Rocket Propulsion

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Lecture No. # 27

Feed System Cycles for Pump Fed Liquid Propellant Rockets

Good morning. Let us continue with liquid propellant rockets.

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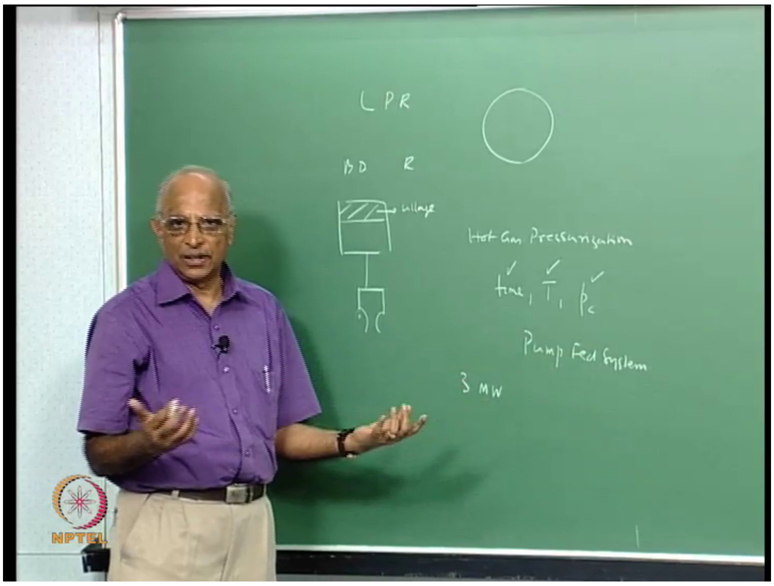

Let us quickly recap where we were in the last class. We addressed the feeding of liquid propellants in a liquid propellant rocket and saw the different ways. One was using a cold gas. The cold gas could be used either in the blow-down mode or in a regulated mode. The difference is that in a regulated mode, we draw gas at a high pressure, maintain a constant lower pressure on the propellant side while pushing the propellant into the chamber. In the blow-down mode, in the tank itself, we have some volume of pressurized gas above the liquid, known as ullage volume, which is pressurized initially and you allow the gas pressure to gradually push the propellant. In this case the pressure at which propellant is supplied into the thrust chamber will keep decreasing with time.

While deriving an equation for the mass of gas required in the gas bottle, we found if the temperature of the gas is higher it is better therefore, we also talked in terms of hot gas pressurization. But we told, even though it is advantageous, it has not been implemented so far. It may be worthwhile to do so. In fact there was one of the French engine, which tried this; but they did not follow it up. But I think it is a strong contender.

We also found out that when the time of operation of a rocket is small or when the thrust is small or when the chamber pressure is small we can go for something like cold gas pressurization in either the blow-down mode or a regulated mode or even a hot gas pressurization mode. But the moment the pressure of the chamber is large or the thrust is large or the time is large, we necessarily have to go for a pump fed system.

We also found that the amount of power required for the pump is enormous like for a 600 kilo Newton engine we found that power required is about 3 Megawatt. And we said even a large power plant like the Ennore power plant at Chenai generates about 450 megawatts of power therefore, we are talking of huge power. And therefore, we cannot drive the pump using a battery, or an electrical supply.

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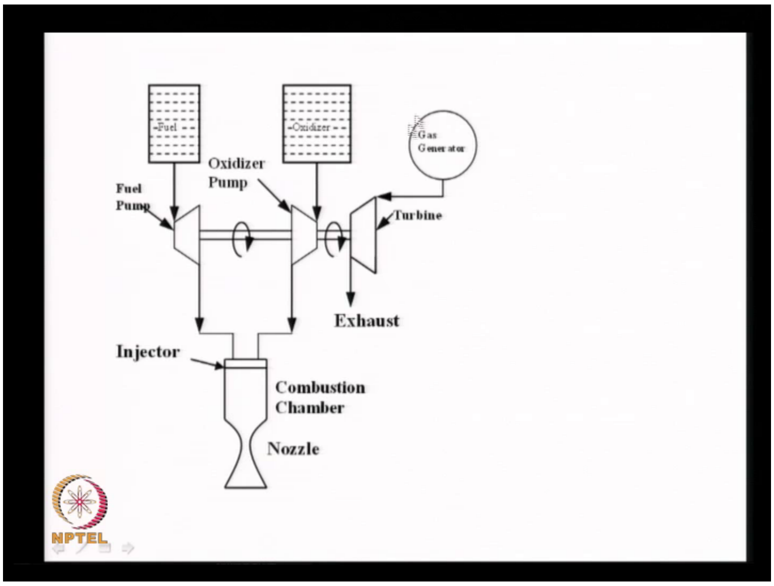

Therefore, in order to drive the pumps, we looked at this figure in the last class also. We have a fuel tank, we have an oxidizer tank and both are liquids. You sort of push the propellants into a pump maybe we need a gas bottle which pressurizes it initially to a small value and pushes it into the pump. We need power or work to rotate these two pumps for which we require a turbine. The turbine generates power and drives the two pumps. We needed a gas generator or a hot gas source, which could power the turbine and the spent hot gases are exhausted out. The pressure built in these two pumps pushes the liquid oxidizer, pushes the liquid fuel into what we said was the thrust chamber. Now, you know to have an additional device for a generating gas and we need another set of lines for the fuel and oxidizer for the gas generator.

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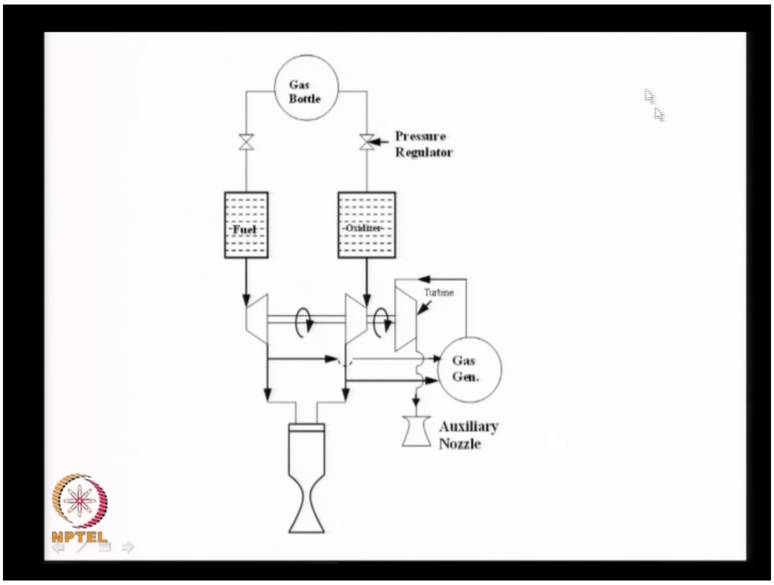

And the normal practice is to make use of the existing fuel and oxidizer itself. How do we do it? We also looked at it in the last class. After pressurizing the liquid fuel, we take a little bit of the liquid fuel into something like a pre burner or a burner, which we call as a gas generator. We also take the oxidizer also mix the two together in the burner to generate hot gases, push the hot gases into the turbine and exhaust the output from the turbine into the ambient. Therefore, you have something like a gas generator, which is driving the turbine, which in turn drives the fuel pump and the oxidizer pump and supplies the fuel and oxidizer into the combustion chamber.

Such a feed system is essentially configured around the gas generator and how does the generator get the fuel and oxidizer? Part of the fuel and oxidizer is drawn from the high pressure lines and is used to generate hot gases. There is a problem with this arrangement. The turbines consist of blades and it could be impulse turbine, reaction turbine maybe we have to look at it, but whatever said and done, whenever we admit very hot gases into the turbine from the stoichiometric combustion of the fuel and oxidizer, the gases are at high temperatures around 3000 to 3600 K.

And therefore we cannot supply these hot gases for running the turbine. The output of the gases from the gas generator must essentially be at a much small temperature maybe around an upper limit of about 900 K. Therefore, how do we get these low temperatures in the gas generator or burner? We use very fuel rich mixtures or very oxidizer rich mixtures to the gas generator and supply the moderately hot gases to the turbine. The work done by the turbine is equal to the work done by the two pumps. This cycle is what we called as the gas generator fed or gas generator cycle for feeding propellants into the combustion chamber.

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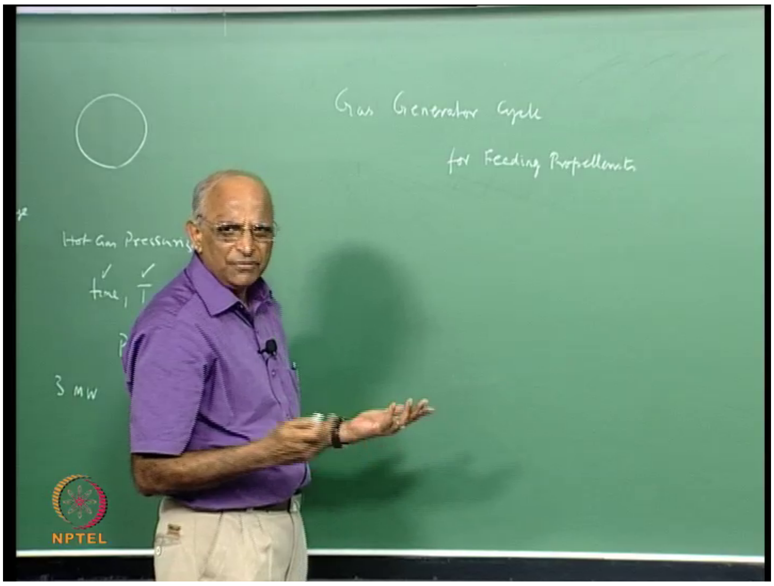

Unfortunately, in thermodynamics we say cycle is something wherein during a process the medium comes back to the initial state. But in the present case, we are feeding propellants and the word cycle is adopted here. It is not related to the thermodynamics cycle such as Otto cycle or Joule cycle or Brayton cycle. It is just a gas generator fed system, but conventionally it is known as a gas generator cycle for feeding propellants and we use the same terminology.

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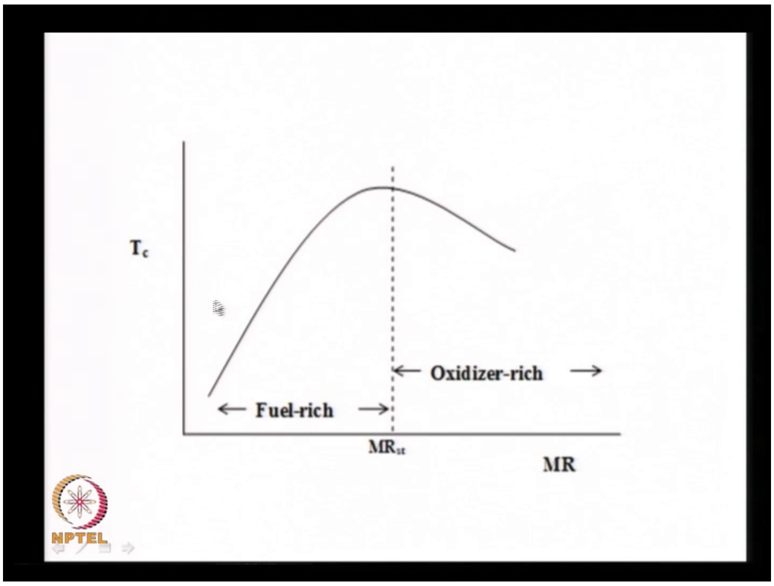

The temperature of the gases as a function of mixture ratio is such that the maximum temperature occurs just below the stoichiometric. We had a slight shift; instead of maximum temperature occurring at stoichiometric it occurs at slightly fuel-rich conditions. But when we need a much lower temperature maybe we have to operate the gas generator in a very fuel-rich region such that we get only around 800 to 900 K i.e., a very oxidizer-rich region wherein I get a low temperature.

Oxidizer-rich region is seldom used, but some of the Russian engines do use oxidizer-rich mixture. What is the reason? Why oxidizer-rich is not conventionally used this? If you have something rich in oxygen, it can always oxidize a metal; whereas, if the mixture ratio is fuel-rich, it cannot oxidize a metal. Therefore, the trend is to use a fuel-rich mixture. This was in the context of the choice of the mixture ratio for the gas generator. We reiterate that a fuel-rich mixture is used for the gas generator and the maximum outlet temperature from the gas turbine or the inlet to the turbine is around 900 K.

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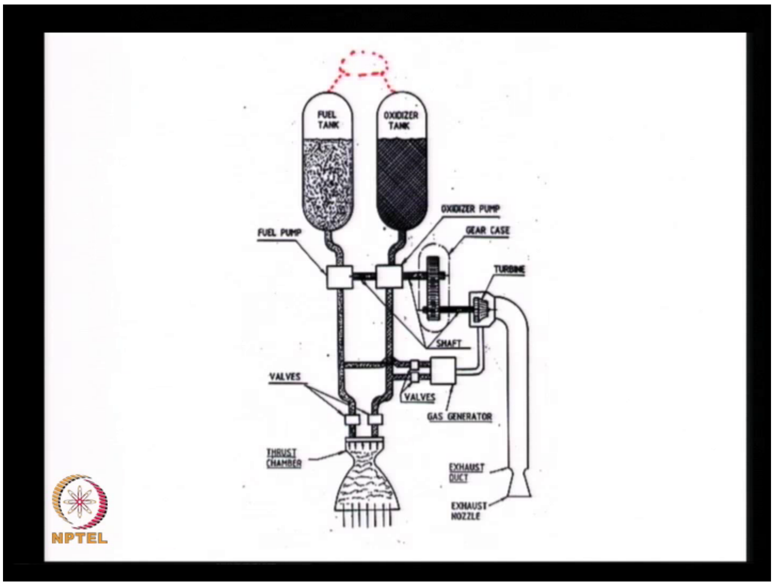

The above figure shows a schematic of a gas generator cycle. We have a small gas bottle, which pressurizes the fuel and the oxidizer. The fuel and oxidizer enter the pumps. We also bleed a little bit of fuel, a little bit of oxidizer after the pump and burn it in a gas generator. We generate hot gases and these hot gases drive the turbine. And when they drive the turbine in this particular schematic you have a gear train for adjusting the speed of the pumps. These two pumps supply the propellant to the engine and the same pumps also bleed or supply propellants for the gas generator.

You may ask how do we start the engine in the first place. After all we need a higher pressure to supply the propellants to the gas generator and the engine. A bottle of high pressure gas could be used to initially run the turbine or a slug of the hot gas can be used to initially use rotate the turbine and once it rotates, it starts pumping and then we shift to the present arrangement after the initial transient. Well this is all about the gas generator cycle. We will get back into it in some detail to find out what is it is performance after we have discussed the other feed system cycles.

Though the term cycle is a misnomer while discussing feed systems as it usually refers to a thermodynamics cycle, there is however a thermodynamic cycle known as Topping cycle used in power plants and followed in feed systems of liquid propellant rockets.

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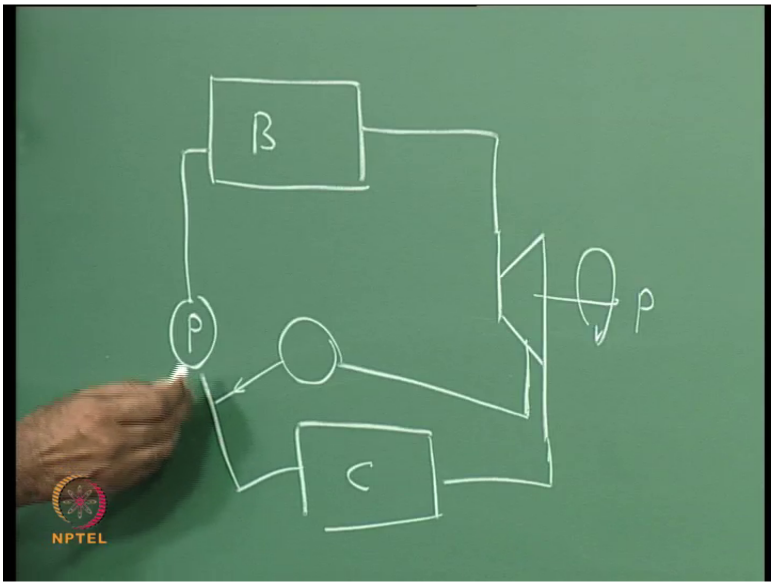

The Rankine cycle, which is used in power plants, has a boiler, which generates steam. The boiler runs a turbine that generates high pressure steam that runs a turbine and that is what generates the power. And then the outlet from the turbine is fed into the condenser, wherein water is condensed and then you have a pump, which pushes back the water into the boiler. The water is heated to form steam in the boiler. Very often what is done to improve the efficiency is, we know Carnot efficiency is maximum when the upper temperature is the highest; we increase the temperature of the cold water by supplying heat from the steam. If we were to bleed part of the steam after a part of the expansion, we bleed some of the steam and take it into a feed water heater. And use this for heating the feed water in a heater. In this way we supply heated water to the boiler. Well the steam temperature will go up and the efficiency of the Rankine cycle will increase.

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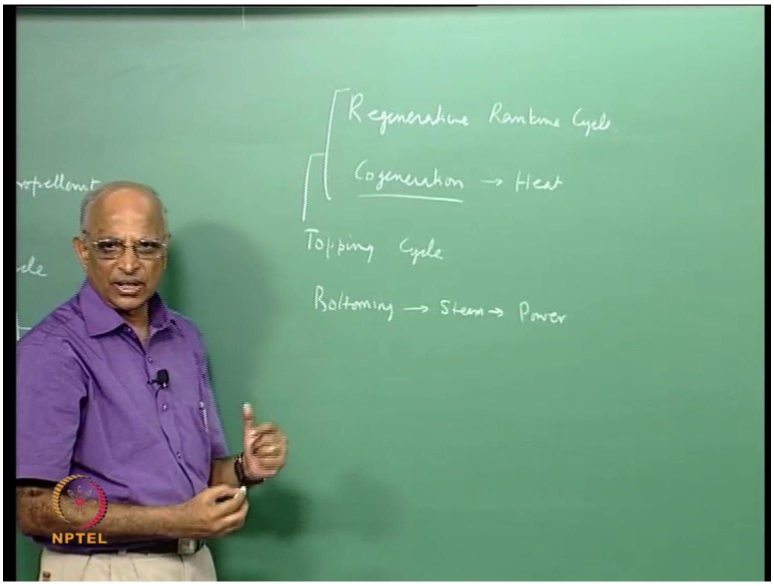

And this is what we call as the regenerative Rankine cycle. Just like you have regenerative process; you heat the water before the boiler using the heat, that is normally wasted, in the process itself; and how do you heat it? We bleed some of the partially expanded gases in the first turbine, allow it to heat the feed water and then you pump the feed water. Therefore, you are essentially able to get a higher efficiency and these cycles are known as regenerative cycles. We also learnt in terms of cogeneration cycles. And what do you mean by cogeneration? We bleed some of the hot gases from of steam coming over here and use it to heat houses or use it for some other purposes than power generation. We use the heat effectively instead of wasting it. This is what is known as cogeneration.

And how do we dissipate heat in a condenser. We have something like a tower, in which we allow the water to drip, we allow the hot steam to go through the tower; the hot steam condenses and it heats the environment of water. Instead of doing this and loose the energy, we could allow the steam to be usefully used for heating houses, for heating some places maybe some industry for heating and that is known as cogeneration. The regenerative cycle is also known as the Topping cycle. Why do say topping? Because we use some of the process heat for "topping up" the temperature of steam in the regenerative Rankine cycle. The cogeneration cycle is known as "bottoming" cycle.

We talk of “Topping” cycle; we also talk in terms of “Bottoming” cycle. If we have a furnace, which is generating very high temperature gases and we use this furnace to heat some product such as for tempering steel or maybe steel making. And instead of allowing the exhaust gases after the useful work to be dissipated into the environment, we use it to generate steam; then it is a bottoming process. But we use the waste heat from the furnace to generate steam and therefore to generate power or to generate electricity. This is known as the Bottoming cycle.

Can we use some element of Topping cycle in the gas generator cycle, which we just discussed. Let us be very clear about it because the feed system cycles are supposed to be very efficient.

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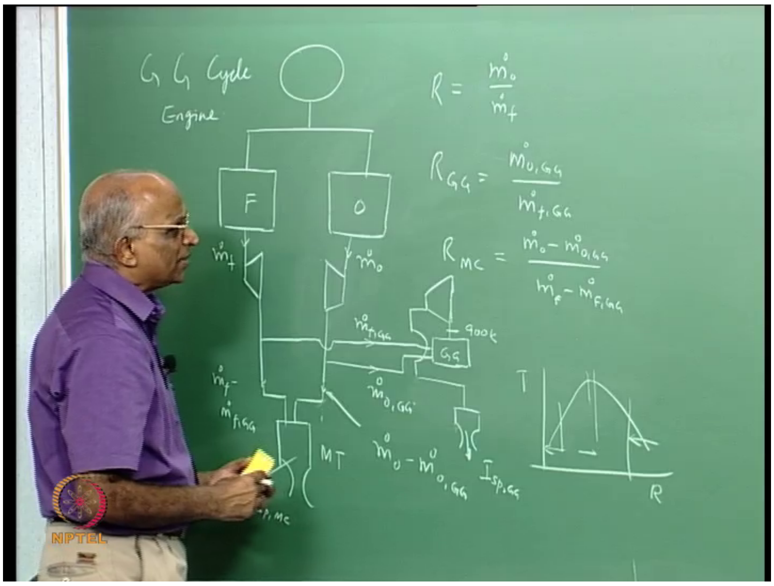

We have a gas bottle, two tanks one for fuel and the other oxidizer. We take the fuel into the pump, the oxidizer into the oxidizer pump and then we push it out at higher pressures. Why is this gas bottle required? We need some minimum pressure to push the liquid into the pump. Therefore, now the requirement of gas is much smaller because my outlet pressure is going to be smaller. And then we take the pressurized fuel and oxidizer, part of it, into something like a gas generator or a pre-burner, or something which generates hot gases at a mixture ratio which produces not a very high temperature. The balance of the propellants is supplied into the main combustion chamber.

Therefore in the process, we have a given mass of fuel m°f, which is supplied by the tank and mass of oxidizer m°o, which is also supplied by the tank. Part of it is bled into the gas generator. The part of the fuel bled into the gas generator is m°f gg and the part of the oxidizer m°o gg is taken to the gas generator. Now, the balance what comes over to the chamber here is m°f − m°f gg, that has been supplied to the gas generator. Similarly, m°o that is the main oxidizer supply − m°o gg into the gas generator is what gets into the main combustion chamber. We have the overall mixture ratio R equal to m°o/m°f viz., the mass of oxidizer divided by mass of fuel.

What is the mixture ratio of the gas generator? R for gas generator is equal to m°o gg which gets into the gas generator ÷ m°f gg . This mixture ratio must be terribly on the fuel rich side so that the temperature is low. We had plotted the temperature versus mixture ratio. If we were to denote the mixture ratio in the gas generator by Rgg; if R gg is stoichiometry we get about the maximum temperature. When we want a reduced temperature, we operate in a fuel-rich region or in the oxidizer-rich region. The value of mixture ratio in the main chamber which generates the thrust R_{MC} is equal to (m°o − m°o gg) ÷ by (m°f − m°f gg).

Therefore, we have three mixture ratios to contend with: R overall, Rgg and R_{MC}. We would like to choose the mixture ratios such that we get maximum specific impulse. But, if we look at the overall system, we would like to choose the mixture ratio R such that we get a good performance. We have to calculate this a little more carefully.

With this background, let us finish the Topping cycle and then determine the value of mixture ratio in a gas generator cycle engine. What did this gas generator do? It draws in part of the propellants, generates low temperature gases for running a turbine. In the turbine we have high pressure gases and these are expanded to low value and work is done. Then we exhaust it out through a nozzle. The nozzle is an auxiliary nozzle.

The temperature at the inlet to the turbine or outlet of the gas generator we said is typically less than about 900 K. Therefore, the temperature at the outlet of your turbine will be even less maybe around 400 K. And therefore, the type of specific impulse, which this expansion in the auxiliary nozzle can give is going to be a small number. Let us call it as specific impulse from the gas generator, Isp,gg which comes from the exhaust of the turbine which is expanded through a nozzle. Whereas, the specific impulse corresponding to the main chamber, we call it as specific impulse of the main chamber is denoted by Isp,$_{MC}$. Therefore, what will be the total impulse of the engine? It will be the fraction of propellant at this specific impulse Isp,gg plus fraction of the propellant in the main chamber at this impulseIsp,$_{MC}$. And therefore the net specific impulse Is,net is calculated.

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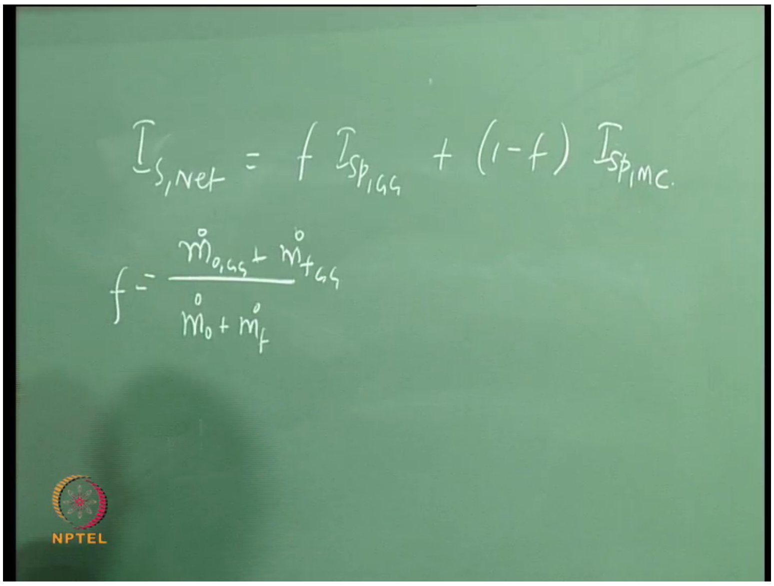

$$I_{s,Net} = f\, I_{sp,GG} + (1-f)\, I_{sp,MC}$$

$$f = \frac{\dot{m}_{o,GG} + \dot{m}_{f,GG}}{\dot{m}_o + \dot{m}_f}$$

The net value Is,net is going to be fraction of the gases f of the total that means m°o + m°f which is the total supply in the denominator and m°o corresponding to the gas generator plus m°fuel corresponding to the gas generator in the numerator. This is how we define the fraction f. And therefore, this fraction × Isp of expansion in the gas generator exit from the turbine + (1−f) that is the part of the propellant which flows through the main engine and has a specific impulse Isp,$_{ME}$ × Isp,$_{ME}$. We noted that the Isp from the gas generator side would be small because the supply temperature is small and the expansion ratio of the gases is small. And therefore, the net specific impulse of this gas generator fed engine is going to be less than the specific impulse what we would have otherwise got had we not had this gas generator, turbine and auxiliary nozzle arrangement.

Therefore, the question is whether this heat, which is being wasted in a poor way, can somehow come into the chamber, to do useful work? This could be very similar to the regenerative process in the Rankine cycle. It could be a Topping cycle instead of the gas generator cycle.

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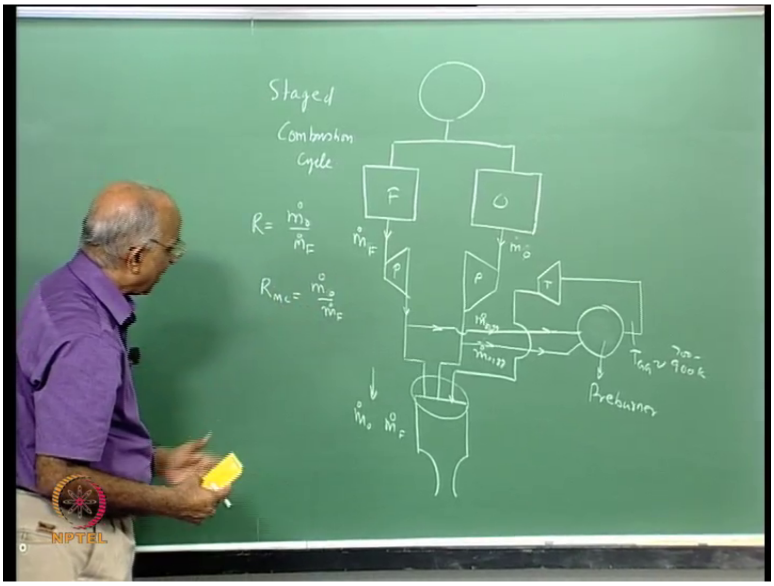

We again have a gas bottle. It supplies the two tanks: a fuel tank may be an oxidizer tank. And then we take the propellants out to over fuel pump increases the pressure and an oxidizer pump, which again increases the pressure. The lines now are at high pressure following the two pumps. And then what is it we do? We take the some of the fuel at high pressure over here take it into the gas generator. We also take a little of the oxidizer and allow it to come into the gas generator. And then we use the hot gases generated to drive a turbine by the expansion of the hot gases to low pressure.

Now, the temperature of the gas generator is typically around 900 K, (80 to 900 K). 900 K is the upper limit; let say 700 to 900 K. And what do you do with the turbine exhaust? Let us see how to regenerate it?

The balance of the fuel and oxidizer come into your main chamber. Now, what do you do with this turbine exhaust? We bring the turbine exhaust and admit it into the main chamber. That means the heat which is left out in the turbine is again re generatively used here and this is what constitutes the Topping cycle used for a feed system.

In other words the exhaust is not wasted with low impulse as in a gas generators cycle, but is fed back into the main chamber for secondary combustion. First combustion takes place here at low temperature then turbine expansion is then mixed with the balance propellants in the main chamber and burnt again. And therefore the combustion now takes place in two stages first in the gas generator and then in the main chamber and therefore the Topping cycle is also referred to as combustion taking place in stages, or a staged combustion cycle.

Let us find out what is the value of the over all mixture ratio and values of mixture ratio of the gas generator and the value of the mixture ratio in the main engine? Just like we did for gas generator, let us do it for the stage combustion cycle.

Therefore in stage combustion cycle, we have m° oxidizer, m° of the fuel, we supply m°o gg to the gas generator and m°f gg into the gas generator. And then for the balance propellant in the feed line to the main chamber, we have m°f − m°f gg and m°o − m°o gg. However, this m dot o, gg and m dot f, gg is coming back into the main chamber and therefore the net value what enters in the main chamber is still m°o and m°f. Or rather the overall mixture ratio is going to be m°o/m°f. The mixture ratio in the main engine or main chamber is again the same value as the overall mixture ratio. And what is the mixture ratio in the gas generator? It is equal to m°o gg/m°f gg.

This is the main distinction with a gas generator cycle and we call it as a stage combustion cycle or topping cycle. And the gas generator in the stage combustion cycle is very often it referred to as a pre-burner because it first burns the propellants and again we have a second burn in the main chamber.

Well! we need not even have a stage combustion cycle when we talk of volatile fuels; we mean fuels like liquid hydrogen or let say liquid methane or liquid propane. It is possible for us to have another cycle. Let us quickly investigate it.

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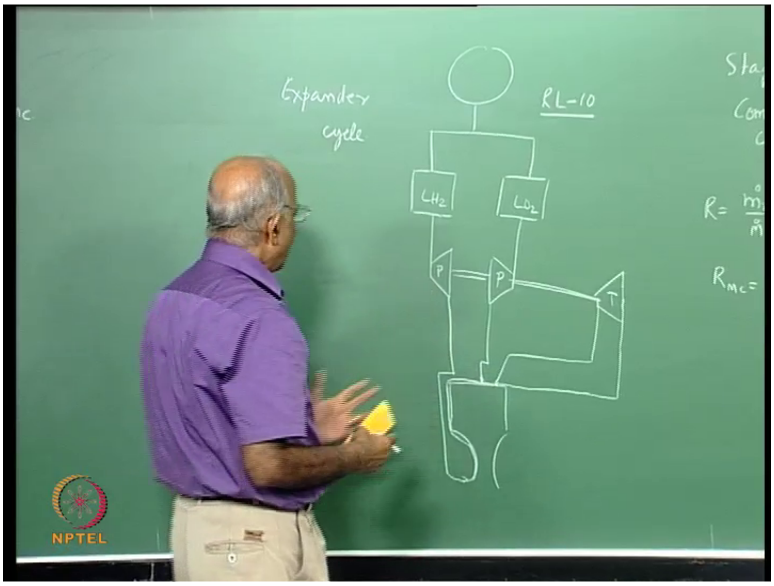

We again start with a gas bottle. We have volatile fuel, let say liquid hydrogen or liquid methane and we have oxidizer like liquid oxygen. In this case we have a pump for the fuel and another pump for the oxygen. Now, what we do is this particular liquid hydrogen, which we have to supply to the combustion chamber, wherein it burns with oxygen and the combustion chamber runs hot, we use it for cooling the chamber. And therefore the hydrogen flows and cools the chamber and gets heated. And this heated hydrogen is in the form of a gas. We use it to run the turbine. The hot gases such as the hot hydrogen vapor, generated during cooling drives the turbine.

And after driving the turbine, we take the exhaust gases and put it into the chamber. We also introduce the oxidizer into the chamber. And in other words what is it we have done? We have turbine, which is run by the heated fuel or heated oxidizer, whatever be it, as long as it is possible to generate a vapor. We generate power in a turbine and drive the pump of the fuel and the pump of the oxidizer. And in other words just by using the hot chamber, we generate hot gases for expansion in a turbine and we run the pumps. Such a cycle is a derivative of stage combustion cycle, but without a pre-burner and is known as an expander cycle.

Therefore, the pump fed systems are basically classified as belonging to gas generator cycle in which case we allow the exhaust from the turbine to be expanded into the ambient. The next is stage combustion cycle wherein we have a pre-burner and take the combustion to occur in two stages. If not, we use the hot vapor generated on the outside of the chamber to run the turbine and introduce the exhaust into the main chamber. May in the next class we will do the regenerative cooling and it will become little more clear at that time. We use the vapor formed during heating of the volatile fuel while cooling of the chamber to run a turbine. This turbine then runs these two pumps. This is an expander cycle.

We could have combinations or variations of some of these cycles. We could allow the exhaust from the gas turbine to be introduced in the nozzle divergent and thus generate more thrust instead of being expanded in an auxiliary nozzle. In this case the cycle known as gas generator with bleed; what is bleed? We allow some of the outlet gases from turbine to come and generate little more thrust by injecting it into the nozzle here. You could keep on devising cycles or maybe we could have something like a combustion taking place in the chamber. We allow the gases to come from the chamber and drive the turbine and this known as the combustion tap off cycle, but it has not been used in practice. What has been used in practice are expander; you will recall the cryogenic engine RL 10 and this uses the expander cycle. We said this was the first engine, first cryogenic engine developed in US.

Many liquid propellant engines use the gas generator cycle but the stage combustion cycles are more powerful and have very high performance. Let us take a look on the merits and performance and see what is normally preferred.

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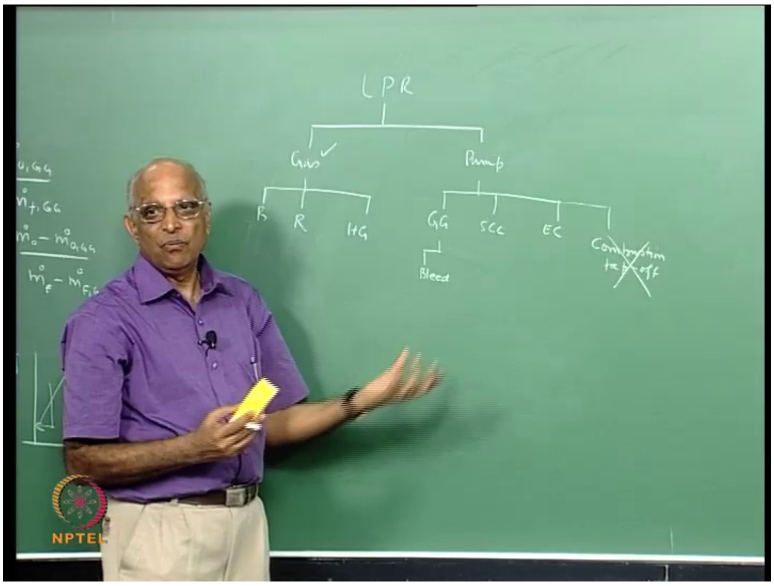

But, to be able to surmise the limits and performance, let us summarize whatever we have understood so far. We said a liquid propellant rocket could either be pump fed or be gas fed. For gas fed we know how to calculate the amount of gas either in the blow-down mode, regulated mode or in the hot gas mode. When we talk of pump fed we found it could operate as gas generator cycle, stage combustion cycle or topping cycle and the expander cycle. You would like to know under what conditions and when we could use these cycles. We could have derivatives gas generator with bleed and variations of the staged and expander cycles.

If we allow the products of combustion from the main chamber to drive the turbine and expand the turbine gases, we have a combustion tap off cycle. We use instead of having a separate pre-burner, some of the products from the combustion. We need to have reproducible temperatures with the same consistent mixture ratio, which is difficult. We have several other cycles, which use part expander and part of different cycles. We must keep our minds open and try to see how best we can improve these cycles. If this part is clear, we can go back and analyze what cycle should we use and when and the merits? Let us do this exercise.

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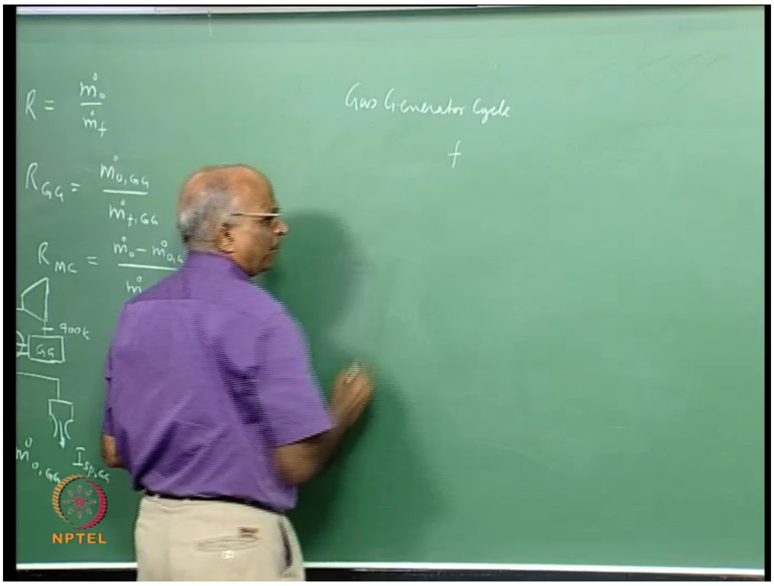

We find that in the gas generator cycle, a fraction f of the propellant in the gas generator is not very efficiently used. Can we calculate the value of f?

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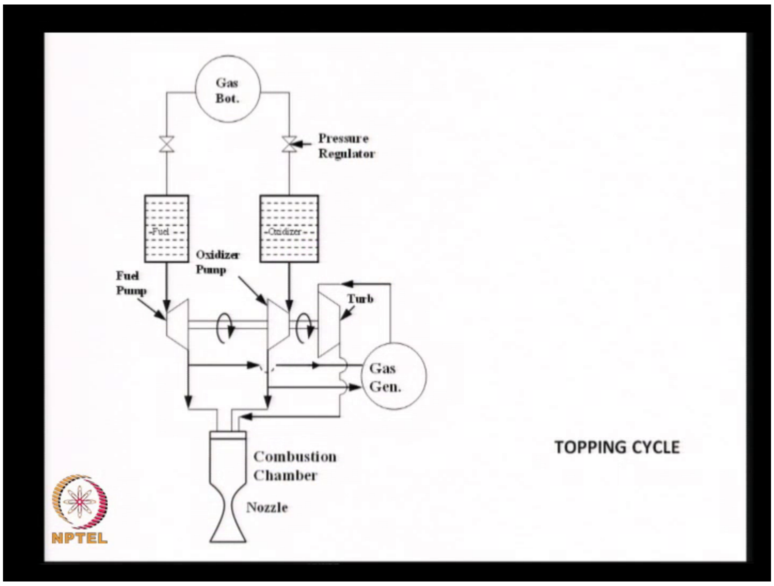

TOPPING CYCLE

But, before we do this let us realize that whatever we are doing is related to the feed system and this is important in the topping cycle as well.

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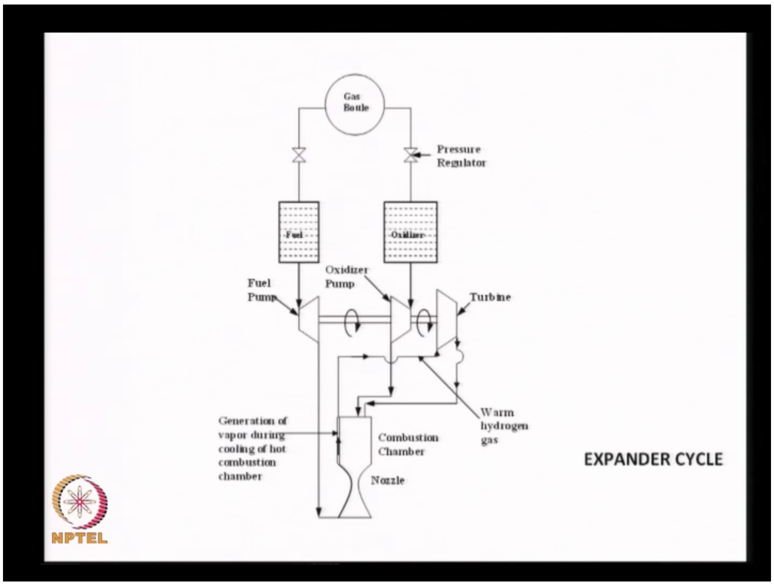
EXPANDER CYCLE

In the expander cycle you have fuel and oxidizer. The oxidizer is pumped into the chamber. The fuel is the volatile fuel. Therefore, while it cools the chamber, the hot vapor so generated runs the turbine and the exhaust from the turbine is fed back after the expansion process in the turbine and that is why we call it as the expander cycle. The vapor is generated during cooling of the particular thrust chamber. Well this is gas generator with bleed, the exhaust instead of being expanded through an auxiliary nozzle comes back into the nozzle in the divergent portion to enhance the Isp.

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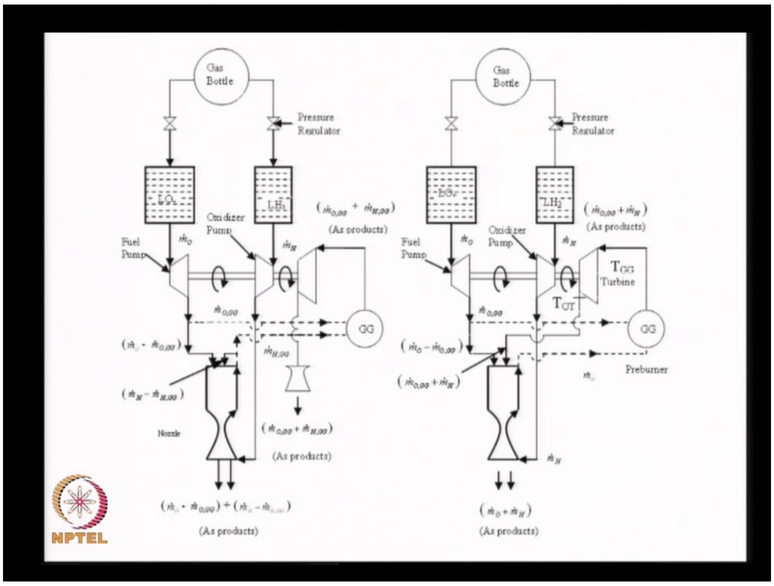

Let us now put things together. This figure looks clumsy but it is exactly what we have been writing on the board. The mass of fuel, mass of oxidizer in different parts is shown in this particular figure. This we show for the gas generator cycle on this left side; you have mass of oxidizer coming into the turbine, the part is taken to the gas generator. We consider this specific case of liquid oxygen, liquid hydrogen as the propellants. Mass of hydrogen is coming into the pump over here. This is taken into the gas generator burns here and drives the turbine. The balance what comes in here is only fuel that is m° hydrogen − m° which goes into your gas generator. Similarly, you have oxygen, which comes in here which is equal to m°oxygen − m° which goes into the gas generator.

In the GG cycle, to repeat again, you have three mixture ratios; an over all mixture ratio, mixture ratio for the gas generator and another mixture ratio for the main chamber. In the stage combustion cycle, the overall mixture ratio is same as the mixture ratio for the main chamber with the mixture ratio R for the gas generator being different. Is this part clear?

If this part is clear, let us quickly do this exercise of finding out what will be the value of f. The turbine generates power, is rotating; the turbine rotates the oxidizer pump the turbine rotates this fuel pump. The power is mechanically transmitted; the power generated in the turbine is running these two pumps.

Therefore, let us find out how what is the fraction of propellant which is required to drive the turbine. We need to be able to find out how much pump power is required. Therefore, let us put it down.

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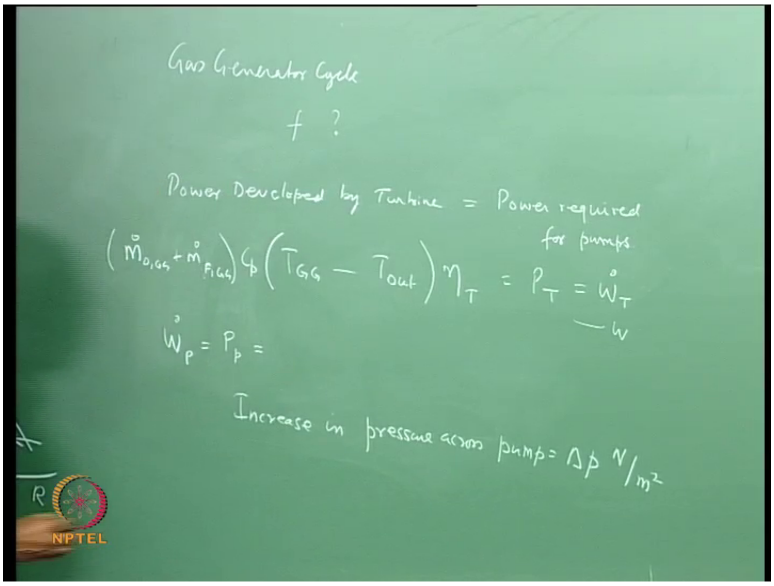

Gas Generator Cycle

f ?

Power Developed by Turbine = Power required for pumps

$$\left(\overset{\circ}{m}_{O,GG}+\overset{\circ}{m}_{F,GG}\right)C_p\left(T_{GG}-T_{out}\right)\eta_T = P_T = \overset{\circ}{W}_T$$

— W

$$\overset{\circ}{W}_p = P_p =$$

Increase in pressure across pump $= \Delta p$ N/m^2

What is the power developed by the turbine and where does the power developed by the turbine go? It runs these two pumps. The total power required of the two pumps equals the power required for fuel pump and the oxidizer pump. What is the power developed by the turbine? We told said that the outlet temperature from the GG is let us say eight hundred to nine hundred Kelvin. We call the outlet temperature as T_{GG} and this is the temperature of gases, which enters into turbine. Let us take the outlet temperature from the turbine after the expansion as T_{out}. We again repeat T_{GG} is the temperature at which the hot gases from the gas generator enter the turbine, some power is generated in the turbine during the expansion of the gases. The turbine may be an impulse turbine. We are expanding the gases and at the outlet of the turbine, the temperature falls to a value T_{out}.

Therefore, what is the work done by the turbine? Rate of work is equal to m°o gg the mass flow rate of oxidizer + mass dot of fuel m°f gg into the gas genrator that is the total propellant flow rate into the GG. The total propellant flow equals the gas flow rate generated m° gg and this into the value of C_P into the temperature is the enthalpy and therefore, this is going to be so much Watts. And m° × C_P × temperature difference between the inlet of the turbine and the outlet is the rate of work done in the turbine. And since, we are talking of mass flow rates, we talk in terms of the rate of work done i.e., power. The turbine has some efficiency. Let us assume that the efficiency of the turbine is η_t and therefore (m°o gg + m°f gg) × Cp × temperature drop in the turbine $(T_{GG}-T_{out})$ $\times\eta_t$ is the useful power produced by the particular turbine. Let us make sure that whatever we are writing is the total mass flow rate into C_P into delta T across the turbine, which is the enthalpy change.

The rate of enthalpy drop in the turbine, which is the rate of work done by the turbine and we can also write as W°t. Can you tell me what is a work done by two pumps? We have been doing it in the last class also. Rate at which work is done by the two pumps is equal to the power of your two pumps is watts, let say the unit is watts viz., joule per second. How do we write it? Let us take a look at this pump; it takes fuel increases the pressure from this value to this value, let us say that the increase in pressure is ΔP, so many Newton per meter square. This is same as Pascal. And what is the rate of work done by the two pumps?

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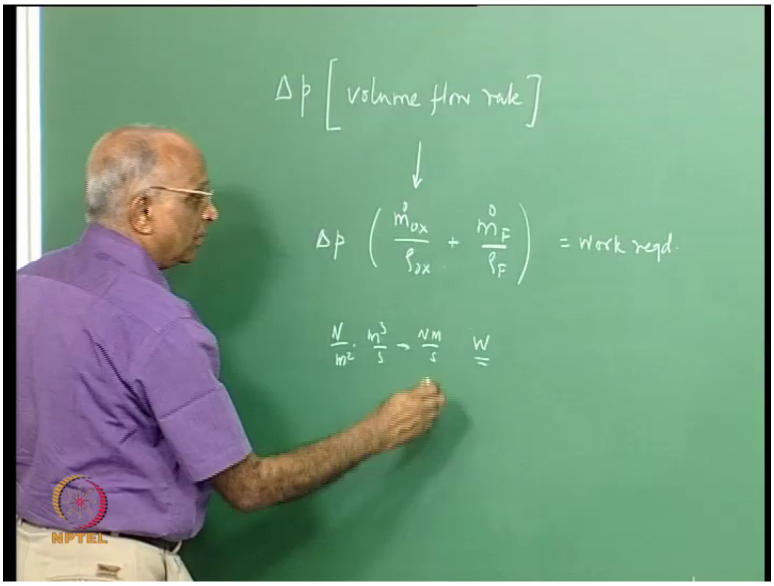

Now, the rate of work done by a pump can be written as ΔP into the volume flow rate, where ΔP is the pressure rise across the pump. The volume flow rate through the pump corresponds to the flow of the oxidizer and the flow of fuel namely, m°o as so many kilograms per second divided by the density of the oxidizer + m°f divided by density of the fuel. This is the volume flow rate of the fuel, this is the volume flow rate of the oxidizer and this therefore, multiplied by ΔP across the pump is equal to the work required to run the pumps per unit time. If we look at the units; ΔP has the units of Newton per meter square and m° have units of kilogram per second, and density has units of kilogram per meter cube.

That is equal to Newton meter per second or joules per second or it is equal to Watts; this is the rate of work required for the pumps. For the present analysis, we have assumed that the pressure increase in the two pumps is the same; otherwise we have to have a separate expression for the oxidizer and a separate expression for the fuel pump. We are just trying to illustrate the method. We have assumed that the pressure increase across the two pumps is the same, which need not be true.

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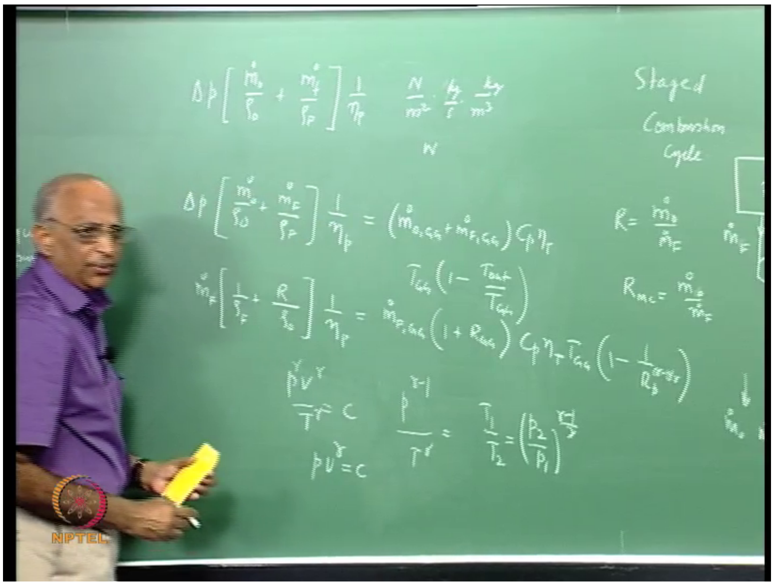

If the efficiency of the pumps is η_P and the total work required would be more so we have to divide it by the efficiency. And therefore, now we equate the rate of power required for the pump with what is the work which is actually done. And therefore, now we write ΔP × [m°o/ρo + m°f/ρf]×1/η_P = rate of work required by the pumps and is equal to whatever we got for the turbine. Let us simplify it and write it is equal to (m°o gg + m°f gg) × C_P × η_T × T_{GG}(1 − T_{out}/ T_{GG}.

And this becomes my expression. How do we get the fuel fraction from this? Fuel fraction f was equal to the fraction which is going into your gas generator divided by the total. Let simplify this little expression further. Let us rewrite it in the following way:

m°f [1/ρf + (m°o/m°f)/ρo] /η_P = m°f [1/ρf + (R /ρo] / η_P where the overall mixture ratio R = m°o/m°f is your overall mixture ratio. We follow the same procedure for the rat eof work in the turbine and get: m°f going through the GG × (1 + R_{GG}) × C_P × η_T × temperature difference ($T_{GG} - T_{out}$).

If we assume isentropic expansion in the turbine T_{out}/T_{GG} and since for $T_1/T_2 = (P_1/P_2)^{(\gamma-1)/\gamma}$ the temperature difference term becomes (1 − the pressure ratio to the power γ−1/γ. How did we do this? We know, that PV by T is a constant from the equation of state for an ideal gas. For isentropic process we have PV^γ is a constant. From the gas equation we get $P^\gamma V^\gamma / T^\gamma$ is a constant. And if we divide one by the other and eliminate V, we get $P^{\gamma-1}/T^\gamma$ is a constant or rather $P_1^{(\gamma-1)/\gamma}/T_1 = P_2^{(\gamma-1)/\gamma}/T_2$.

Therefore, if now we have to simplify this expression we get the value of m°f/m°f gg as equal to the expression in the following slide. We take η to be the product of the turbine efficiency and your pump efficiency, i.e., efficiency of the turbopump.

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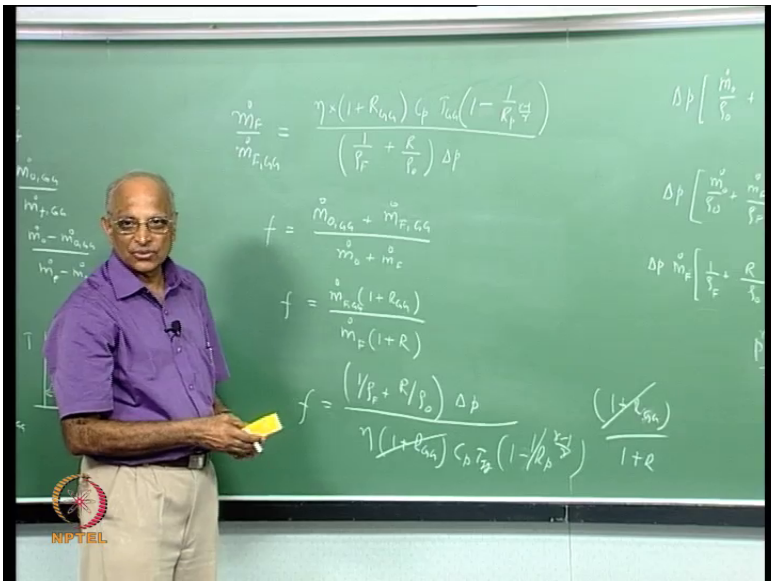

$$\frac{\overset{\circ}{m}_F}{\overset{\circ}{m}_{F,GG}} = \frac{\eta \times (1+R_{GG})\, C_p\, T_{GG}\left(1-\frac{1}{R_p^{\frac{\gamma-1}{\gamma}}}\right)}{\left(\frac{1}{\rho_F}+\frac{R}{\rho_o}\right)\Delta p}$$

$$f = \frac{\overset{\circ}{m}_{O,GG} + \overset{\circ}{m}_{F,GG}}{\overset{\circ}{m}_o + \overset{\circ}{m}_F}$$

$$f = \frac{\overset{\circ}{m}_{F,GG}(1+R_{GG})}{\overset{\circ}{m}_F(1+R)}$$

$$f = \frac{\left(1/\rho_F + R/\rho_o\right)\Delta p}{\eta\,\cancel{(1+R_{GG})}\, C_p T_{GG}\left(1-1/R_p^{\frac{\gamma-1}{\gamma}}\right)} \quad \frac{\cancel{(1+R_{GG})}}{1+R}$$

That means mf/m°f $_{GG}$ is equal to η and η is equal to pump efficiency into the turbine efficiency × (1+ R_{GG} viz.,mixture ratio in the gas generator) × C_P × T_{GG} × (1 − 1÷the pressure ratio in your particular pump to the power gamma minus 1 by gamma) and this is divided by (1/ρf + R the overall mixture ratio/ ρo) × ΔP.

We are able to find out the value of m°f/m°f gg through the gas generator. But what is it that we want? We want to find out the fraction of the fuel and oxidizer, which is going through the gas generator. We need m° oxidizer which is going through the gas generator + m° fuel which is going through the gas generator divided by m° o + m° fuel. That means, the fraction propellant that is going through the gas generator divided by the total. Now, this we again simplify as [m°f_{GG} × (1 + R_{GG})] / [(m°f × (1 + the overall mixture ratio R)]. Please let us be very clear about it and this is equal to the fraction f. But what is it we have got here? We have got m°f_{GG}/m°f. Hence we substitute this value of m°f gg/m°f and get the value of fraction f = by m dot o, which can be written as $(1/\rho_f + R/\rho_o) \times P \div \eta\ (1 + R_{GG}) \times C_P \times T_{GG} \times (1 - 1/r_P^{(\gamma-1)/\gamma})$ and is multiplied by $(1+R_{GG})/(1+R)$. We find that 1 + R_{GG} and 1 + R_{GG} gets cancelled and therefore, f is equal to $(1/\rho_f$ + mixture ratio $R/\rho_o)$ × the pressure increase in the pump ΔP ÷ the net efficiency of the pump and turbine into $C_P\ T_{GG}$ × (1 − 1 by pressure ratio in the turbine to the exponent.

Now, we need to discuss these results. We will do it in the next class. What is it we did in this class? We looked at the fraction of the propellant, which flows through the gas generator and we have got an expression. We also addressed the different feed system cycles.