Rocket Propulsion

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Lecture No. # 23

Burning Surface Area of Solid Propellant Grains

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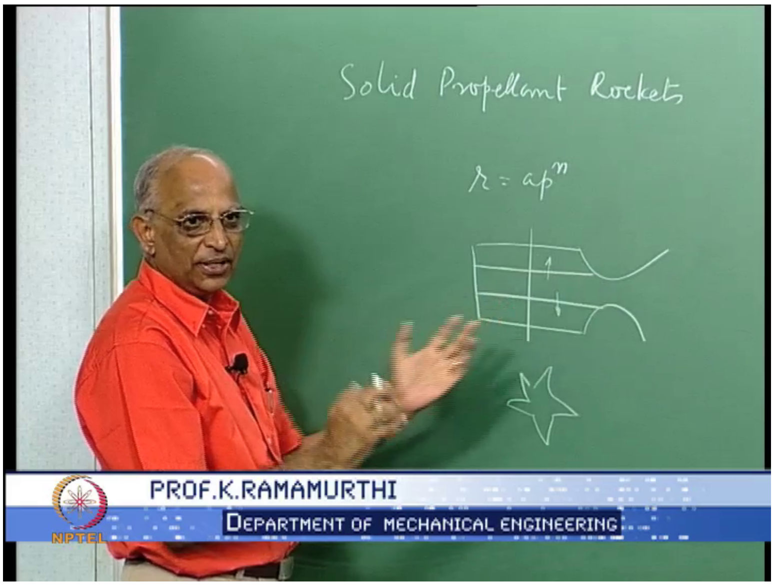

In today's class, we continue with our discussions on solid propellant rockets. By now, we know what the burning rate is; it is the linear regression law given by $r = a\,p^n$; this is applicable for both composite and double base propellants and also the composite modified double base and also nitramine propellants. What we were discussing is how do we assemble the propellant grain and what should be the configuration of the grain. We saw in the last class was if we have something like a radial burning grain, we would like to wrinkle this surface into something like a star or some other configuration, such that we get increased burning surface area and therefore, increased pressure and therefore increased thrust. The aim is to get a large thrust.

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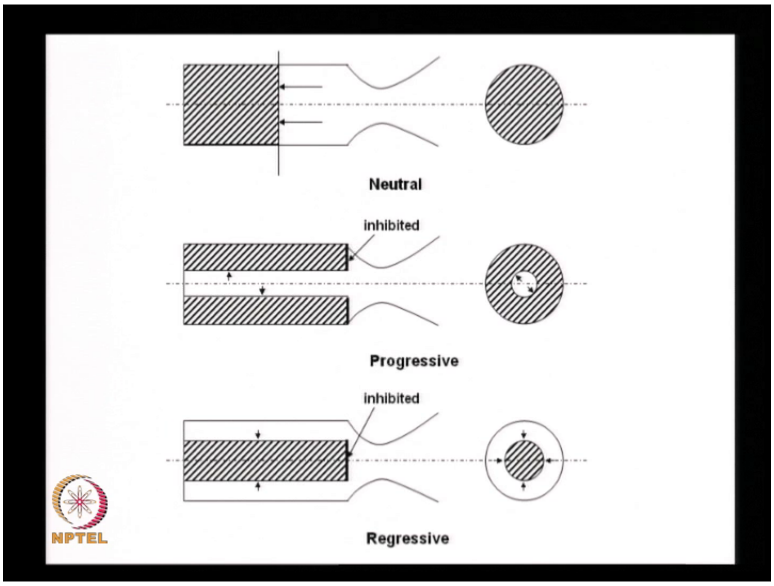

Therefore, let us quickly recap where we were. We talked in terms of let say neutral burning grain; that means, the burning surface area is constant as it keeps burning. We called it as neutral burning, because both the pressure and the thrust were constant at all times. We also discussed about radial burning grain, burning from inside to outside and what did we find? We found that the pressure progressively increased and the thrust progressively increased. In other words, burning started at the inner surface and progressed to the outer surface; we called this as progressive burning. And, if we could somehow get the burning to start from the outer and progress inward, the pressure will keep falling with time, we called it as regressive. Therefore, we talked in terms of neutral burning, progressive burning and regressive burning.

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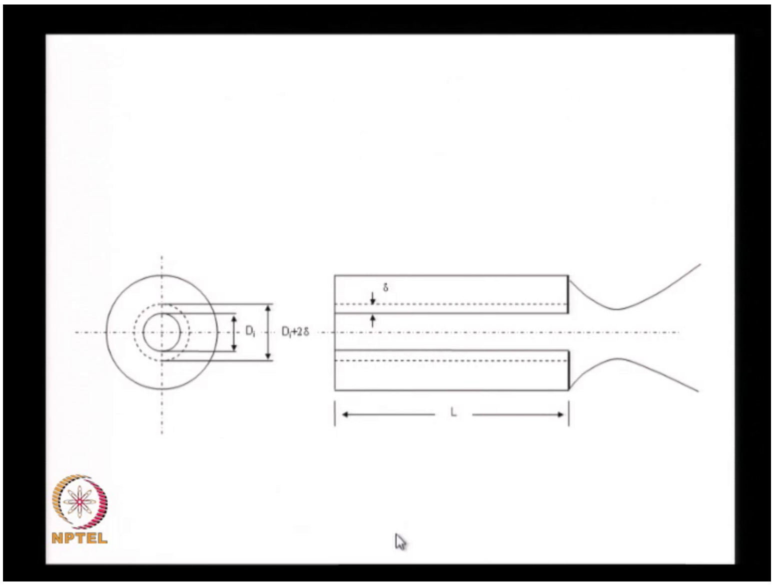

And, thereafter, we also evaluated how we could determine the pressures at different instance of time for the radial inward burning grain and also the time taken to consume the propellant from the inner diameter Di to the outer diameter Do.

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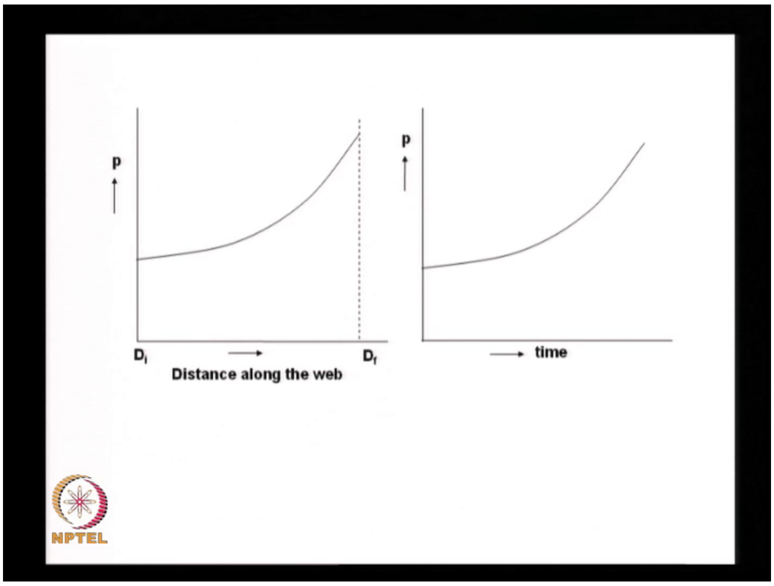

Having said that it is not possible for us to determine the pressure variation along the grain analytically for non-circular shapes, let us look at a few definitions. The distance between the inner surface and the outer surface is what we call as the thickness of the grain. The minimum thickness is known as a web thickness. And, if we say this is the inner diameter this is the outer diameter, the pressure keeps increasing and therefore, as distance along the grain increases, the time increases. Therefore, we were able to plot pressure as a function of time and we found that the thrust of this rocket, thrust going as C_F into p into At keeps increasing, with time as is shown here. This is the progressive burning grain.

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We had also addressed wrinkling of propellant surfaces. I show a picture of a wrinkled propellant grain burning radially outwards. Instead of having the grain, which was circular on the inside, we sort of wrinkle the surface such that we have something like a star shape and this star shape is throughout the grain surface. And, now what is going to happen as the burning progresses? The inner burning surface area Sb is going to be much larger, in other words Sb is going to be the perimeter of this particular star shape multiplied by the length along the grain that is going to be the initial burning surface area. And, therefore, the pressure will be much higher, the thrust will be much higher compared to what it would have been had it been for a circular geometry of diameter D. These are wrinkled surfaces. And, in today's class we will calculate how the burning surface will evolve for this particular star grain.

But a wrinkled radial burning propellant grain need not be a star alone.

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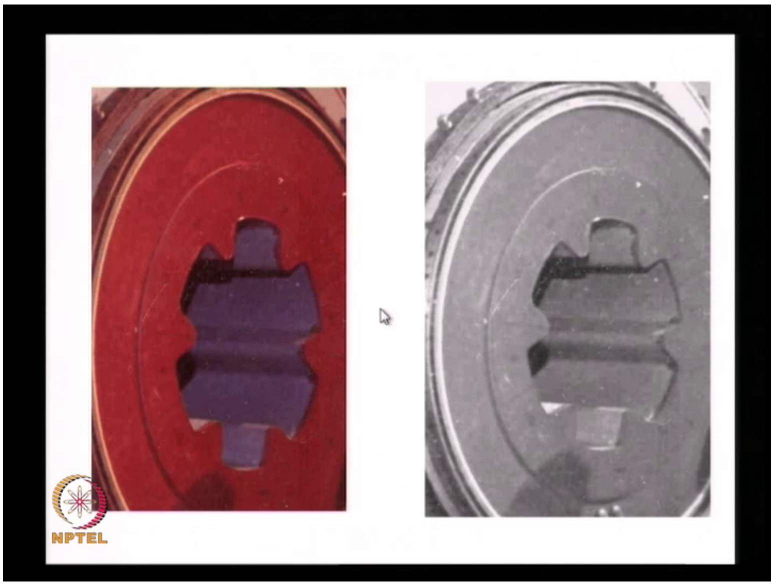

We could have grains of different shapes; maybe we could have lobe like this instead of having a star over here. The burning surface area, this is the inner surface of the grain, this is the length of the grain in this direction, the evolution of surface area would keep on evolving like this until it touches the outer surface of the grain.

The minimum thickness from this root to the case is what is known as the web thickness. Mind you, there are several thicknesses. It could be from the vertex to the outer diameter or it could be from the root or bottom part to the outer diameter. The minimum thickness is what we call as the web thickness. The same grain I show over here just to make sure we understand. We find that the inner surface has something like a projection here, a valley here, again a projection here; this projection is all along the surface and the grain burns radially. That means, it burns into the grain in this particular fashion and all along the surface, the surface keeps regressing like this. And, we will consider a few examples such that we are very clear how to calculate the burning surface areas.

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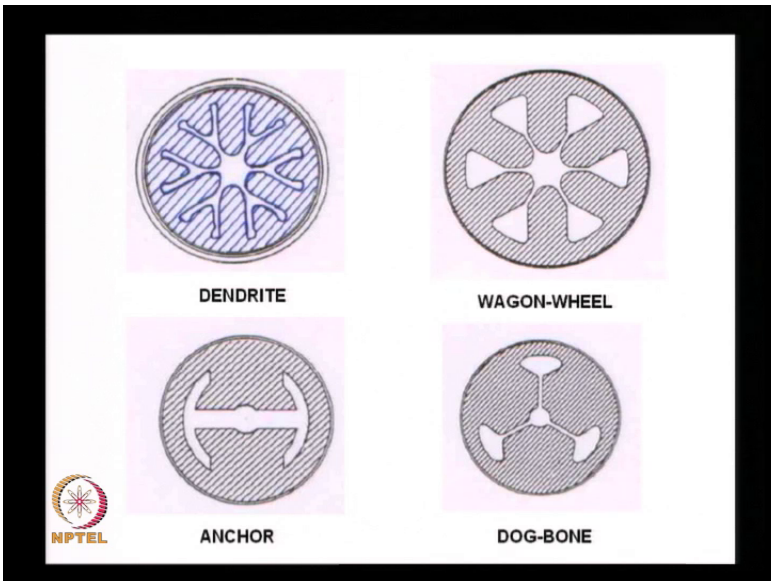

The grain surface need not always be a star or a shape, like what I just now showed which was a lobe like a wagon wheel. But it could be any shape other shape; could be a dendrite, dendrite is a crystal shape which is like this. And, burning surface starts or the inner burning surface to begin with; this is the perimeter into the length of the grain over here. And burning proceeds in this direction; this is known as a dendrite grain. You have a wagon-wheel in the shape of the wheel of a wagon, see you have the inner surface and the length of this is along this particular direction. Therefore, the perimeter along this is not quite large compared to what would have been for a circular diameter over here, multiplied by the length is what gives me the burning surface area and this is known as a wagon-wheel.

We could have the shape of an anchor, instead of having a circle like this, I sort of extended make it shape like an anchor and what is an anchor? You drop an anchor when a ship is not sailing and this is the shape of an anchor. And if this burns, well the burning shape will keep on evolving along the anchor surfaces and I can find out how the burning shape evolves with time and calculate how the thrust as keeps burning. I could have dendrite, I could have wagon-wheel shape, I could also have the shape of a bone, the dog-bone where this is the shape of the bone which is shown. This is the inner diameter. Now, how is this grain going to evolve? It is going to burn perpendicular to the face of the bone shape. We should be able to calculate it progressively till this the surface touches here and thereafter the burning surface area decreases.

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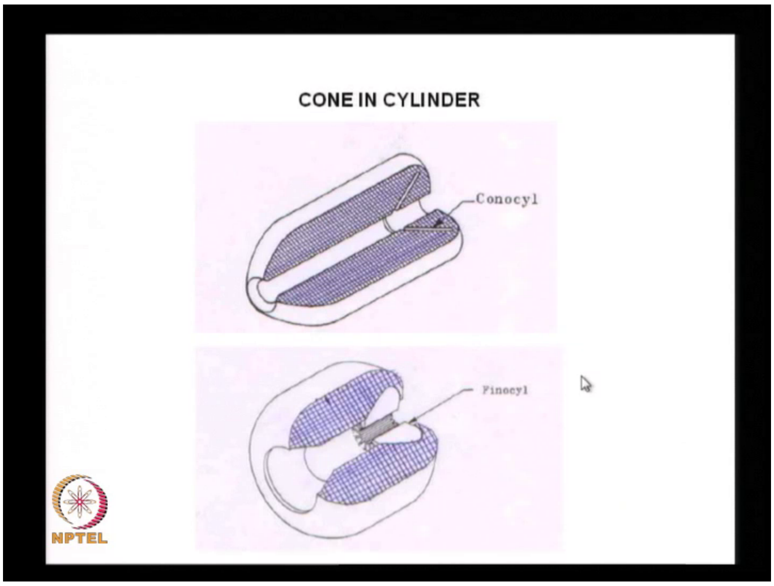

We could also have something like a cylindrical shape and at the end of which we have a cone; that means, I have a cone within a cylinder and this is known as conocyl; that means, I have a cone within a cylinder. How is this going to evolve? The surface would keep on evolving parallel to the initial surface. And it is the evolution of burning surface area what we are basically interested. In addition to having sort of a cone in a cylinder, I could also have a cone like this and this cone has this cylindrical portion; in the cylindrical portion I make ribs like what I show here.

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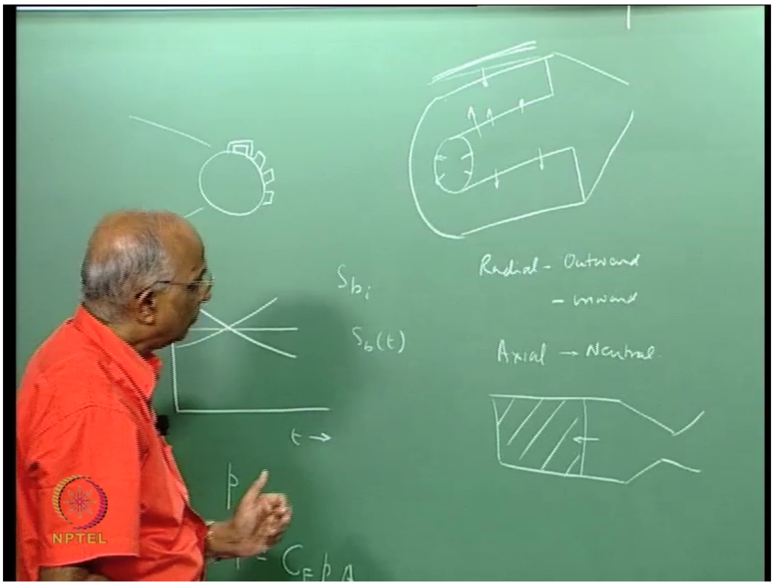

In the section shown, we make ribs like this; something like fins. Therefore, in addition to having a cone on which this is situated, we have something like a cone over here, coming to a cylindrical portion on which we have ribs; therefore, here also burning could evolve along the cylinder, cone and fins and we would like to maximize the perimeter and the burning surface area. When we have a fin in a cone with cylinder arrangement, the grain is known as Finocyl; that is we have fin in the cylindrical portion. Therefore, there are various grain shapes, but what is maximally used amongst these is a star grain. We will see the reason in this class. We note that different grain shapes are used.

Therefore, we could have any shape of the inner surface? We want to increase the burning surface area to the maximum possible extent, initial burning surface area and the progression of the burning surface with respect to time. In other words if we are interested as time or as burning proceeds, the burning surface area should initially be large such that I get a thrust; you could either evolve progressively increasing like this, or it could decrease or it could be a constant. In other words the shape could provide progressive, regressive or neutral burning. The burning surface area directly translates into pressure and pressure directly translates into thrust F which is equal to C_F × chamber pressure × At. And, what was chamber pressure p? We got an expression in terms of

$Sb^{1/(1 - 'n')}$ and therefore, it was directly connected.

Let us now go to the star grain, which is of primary interest. I forgot to mention one grain, which is known as a slotted grain. See, so far when we considered these different grain shapes; we essentially considered let say a cylindrical grain; wherein I have something like a cylinder, this is the length, may be this is my outer case over here and here I put my nozzle over here, we told ourselves this end is insulated it does not burn over here. Burning takes place along this cylindrical surface. Therefore, the burning surface area progresses radially and it is outward from the centre. Burning takes place from the inside surface to the outside. If we ignite it on the outer surface may be we have to allow a gap between the case which is insulated and we allow it to burn inward; therefore, we say radial inward. Radial outward was progressive and radial inward was regressive.

We also had axial burning; burning takes place along the axis of the grain. We had something like a case in which I put the grain over here; we allow the burning to take place normal to this surface axially; here also the burning is normal to this particular surface over here. But it is in the radial direction while this is in the axial direction. And, for the neutral burning, that is end burning i.e., regressing from from end to end, it is sort of end burning which is neutral. It is also possible to have some configuration like combining radial and axial. Let slightly modify this configuration.

If we make some slots on a cylindical grain. How is burning going to proceed? Burning goes radially here, burning progresses axially here; that means, at the next instant of time the surface is going to be something like this, coming over here radially, surface comes here axially. It goes both axially and radially and these are known as radial cum axial burning or three dimensional burning surfaces. In other words, it is both axial and axisymmetric and therefore, these are known as three dimensional burning surfaces. And we show a three dimensional surface over here which is a slot. It continuous to burn in this direction along the slot and cylinder, but the problem is a simple geometric problem.

We want to find out how this initial burning surface area which is the perimeter into the length keeps increasing as the regression of the surface continues? This is the slot. If we put a number of these cylindriacal grains together and allow them to burn both on the inner and outer surfaces we are no longer having a particular burning. The burning is unrestricted because it is burning from radially inward and radially outward and we get a large burning surface area. Control of burning is difficult with unrestricted burning but is used in practice for small duaration of large thrust requirements such as rocket assisted takeoff for planes.

Therefore, to summarize we talk in terms of neutral burning, progressive burning, regressive burning, unrestricted burning and everything decides on how the surface area keeps changing. Having said that let us come to this particular problem of a star grain. See, here I show in the end view a star grain.

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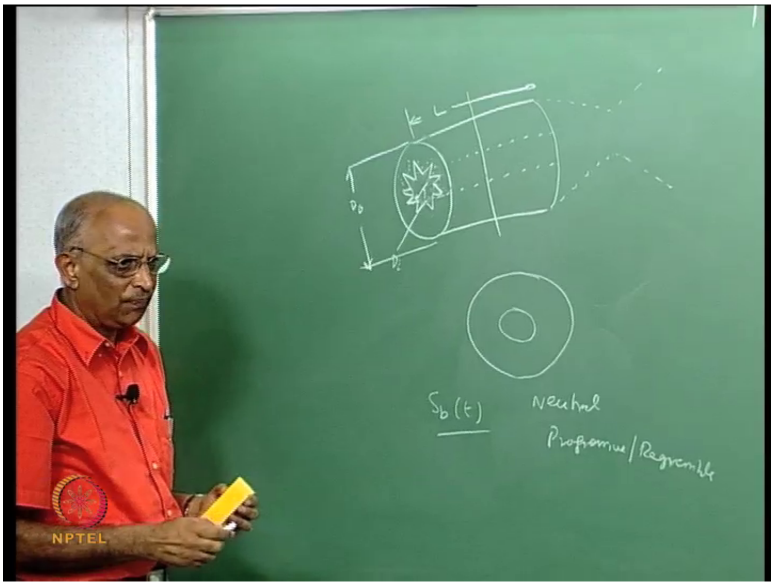

We have a grain like this; this is the length of the grain, propellant grain over here and the nozzle is integrated to give me the thrust. We were able to calculate the pressure and thrust when we have a circular section corresponding to the outer diameter and a circular hole at the centre. And now the initial burning surface area is in the form of a star, this is the length of the grain L, this was the mean inner diameter and this was the outer diameter Do.

Now, if we sort of wrinkle this surface and we showed how wrinkling is done, instead of having this surface we have a multi star configuration. That means, I have 1 2 3 4 5 6 7 8 or 9, 9 star, 9 vertices of the star. We remove the inner diameter and introduce the 9 vertices star. And we find that we have a much larger burning surface area and how is it going to evolve at the next instant of time as the propellant burns? The flat surface evolves as such with the vertex evolving as a circle. And, therefore, we find it regresses in a slightly different shape and we are interested in finding how the perimeter and burning surface area evolves with time. Question is whether we will get progressive burning, regressive burning or neutral burning. It is possible to configure the star grain to give both neutral as well as progressive or regressive burning. Let us do it.

And, how do we predict the thrust developed by a star grain? If we take a look at this figure again, what is it that we see? This is the star grain. In practice to get a point is difficult and therefore it is slightly curved at the vertex. And, therefore, the next line over here shows, after sometime when the burning has progressed it goes like this. And, therefore, I need to calculate this perimeter and if I calculate this perimeter and multiply by the particular length, I get the burning surface area Sb after some time t. At the next instant of time, well the surface is again evolving; we can calculate this perimeter and so on. Towards the end, this is the shape; see, initially I have these surfaces, but as it progresses it becomes something like a circle. And, why does it become a circle, because a point when it evolves, it progresses into a circular fashion. A normal to the circle is along all directions. And, therefore, to be able to understand the evolution of the star surface, we first deal with a simpler case of a square opening in a cylindrical grain.

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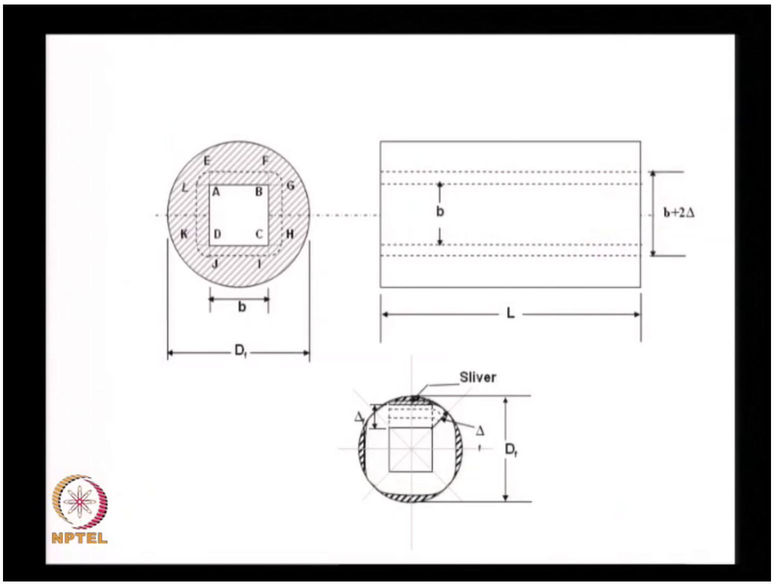

We will calculate how the surface evolves with time and based on the experience that we gain in this particular case, we will take a look at how a burning in a star propellant grain takes place.

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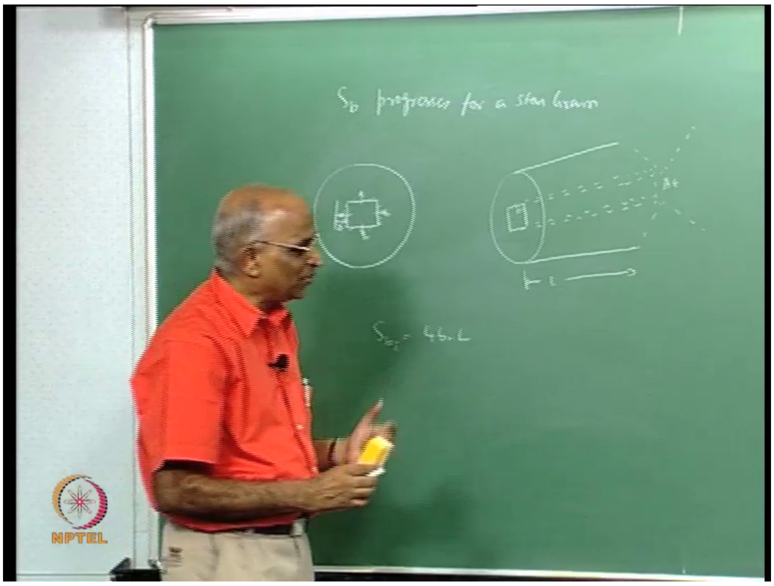

We would like to find out, how the burning surface area progresses for a star grain; what is a star gain? It is one in which the internal surface is sort of wrinkled such that we have larger number of surfaces. If we can estimate the how to find out the evolution of the burning surface for a star grain and if we can do one or two cases, we can find out how the thrust of a solid propellant rocket can be varied.

Let us do this simple configuration of a square in a cylindrical grain. We show a cut view of the grain Instead of having a circle at the centre, which we have already done; radial burning and radial burning is normal to the circle, we now consider the case wherein at the centre we put a square hole instead of a circular hole. We have a square hole of dimension b. The grain is of length L. We have a nozzle of throat area At connected to the case. We would like to know, how this square surface evolves? How these four straight lines which constitute the perimeter evolve?

We said that the length of the sides are b. Therefore, the surface area of each side is L × b. The total surface area at the start of burning will be 4 b which is the perimeter × the length L. What is the surface area which we are igniting; the inner surface area corresponds to b into L, b into L, b into L, b into L or rather perimeter is 4 b × length; that is the initial surface area which begins to burn.

How does burning proceed? Burning takes place linearly at the surface. I want to find out, what will be the burning surface area after a certain time; let say when the surface moves through a distance let us say Δ. What would be the burning surface area? Initial value is 4 b into L. Is this clear, are there any questions?

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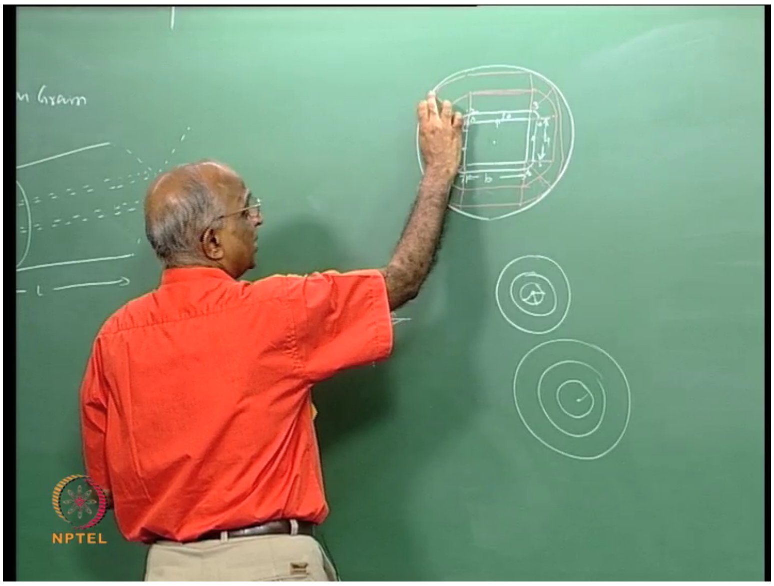

Let me draw a slightly bigger figure. Now, this is the centre, all sides are b. Let us assume that the grain moves through a distance or regresses through a distance Δ. Therefore, this straight line shifts by Δ, this straight line moves normally by Δ; all four lines of the perimeter get shifted by Δ.

How does this corner point evolve? We said burning is always normal, therefore, how should it evolve? Let say, we have this vertex here, burning surface is normal here, burning surface is normal here; this is the surface which burns. How will this point evolve? Normal? Point will evolve like what? How would you look at this problem?

Two things; this surface goes straight normal to the surface. How will a point burn as it proceeds? When we have a point then it should be normal to the point; that means, burning should take place here, what happens over here? That means, a point will go as a curve of particular radius; that means, when this flat surface moves through a particular distance Δ, it will form at the corner a quadrant because for a point we say burning is always normal to it. Normal to the point could be in any direction, therefore, normal to the point will go as a circle, but if we look at the flat surface, it evolves parallel to itself.

Similarly this point will evolve as a circle over here; that means, only a quadrant till it meets lines. What is that radius of this quadrant? The distance moved by the flat surface. Therefore, radius is delta again. That means, if we have a point and the burning rate is r, in the first second it comes to r, second second it comes to 2 r, third second it comes to 3 r and so on. Therefore, this has now regressed by delta in all the directions delta, delta over here. Therefore, when the surface has sort of gone from the initial point to the final point which is delta away, what is the value of the burning surface area?

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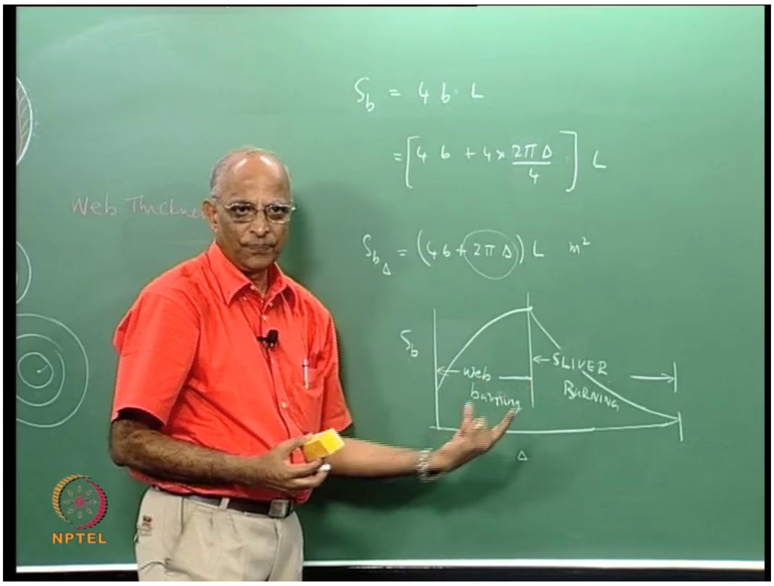

We said that at the beginning the perimeter was just 4b and the burning surface area was 4Lb. Each b remains as it is and what is the additional value perimeter that we now get? We get four quadrants, four into each quadrant with radius Δ; that means radius is Δ; therefore, 2πr, 2πΔ divided by 4 is the perimeter of each quadrant. We get the burning surface area, when the grain has regressed by delta as equal to (4 b + 2 πΔ) × the length L, so many meter square. If we have something like a square with a vertex over here, the vertex evolves as circle when it regresses. But the flat surface evolves as a plane because burning is normal to it.

Therefore, the vertex as it regresses meets it - meets these two adjacent surfaces. Therefore, we have a quadrant, the perimeter of this quadrant is equal to 2 π × radius or 2πΔ/4 is the length of the quadrant. let say from 1 to 2, from 3 to 4, the length is 2 π Δ/4, for 5 to 6 and for 7 to 8; whereas, surfaces 2 -3 is equal to the initial perimeter, giving the total sutface arae as $(4\ b + 2\ \pi\Delta)\times$ L meter^2.

Therefore, we find that the burning surface area now keeps increasing for each additional value of delta and how long will it keep increasing? We find that the surface for some distance keeps evolving in the same way; we are very clear about it. If the burning process let say by some amount Δ over here, now my new Δ is Δ_1, we have another arc of a circle, another arc of a circle, straight line, arc of a circle straight line, straight line over here, arc of a circle. And, therefore, the perimeter corresponding to burning surface area in this case is 1 2 3 4 which is same as b + the four arcs of the quadrants.

And, we find Δ keeps increasing as the burning proceeds. Therefore, Sb now keeps increasing with time; rather instead of having the grain wherein diameter, if I had something like a circular hole, you know the diameter directly increases. In this case, we get smaller effect. The burning surface area would keep on increasing still further till it reaches the limit when this particular arc or vertex comes and hits the outer diameter. Then we have a circle of diameter over here and the plane surface comes over here.

Similarly, the circumference of this quadrant comes and touches the outer diameter over here, for I have a circle like this; plane surface continues. And, this is the limit till the burning surface area keeps increasing as the perimeter 4b plus $2\pi \times$ value of Δ or portion of the propellant which gets burnt. Once, this happens, the perimeter and the associated surface area begins to decrease. This is because the propellant surface has already reached the case and there is no propellant left in the quadrant to further evolve. The rate of increase first decreases and therafter the value decreases. Therefore, once the vertex corresponding to the middle of the quadrant comes and hits the case, thereafter the burning surface area would decrease. After this the perimeter corresponding the the evolving quadrant decreases. And, therefore, the burning surface area may be decreases like this and ultimately becomes 0. This is the way they burning surface area evolves or changes.

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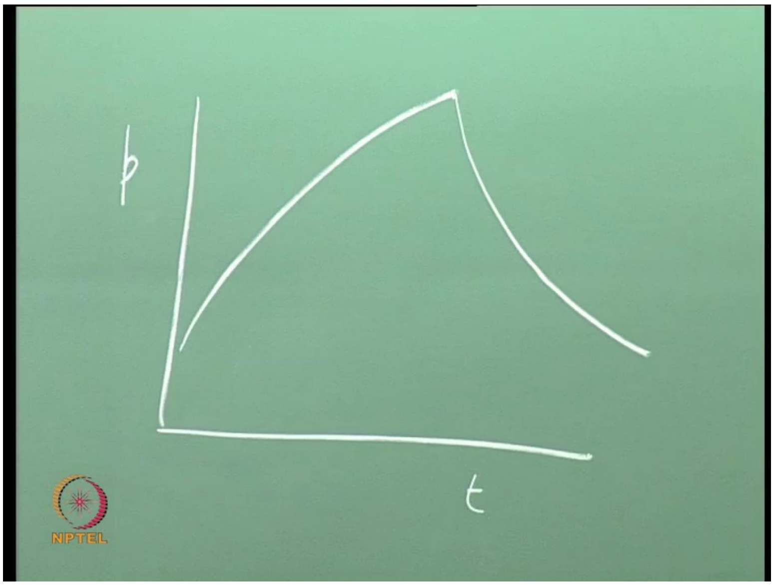

And therefore, the pressure in this particular motor which has a square cavity in it will evolve with time something like shown in the figure. But the minimum distance between the surface or a point on this surface and the case we defined as the web thickness. Let us again take a look at what I mean by this web thickness, I will show it in this figure again. If we have something like a outer diameter over here and the inner diameter over here Di and the outer diameter is Do, the thickness of the grain is equal to (Do − Di)/2. This is the thickness of the grain; that means, the grain starts burning here, when the thickness is so much it gets totally consumed.

Now, if instead of a circle of diameter Di, supposing we were to put a square over here at the centre. In other words, now the grain is of this shape. Now, what is the web thicknesses that I can talk of? This thickness is a little larger, this along the vertex if I join a line from the center to the vertex and then to the case, the distance between the vertex and the case along this line will be the minimum distance. As the grain evolves, it goes as a circle and first touches the outer daimeter of the grain. The thickness of the propellant is more along the other lines from the center. And therefore, this thickness between the corner point or vertex to the casing is the minimum thickness and this minimum thickness is what is known as a web thickness. Why do we call it as web thickness, because till the web thickness is consumed, the burning is progressive or the burning surface area keeps increasing and after the surface comes over here the burning surface area decreases.

Thereafter, what is going to happen this particular perimeter? It keeps decreasing once the point reaches the outside diameter; whereas, the straight line portion is still constant. Therefore, the burning surface area will begin to decrease and therefore the progressive burning is up till the web gets consumed. The region of incresing pressure is known as web burning and this progressive part corresponds to the web being consumed. However, some propellant is still left and is known as sliver or left over sliver burning. Therefore, in such square hole grains, you find that sliver burning occupies quite some time and leads to low pressure and low thrust. The low pressure comes from the smaller burning surface area. This is the reason why such square cavity grains are not used in practice.

If now we go to a star propellant grain with which we are interested, we had something like a circle, we wrinkle the circle such that we get a star shape. We have a number of star points over the circle. The web thickness will be from the vertex to the outer diameter of the grain. This is the minimum thickness and is the web thickness. At the other places, the thickness is much larger. Therefore, I am interested in making sure that the web burning distance is large with respect to the propellant size so that the sliver, sliver is the length left over after the burning, is quite small. Inadvertently we introduced some words like web burning and left over propellant after web burning as sliver. With this background, I think we are in a position to be able to calculate how the burning should proceed for a let say a star grain.

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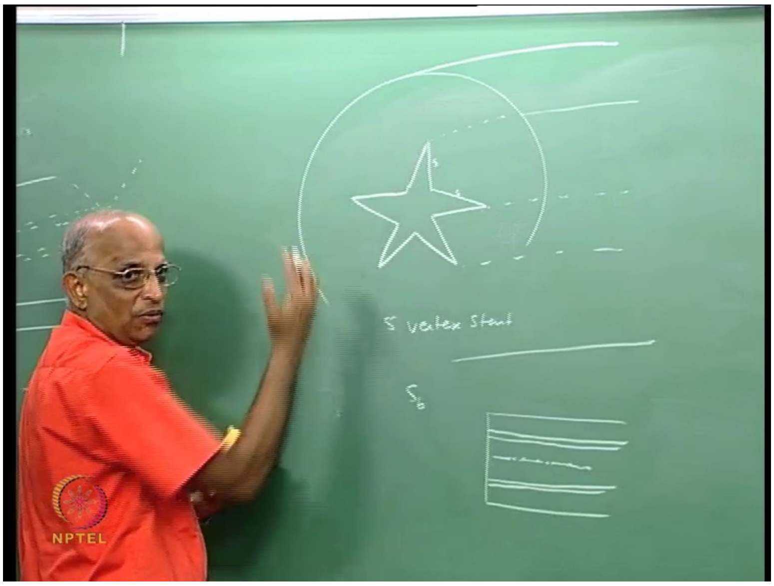

We wrinkle the surface such that we get something like a star surface; let us have 1 2 3 4 5 that is of 5 vertex star. I could choose any number of vertex in the star, it could be 7, it could be 8. How do I characterize the evolution of the burning surface from the star surface as the burning progresses towards the outer diameter of the propellant grain or the case diameter? We are interested in finding out how the burning surface area keeps evolving with time. Let us say that the sides of the star are of length s and how does the grain look in the three dimensional plane? Well, this is my outer surface; all these are points here. And, therefore, if we were to make a plan view of this, we get a cylinder, we get this as the center line, corresponding to one vertex we get a line over here it should be dotted, we get another line over here corresponding to the other vertex. We are interested how the surface keeps evolving? Now, you could tell me, the initial value burning surface area.

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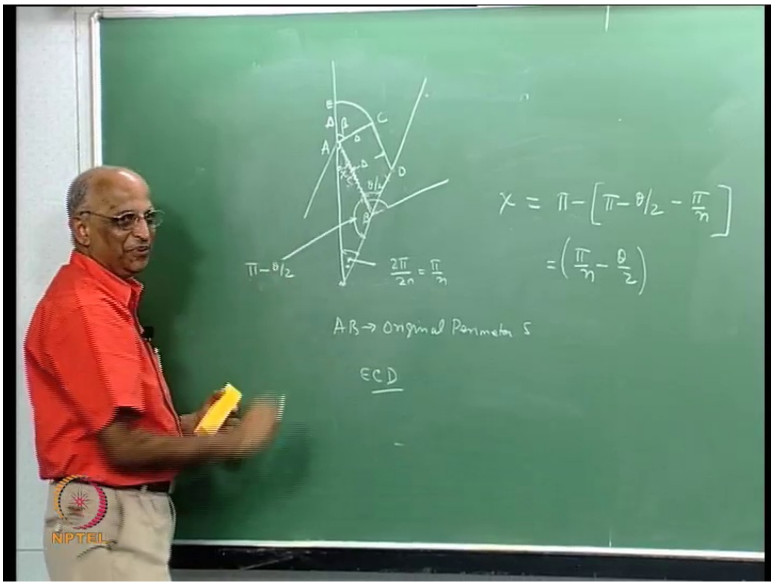

Let us have a star grain with n vertices. We have something like n of these vertices. The vertex A is between the two straight lines of length s. Therefore, what is the initial burning surface area? It is an n pointed star. The length of the grain is L.

The initial burning perimeter is 2sn and the initial surface is 2nsL. You are correct.

We want to know what will be the shape of the perimeter or what will be the value of this length s when the regression progresses through a distance Δ. Let say the burning surface area now comes over here. It moves through a distance Δ. We want to calculate what will be the configuration of this particular point? Can we choose some axis of symmetry and do the problem? Let us examine this figure again; may be it will become a little more clear. We say this is the point A, the vertex; it will evolve as circle. Here on the flat surface the regression will be normal to the surface. It means that this perimeter should become like this. But then the length will change as we cannot have interference. But this is going to be my access of symmetry here; and it is sufficient if we consider the evolution of a single line of initial length s.

Therefore, we have an axis of symmetry here and now with this axis of symmetry the center is over here. All sides of the star are symmetrical. Therefore, if we can determine the changes for this single line when the burning progresses by a distance delta, the length of the line can be calculated. The burning surface area will equal the new value of s into L into now we have 2 n of these surfaces. How do we calculate the length of s as it regresses?

The change is Δ normal to the surface and the line moves by distance Δ. Therefore, if AB is the original length, when it moves by delta normal to itself, it comes from AB to CD as shown in the figure. What happens to this particular point viz., the vertex? Again, we find this is going a circular arc. The new length of this line multiplied by 2 n times, multiplied by the grain length is the new surface area.

How does this point A evolve for the regression by delta? It evolves as a circle of radius delta. It evolves as a circle. This becomes delta here; the arc is EC. Therefore, I find that this particular line on the inner surface now becomes partly the same straight line parallel to AB and partly this circular arc.

How do I calculate this particular total length E C to D? Is the problem clear? We have n vertices. We choose a symmetry line so that we need to examine a single line and multiply it by 2n to get the perimeter. And, whatever happens to this line will happen to all the other 2n minus 1 lines. We would like to find out the length of ECD.

It is a geometric problem; there is no burning. How do we determine the value of E C? I know that the radius is delta. If we specify this angle as β radians we can now say EC is equal to delta into beta. And, we should also calculate the value of CD. To get the value of EC, we need the value of angle β?

For the particular star let the total angle between the sides of the star be θ, in other words we have this star; let this angle between its sides be θ; therefore, this half angle which is bisected by the line of symmetry is theta/2. In other words this small angle is equal to theta/2.

What is the angle at the center, which is included by this particular side AB of lengths? The n vertices have an angle equal to 360 degrees at the center, that is 2π radians. We have n vertices and now each of the n vertices consist of 2 lines; therefore, the angle included is by line AB is $2\pi/2$ n or rather π/n is the included angle for this side at the center. What is the value of this angle adjacent to $\theta/2$? This angle we said $\theta/2$ giving the angle as 180 or $\pi - \theta/2$. If this angle is $\pi - \theta/2$ and this angle is π/n, what is the value of this angle χ in the triangle?

The sum of the angles in a triangle is 180 degrees or pi radians. The angle $\chi = \pi - (\pi - \theta/2 - \pi/n) = \theta/2 - \pi/n$.

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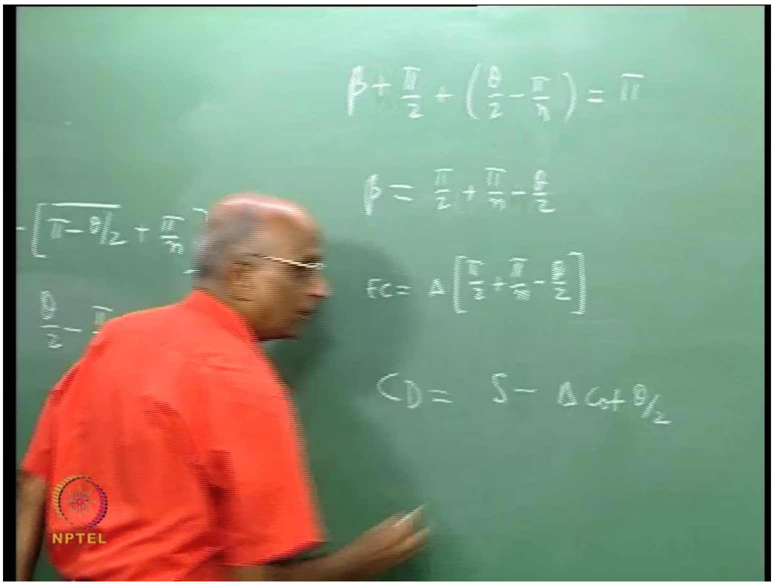

This is right angle adjacent to χ; therefore $\beta = 2\pi - \pi/2 - \chi$. Since χ = was equal to pi by n minus theta by 2, beta is equal to $\pi/2 - \pi/n + \theta/2$. Therefore, we have found out this angle β and therefore, the length of the line EC is equal to $\Delta \times (\pi/2 - \pi/n + \theta/2)$.

Let us check the angles. The angle made at the center is π/n, this angle becomes $\pi - \theta/2$; therefore, the sum of these angles become $\pi - \theta 2 + \pi/n$. See, this value of $\chi = \pi - \theta/2 - \pi/n$. And, this is $\theta/2 - \pi/n$. The angle beta therefore is equal to $\pi -$ (pi/2 $+ \pi/n - \theta/2$).

What is the value of the straight part CD? How do I get this value? We find that this is also right angle and we can have a right angle for which the perpendicular distance is delta. The base of this triangle, is equal to $\cot \theta/2 \times \Delta$ to give $\Delta \cot \theta/2$. The value of AB minus $\Delta \cot \theta/2$ would be the length of the straight line CD. Let us make this clear. The perpendicular distance is delta, this angle is $\theta 2$; therefore, the base is equal to $\Delta \cot \theta/2$.

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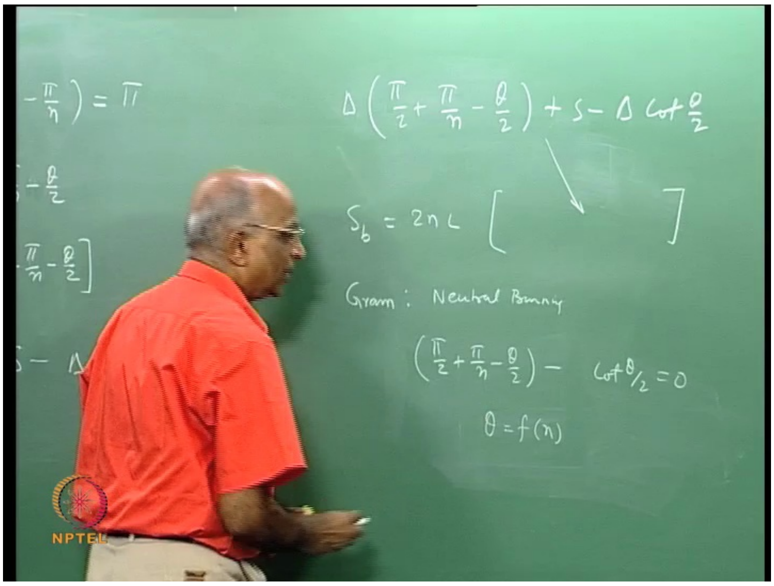

The length of single stretch of line ECD equals $\Delta \times \pi/2 + \pi/n - \theta/2 + s - \Delta \cot \theta/2$. Therefore, what is the new surface area Sb when the grain has receded by delta? Sb is equal to 2 n of the s perimeters × the length and this particular value is what gives me the new burning surface area.

If we want to form a grain a particular star grain, which should have neutral burning, the evolution of the line should not change their length. It must always be s; in other words for neutral burning it is necessary for us to have $\Delta \times \pi/2 + \pi/n - \theta/2 - \Delta \cot \theta/2$ is equal to zero. For this condition, we will have neutral burning. And, what is it we get? We solve this equation, we find Δ and Δ gets canceled and rather we get $\cot \theta/2 - \pi/2 + \pi/n - \theta/2 = 0$.

And, since it involves a cot term we cannot do it directly. We do it numerically. May be we use the method of steepest descent or a Newton Raphson scheme. We find that theta is going to be a function of the number of vertices in the star.

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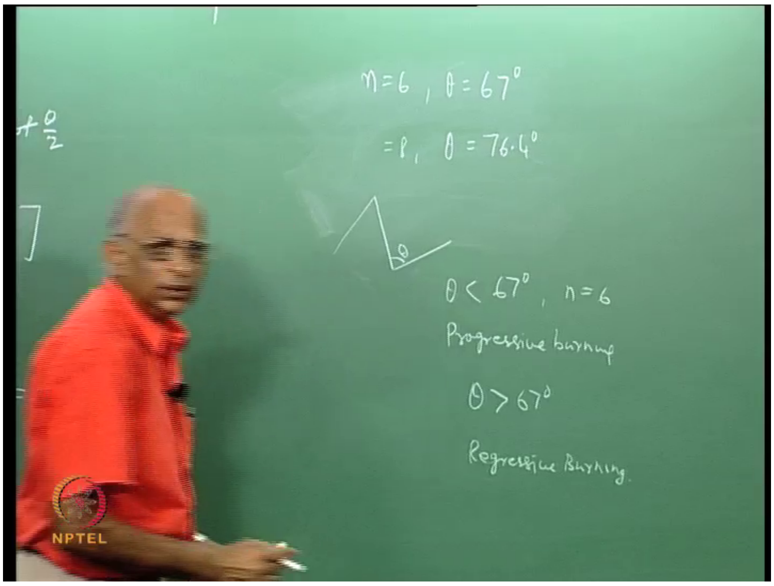

If number of vertices are 6, the value of $\theta = 67$ degrees. If the number of vertices is 8, the value of $\theta = 67$ degrees. In other words depending on the angle θ, we could have neutral burning. And if the angle $< \theta$, what happens? If the angle is less than θ, the surface area keeps decreasing because we are subtracting a larger quantity corresponding to cot $\theta/2$. Therefore, if theta is less than 67 degrees for something like a 6 vertex star, we get regressive burning.

Whereas, when theta is greater than 67 degrees; that means for n is equal to 6 we get progressive burning. Therefore, a star grain depending on the value of θ and the number of vertex, we could have either neutral burning or progressive burning or regressive burning and that is why star grains are quite useful. By suitable wrinkling, we can make it burn the way we want.

Let us just conclude. For a grain with a central square cavity, when the corner of the square meets the outer grain diameter then the balance is what is known as a sliver and you have web burning up to this particular time. We thus differentiated between web burning and sliver burning. And, then we analyzed the star grain, we determined the evolution of the burning surface. We found that this is equal to the length of each side of star $(s - \Delta \cot \theta/2 + \Delta \times \pi/2 + \pi/n - \theta/2) \times 2n \times$ length L of the star grain. Using the burning surface area, the equilibrium pressure and the thrust of the rocket are calculated.

And, then we got the value of θ for neutral burning as equal to 67 degrees for neutral burning when the number of vertex n the star was 6. For larger values of theta we get progressive burning while for smaller θ we get regressive burning. Therefore, a star grain could be designed for whatever be the type of burning we desire. This is all about evolving burning surface area in different propellant grains. In the next class, we will look at the other elements of solid propellant rocket.