, and therefore now I must be able to evaluate how the burning surface comprising the wrinkled surface will evolve as it continues to burn.

In other words this my outer diameter; the outer diameter will come over here, and the burning surface will evolve along these surfaces, and this is the problem which we must do. However, before doing this problem, let us do the simple problem of may be a radial burning grain burning from inside to outside in a cylindrical configuration. What is the value of pressure and what is the time required for burning this cylindrical grain. In the case of end burning grain, we got the burn time t_b = length ÷ a p^n; we knew how to calculate the pressure and hence the duration of burning. We wish to follow a similar procedure for radial burning.

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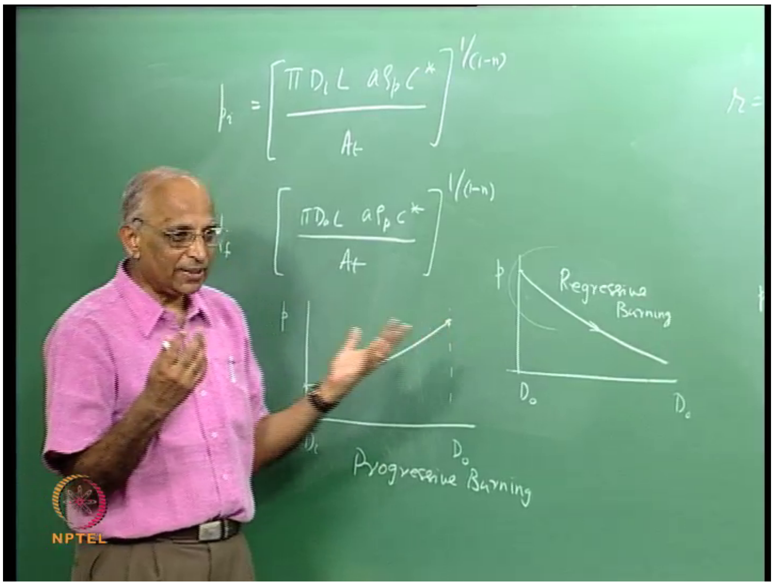

Let us also find out how the pressure will change since the burning surface area is changing. We have the initial value of pressure is equal to whatever we have written here, let us rewrite it : (πDi which is the initial perimeter × L and this gives the initial burning surface area × ‘a’ × ρ_p × C* / At) to the power 1/(1 – n). Now, what is the final value of pressure when the burning has progressed? In other words the burning progress from the inner cylinder and reaches the outer cylinder. What is the value the final value of burning surface area: πDo is the perimeter into L is the final surface area. This multiplied by a × ρ_p × C* / At to the power of 1/ (1 – n) gives the final value of pressure.

Now, is the pressure constant like as in the end burning rocket grain? It is a variable. Di has increased to Do and so the pressure also increases. If we were to plot the pressure, the pressure initially corresponds to diameter Di, while when the rocket burns out, the diameter is Do. When the diameter is Di, the pressure is pi and when the diameter is Do the pressure is po. The value of po > pi.

We know that Di < Do; therefore, initially the pressure is less, let us say when it reaches the final value the pressure is higher, therefore the pressure increases. In the end burning grain we had the same pressure throughout.

If instead of having the grain start burning at the inner diameter and progressively burn to the outer diameter, we somehow put the case over here and ignite the outer surface, and then the flame propagates inward.

Then what is going to happen? We will get just the opposite evolution of pressure. We start with a value of po of pressure; that means the initial value is now higher and the pressure drops to the value pi. In other words while the end burning grain gave us a constant pressure; and therefore a constant value of thrust, a cylindrical grain burning from inside to outside gave a progressive increase in pressure or a progressive increase in thrust. A cylindrical grain burning from outside to inside gives a progressive decrease of pressure and thrust.

This means that a cylindrical propellant grain burning from outside to inside gave us a falling pressure; and therefore, a decreasing thrust. We could have three types of thrust evolution in such rockets. When the pressure is constant, we call as neutral burning. What should be the name for this progressive increase of thrust? This is progressive burning. If the pressure and thrust keeps decreasing; that is regressive burning.

If we have a propellant, which burns from end to end viz., end burning, the type of burning is neutral and we get constant pressure constant thrust. If the burning is radial from inside to the outside the pressure keeps increasing and the burning is progressive. While if the radial burning is from outside to inside, the burning could be regressive.

Therefore, we could think in terms of three types of burning; neutral burning rockets, progressive burning rockets and regressive burning rockets. If this is clear, the question is what type of burning is required for solid propellant rockets? We cannot think of a rocket, which is progressive burning as when the rocket takes off the thrust must be high and as it goes up the thrust can come down.

Therefore, may be something like this with higher initial thrust may be better than progressive or regressive, because when the rocket goes up it is mass is higher and it flies in the atmosphere where there is drag. We cannot expect it to go up with a higher acceleration in the beginning itself. Therefore, we have to somehow get a thrust pattern, which is desirable. We also require higher thrust to begin with and for which we just took the inner surface and wrinkled it. The wrinkled shape was in the form of a star. We could have wrinkled into some other shape instead of giving the shape of a star. We could have given a different shape something like this. We could have given any shape and we want to calculate, how does the burning rate evolve around this surface. We find out how the burning surface Sb changes with time. And once we know how the Sb changes, we know how the pressure changes with time. We can find out how the force or the thrust which the rocket will develop with time, and that is all what we need to do in the design of propellant grains in solid propellant rockets. Therefore to be able to pursue on this, let us start with a simple example.

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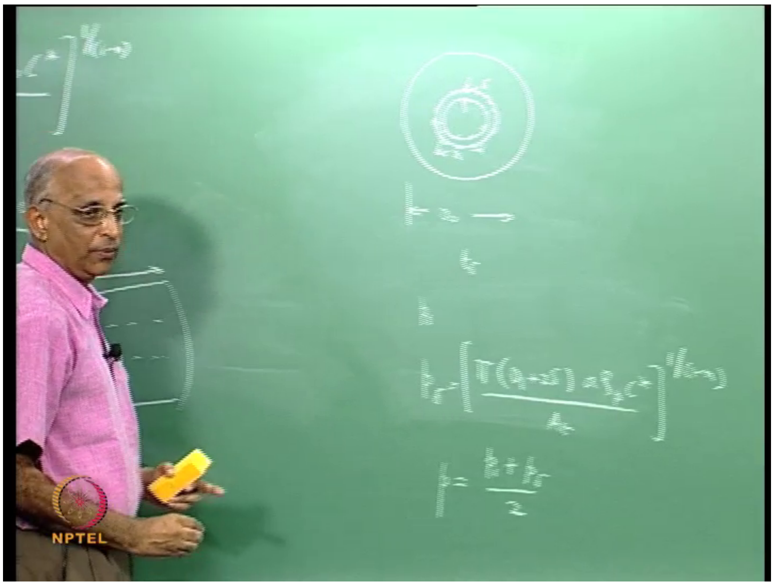

Let us find out the time taken for a cylindrical grain to burn. What do we mean by a cylindrical grain; the first example; we have an end as shown. We make a hole here I have radial burning from inside to outside. It burns through from inner surface to outer surface. The inner diameter is let us say Di, the outer diameter is Do, the length is L. We are repeating this figure. The question is can we predict how the pressure will change with time and the time for burning. And how do we do it? It is a simple problem.

Let us consider a small part of the propellant between Di and Do gets burnt; and let the small part of thickness δ over a time; let us say small time dt. At the beginning of dt let the pressure be p corresponding to the initial burning surface area which is πDL. At the end of dt, we will calculate the pressure again. The value of diameter at the burn out of this element will be the original diameter plus twice delta. In this way we can progressively calculate the pressure for each time step and the new value of the burning surface.

We know the pressure to begin with. It is this value at the burning surface given by the surface at Di. What is the pressure when the diameter is Di plus 2δ. We know the new value of perimeter and the length and hence the new value of the burning surface area. We can determine the value of p at Di+2δ. What is the value of p delta. The value is Di plus 2 delta into these terms viz., 'a' burn rate constant, rho p, C star divided by throat area to the power 1 by 1 minus n. We know the value of pressure this point. We want to know the time taken for consuming delta of the propellant.

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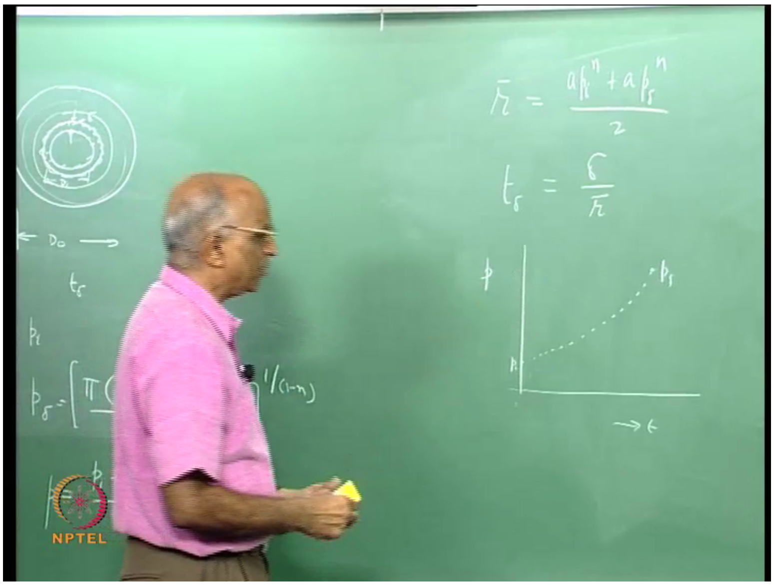

Therefore, the mean pressure between this and the earlier value pi which is (pi + p delta)/2. And therefore, the mean burn rate between the initial at Di and Di plus 2 delta is determined equal to r bar, the mean value. And what is the mean value of burn rate equal to? $a\,pi^n + a\,pdelta^n / 2$.

When the burning has progressed by a distance delta from the initial diameter Di, we find that the pressure has increased. The burn rate has also gone up. The time taken to burn the small quantity between Di and Di plus 2 delta is t delta and is equal to delta divided by r bar. We continue with the same process further. We now take this as initial condition and go to the next step of delta and find the value the value of pressure and the time taken to consume the element. In this way we march ahead till we reach the outer diameter. We can also determine by summation the total time required for the evolution of pressure starting from pi to pf.

In an inner burning rocket we can follow this procedure, but this is numerical way of doing the problem. There is no other way of doing when we have complex configuration with wrinkled inner surface. We can find out how the surface should evolve with time. And this is the method to calculate the variation of pressure with time. Once we know the variation in pressure with time, we can readily go ahead and determine the thrust variations with time. This is how a solid propellant rocket propellant grain is analyzed.

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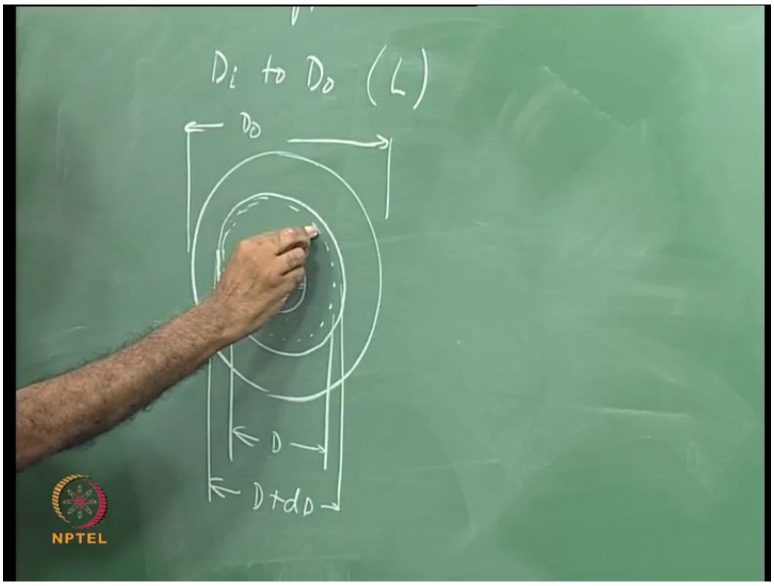

Let us do one small problem. Suppose we are asked to find the time taken to burn a propellant that burns radially outward. Now we are using radial grain between diameter Di and Do, the length of the grain being L. What we consider is the initial diameter of the cylindrical grain is Di; the final diameter or the outer diameter is Do and the length is L. We want to find the time of burn time when a nozzle of throat diameter At is connected to it. We can follow the procedure outlined earlier. But in this case a simple analytical solution is possible.

Let us consider any diameter D between Di and Do. Let us find out the time taken for the this diameter D in between Di and Do to increase from the value D to a value D plus a small change over here say dD, and this we say the diameter has increased to D plus dD.

If we can find the time taken to burn this part we can integrate out between the initial Di and the outer Do and find the time taken. And that is what we are going to do.

Let the time required to burn the grain between diameter D and diameter D plus dD be dt. Since the distance dD is very small, the variation in pressure while burning between D and D plus dD will be very small. . Therefore, the pressure could be assumed to be the value at D; and therefore, the pressure is equal to. Sb is equal to pi×D, the perimeter × length L is the burning surface area × ρ_p × ‘a’ × C* / At to the power 1/(1 – n).

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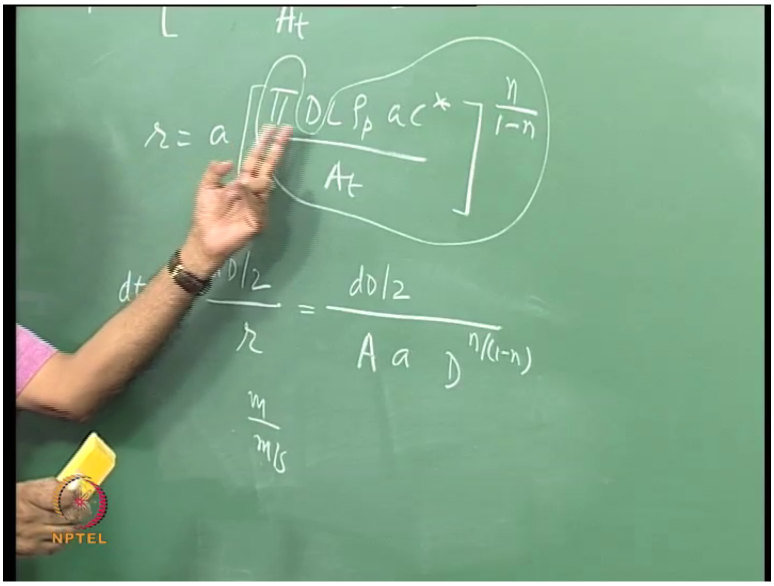

Therefore, what is the burning rate r given by the expression? It is to a p^n. We substitute the value of p as a function of D in the burn rate law to give ‘a’ into the factor pi into D into L into rho p into a into C star by At to the power n by 1 minus n. The time taken to burn a small distance dD by 2 at the diameter D i.e., time taken for diameter to burn from D to D plus dD is therefore this small thickness is equal to dD /2 divided by r which is the time taken dt. And what does is this come out to be.

We find π is a constant, length L is a constant, D is variable, ρ_p is a constant, C* and At are also constants and we can write this as equal to (dD/2) /constant A × D to the power n /(1−n). We have this a in the denominator. And then we write it as D to the power n by 1 minus n. We first check, if what we are expressing has units of length say meters divided by meter per second and this is the time taken. Here you have dD by 2 is the distance propagated divided by a into all the factors equal to π L C* / A to the power n/(1 – n) that is p to the power n. Let us say the time taken to burn is dt. The total time to burn from diameter Di to Do is denoted by tb

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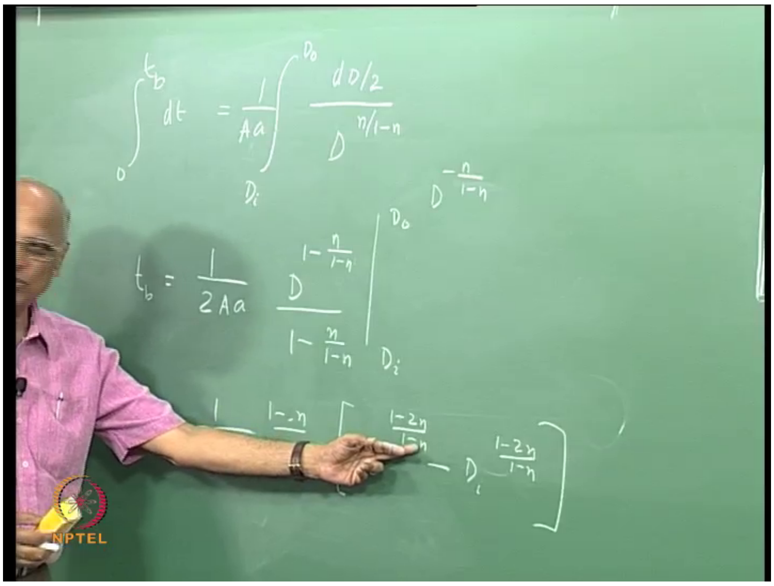

We get dt as it goes from start to the end, which is the burn time from zero to tb which must be now equal to the diameter going from initial diameter Di to the outer diameter Do. This equals dD/2 divided by D n/(n−1) and we can take A since these are constants. The limits of integration are that the diameter varies from Di to Do. What do we get on integration, tb minus 0; therefore, tb is equal to 1 over 2 (A × a the exponent of the burning rate law). Now we integrate $D^{-n/(1-n)}$. This becomes 1 – n/(1−n). We have $D^{1-n/(1-n)}$ and divided by 1 − n / (1 – n).

What is the final value? Therefore, this is equal to 1 / (2 A a) ×(1 – n)/ (1 − 2 n) × $Do^{(1-2n)/(1-n)} - Di^{(1-2n)/(1-n)}$. This is the time taken tb for the burning.

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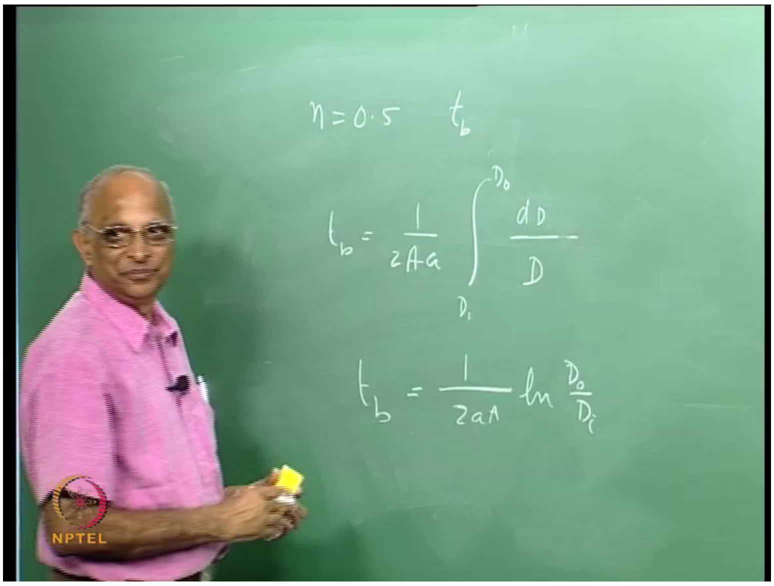

Therefore, we are able to find out the burn time of the cylindrical grain. If we have for the exponent 'n' in the burn rate law n = 0.5, what is the time for burning? We said that as long as n is less than 1, it is usable. If n equals 0.5 what would be the value of burning time? Lets do it 1 minus 2n is 0, so there is a problem. What is wrong? The expression is derived correctly.

But it is not working at n = 0.5. Can somebody come out with an answer and sort out the problem. I think we should be able to analyze it. Let us write it as 1 / Aa × integral of Di to Do of dD / 2 D to the power 0.5 divided by 0.5; and therefore this equal to dD/D. We take 2 outside, and get for n is equal to 0.5, tb is equal to 2aA ln of Do by Di. That means what happens for 0.5 is that we need to use a logarithmic form for n is equal to 0.5. The problem is not physical like for n equal to 1 but only mathematical.

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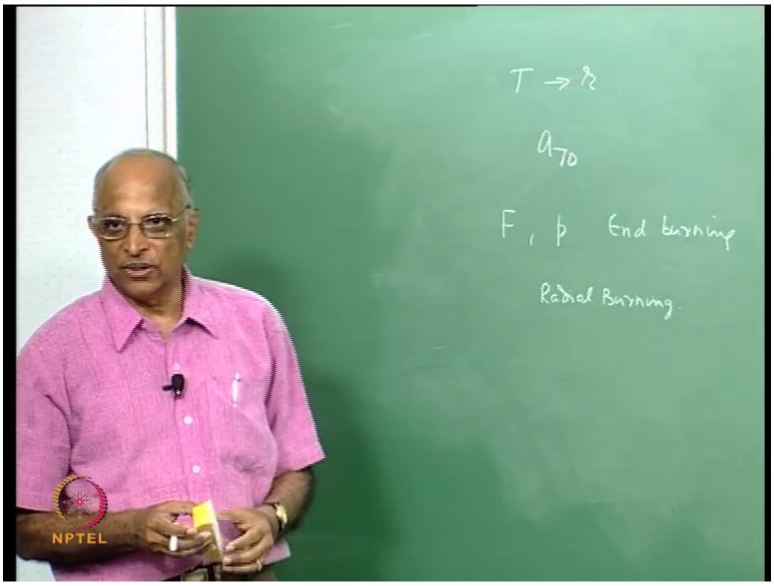

We are able to find the time of burning and this is how a cylindrical grain will be analyzed and designed. What is it we have done in today's class? We looked at the effect of temperature on burn rate, we also defined the value of the constant in burn rate law a as a_{70} at a reference pressure of 70 bar. Then we learnt how to develop an equation for thrust and pressure for an end burning grain. We also determined the pressures and burn times for radial burning. We found for radial burning grain, the pressure evolution had to be done by in increments as the burning progresses.

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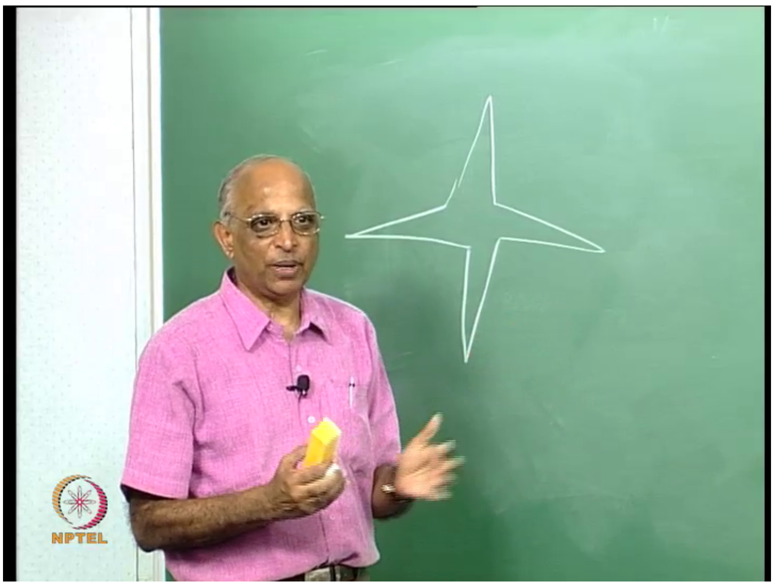

We could solve analytically for the cylindrical radial burning grain. We shall continue with this in the next class and look at the evolution of burning surface area from something like a star grain, which was wrinkled to give a large burning surface area. We will also look at the different forms of grain shapes, which are used in practice, and the reasons for it. After that we will summarize the solid propellant rockets by incorporating the igniter in it along with the other aspects.