Rocket Propulsion

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Lecture No. #03
Rotational Frame of Reference and Orbital Velocities

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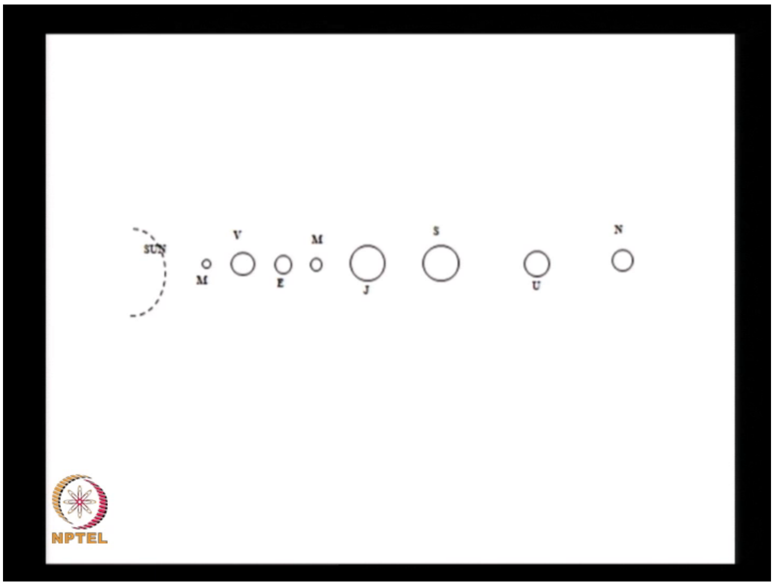

Good morning. In today's class we will address the orbital requirements of satellites and therefore, determine what rocket should be doing. In last two classes we learnt that there are something like eight planets revolving round the sun. We said these planets consisted of Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. We also said that there was one planet called Pluto which is no longer considered to be a planet because it is not dense. Having said that you know there is lot of interest in exploring the different planets and I will take two examples

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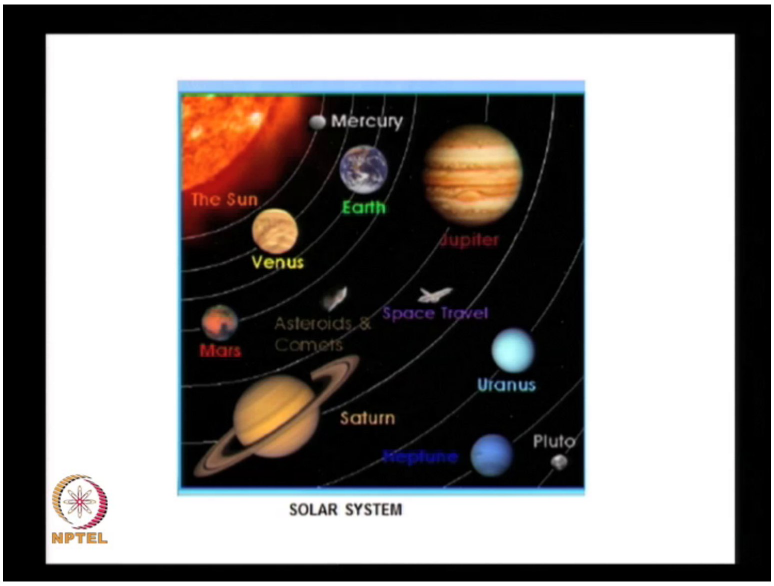

However, before doing so let me repeat of what I just now told. The Sun which is shown in red here; you have about it may be mercury as after that Venus then Earth and the different planets. Seen in between you also see some asteroids which we talked about in the last class I will come back to this point a little later,

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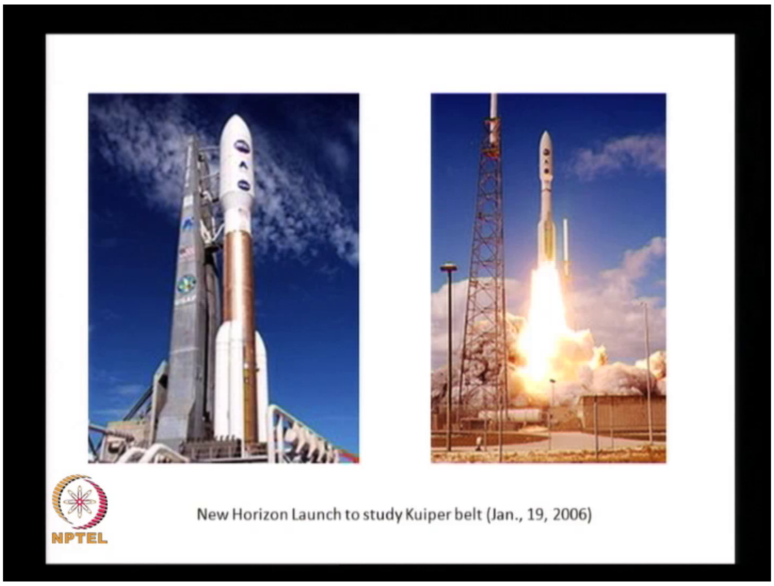

You have a rocket launch vehicle known as New Horizon; it is a rocket which goes and finds out what is happening in the Kuiper belt beyond Pluto. If you take the time taken to go to Pluto - this was launched in January 2006 and it is supposed to reach there in another 4 or 5 years; something like it takes almost ten years to reach Pluto and what does it consists of? It consists of series of rockets one about the other and the purpose of this course is to be able to size the rockets required. We must be able to have rockets such that we can achieve a specific mission and that is why I showed this rocket.

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And then next slide shows the space craft which this particular rocket launches and it goes round and round the planet in which we are interested. This spacecraft should orbit the Kuiper belt that is beyond Pluto

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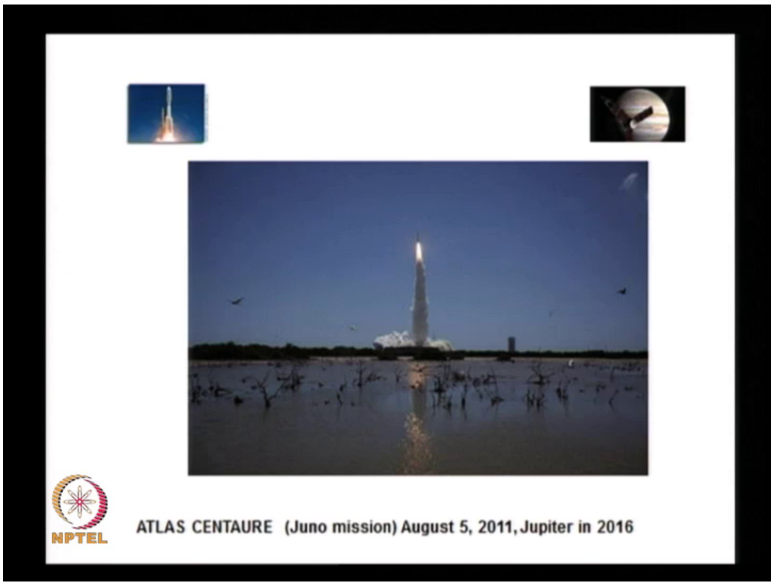

The next slide that I show is a launch of a rocket which took place some on August 5, 2011; it was an Atlas Centaure rocket and this is again from U S. It is supposed to be for a Juno mission i.e., go to Jupiter. As per Greek mythology Jupiter is supposed to be a God and his wife is known as Juno. Therefore, they have named it as Juno mission.

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Planet	Mass(kg)	Diameter(km)
1. Mercury	3.302×10^{23}	4,880
2. Venus	4.869×10^{24}	12,104
3. Earth	5.974×10^{24}	12,756
4. Mars	6.419×10^{23}	6,805
5. Jupiter	1.899×10^{27}	142,984
6. Saturn	5.685×10^{26}	120,536
7. Uranus	8.683×10^{25}	51,118
8. Neptune	1.024×10^{26}	49,528

We also looked at the different planets. We looked at the mass of the different planets the diameter and the distance from the Sun and so on. One thing we must keep in mind is the Earth's mass is around 5.974×10^{24} kilograms and its diameter is 12,756 km. The smallest planet is Mercury whose diameter is about 1/3 of Earth. The largest we said was Jupiter. We also talked of moons and the moon of Earth is smaller than Mercury. There are 31 moons which are available in the solar system

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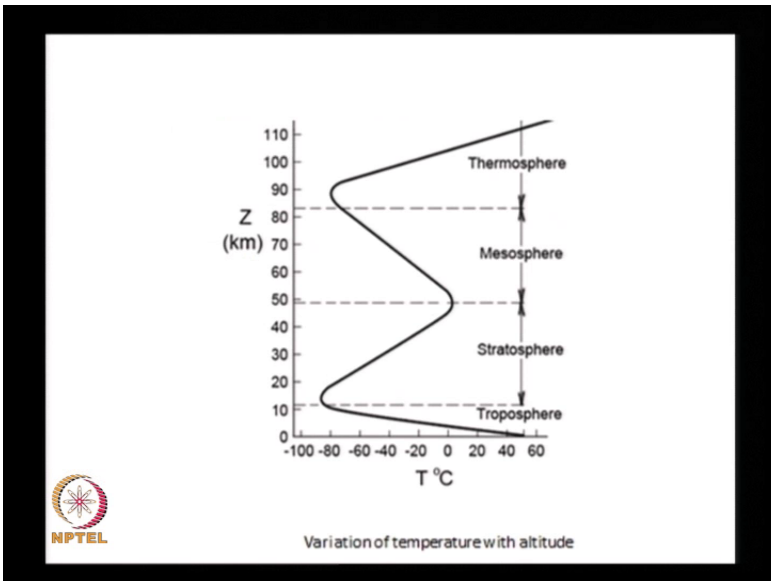

We also looked at the atmosphere of the Earth and the temperature variations in it. When I wanted clarifications on what is the difference between a rocket and an aeroplane and other modes of propulsion, you told me that a rocket flies in vacuum; it is really not a must that it travel in vacuum. Anything which goes in space is what we called as a rocket and we talked in terms of the temperature variations above the surface of the Earth. The temperature in the plot is given along the X axis and altitude along the Y axis. We said because the Earth gets heated by the Sun, the layer of air above the Earth gets heated and therefore, the temperature drops from a high value of around 40 degree Centigrade at the surface of the earth to a low value around minus 60 degree Centigrade at a height of around 10 kilometres

It is at this height that a jet aircraft flies (at altitude between 8 and 10 kilometre). Above this height the temperature increases. In the troposphere just above the surface of the earth the temperature drops and after some particular height between 10 and 15 kilometres the temperature increases again. The increase is because the ozone gas which is available there absorbs the heat energy radiating from the Sun and dissocites giving heat.

But then you keep on going to yet higher and higher altitudes, say around 50 kilometres; beyond that you know the amount of air available is negligibly small and therefore there is nothing really to absorb the heat from the Sun. The temperature begins to rise again.

Whereas, when you go to extremely high values around 80 to 85 kilometres and so on, the individual atoms like oxygen atom combines with each other to generate heat and again the temperature increases. But then the pressure monotonically falls and in thermosphere there is hardly any pressure therefore, the concept of temperature no longer holds good.

It is the individual molecules which are at high temperature. There is no continuum; we cannot even talk of temperature. The rockets goes through the troposphere, stratosphere, mesosphere into the thermosphere and further.

I think this must be clear to each one of us. Why does the temperature decreases above the surface of the Earth, the height at which a jet aircraft flies is around 8 to 10 kilometres where the temperature is around minus 45 to minus 50 degree centigrade. Then you have the stratosphere, mesosphere where the temperature of air again increases and decrease.

What else did we do in last class?

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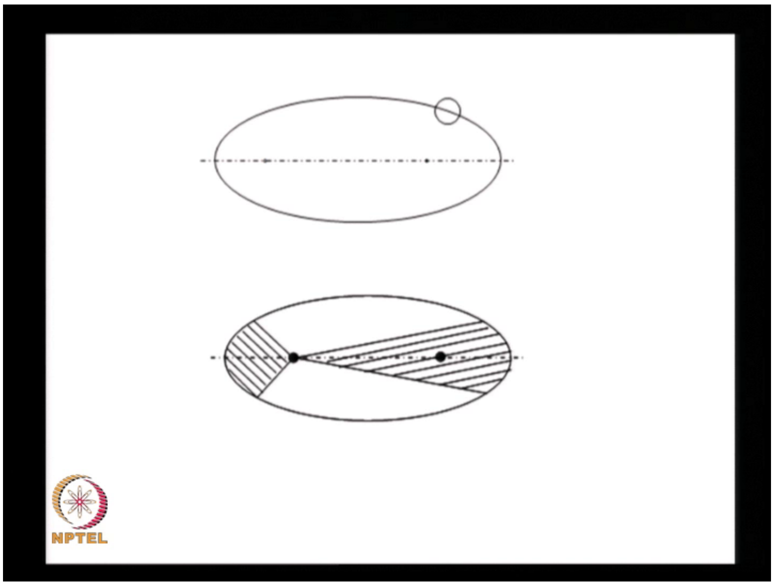

We said that planets move in elliptical paths around the Sun and they trace out the equal areas in equal times.

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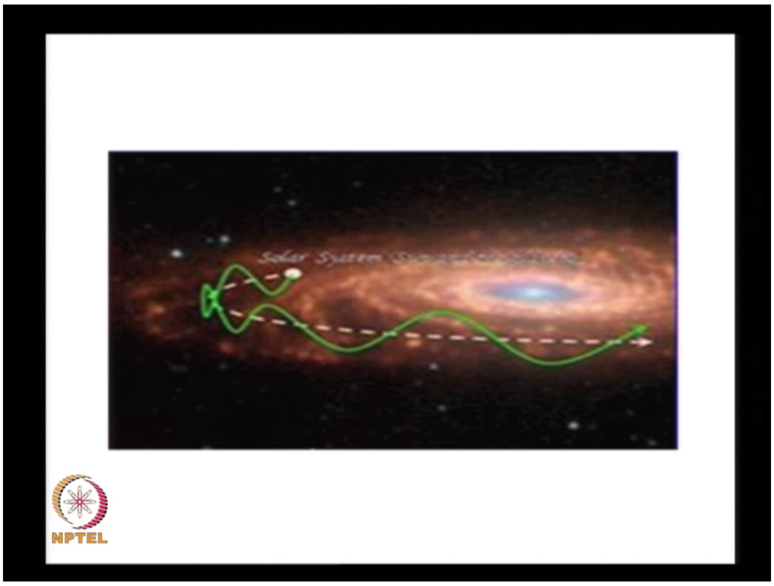

But there are reports in the science magazines over the last two three months about undulations which are there around the elliptical orbit. The green colours shows the real orbit.

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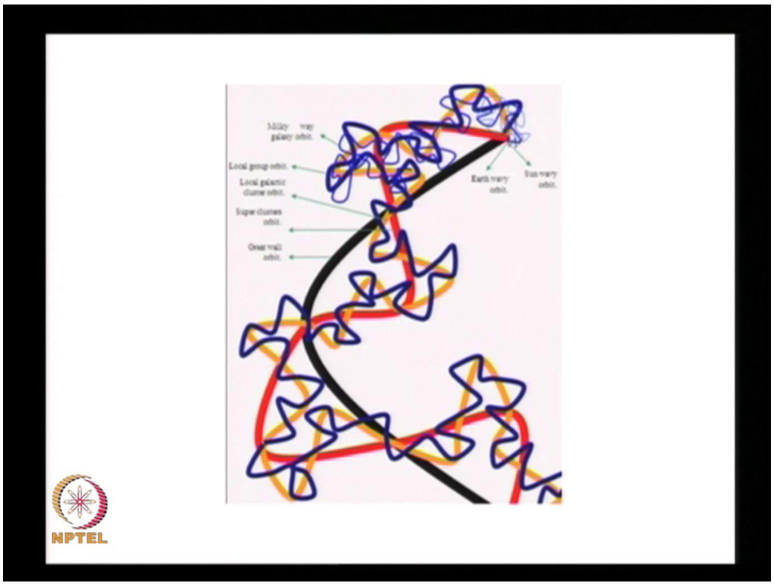

Even within the undulations there are further undulations like turbulence.

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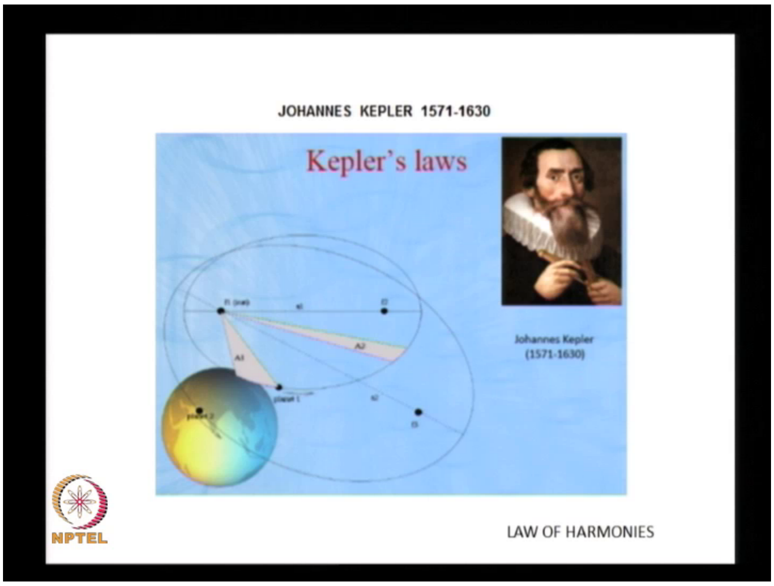

And we had Johannes Kepler who proposed three laws describing the motion of the palnets — laws for planetary motion. The first law states that all planets move in elliptical orbit around the Sun; the second law said that equal areas are traced in equal amount of time in this figure the sun is at one of the focus and therefore, you see the focus on the left hand side around which those white patches show equal areas in equal time. The third law which is law of harmonies related the time period t divided by R namely, t^2/R^3 is a constant. Therefore you have very well synchronized motion of the planets around the sun and therefore, this is useful. Newton formulated the universal law for gravitation based on the planetary motion.

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How did he go about doing it? Before Newton we said that Johannes Kepler was in the period 1570 to 1630 and then comes Galileo Galilee who was also interested in looking at the planets; You see him in this slide on the left hand side gazing at the planets and he is supposed to have done an experiments in which a feather and a steel ball fall together under evacuated or vacuum conditions. Newton did not do any experiment but relied on these data.

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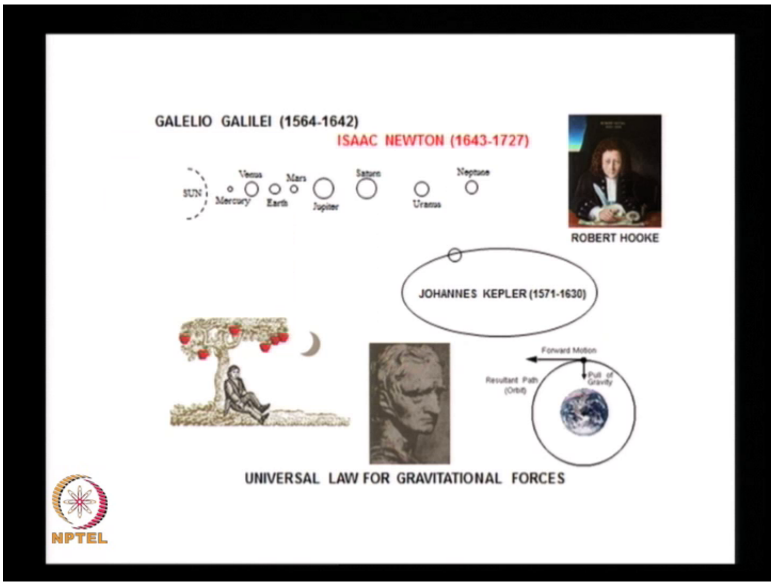

He relied on these observations and it is stated that he sees an apple falling from a tree he uses the information which Johannes Kepler gathered (mainly you have elliptical motion you have equal areas you have t^2/R^3 is a constant) and then there was this person Robert Hooke. You have all read about Hooke's law in Mechanics (stress – strain) and Hooke also was addressing the free fall of planets and their motion. Newton put everything together and he said well an apple gets attracted by Earth because Earth is a large mass just like the Sun which attracts the different planets and he formulates the Universal Law for Gravitational Forces. Planets fall freely onto the Sun just like a stone falls on the Earth.

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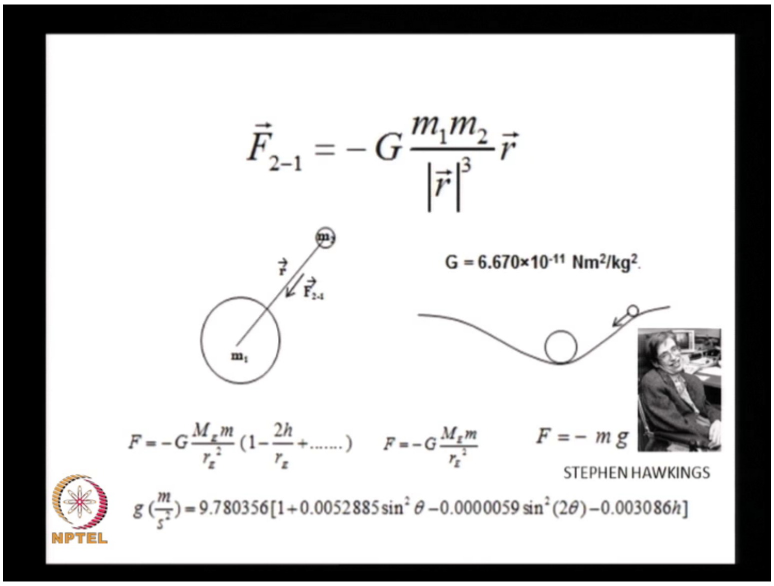

To summarize again, we said that the Law states that if I have a heavy mass m_1 as shown over here and then I have a light mass m_2 which is at a distance r away then I have the attraction force with which the heavy body attracts the lighter body as F as equal to G into m_1m_2 by R square. This is the universal law for gravitational forces. We said G has units of Newton meter square by kilogram square; the unit is 6.670×10^{-11}. But then we were not clear and why this gravity force must exist. We talked of very powerful minds like Stephen Hawking looking into the reasons or theoretical derivation for the Law. As a lay person, we said that if I stretch a sheet and if I put a heavy mass over the stretched sheet, the heavy mass deflects the sheet. If we now put a light mass, it rolls towards the heavy mass. This we said could be due to the field from the heavy body. And we went further; we expanded the value of the universal law $F = Gm_1m_2/R^2$ in terms of the heavy mass being the Earth and R being the radius of the earth plus the altitude h and we found that F= − mg.

But you know the gravitational field g is not constant all along the surface of the Earth because Earth is little bit chubby in shape. It is not a pure sphere. Therefore you find that with the angle of inclination or the latitude and the height, the gravity or the gravitational field g keeps changing. The Universal Law $F = G\, m_1 m_2/R^2$ remains the same and it is the gravitational field of a particular planet that varies. The units of g was derived as meter per second square (m/s^2) which why we call this as acceleration due to gravity.

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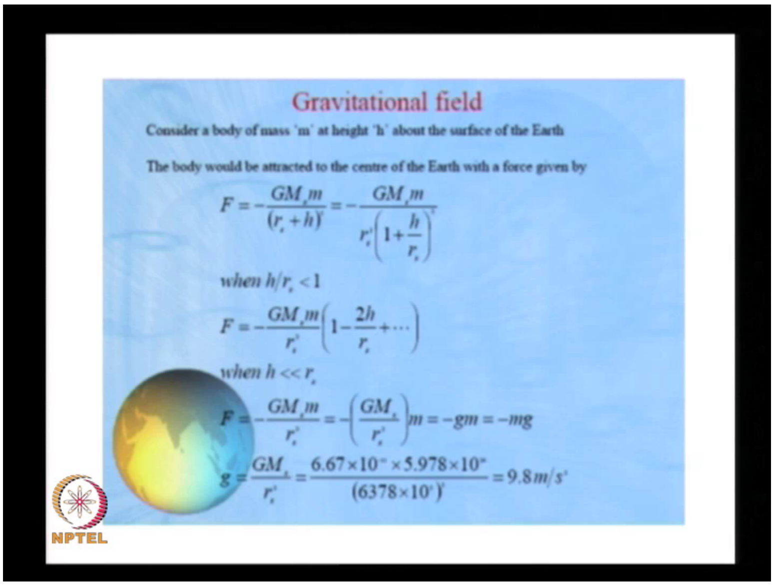

The derivation of F = − mg from the universal law for gravitational forces is shown above. It came out to be 9.8 m/s^2. We had also derived this in the last class.

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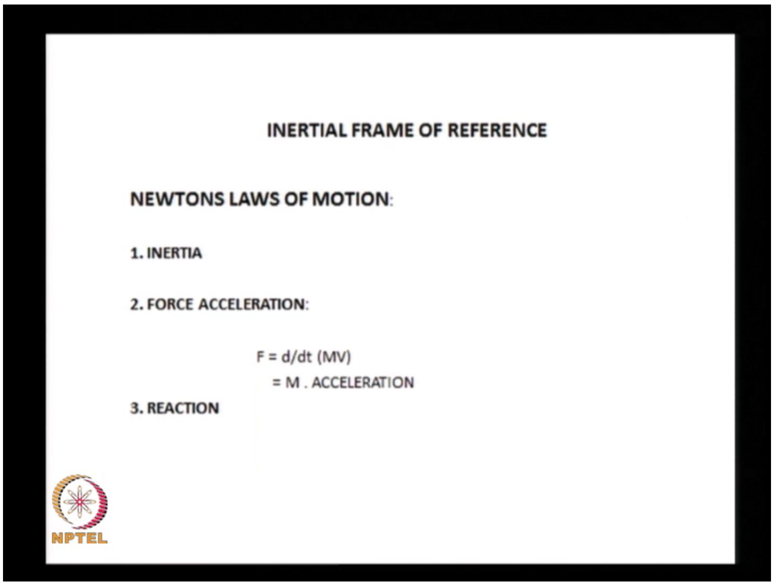

We defined an inertial frame of reference since all of us are moving, the Earth and all the galaxies are moving, the Sun is also moving. Therefore we said either the Frame of Reference must be stationary or must move at constant velocity along a straight line for the change in the in motion of the body to be determined.

The inertial frame of reference is a frame of reference which is either stationary or it moves at constant linear velocity. Based on this inertial frame of reference we talk in terms of Newton's laws of motion which is precisely what we have been talking all along namely inertia i.e., a body continues to remain in a state of rest or of uniform motion unless it is constrained otherwise by an external force and this makes sense.

The second law of Newton we say is rate of momentum is proportional to force or force is equal to rate of change of momentum. Force F is equal to d/dt (mv). You take constant mass outside you have d/dt of v which is the acceleration. Therefore, second law of Newton can be expressed either as a relation between force and change of momentum or between mass and acceleration. We can state that acceleration goes inversely as the mass of the body and directly as the force on a body.

We will be using these laws. The third law states that action and reaction are equal and opposite; but mind you the important qualification is inertial frame of reference. These laws are valid only if the frame of references is inertial. As an example, I want to find the temperature distribution in a rotor in a machine; therefore, I am looking at the blade as it is rotating; to be able to measure the temperature of a blade, I will be sitting on the blade and monitoring the temperature. The rotating blade need not be in an inertial frame of references. Therefore the momentum equation for a rotating body need not follow the Newton's law.

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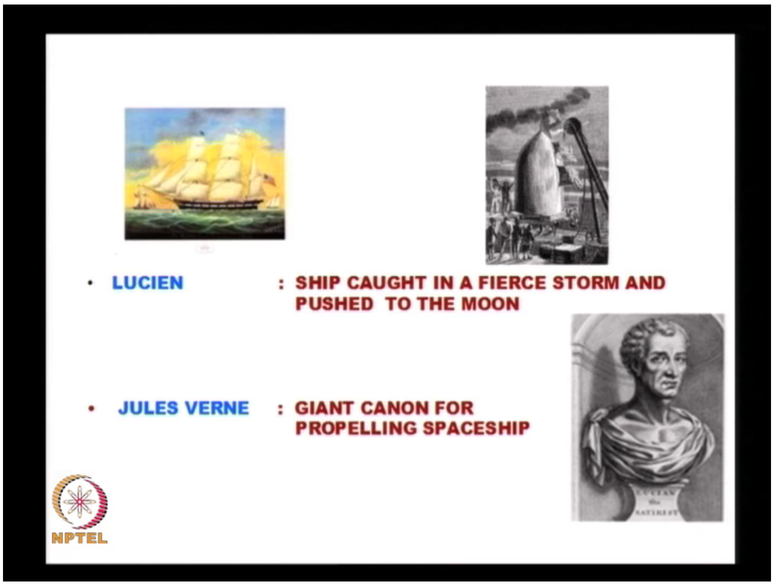

Now let me just get into this last slide corresponding to what we did in the last class. We said to be able to put anything in space you need to give a force. Lucien, around 40 BC fancied a giant storm pushing a ship to the moon. But when we looked at the Science fiction author Jules Verne; he wrote about a capsule which is contained in the barrel of a canon and forced through to space.

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We also talked in terms of asteroids which are loose bodies in space; one such asteroid missed collision with the space station around two months back. The space station is a satellite which is in low earth orbit and it is used for scientific experiments. Whenever some asteroid is likely to hit it, the position of the space station slightly shifted so that while in orbit the asteroid does not hit it. You could have these misses and collisions with bodies in space

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But something which is disturbing and about which we talked in the last class is an asteroid hitting the Earth. I took this slide from the Newspaper “The Hindu”. The incident was in the news in February this year. It says that on April 13, 2036 one asteroid by name Apophis is likely to hit the earth. And if this asteroid were to enter the atmosphere of Earth and hit it, the asteroid would get rapidly heated due to friction; it would explode and form a blast waves and the resulting severe wind and extremely high temperatures might cause the end of civilization itself. And in fact some 6 million years ago the extinction of dinosaurs was because of an impact of an asteroid with the Earth.

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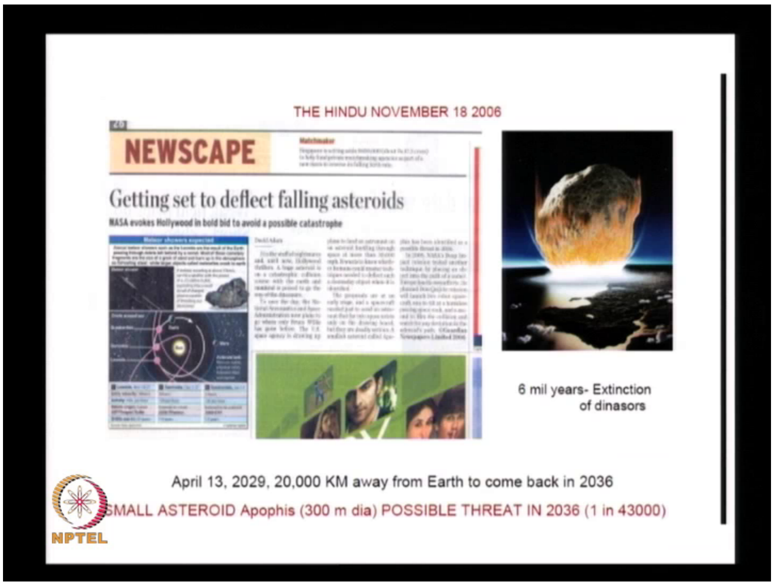

Therefore, the question is how to deflect away the asteroid away from the Earth. One of the things which are talked of is to put up something like a like a heavy satellite near to the asteroid and now the asteroid has a certain mass and because of the gravitational force exerted by the heavy satellite, the asteroid slightly deflects away.

This summarises what we have discussed so far.

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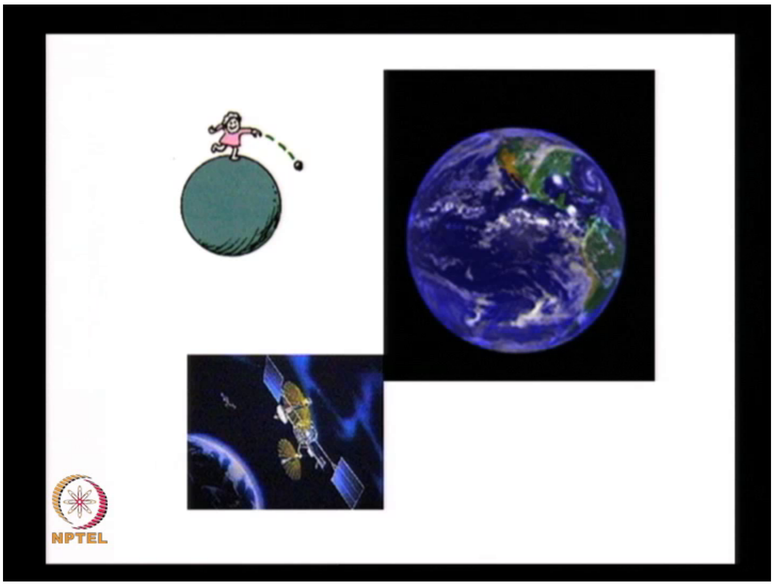

If we were to consider an orbiting spacecraft about the Earth, i.e., revolving round the Earth as it were; the bottom picture shows the INSAT (Indian National Satellite) satellite going around the earth. If we throw a stone with a particular velocity the stone would keep on rotating round and round the earth. It is like the case of the planets going round the Sun. Now, the pull of gravity attracts it and causes it to fall freely while the forward velocity component shifts it away from falling onto the earth and the body gets into a circular orbit for a given value of the velocity.

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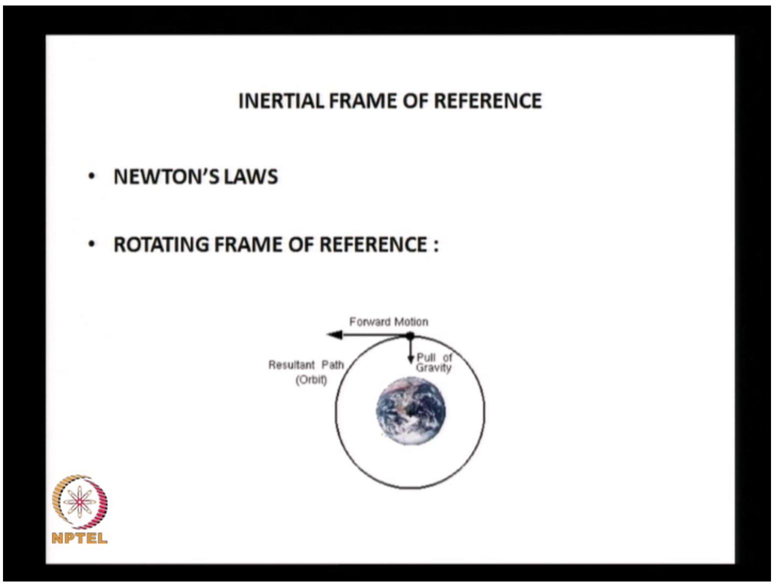

Now to describe the motion of the body as it is going around the Earth, the frame of reference could be inertial like when I am standing on the ground and am watching the body revolve around the Earth. I could also sit on the body and describe the motion in which case the frame of reference for describing the motion is not along a line and is not inertial.

Therefore, I would like to get back and ask a question; can I get out of this inertial frame of reference and describe the motion with the frame of reference being the body?

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Let me put the argument through with an example. Many of us go to watch circus and you know one of the things which is sometimes shown in a circus is a motor cyclist or a few of the motor cyclists going round and round in a spherical cage. How does he support himself when he is on top of the cage in an inverted position? He does not fall down. If I were to consider myself sitting along with the cyclist and I want to describe my motion and that is how a motion of a satellite could be described in the frame of reference of the rotating satellite. I would like to know what is the type of forces which are acting on me and with this I should be able to correctly describe the motion of bodies as they are rotating when I am on the body itself.

We say that in the frame of reference of the rotating body; it is not something which is moving at constant velocity or which is stationary i.e. we have a non-inertial frame of reference with which I would like to describe the motion of the body. In the inertial frame of reference, (with the observer on the ground), the body has a velocity dx/dt in the x direction and dy/dt in the y direction.

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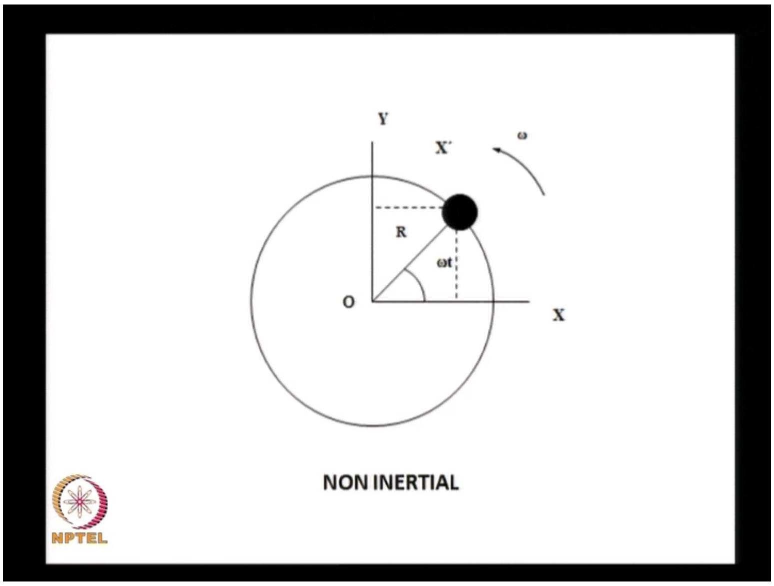

We will address this problem of describing the motion in the rotating frame of reference along with that we will be able to find out what we mean by orbital velocity.

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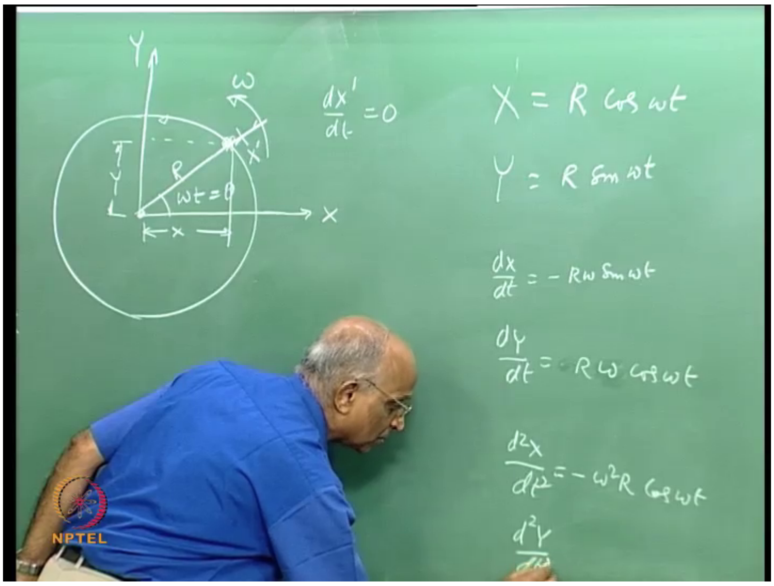

Therefore, I have something like an orbiting mass something is going round and round about the centre over here; alternatively I have the centre over here let us say a ball is going round and round. It is in this rotating frame of reference rotating at a speed omega, that I am interested in finding out the equation of motion. Is there something different that I have to do when I consider the rotating frame of reference? As long as I considered myself to be stationary on the ground, the frame of reference was inertial and the Newton's laws of motions could be applied. Now I am rotating with the rotational velocity omega; I am no longer looking at the body from the inertial frame of reference since I am on the body; I want to look at the body whose cordinate I call as x' which is a rotational frame of reference. In other words I also sit on this body and I want to describe my motion properly.

Let me say that instead of me standing here and watching this mass go round I tell myself well I am on this body X' and I want to describe my motion properly. In other words you know my motion about the body is zero since I am on the body. With respect to the body; my displacement or velocity of motion is dx'/ dt is 0. There is no change; but how do I describe this motion?

Let us say that I move with an angular velocity ω and let us say at time t my position is X'. At this particular point ωt is equal to the particular angular movement θ. I am here at X' at a time duration t. I have reached this particular point and therefore, I can write the x coordinate and the y coordinate. Now x prime is equal to the radius of the circle R. I can write x as equal to R $\cos\omega t$ and y is equal to R $\sin\omega t$.

The x component of radius is x $\cos\theta$ and the y component of the radius is $y = R \sin\theta$ and theta is equal to ωt. I differentiate it to get dx/dt is equal to $- R\omega \sin \omega t$ and I get dy/dt is equal to $R\omega \cos\omega t$. Instead of taking the velocity along x and along y, if I want to find the acceleration along x and y. I get $d^2x/dt^2 = - \omega^2 R\cos\omega t$ and the second equation I get is $d^2y/dt^2 = - \omega^2 R\sin\omega t$.

In other words when a body is rotating I find I can describe the acceleration along x is pointed towards the centre; it is minus. It is in this direction equal to $- \omega^2 R \cos\omega t$. I also have acceleration in the y direction is equal to $- \omega^2 R \sin\omega t$ and the net acceleration is:

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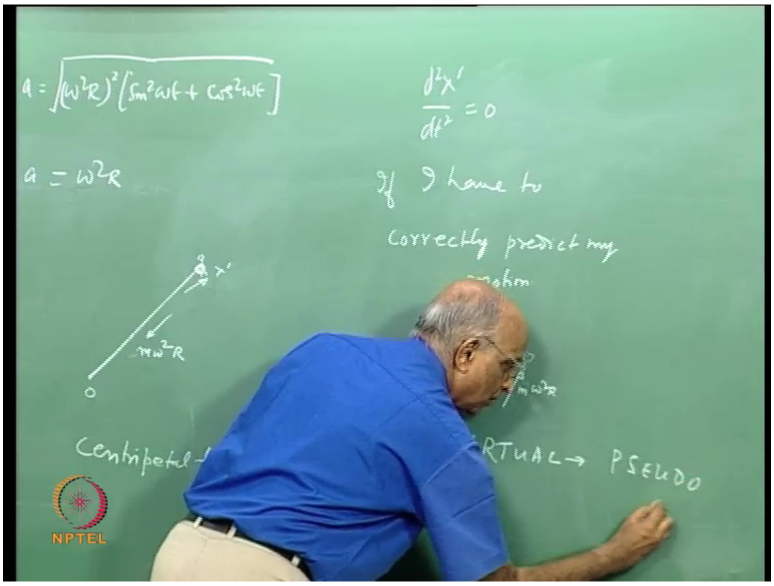

If we were to write the net acceleration (a), we get it equal to the under root of these two terms squared added together, namely ω^2R. This is in the inertial frame of reference. The sum of $\sin^2\omega t + \cos^2\omega t = 1$ and therefore this is equal to ω^2R. This is the acceleration. That means when the body is rotating, I have a net acceleration which is along the radius pointing towards the centre O.

Now from second law we know if a rotating body has an acceleration in some direction, it should have a force. The force is in the direction of the acceleration and equal – $m\omega^2R$. The force is towards the centre O.

What is the name of this force? Centripetal force.

What we find is we are considering a body which is rotating and we find that there is a net force which act towards the center and this force is equal to $m\omega^2R$. We see this in the inertial frame of reference.

However, when I am sitting on this body, i.e., in the frame of reference of the rotating body X' itself, I am not moving with respect to the body. Since my movement with respect to the body is zero, my velocity dx'/dt and my acceleration d^2x'/dt^2 is zero. This means that the acceleration of the body in the frame of reference of the body is zero. But then I find in the inertial frame of reference that I am talking of a centripetal force which is acting on me. Therefore, if I have to describe my motion correctly, i.e., correctly sort of predict my motion when I am on the body, what should I do? It is necessary for me to put a force on the body equal and opposite to – $m\omega^2R$. This force is not real but is one which is required to correctly predict my motion in the rotating frame of reference. It is something which is virtual or which we call as a pseudo force. And why do I have to put this force? In order to be able to correctly describe the motion of the body in the context of the body itself so that there is no net force because with respect the body; the body is not moving. I am sitting with the body. If I say that I am not moving with respect to the body, it is only possible when I put a force like this such that the centripetal force and this pseudo force are same and opposite to each other. This pseudo force is what we call as centrifugal force.

Let us repeat the requirement of the pseudo force in the non-inertial rotating frame of reference. In an inertial frame of reference Newton's laws are valid. Now I consider a frame of reference which is not an inertial frame of reference, but it is a rotating frame of reference. Now I am looking at the motion from the perspective of the body which means is I am sitting on the body. If I am sitting on the body and I am rotating then

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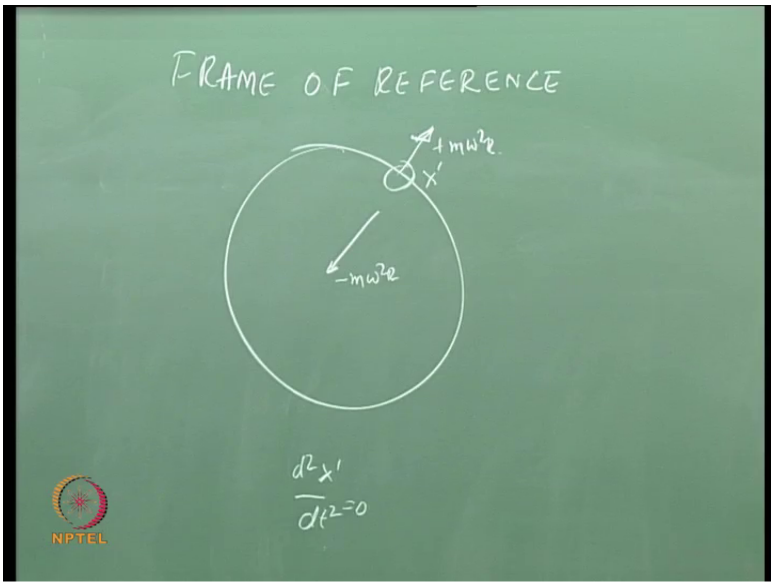

to be able to correctly define my coordinate, i.e., a pseudo force has to be put on me to be able to correctly describe my motion, because I find that d^2x'/dt^2 is equal to 0.

In other words I have a centripetal force minus omega square R and I have to put a force equal to plus $m\omega^2R$ to be able to correctly determine my motion. We call this as a pseudo force since it is not real, but it helps me to define my motion corectly in the in the non-inertial frame of reference.

With this background, let us find out velocities in orbit.

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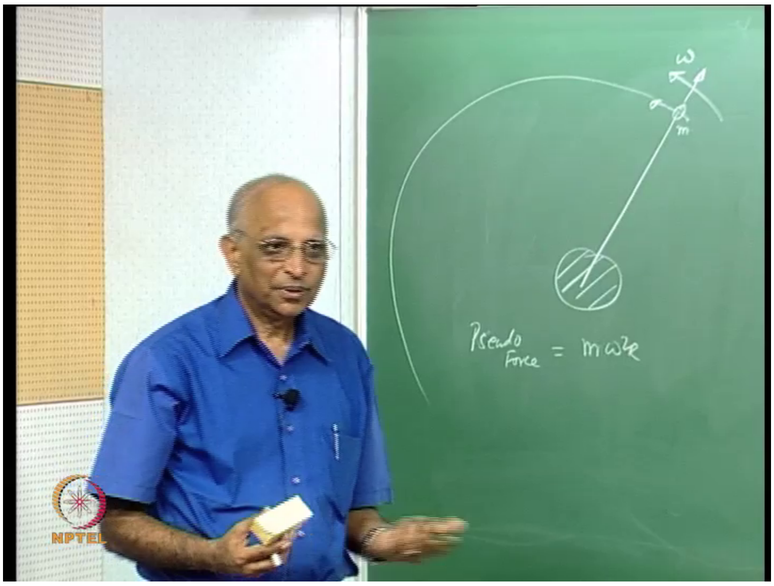

Let us consider the Earth to be here let us consider a satellite or some other body of mass m going round earth in a spherical orbit of radius R. Now, I want to find out the motion of this particular body. We assume that the body rotates with an angular velocity ω. With respect to the body, i.e., in the rotating frame of reference we need to introduce a pseudo force which we call as centrifugal force to correctly define its motion in its own frame of reference . This pseudo force is $m\omega^2R$ which is acting along the radius in the outward direction.

But what could balance this pseudo force the application of which correctly defined its motion?

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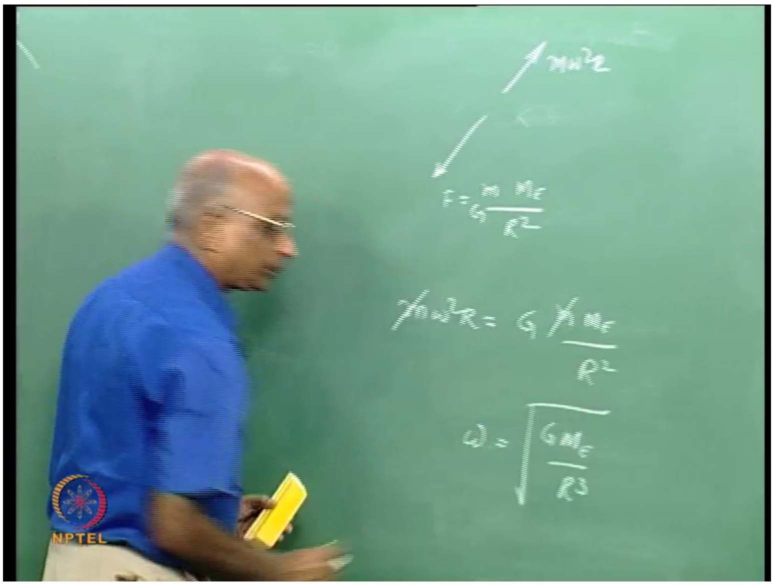

Gravitational force of attraction by the Earth of the body of mass m. The gravitational forces balances the pseudo force. In other words the pseudo acts in this direction equal to $m\omega^2R$. And what is the force in the opposite direction? The gravitational force which is given by universal law for gravitational forces and is equal to the mass of the body into mass of the Earth divided by R^2 into the gravitational constant G.

We take the force balance. I have the pseudo force or centrifugal force of $m\omega^2R$ equal to G into m (mass of the body) multiplied by mass of Earth divided by the square of distance from the centre of earth to this particular body. And now I find out the angular velocity omega. m and m cancels you find that irrespective of mass of body, the value of the angular velocity is the same and we get it equal to $\sqrt{GM_E/R^3}$. This is what gives us the angular velocity of rotation of a body around the Earth. If I were to consider instead of the earth that I go around the Sun, let us write the equation for the angular velocity of orbit.

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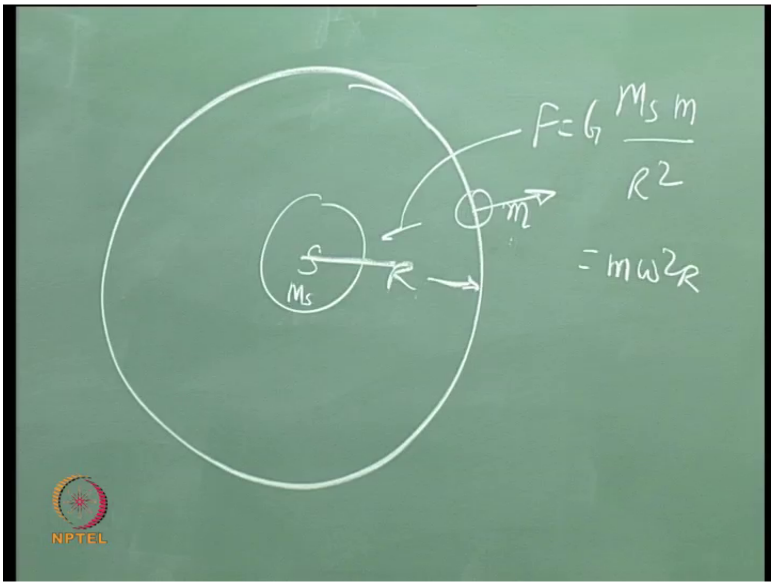

We have the sun of mass Ms over here I have a body of mass m going round the Sun at a distance R from the centre of the Sun. Then I have in the frame of reference of the mass m, the gravitational force that is F = GmMs/R^2 and I have the pseudo force which is equal to $m\omega^2R$. Therefore, I get the angular velocity ω^2 as equal to G into mass of Sun divided by the cube of the distance from the canter of the Sun and the body. (G Ms/R^3)$^{1/2}$.

Once I know the angular velocity I can readily find out velocity of orbit.

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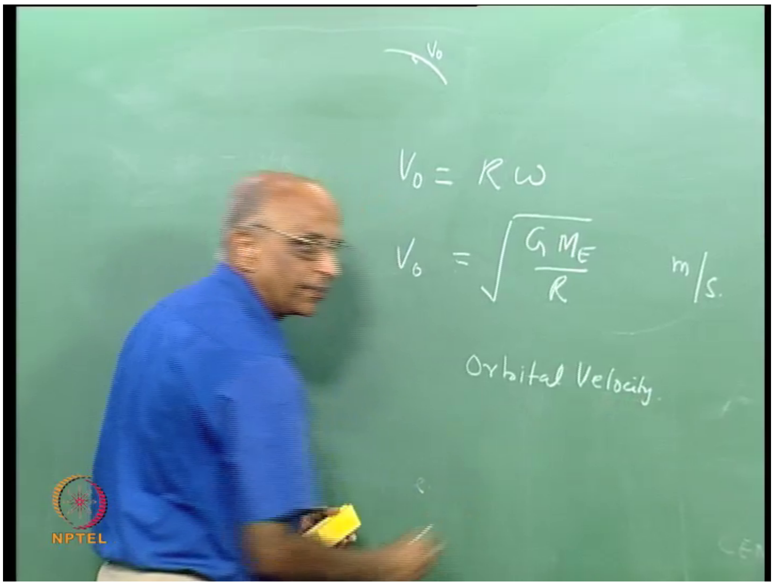

The velocity of the orbit is the velocity with which it is rotating viz., V_0. This equals Rω. The orbital velocity becomes equal to $\sqrt{GM_E/R}$. The unit is metres per second. This is how we calculate the orbital velocity of anybody like the moon is going around the Earth we know the mass of the moon we know the mass of the Earth we know the distance between centre of moon and centre of Earth. We can find out the orbital velocity i.e., the velocity at which the moon is orbiting or travelling provided that the orbit is circular.

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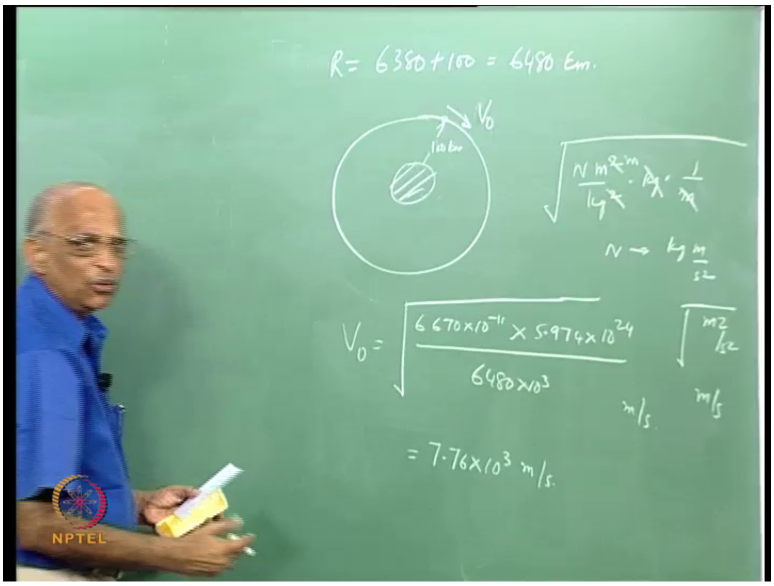

Let us do a simple example to illustrate this. Let us find out the velocity of a spacecraft orbiting the Earth in a circular orbit at a height let us say of 100 km above the Earth. I want to find out velocity of the orbit V_0. I make use of this equation $V_0 = \sqrt{GM_E/R} = \sqrt{6.670 x 10^{-11}}$,the unit Newton meter square by kilogram square × the mass of the Earth viz., 5.974×10^{24} kg divided by the radius R we said was hundred kilometre above the surface of the Earth; therefore, R is equal to the radius of the Earth which is 6380 kilometre plus 100 kilometres which is equal to 6480 kilometres into 10 to the power 3 meters. I am left with kg multiplied by kg. Newton is equal to kilogram meter per second square. Therefore, this becomes under root meter square by second square and the unit is meter per second.

And if you calculate the value, this will come out to be about 7.76×10^3 meter per second or 7.76 kilometres per second. This is typically the velocity of a body orbiting the earth at a distance of 100 km above the surface of Earth. This is how we calculate the orbital velocity.

Can we find anything elese from the expression for orbital velocities?

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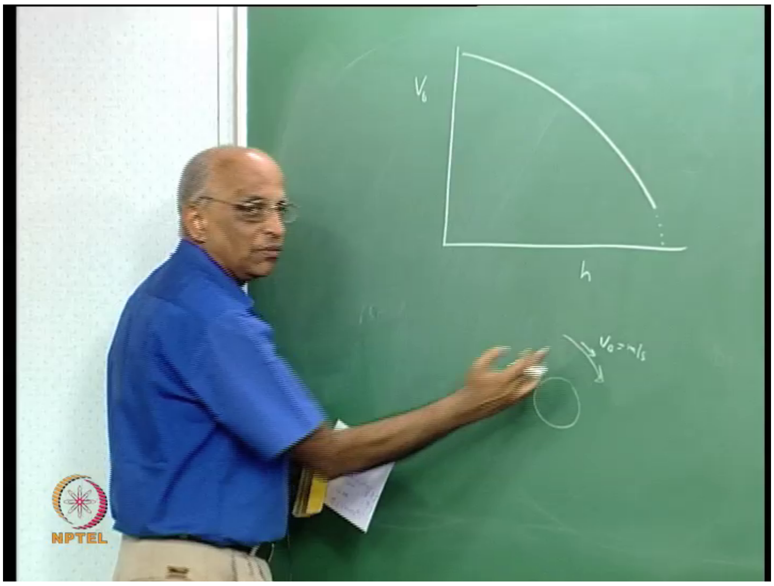

We find that as the height above the Earth increases the value of orbital velocity will keep coming down. When I reach infinite height the orbital velocity is zero.

In other words whenever we are considering any planet and a body is orbiting around a planet, we can calculate the orbital velocity V_0 in meters per second i.e., the orbital velocity of the rotating body. We talked in terms of INSAT spacecraft which is going round the earth and therefore we say well, it is at this height and therefore it is rotating at this particular speed.

In the next class we will go into some details of the total velocity required to orbit a spacecraft at a given height. You can see from this figure that the orbital velocity keeps falling as you go higher up but then you expect more velocity should be required to go to higher orbits. We will also go into more details of different orbits and then fix the requirements for a rocket to be able to put a spacecraft into different orbits.