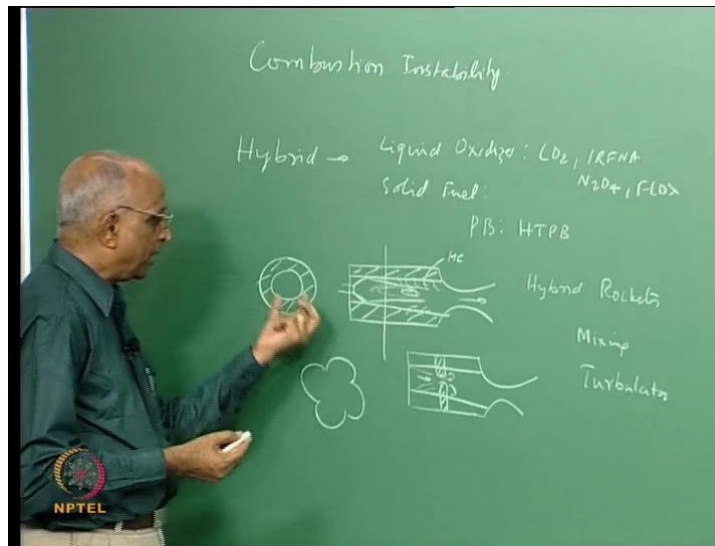


**Rocket Propulsion**  
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**Lecture 33**  
**Combustion Instability in Rockets**

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We will talk on combustion instability in chemical propellant rockets. Namely, combustion instability in solid, liquid and other types of rockets, but I thought in the last class we dealt with hybrid rockets and therefore, let me spend a couple of minutes on the hybrid rockets. The very name hybrid means, it is a combination of two different phases, maybe a liquid and a solid as propellants, or a gas and a liquid, etc. But what is normally used in hybrid is a liquid as an oxidizer and solid as a fuel, and we know that a solid fuel is like polybutadiene HTPB, PBAN, PVC, etc., like hydroxy-terminated polybutadiene, which we used as a binder in solid propellant rockets.

The liquid oxidizers which are used could be liquid oxygen, could be nitric acid, inhibited red fuming nitric acid, could be  $N_2O_4$ ; we talked in terms of FLOX - liquid oxygen to which was added fluorine to make it more powerful and these are the liquid oxidizers. But the solid fuel is essentially polybutadiene HTPB, which has more hydrogen. How will the construction look like? We have a cylindrical case, a nozzle, and the solid fuel

block, which is attached to the case. It could be any configuration depending on the thrust requirements.

Let us take a star configuration or a circular configuration. We take a section over here this is the solid fuel as shown. And what is it we do? I spray the liquid oxidizer onto the fuel, and in case the liquid fuel like let us say RFNA is hyperbolic with respect to the fuel then it begins to react at the surface. Vapor is evolved at the fuel surface from the heat transfer and we have vapor, which keeps coming out. Since we have the oxidizer adjacent to it, a stoichiometric mixture or a mixture between the oxidizer and the fuel vapor gets formed, it burns and then you get the thrust. This is the principle of a hybrid rocket.

Essentially the hybrid rocket is somewhat in between a solid rocket and a liquid rocket. In the case of hybrid rockets, what happens is that provided the fuel surface is hyperbolic with respect to the oxidizer, the moment we inject liquid oxidizer on to the solid fuel surface, the chemical reaction begin to occur. The heat feedback to the surface generates the fuel vapor say hydrocarbon vapor, and that hydrocarbon vapor mixes with the oxidizer vapor and you get combustion taking place The combustion products are exhausted through a nozzle, and this is the principle of hybrid rockets.

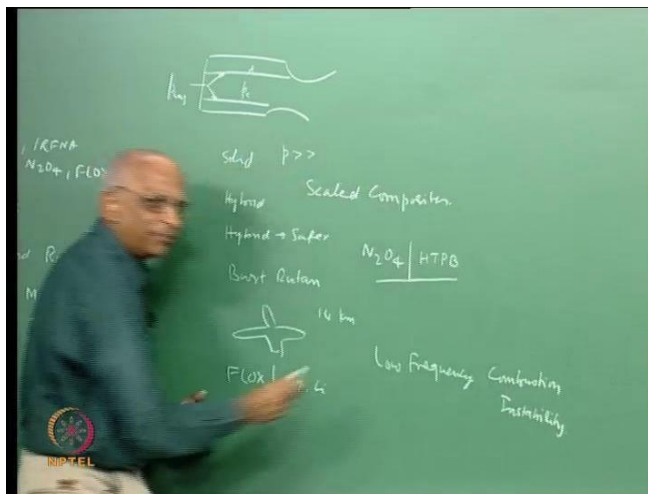
In case, the surface is not hypergolic with respect to the oxidizer, we have to coat on the surface a substance which initially will start combustion when it comes into contact with the oxidizer. It could be a paste containing amines, which is hypergolic with respect to the liquid oxidizer heat say  $N_2O_4$ . Once combustion starts, the heat is getting generated and it is the heat transfer namely the convective heat transfer, which further evaporates the solid fuel. The fuel vapor mixes with the oxidizer vapor formed from its droplets, and combustion progresses.

Therefore, the controlling event in these hybrid rockets is generation of fuel vapor and essentially mixing of it with the oxidizer vapor. The oxidizer vapor is formed by evaporation of the oxidizer droplets formed in the spray. What is mixing? At the surface you have hydrocarbon vapors being formed; it mixes with the free stream of oxidizer vapor, because when we inject the oxidizer as a spray and it also vaporizes. Mixing is sort of a problem in the short length of the combustor. You can recall the stream tube mode of

combustion in liquid propellant rockets. There was this researcher by name Professor Dadiou in Germany who suggested introducing a turbulator in the hybrid chamber. Since mixing is a problem, he put something like a turbulator or turbulent generator in the hybrid combustion chamber. What is this turbulator? It is used in many engineering situations. Namely, when we look at this particular scheme of hybrid rockets, fuel is getting evaporated at the surface; oxidizer is coming over here as shown. We make a constriction over here in the form of let us say petals. We put this configuration or constriction over here in the port volume. When the gas is flowing through it, eddies of different sizes are formed behind it and it helps to mix the gases.

A turbulator would provide for mixing the fuel and oxidizer vapor so that combustion gets completed in the chamber. Therefore, you know in the early days when people were working with hybrid rockets, they never really used turbulator; they got very low performance. After incorporating a turbulator mixing is better, but still the performance is not as good as in liquid propellant rockets but it is better than solid fuel rockets. This is all about hybrid rockets. But there is one distinct advantage in using hybrid rockets. What is this advantage?

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Let us say you have a defective solid fuel grain; the solid fuel has a crack in it or some opening in it. Therefore, the surface area increases over and above the design value. And you are spraying liquid oxidizer onto the surface. Because the surface area is increasing, more pressure gets generated in the combustion chamber. When pressure gets

generated, automatically the pressure drop across the injector gets decreased, because pressure here is higher while the injection pressure is at constant pressure. The pressure drop across the injector decreases and therefore the flow rate of oxidizer decreases. This decreases the chamber pressure and therefore the chamber pressure is self-regulating.

In solid propellant rockets if there is a crack, if surface area increases pressure continues to increase as the burning rate increases with pressure. In the case of hybrid if there is some surface defect, what happens is pressure gets generated; once pressure gets generated automatically the pressure difference at injection decreases, and pressure in the combustion chamber being higher decreases the flow rate, the flow rate of oxidizer decreases automatically and the thrust gets regulated. This makes the hybrid rocket much safer than a solid propellant rocket as it can accommodate certain amount of surface defects.

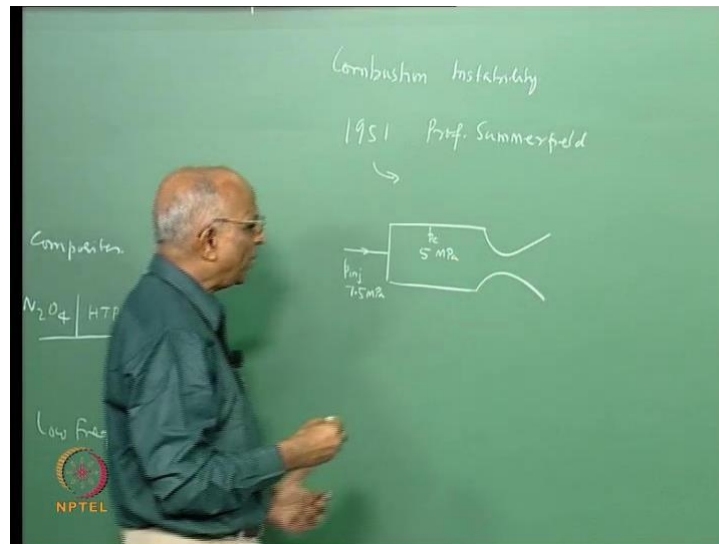
A British millionaire by name Burt Rutan formed a company known as Scaled Composites, and he used hybrid rockets for ferrying people to space. He believes that space can be used for tourists who would like a voyage in space and view the Earth from space. He used an aircraft White Knight, which I showed you as a slide in the last lecture. In the belly of the aircraft he mounts a space capsule powered by a hybrid rocket. The White Knight aircraft goes up to 15 kilometers height, and from there the hybrid rocket takes the space capsule into a sub-orbital flight in space. The purpose of the hybrid rocket is for ferrying people from 14 or 15 kilometers to space and return to the ground. This is how the hybrid rocket is used.

The company, which does it is a private company known as Scaled Composites. Last week the launch pad was officially inaugurated in the desert in California. I think we should keep such developments in mind. Therefore, we would conclude by saying that a hybrid rocket has not been very much used in practice. It uses convective heat transfer for vaporizing the solid fuel, the fuel vapor mixes with oxidizer vapor and burns. It has lower performance than liquid propellant rocket; however, it is safer than many of the other types of rockets because it is self regulating and it is finding increased applications. I do foresee much more applications for hybrid rockets in the years to come.

We can even think in terms of other propellant may be FLOX if you want very high performance, maybe with metal embedded in the fuel, like maybe you could have some light

metals say magnesium or lithium. What is used by Scaled composites is  $N_2O_4$  as oxidizer and HTPB as fuel. I think this is all about hybrid rockets, but there is one problem with hybrid rockets because of the slow rate of reactions. It is very susceptible to combustion instability in the low frequency mode, and I am going to talk about it in the following. The subject of combustion instability is new for us.

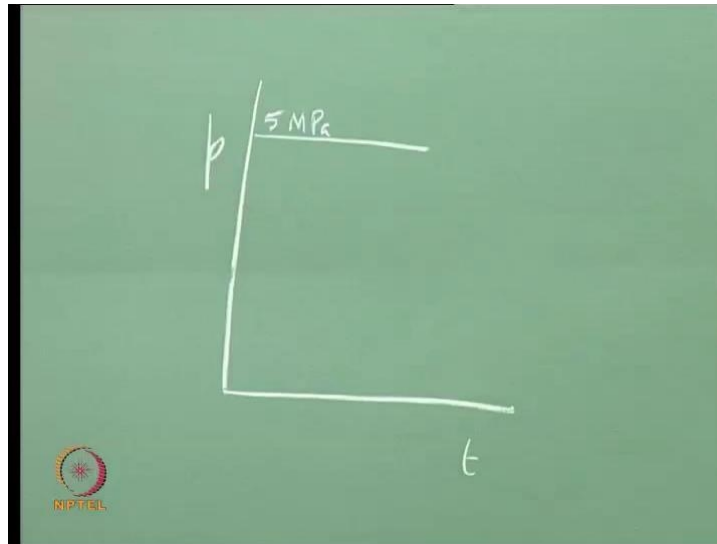
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I will start with a very illustrative example and this example is not mine. This example was given in 1951 by Professor Summerfield; he was at Princeton and he published a paper in 1951. I think it was in Rocket Society Journal; AIAA journal was not there and the American Rocket Society Journal preceded the AIAA Journal. In this paper he gives an example, I modify the numbers because his numbers were in foot pound system of units - FPS system and not that that easy to work on the board. I take a typical liquid propellant rocket and follow his line of arguments presented in his particular paper. Let the chamber pressure of a liquid propellant rocket be 5 MPa; 5 mega Pascal. 5 mega Pascal is something like 50 bar pressure.

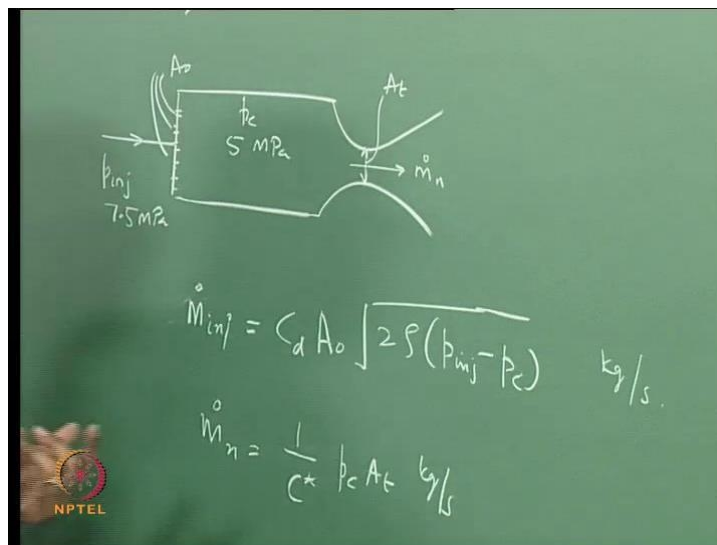
Let the fuel and oxidizer, both of them be injected into the chamber at a pressure of 7.5 MPa. I just choose the numbers to illustrate the phenomenon; that means, propellant is injected at a pressure of 75 atmospheres, and the chamber pressure  $p_c$  is equal to 5 MPa or 50 atmospheres.

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Now, let us put it down on the board. Well, we expect the chamber pressure with respect to time if it is burning steadily to be always 5 MPa and this is what I show here. We start the burning by injecting propellant at 7.5 MPa and hot gases steadily leave through the nozzle.

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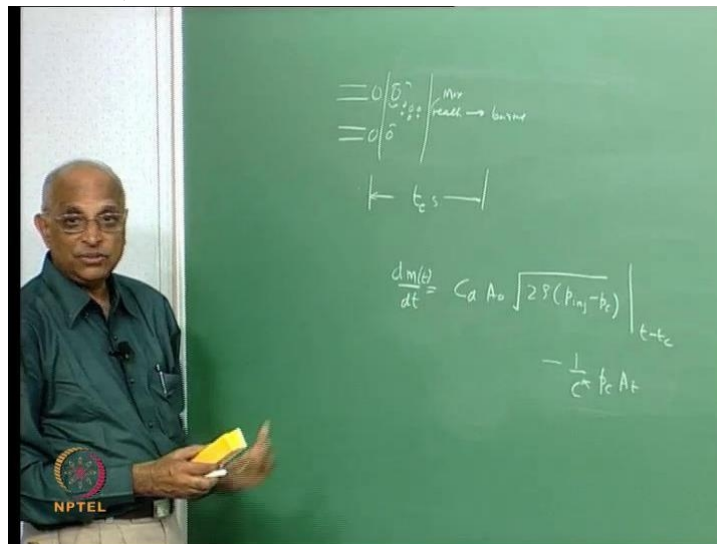


And why do you get a chamber pressure? After all, you add some mass let us say a mass of propellant is injected, and how do you calculate the mass of propellant which is injected? You inject something; that means, you have injector orifices having total area  $A_0$ , let the discharge coefficient  $C_d$  of the oxidizer and fuel orifice be the same;  $C_d \times A_0 \sqrt{2 \rho \times (p_{\text{injected}} - p_{\text{chamber}})}$  is the rate at which mass is injected; so many kilograms per

second. This is under steady conditions.  $C_d \times A_0 \times \sqrt{2 \Delta p / \rho}$  is the volume flow rate and multiplied by density gives the mass flow rate. And what is the rate at which gases are leaving  $m^\circ$  through the nozzle? It is leaving at the rate  $p_c \times A_T / C^*$ . This is the rate at which gases are leaving: so many kilograms per second.

Therefore, we now say this is the value of  $A_T$ , and the number of holes for the fuel and oxidizer which gives the total area of flow for injection is  $A_0$ . Now, we one question. What will be the equilibrium pressure i.e., the mass injected and burnt is equal to the mass, which is leaving. But, we have a strange problem. The fuel and oxidizer viz., the propellant, which is injected takes some time to burn. Suppose, I inject a parcel of propellants at a rate here, it is going to start burning after sometime because there has to be a delay; there has to be something like a combustion delay. Let me go back to the previous discussions again, because this point is important and this will be central to all our discussions.

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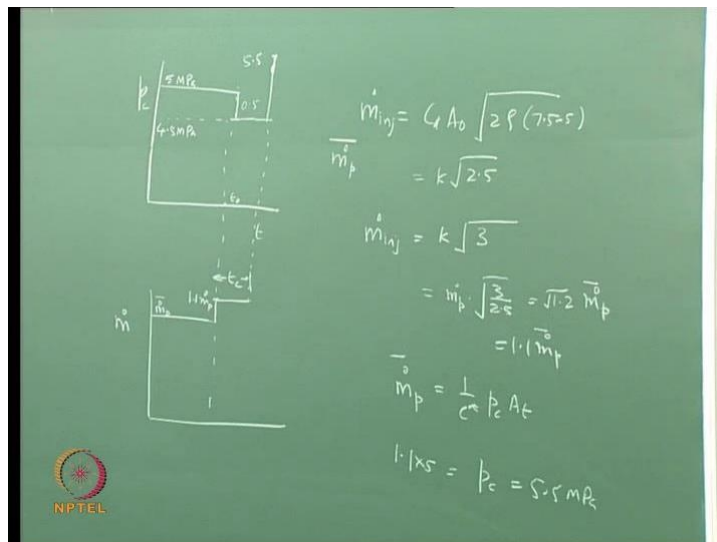


All what we are saying is there is a certain delay, because whenever we inject something maybe we inject it as a liquid, it breaks into droplets, then it evaporates, the fuel and vapor mix together, then they react to form the burnt gases. That means, the process of combustion takes a finite amount of time, the vaporization takes time, atomization is fast, reaction maybe fast, mixing before the chemical reaction also takes sometime. Therefore we conclude that the entire process of injection to burning takes some time due to combustion delay and is  $t_c$  seconds.

Then we need to make some changes in this equation for the equilibrium pressure. What is that we are telling? Something is entering, it has to form gases before it leaves the nozzle and therefore, if we were to write an equation which takes care of the rate of mass variations, what is it we would be writing? I would be writing under normal circumstances  $dm/dt$  the rate at which mass is getting accumulated in the chamber =  $Cd \times A_0 \sqrt{2} \times \text{density} \times (p_{\text{injected}} - p_c)$ , this the mass which is coming in, and what is going out?  $-1/C^* \times p_c \times A_t$ . But now we say this equation may not be really correct. Why? Because, what comes out can happen only  $t_c$  later after the injection, or rather I must say the quantity which comes out through the nozzle at time  $t$  is equal the injected flow rate at  $t - t_c$ . This is what goes out at time  $t$ . The equation let us say  $dm/dt$ , mass at time  $t$ : this leaves at time  $t$ , but what should really burn is what is injected  $t_c$  time earlier. This is my dynamical equation or this is my actual equation. To solve this equation is difficult.

But, we must have a procedure and that is what we will be doing in this class. Therefore, we find that there is a time delay before something happens and let us get back to this example. Mass is injected. Therefore, initially let us say chamber pressure is 5 MPa, which is the steady value of the chamber pressure.

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We can now write the mass which is injected  $m^{\circ}_{inj} = Cd \times A_0 \times \sqrt{2} \rho \times (\text{injection pressure} - \text{chamber pressure})$ . We do not bother about units because we are not concerned with actual values. The chamber pressure is 5 MPa, injection pressure is 7.5 MPa, that means, 7.5 minus 5 is the value of pressure drop across the injector, or rather we say it is



equal to a constant  $k \times \sqrt{2.5}$ . This is the mass flow rate from the injector. I take all the fixed values other than pressure in the constant  $k$ . We may have to multiply by  $10^6$  for MPa, the values of  $C_d$ ,  $A_0$  and density and all these are consolidated in the value of  $k$ . We say this is my nominal value and we call it as  $\dot{m}_{inj}^o$  of propellant which is getting injected. Nominal value is because it corresponds to the steady conditions in the chamber.

Let at this particular instant of time  $t_0$ , shown in slide, let the chamber pressure drops by let us say 0.5 MPa; that means, this magnitude of chamber pressure is 4.5 MPa; something happens in the chamber, maybe there is some problem with the injector or something happens within this chamber and the pressure to fall from 5 MPa to 4.5 MPa. Let the pressure fall instantaneously to this lower value. Therefore, now the chamber pressure, which is  $p_c$  is 4.5 MPa.

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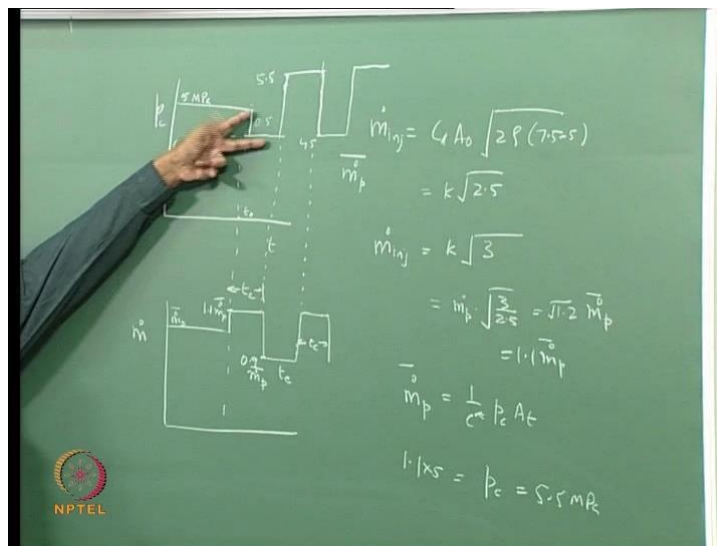
The image shows a green chalkboard with handwritten mathematical equations. The first equation is  $\dot{m}_{inj} = k \sqrt{7.5 - 5.5} = k \sqrt{2}$ . The second equation is  $\dot{m}_{inj} = \dot{m}_p^o \cdot \sqrt{\frac{2}{2.5}} = \sqrt{0.8} \dot{m}_p^o$ . The third equation is  $= 0.9 \dot{m}_p^o$ . In the bottom left corner, there is a small circular logo with the text 'NPTEL' below it.

If chamber pressure is 4.5 MPa, the mass which is injected is going to be  $k \times \sqrt{7.5 - 4.5}$  which is  $k \times \sqrt{3}$ . Therefore, if we were to put it in terms of the steady nominal value of  $\dot{m}_{inj}^o$  which was the steady value, it is going to be  $\dot{m}_{inj}^o \times \sqrt{3} / \sqrt{2.5}$ . This is equal to  $\sqrt{1.2} \times \dot{m}_{inj}^o$ , which is equal to 1.1 times the value of  $\dot{m}_{inj}^o$  which is the nominal value at steady state conditions. That means, all of a sudden we are injecting little more which is now 1.1 times  $\dot{m}_{inj}^o$ . See, since the pressure has fallen, because of the decrease in chamber pressure, what is injected has now gone up by 1.1 times. It takes a certain time delay and let me expand on it. It takes  $t_c$  time to evaporate mix and burn together, and when it mixes and burns what is the chamber pressure that we get? Originally, we had when the nominal value was  $\dot{m}_{inj}^o$ , we

have the chamber pressure  $p_c$  which is given by  $1/C^* \times p_c \times A_t$ , and now I get a value which is 1.1 times and therefore, my chamber pressure will be equal to  $1.1 \times 5$  MPa earlier. After this delay time of  $t_c$  seconds, when the new parcel of gas burns and exhaust through the nozzle the pressure increases to  $1.1 \times 5 = 5.5$  MPa.

At a time  $t_c$  after the chamber pressure dropped to 4.5 MPa, the increased mass flow rate through the injector which burns causes the pressure to increase to 5.5 MPa. What is now the implication? The implication is maybe at this point in time when we said the chamber pressure is 5.5, the mass which is injected is now going to be the value of  $k \times \sqrt{\text{the supply pressure}}$  is still  $7.5 - 5.5$  that is equal to  $k \sqrt{2}$ . Therefore, if the mass injected is now  $k\sqrt{2}$  of the value of mass injected in terms of the nominal value, which was  $m_{inj}^{\circ}$  and which was based on the pressure drop across the injector of 2.5 MPa. The value of the injected mass flow rate is now  $= m_{inj}^{\circ} \times \sqrt{2}/\sqrt{2.5} = m_{inj}^{\circ} \sqrt{2/2.5} = \sqrt{0.8} \times m_{inj}^{\circ} = 0.9 m_{inj}^{\circ}$ .

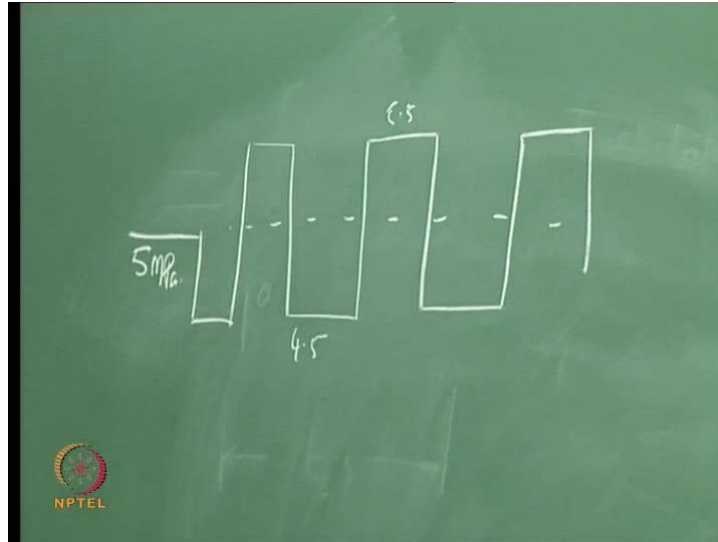
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And therefore, what happens when the pressure increases? The mass flow rate through the injector now drops to a value 0.9 of the nominal value corresponding to  $m_{inj}^{\circ}$  over here. The chamber pressure remains at the higher value of 5.5 MPa till this new reduced parcel of liquid evaporates burns over a time  $t_c$ . During this period of  $t_c$  nothing really happens, and the chamber pressure remains at 5.5 MPa. When this reduced quantity has burnt, the chamber pressure now corresponds to a mass of 0.9 times the nominal and this would be 0.9 into 5 which is 4.5 MPa. This is from equilibrium of  $0.9 \times \text{mass flow rate}$

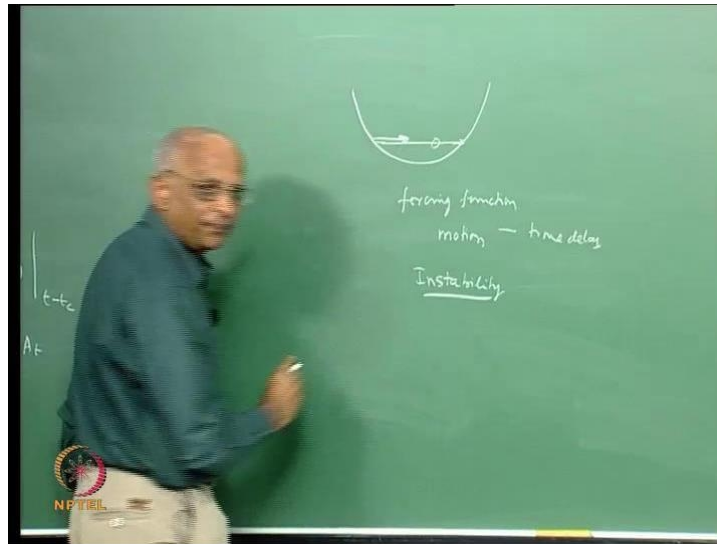
which passes through the nozzle. The chamber pressure now drops to 4.5 again. And this sequence would continue again after a delay time; it increases to 5.5 MPa, falls again to 4.5 MPa and so on. This sequence of events in  $m^\circ$  and chamber pressure is seen to be primarily due to a delay in the burning of the propellant after injection by a time  $t_c$  seconds. And therefore, you find that a momentary drop of 0.5 MPa from a steady value of 5 MPa makes the chamber pressure oscillate between values between 4.5 to 5.5 MPa.

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And out of a steady situation, when you have this delay term and a change in chamber pressure, what happened? You started getting oscillations. Well, the oscillations are neutral in that, maybe it keeps fluctuating between a value of between 4.5 to 5.5 whereas, the nominal value before it dropped down to 4.5 MPa was 5 MPa. Is the process clear? I think if this is clear, we would have understood some part of combustion instability. Namely, a drop or equivalently an increase in chamber pressure whenever there is a delay causes this problem of fluctuations. I will come back to it. Let me give a physical example before we proceed. We would have seen many toys in the market; I just brought one such toy here. It is very illustrative of the phenomenon. What we do is you just take this toy, you sway it once. It will keep on oscillating up and down. I put it on the table. I push it here it goes up and down, it keeps on rollicking up and down, it is in a state of neutral oscillations. Why does it have to do this?

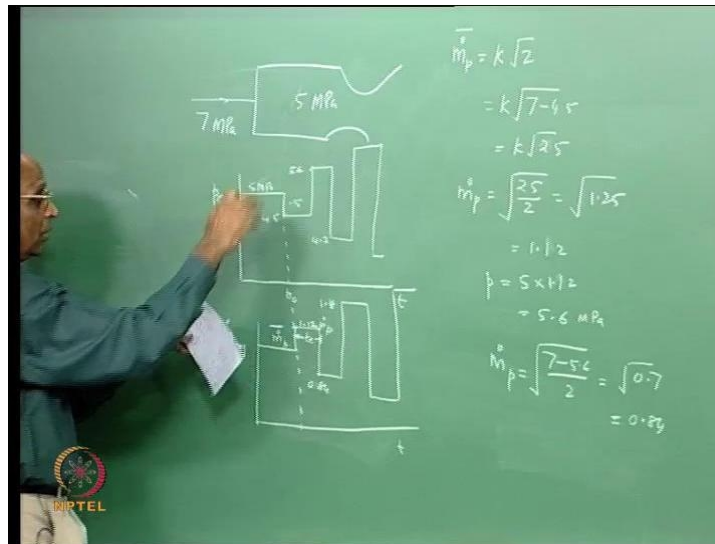
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Let us take a look at the construction of this particular toy. There are various such toys. There is a small marble inside it that is what is making this particular noise. When I shift it, when I shift it there is a sling and a marble, when I shift it, the marble comes and hits over here and rebounds. It goes to the opposite side causing the toy to deflect after a short time. There is a delay between the forcing function and the motion and it is this time delay, which keeps this toy oscillating. Once the oscillations are started it keeps on oscillating.

This particular toy is different from a toy at the bottom of which you have a mass placed and the momentum about the center of gravity governs the motion. But in this case there is a marble inside and it is this delayed response, which keeps it going up and down, until the friction finally stops the motion. Something similar is happening when you inject propellants into the combustion chamber; there is a time delay for burning, the time delay precedes whatever be the change that happens. This is what happens in the case of instability. Can we say this is understood? If this is understood let me go back to the next example. I just do one more example and then we can generalize it.

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Let us now assume that in rocket chamber the injection pressure is now 7 MPa and the steady value of chamber pressure is still the same viz., 5 MPa. The injection pressure is now reduced to 7 MPa, and we ask what is going to happen in this case? Now, we do not have to repeat much. All what I want to do is maybe make these 2 plots for the chamber pressure as a function of time and the mass which is getting injected as a function of time. The initial steady state pressure is 5 MPa, and maybe at time  $t_0$  we reduce the chamber pressure like in the previous example to 4.5 MPa; that means, I decrease it by 0.5 to 4.5 MPa. Now, what is going to happen in this case? Let us say corresponding to 5 MPa, I have a steady value of mass injected corresponding to  $m_{inj}^0$ . This value is  $k\sqrt{7-5} = k\sqrt{2}$ .

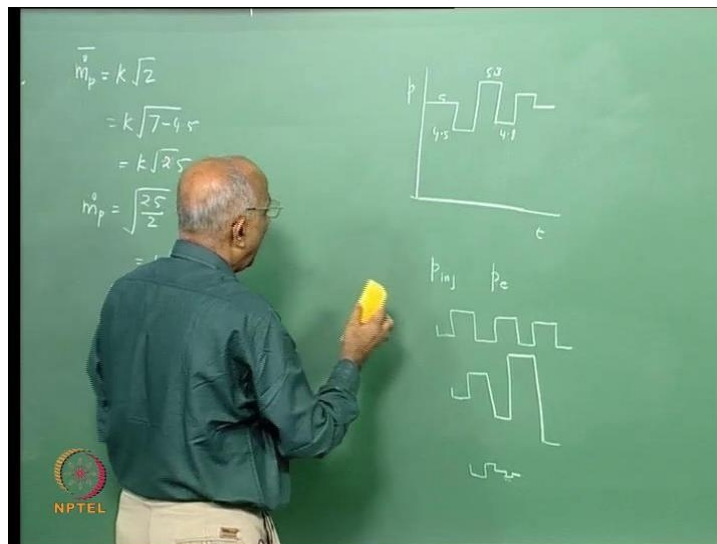
We have reduced the chamber pressure from 5 to 4.5 MPa. Now the mass that is injected would be different from the value under steady conditions of  $k\sqrt{2}$ . When the chamber pressure has got reduced to 4.5 MPa, the mass injected becomes  $k \times \sqrt{7-4.5} = k\sqrt{2.5}$ . Or rather the new value of the injected mass =  $\sqrt{2.5} / \sqrt{2}$  of the original =  $k\sqrt{1.25}$  and  $\sqrt{1.25} \approx 1.12$ . Therefore, now we find that the at this point  $t_0$  the injected mass increases to 1.12 of the value of  $m_{inj}^0$ . And what is the repercussion?

Well, it is going to burn after  $t_c$  time, and once it burns the pressure increases from the steady value of 5 MPa by the corresponding mass ratio. This value will be  $5 \times 1.12 = 5.6$  MPa. Therefore, the pressure now goes up to 5.6 MPa. When the chamber pressure is 5.6 MPa, the mass flow rate injected now decreases and it becomes equal to with respect to the

steady difference of the injection pressure of 2 MPa now it is going to be  $\sqrt{7} - 5.6 = k\sqrt{1.4}$ . Expressed as a fraction of the steady injection flow rate it is equal to  $\sqrt{1.4} / \sqrt{2}$  viz.,  $\sqrt{0.7} = 0.84$ . The flow continues at this rate until after a time  $t_c$  when this parcel burns and the pressure in the chamber reaches a value  $5 \times 0.84 = 4.2$  MPa. The injection pressure drop thereafter increases to  $7 - 4.2 = 2.8$  MPa. The increase in pressure drop causes more flow compared to the nominal chamber pressure of 5 bar and this after the combustion time of  $t_c$  seconds results in a further increase of chamber pressure.

The magnitude of the step from 0.5 bar keeps increasing in this case. The oscillations diverge and keeps on increasing. The oscillations need not be neutral such as it was for an injection pressure of 7.5 MPa. With a reduction of injection pressure to 7 MPa the oscillations diverge out. I now do a third problem in which I choose the injection pressure as 9 MPa, and the chamber pressure is kept the same value at 5 MPa.

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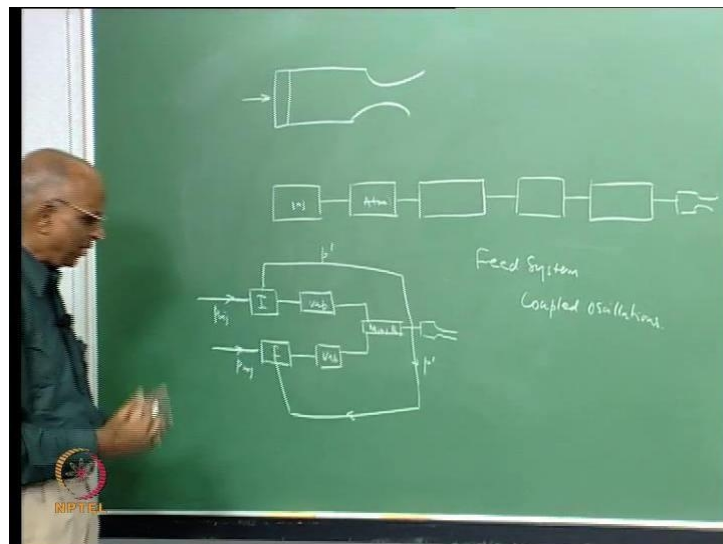


And what do we get for this pressure when we start with 5 MPa chamber pressure and the chamber pressure drops from 5 to 4.5 MPa at time  $t_0$ ? Then as time progresses the steps in the amount of 0.5 MPa decrease as time progresses. The pressure with respect to time follows a decreasing trend. And let us put the numbers because it is important to understand. From the value of 4.5 MPa, the chamber pressure rose to 5.3 MPa, the value comes back to 4.8 MPa. It will be something like 5.1 MPa next.

Therefore, what we have just done is that we have taken a simple case wherein, we varied the injection pressure keeping the chamber pressure constant, and we found after a step drop in pressure the oscillations are so evolving such that it keeps on oscillating in a limit cycle mode with the same amplitude or else the amplitude would diverge and ultimately explode or the amplitude would decay as a stable system. That is, it could be neutral, it could diverge or increase in amplitude or be a decaying oscillations as a stable system. It is possible to get all these oscillations in the rocket under some condition or the other by variation of the injection pressure and chamber pressure.

And as in the example of the toy, which I said is very similar to the chamber.

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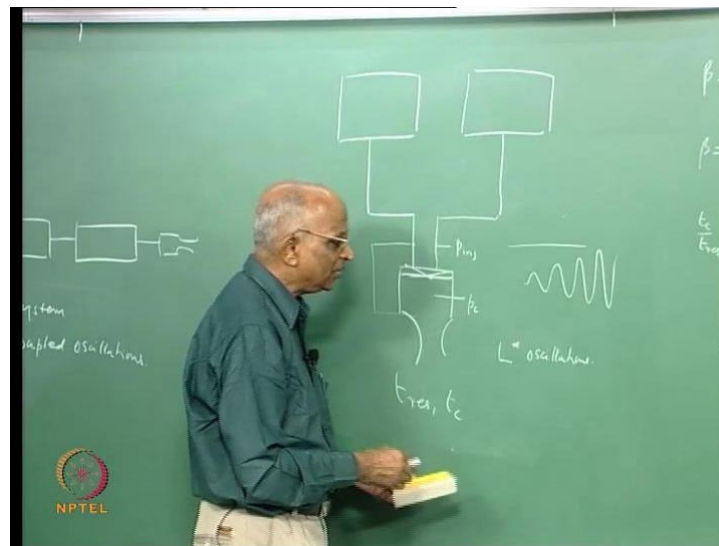


In practice, we have in a liquid propellant rocket an injector, we have a combustion chamber and the nozzle. We are injecting propellants at the injector and what happens after injection. I have the process of atomization taking place atomization then we have vaporization of the propellant taking place after vaporization we have mixing taking place and after mixing we have chemical reactions taking place and then the gases leave through the nozzle. We have fuel and oxidizer being injected; may be we have fuel injector holes, we have oxidizer holes; the fuel injection orifices creates the fuel drops and it vaporizes; the oxidizer orifices creates oxidizer drops which subsequently vaporize. The vapors come and mix in the chamber and reactions are taking place over here. The gases are ejected out through the nozzle.

During this process of combustion, if there is some oscillation namely  $p'$  it gets back into your injector over here; this is where you are injecting at pressure  $p_{inj}$  and this  $p'$  modulates the flow rate through the injector. We obtain a changed value of mass flow rate that is injected, which is coming into the chamber again. The vaporization now gets affected and this becomes something like a feedback circuit. In other words a change in the downstream value of  $p'$  due to mass generation gets coupled to the mass flow rate through the injector. That means, it is a feed system that gets affected and therefore this type of combustion oscillations is known as feed system coupled oscillations.

Let us now physically interpret what little we have done so far in some other way. We tell that we have something like a tank, which supplies the liquid propellants into the combustion chamber.

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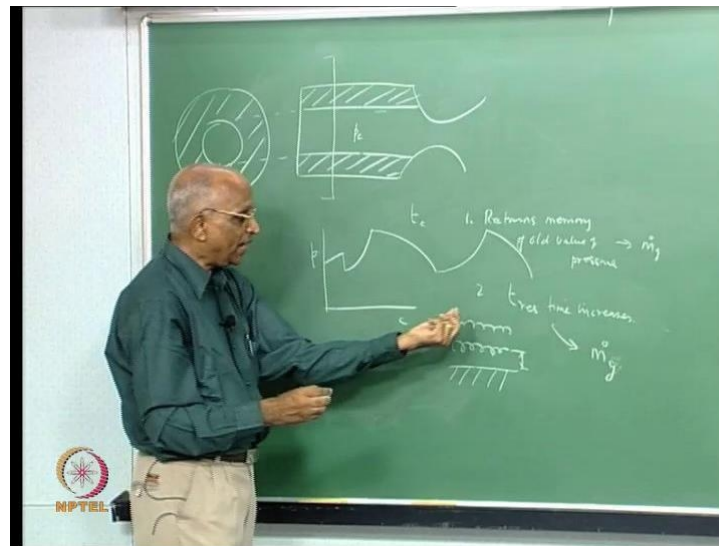
We have an injection pressure upstream, we have the injector spraying the liquid propellants; we have the  $p_c$  over here. We have an injector pressure drop and any pressure change creates a differential flow over here. In other words, this is what gives the feedback that the pressure change in the chamber reflects on the flow of propellant into the chamber from the feed system. This means, feed system is influenced and such type of oscillations are known as feed system coupled oscillations. Therefore, it is known as feed system oscillations. Rather the oscillations depend on the residence and time of delay  $t_c$ . The value of  $t$  residence depends on the  $L^*$  of the motor.

Therefore, some people also call it as  $L^*$  oscillations as the length of the chamber is involved. All what we have seen is it is quite possible when we have the injector pressure



drop, which is less than some threshold value then it is quite possible instead of having a steady value of pressure to get the pressure oscillations that diverge with increased level of amplitudes. These oscillations are linked to the feed system because the feed is what gives you the pressure drop over here. If we have a very high pressure drop at the injector, we will not get these oscillations and therefore, it is known as feed coupled oscillations or  $L^*$  oscillations. Now, let me see whether I can get similar oscillations for solid propellant rockets; let us start with a set of arguments.

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Let us consider the case of a radial burning solid propellant rocket. We have a propellant over here, which is burning. We will try to go through the same arguments as for the liquid propellant rockets. Let the chamber pressure be  $p_c$ . Let us say that the pressure drops as it did in the liquid propellant rocket. If the pressure drops what is going to be the effect? Would we have a cascading effect like what we had in liquid propellant rocket?

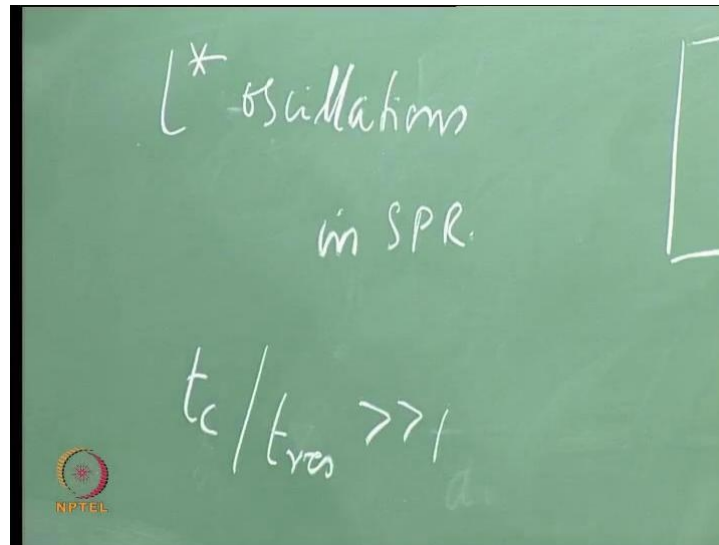
Let us see what happens when pressure drops. The heat transfer from the gases or rather the hot gases are still seeing the propellant; that means, the propellant surface is still hot and it has a memory of higher pressure before the step in pressure drop. Higher pressure means the flame is nearer to the propellant surface. Therefore, more heat is generated and therefore, propellant surface gets more heated than at lower pressure to which the chamber pressure has dropped. Therefore, we find when the heat flux or the heat load on the propellant is higher.

Even though the pressure drops, the propellant still retains memory of the old pressure and it does not immediately relax to a lower heat flux at the surface; that means, we say it retains memory of old pressure. Next, when the pressure has dropped; what is going to happen the flame front viz., the distance of the flame from the propellant surface? Goes a little bit further away since the pressure is directly proportional to the number of molecules, and the rate of reaction decreases as the number density of the molecules decrease. If the rate of reaction decreases well the rate of generation of hot gases decreases or else the velocity decreases and if the velocity decreases the residence time increases. Let me go through this a little more in some more detail.

With a solid propellant burning, we have a propellant surface and a flame, which is standing off at a certain distance viz., the stand off distance from the surface. We reduce the chamber pressure; therefore, the flame standoff increases, but the surface still has memory of the earlier high pressure. Now the stand off distance has gone as the pressure has decreased; if pressure has decreased the value the chemical reaction rate has decreased. If chemical reaction rate has decreased, the rate at which mass of the gases is getting generated has decreased. The velocity has decreased. If velocity has decreased we get more residence time and the residence time has increased. If residence time has increased the chances for chemical reactions to get completed is more resulting in higher value of the rate of the mass generated.

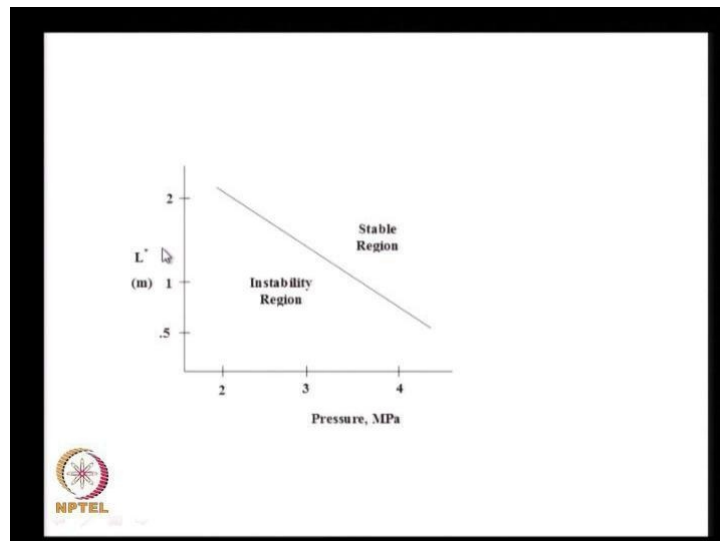
Now, the surface still retains memory of the past it still hot therefore, it is still producing gases at the old rate and therefore, even though the pressure has fallen it takes some time for the surface to come back to the present state. And therefore, what is going to happen? The pressure increases because we increased the mass generation and have a higher mass flux rather than the old value. We therefore get a higher value of pressure. At higher value of pressure what happens is that the memory of the surface is with respect to the older value of pressure for which the mass generation rate is lower, and the residence time is less. If the residence time is less, we have a smaller chemical reaction time to generate gases and therefore the pressure again decreases. The process repeats as in liquid propellant rockets. Whenever there is a delay due to the thermal lag at the surface and we have the residence time getting changed, we get oscillations.

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And this type of oscillation in chamber pressure is known as  $L^*$  oscillations in solid propellant rocket. Why do we say  $L^*$ ? If we have a very small rocket in which the value of volume by throat area is small; the value of  $t_c$  compared to  $t_{residence}$  will be larger and therefore, the oscillations are more profound.

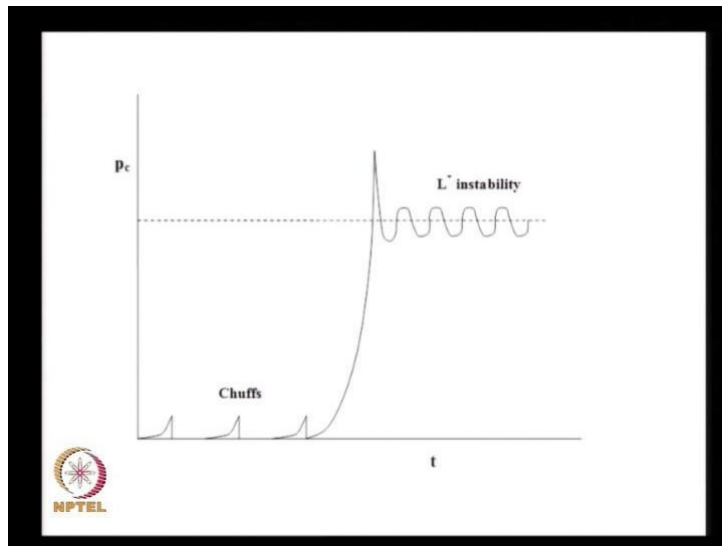
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Let us focus on this particular slide given above. In this slide we show the value of the  $L^*$  as a function of chamber pressure and we define the regions of instability that is  $L^*$  mode of oscillations. The stable region, if the value of the  $L^*$  is small, occurs at larger values of chamber pressures. Well the time of residence of the gas is  $t_{residence}$  of the gases in the chamber is small when  $L^*$  is less. If the value the residence time is small the value of  $t_c$  by  $t_{residence}$  is large and the combustion is more likely to be influenced and unstable and

therefore we have the instability region corresponding to small value of  $L^*$ . If the chamber pressure is smaller the chemical reaction time is larger or chemical reactions take more time to go to completion. And therefore, I have an instability region, which is on the left portion for small  $L^*$ . A stable region corresponding to larger values of  $L^*$  and larger values of chamber pressure. We find that for solid propellant rockets it is necessary to provide either a large value of  $L^*$  or operate it at a large value of chamber pressure for stable operation.

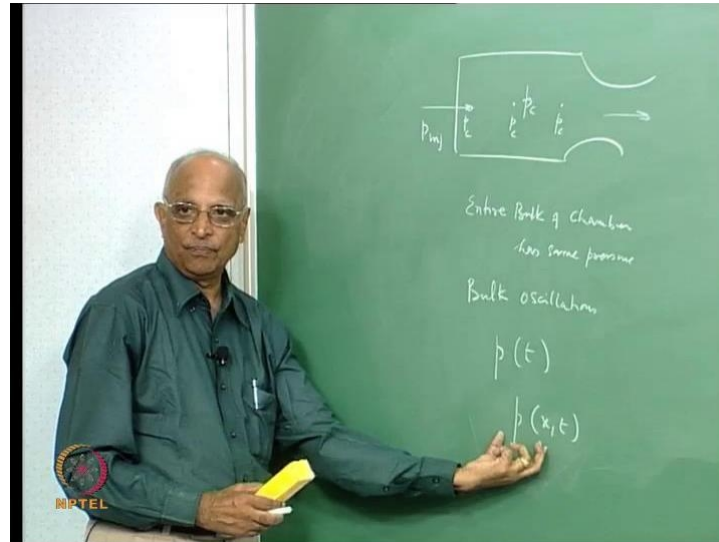
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Let us illustrate this further. If we have an  $L^*$  instability, we have oscillations in chamber pressure, which are seen in this slide. This is  $L^*$  instability. But very often we also find that when a motor is ignited and the ignition is not sustained we sort of get some spikes like chuffs. You get a small spikes in pressure when the motor gets ignited, but the rocket gets quenched soon. Since the motor grain is still hot and thereafter, the heat or the temperature in the grain again ignites the motor and again I get another pressure spike another spike something like a train chuffing along chuff after chuff. You get these small oscillations in pressure, which are due to ignition and hang fire situation rather than  $L^*$  oscillations.  $L^*$  instability, which occurs when the chamber pressure has a finite value, are due to combustion response due to change of pressure while chuffs are essentially an ignition phenomenon i.e., not providing good ignition.  $L^*$  instability occurs because of the incompatibility between the chemical reaction time and residence time. When the chemical reaction time is large compared to residence time and you have these reflexes of pressure and heat release from the propellant, and we get the  $L^*$  instability.

This is how we define the L star boundary; this plot is generally fairly linear and we say well this is the instability region while the region to the right is the stable region. This boundary in the plot of  $L^*$  versus pressure defines or gives us the boundary between stable operation and unstable operation for  $L^*$  oscillations. At this point in time, I should point out that whenever we talked in terms of feed coupled oscillations in a liquid propellant rocket or in terms of  $L^*$  oscillations in a solid propellant rocket,

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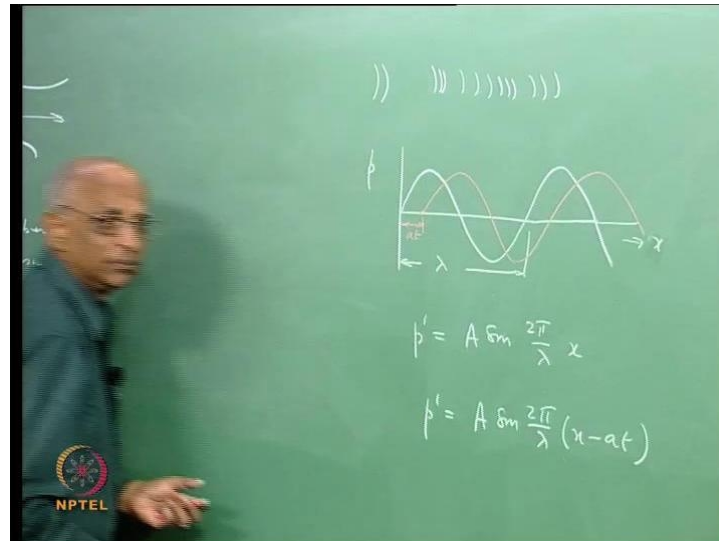


the pressure in the chamber or within the grain is the same at all locations. At this point the pressure is the same as at this other point. Within the chamber the pressure is the same. There is no variation in pressure in the port volume; that means, we are talking of may be the entire bulk (entire bulk of chamber) having the same gas pressure. In the liquid propellant rockets, what did we do? We injected propellants at  $p$  injection; we had  $p_c$  in the chamber and the same  $p_c$  is used for exhausting through the nozzle. That means, the entire volume or the entire bulk of the chamber is at the same pressure and therefore, such oscillations are also known as bulk oscillations. Something very similar to the toy I showed you, the entire body is moving. It is not that one part; not that the hand moves and the other hand is stationary. But in practice what happens? We could have different pressures in the different regions.

And that is what happens when I am talking to you. As I am talking, you know it takes some time for my signal to reach you. Pressure instead being a function of time alone as in the bulk mode of oscillations, could be a function of distance and time. Therefore,

whatever we have discussed so far relates to something like a lump mass assumption wherein, we take the entire volume to oscillate in unison. This may not really be true for all cases. We must move to differentiate between pressure at the different points in the bulk or volume. We will illustrate it through some physical examples.

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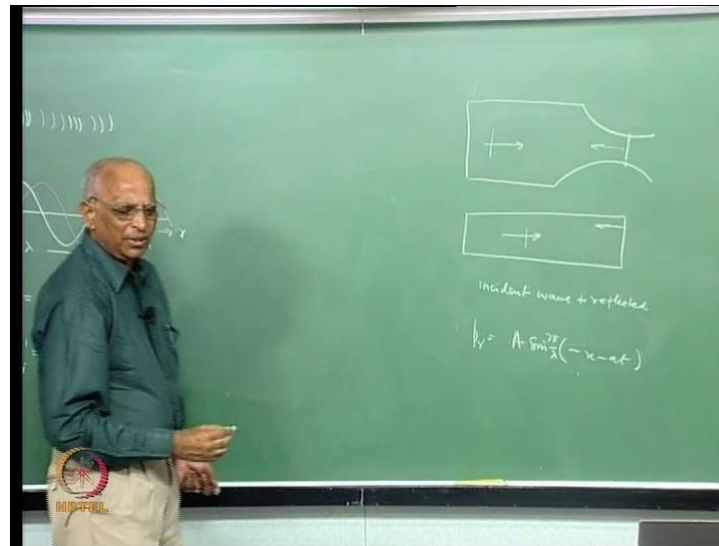


Let us consider the following example. I am talking to you. Some acoustic oscillations get transmitted to you. How does the signal i.e., the sound come to you? Through a series of compressions followed by rarefactions followed by compression followed by rarefaction right? In other words, if we were to plot it what is it we get? We get region wherein I get compression followed by rarefaction followed by compression followed by rarefaction. In fact, this is the sound wave and how do we represent pressure as a function of time. We can write the fluctuations as a sine wave: this is one cycle of oscillation viz., that is one wave length of oscillations and this is my distance. As distance increases my compression- rarefaction sound wave travels and therefore, I can write this as  $p = \text{Some amplitude } A \times \sin\theta$  with  $\theta = 2\pi x/\lambda$ .

This is the equation to a sound wave. The sound wave means, a disturbance wave  $p' = p \sin 2\pi x/\lambda$ . But there is no traveling component in this expression. It is just a wave of compression and rarefaction. We said that the sound wave travels at the speed of sound. If the wave is going to travel at speed of sound that is 'a' meters per second where 'a' denotes the speed of sound. Well after a let us say after a time t after we start the wave should come over here, that means, over a time t the wave would have moved a distance  $a \times t$  where 'a' is

the velocity of sound. If that is the case the equation to a traveling wave should be  $p' = A \sin 2\pi/\lambda(x-at)$ . Now what happens now when we look at this particular sound wave traveling axially in a rocket chamber? A rocket chamber it is something like a cavity.

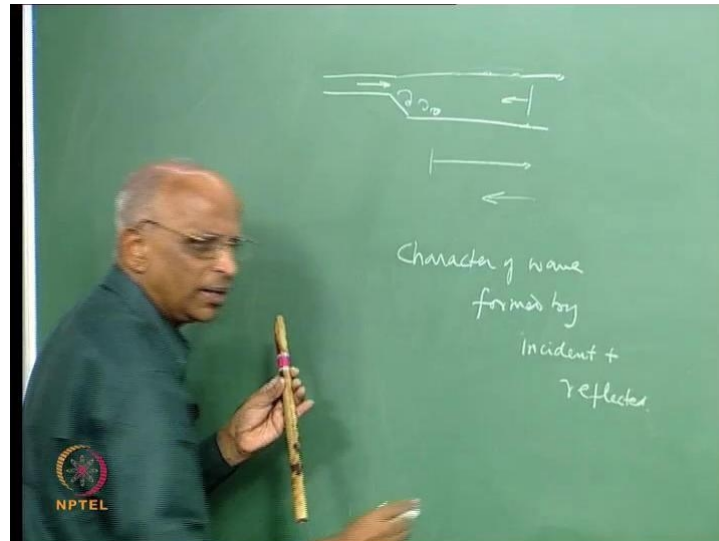
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We have the injector side over here or the head end of a solid propellant rocket. We have the nozzle on the right side. We found very rapid changes in density at the nozzle throat and it acts as if it were a solid surface itself. Therefore, whenever some wave travels up over here this looks at it as it was a solid surface. It reflects back the wave as if it were an enclosure in which wave moves up over here reflects over here comes back. During this period there is another wave coming and therefore there is an interaction of let us say an incident wave plus a reflected wave. But, we know the equation to a wave can be written like what was stated earlier. Now what will be the equation to a reflected wave if the incident wave is given by this equation? The equation to a reflected wave that would be moving in the opposite direction should be  $A \sin (2\pi/\lambda[-x-at]) = -A \sin(2\pi/\lambda)[x+at]$ .

I brought a flute with me and I always illustrate combustion instability problem through this. I have something like I blow into the flute. Why does it make noise? After all when I am freely blow and steadily. Similarly, if I have a whistle. I blow into it and it makes noise. Why does it make noise? Let us try to understand this problem as a prelude to solving this spatial distribution of pressure. And if this part is clear, may be we will be better equipped to relate to the combustion instability.

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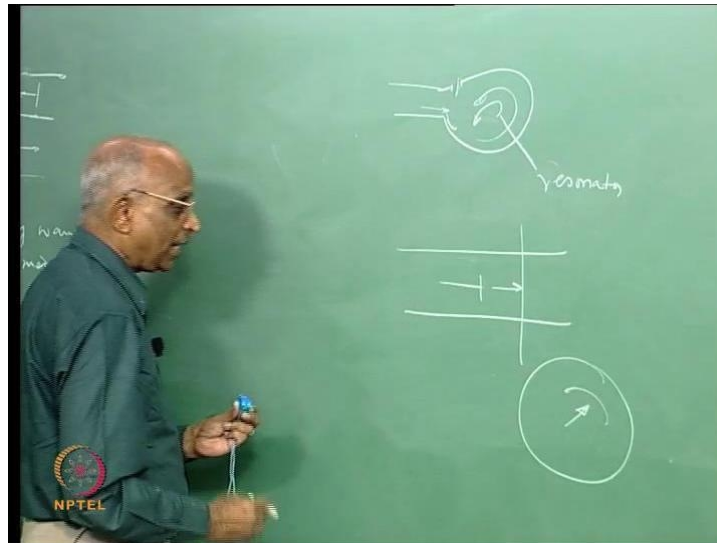


After all we have something in which I am blowing air into. I have something like a step over here in the flute. We have some holes here. What is it I do? I blow air here when I blow air I get some eddies downstream of the step, because all of a sudden there is a change here. Some disturbances are generated and when disturbances are generated they move in a chamber over here it sees an open part here, where it gets reflected back and therefore, an interaction between the forward running wave and backward running wave is created and that creates some resultant wave which amplifies the sound. If I were to plug one or let us say I plug all the holes here I have something like 6 holes I plug it still it makes noise that means, but it makes a different frequency of sound.

In which case I just have a chamber here in which something is happening. When I open some hole my net reflection is somewhere earlier and therefore, I can change the character of wave formed by incident plus reflected. And exactly the same thing happens in the case of a whistle. What happens in the case of a whistle, let us sketch it out.



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You have cylindrical cavity here. What is it I do? I push air it creates some disturbances here. The disturbances are generated. We have the waves moving in round like this and it is this which functions as a resonator. And why does it resonate? I have forward and reflected waves coming. In other words, when I have a chamber I could have waves not only moving in the axial direction, but if I take a section, I have a circular section and waves could also move in the tangential direction. Waves could also move radial direction and this is what leads to disturbances in a chamber and if these disturbances were to couple with the combustion, we could have instability.

In this way, we understand about oscillations in chamber pressure in a rocket and the bulk mode and wave mode of instability. Some of you may like to study further about instability and how to control instabilities. However, we stop here with combustion instability in rockets and in the next class, we will deal with electrical rockets.