Rocket Propulsion

Prof.K.Ramamurthi
Department of Mechanical engineering
Indian Institute of Technology, Madras

Lecture No. # 04
Velocity Requirements

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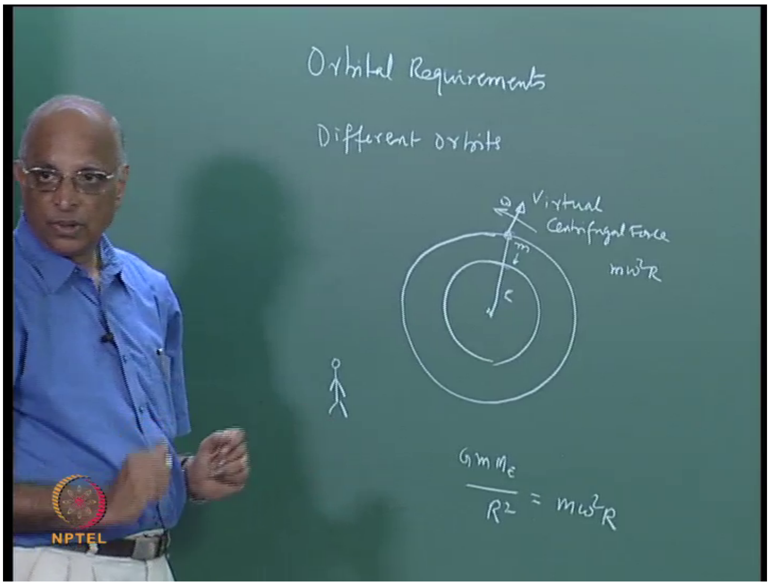

Good morning. We will be looking at the orbital requirements and also will examine the different orbits in the class today. To be able to do this, let us just briefly recap where we were in the last class. We found that a body goes around the Earth, an object going around the Earth for which we would like to have a frame of reference on this body itself.

That means, I am sitting on this body as it were and looking at my rotating body, it is not that I am in an inertial frame where in, I sit here on the ground and watch this. But I have what we said is the rotating frame of reference and what did we find? If I have a rotating frame of reference it is necessary for me to put a fictitious force and this fictitious force we called as a pseudo or virtual force: we call this pseudo force as centrifugal force. We found that this centrifugal force is equal to $m\omega^2R$, where ω is the rotational angular velocity of the body as I am sitting on it and R is the radius from the centre over here. So far so good.

The mass of this object is m. You know this force was required to correctly predict the motion of the body in the frame of reference of the body itself. How did this force arise? Well, in the perspective of me sitting on the body the body is not moving and therefore, I had to put a force to correctly define the motion of the body. Namely, we said X' is the coordinate of the body there is no change in X' and therefore, we had d^2x'/dt^2 was equal to 0. Using this pseudo force, we wrote an equation in which this is balanced, we said, by the earth attracting this body due to the universal law of gravitational forces; we wrote it as G into mass of the body into M_E divided by R squared. M_E is the mass of the earth. We consider the product of mass of body and mass of Earth divided by R^2 into the gravitational constant G is equal to the psuedo force which is acting namely $m\omega^2R$.

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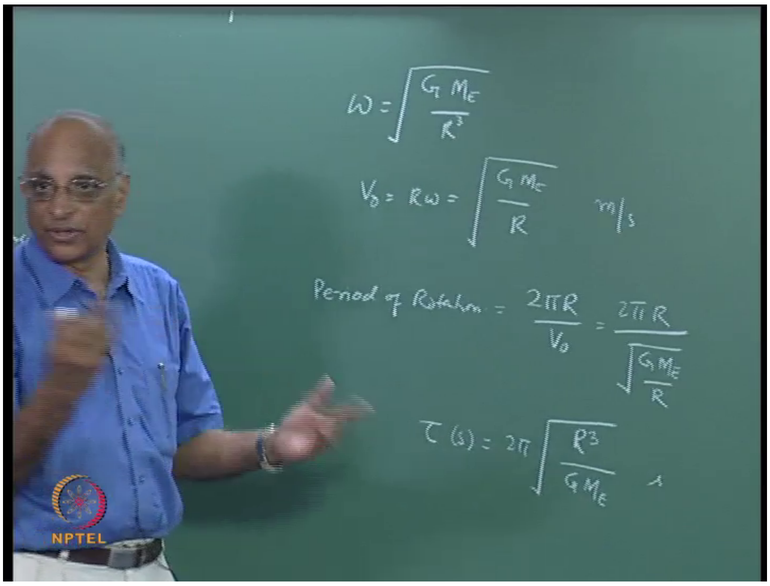

And therefore we were able to get the angular velocity of rotation, $\omega = \sqrt{GM_E/R^3}$. So many radians per second and this is what we did. We also went one step further wanted to find out what is the velocity V_0 of this object as it is rotating around. We told that V_0 is equal to $R\omega$ and therefore, $V_0 = \sqrt{GM_E/R}$. We had $\sqrt{R^2}$ in the denominator cancelling with the R in the numerator. If I express G in Newton metre square by kilogram square, mass of the Earth in kilograms and R in metre, the unit we got was metre per second. This is the orbital velocity.

Now, we go one step further. Supposing this body is rotating, what is the period of rotation? What is meant by a period; time required to complete one rotation. I find that it travels through 360 degrees or 2π radians and therefore, I say the period of rotation must be $2\pi R$ is the total distance it travels divided by V_0. Therefore, the distance travelled in one orbit is equal to πd or $2\pi R$ divided by V_0. And therefore, the period of rotation will come out to be equal to $2\pi R$ divided by $\sqrt{GM_E/R}$. And what does that give you? The period of rotation which let us call tau τ is equal to so many seconds. So many seconds is the period of one rotation. Therefore, what is it you find from this? Let us do one or two small examples. Let us find the period of rotation for two distances:

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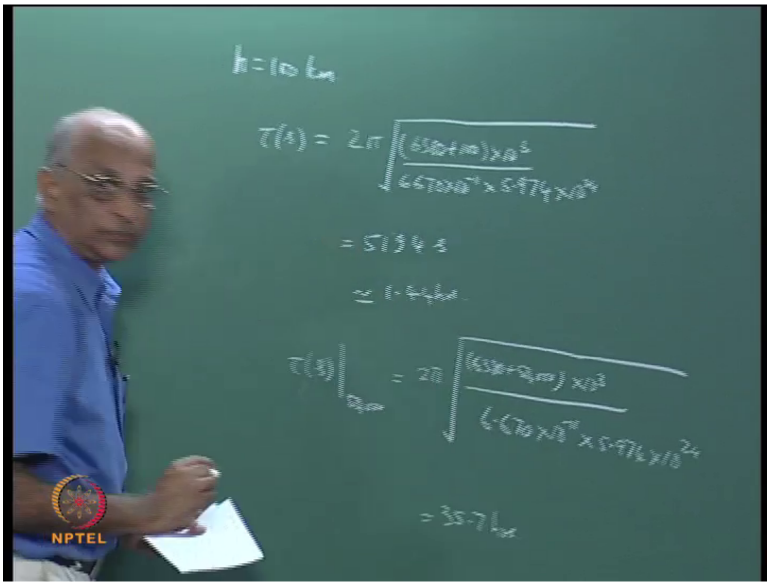

Let us take the problem which we did the other day. When the height above the earth is 100 kilometres we found out the value of the orbital velocity is 7.4 kilometres per second. I want to find out what is the period and therefore the period in seconds $\tau = 2\pi R$ divided by the velocity V_0. 6380 km is the radius of the earth. Let me write the radius R_E as radius of the earth is 6380 plus, I have 100 km as height above the Earth, so R is 6480 kilometres into 10^3 in meters divided by G the value of G the gravitational constant is 6.670×10^{-11} and the mass of the earth is equal to 5.974×10^{24} kg. This is the time taken to complete one orbit at a distance of 100 kilometres above us. When I calculate the value I find this comes out to be something like 5194 seconds or something like equal to 1.44 hours.

Instead of a circular orbit of 100 kilometres height I go to a higher orbit, instead of being 100 kilometres above the earth, I go to a distance let us say 50,000 kilometres above the earth. And now, I ask what is the value of the time period? If I do the same set of calculations, τ in seconds at a distance of h equal to 50,000 kilometres will give me a much larger value. Again we put 2π into 6380 radius of Earth plus 50,000 in kilometres into 10^3 divided by $\sqrt{}$ 6.670×10^{-11} into the mass of the earth which is equal to 5.974×10^{24} ÷ the radius. And what does this come out to be? This comes out to be something like 35.7 hours. All what we are doing is, we wanted to find the time for going through one orbit at different heights over the Earth.

At a height of 100 kilometres we find the period is something like an hour and odd and if have the orbit which is at a height of 50000 km height, it is something like 35 hours. And if I want to plot this what is it that we get? I keep this figure and erase this part.

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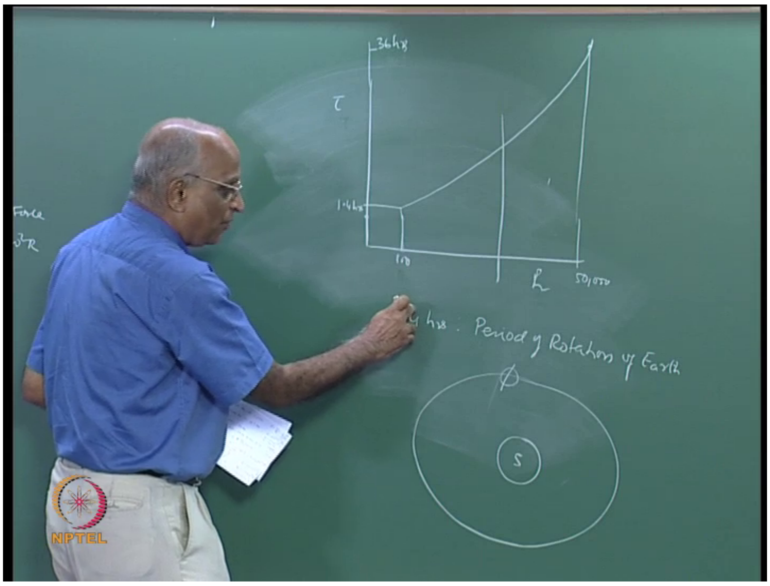

Now I plot the radius or height above the earth as a function of time period. On the surface of earth which is something like 6300 km, it will be little bit lower than at a height of 100 kilometres where we got a value equal to 1.4 hours or so. At a height of 50,000 kilometres I got it as equal to something like 35.7 or 36 hours.

And the graph shows that the time for one orbit is $R^{3/2}$. Therefore, the graph shows this somewhere in between 1.4 hours and something like 35.7 hours. We have the value corresponding to a single rotation of the earth and that is 24 hours. That means, I have 24 hours is the period of rotation of the earth. How do you define the period of rotation of the Earth? We define it as the time between when the sun appears vertically overhead at a given time (midday) today and sun is vertical at the same time (midday) tomorrow (the next day); that is the period and that we say is one day or 24 hours. This is known as solar day.

But actually what is happening? You have the Sun at the centre of the solar system, you have the Earth rotating in an elliptical fashion and the earth is also revolving on its axis as it is rotating around the Sun. Therefore, the solar day is going to be different from the actual rotation time. The period of one rotation is going to be slightly less than 24 hours and therefore, because it is revolving and as it is moving it is rotating and therefore, the period cannot be 24 hours.

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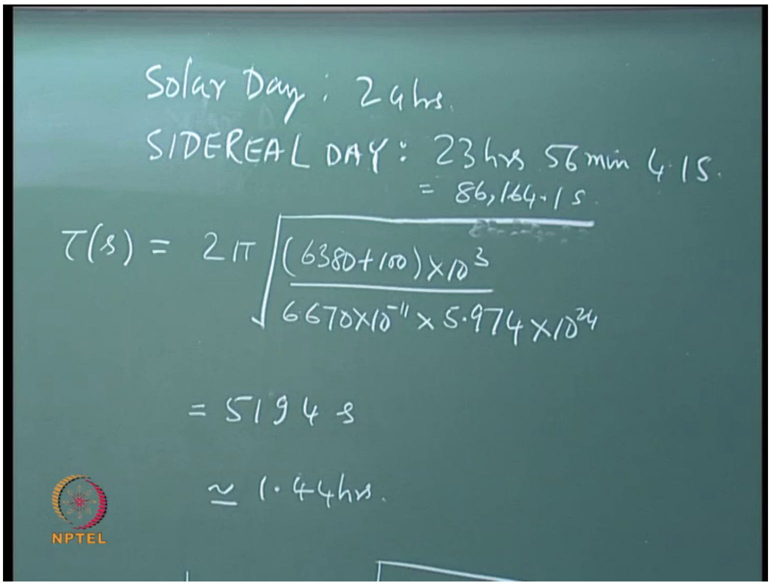

And therefore we define a sidereal day instead of a solar day; that means, 24 hours is the period corresponding to solar day while the real actual time taken for a rotation which we call as sidereal day is slightly less; it is something like 23 hours 56 minutes and 4.1 seconds. Therefore, whenever we say one rotation; what is happening is the Earth is rotating on its axis and as it is rotating it is also revolving and therefore, one rotation corresponds to not one solar day. Therefore one works with what we call as a sidereal day which is, 23 hours 56 minutes and this works out to be something like equal to 86164.1 second.

I do not think we are going to get into that depth of trying to find out the difference between the solar day and the sidereal day and we will assume that the Earth rotates once in 24 hours. And having said that you know somewhere in between 36 hours and 1.4 hours we will find that you have the time of one orbit as 24 hours.

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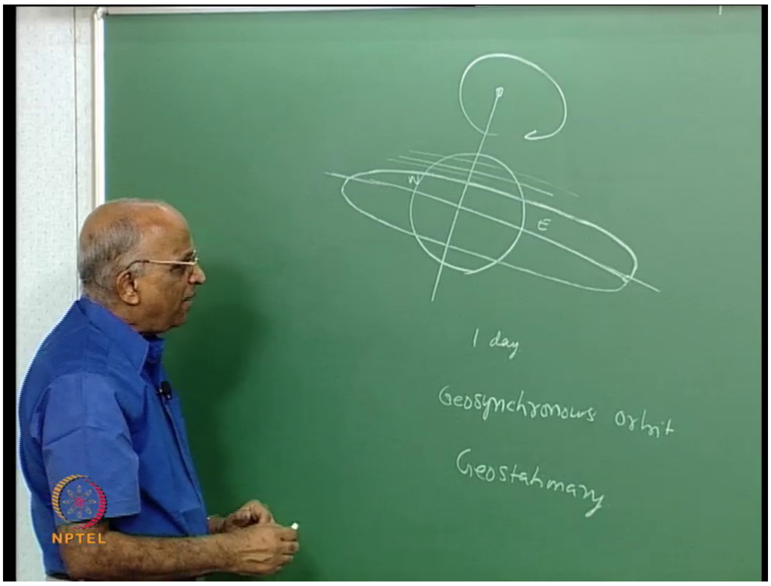

And at this point when the orbital time is 24 hours what is going to happen? You know we showed the Earth over here; the Earth rotates on its axis and you have the body. When the body or the satellite moves with respect to the earth and the rotation of the body with respect to the earth is the same, in other words the time taken for one revolution let us say of the body is on this plane the body is moving, let us say from east to west and the time taken for one orbit as it goes around is one day which is the same rate at which the Earth is rotating on its axis. In other words the period of rotation of this body is synchronous with the rotation of the earth and we call this orbit as geosynchronous orbit.

In other words the Earth rotates from east to west and if the body rotates in a plane which also rotates once per day we say that the rotation of this orbiting object or body or satellite is synchronous with respect to the Earth, we call it as geosynchronous orbit. If now the body is rotating on the east west axis namely on the equatorial plane of the Earth and the period is 24 hours just the same way as the Earth rotates in 24 hours, then any point on the surface of the Earth since the Earth is also rotating once in 24 hours and this is rotating in the equatorial plane at the same rate as once in the 24 hours, the satellite will always appear stationary at all points on the surface of the Earth. And such an orbit is known as geostationary orbit. We call is as geostationary because for all points on the surface of the Earth, the satellite appears stationary. But if by chance the orbit is not along the equatorial plane, but it is in some other latitude or in some other plane, we call it simply as geosynchronous, but not geostationary. Therefore, the distinction between geostationary and geosynchronous is that the geostationary is in the equatorial plane having a rotational period same as the rotational period of the Earth whereas geosynchronous could be at any other plane with a period of 24 hours.

Having understood this, it is not necessary that, we should have Geosynchronous and geostationary orbits only for the Earth. It is possible to have the stationary and synchronous orbits for any other planet so long as the planet is rotating at a reasonable speed. Any other heavenly body if it is rotating and the satellite or object is moving at the same rate as the rotation of the given planet, we could have a stationary or synchronous orbit.

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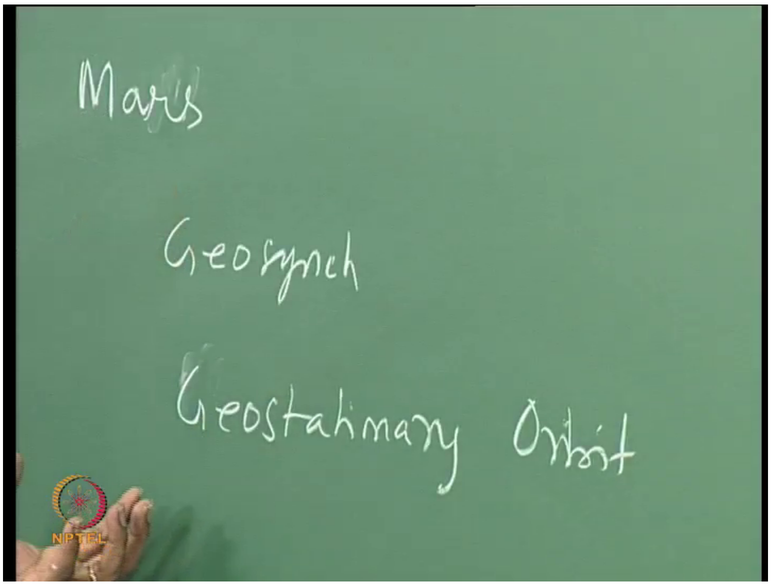

For instance, with planet Mars we could have a synchronous and a stationary orbit.. You know, all what I am I am trying to say is the value of the height and I call this height plus the radius of Earth as radius of geosynchronous orbit which is equal to the radius of the Earth plus the height of the orbit at which, the period is going to be 24 hours. I also qualify by saying 24 hours is the solar day and I must distinguish it from the sidereal day which is slightly smaller, which is what you must be actually using. May be for a person who works on the mission he must take into consideration the sidereal day of 23 hours and may be 56 minutes and 4.1 second.

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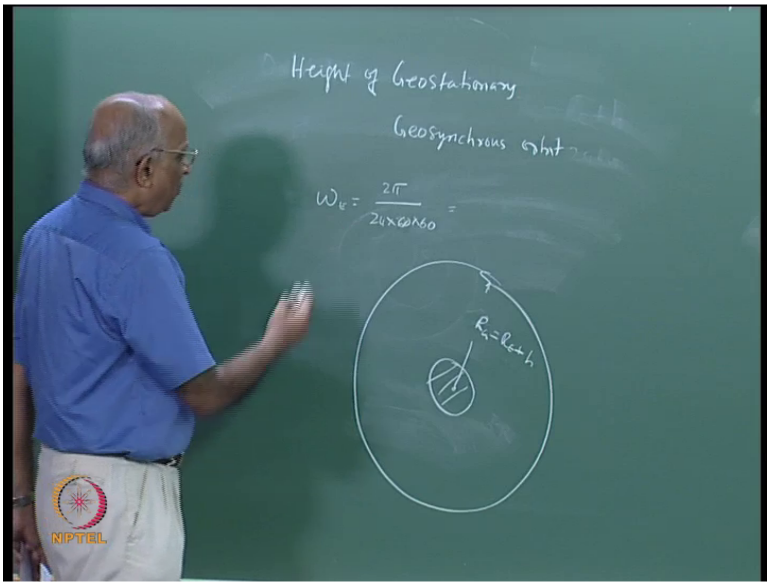

Now, I want to find out the height of the geosynchronous orbit or geostationary orbit? The difference between the two is in the plane of the equator (equatorial plane) for geostationary whereas, geosynchronous could be in any plane. We do the same thing as before viz., rotational period of the Earth is through 360 degrees that is 2π radians in 24 hours that is, 60 into 60 seconds is the angular velocity and what is the angular velocity at a height which will be R_G; that means, I am considering the Earth over here I am considering orbits going round over here may be on the equatorial plane as it were. I am considering the height as hg and the radius if I consider the radius say R_G. R_G is equal to the radius of the earth plus the value of h corresponding to the geostationary orbit. And therefore, we have the expression for ω corresponding to 24 hours for one orbit.

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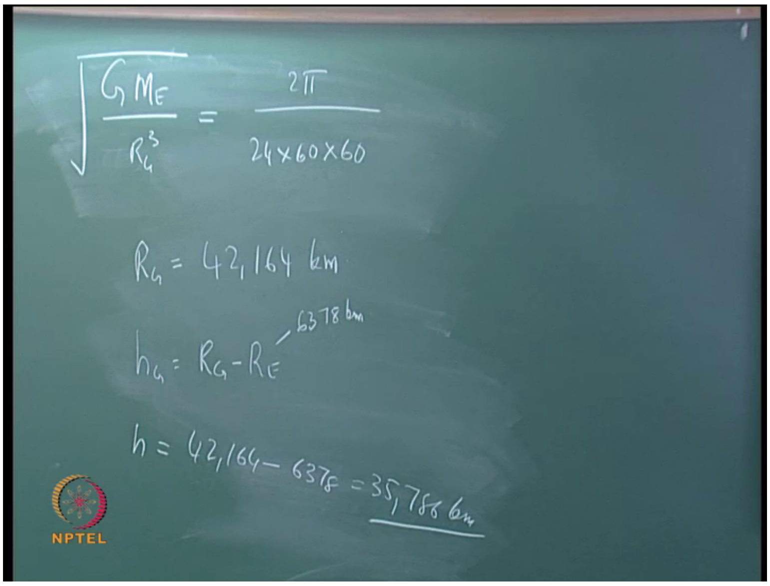

I get $\sqrt{G M_E / R_G^3}$ cube is equal to 2π divided by 24 into 60 into 60. And you know the value of the gravitational constant G is 6.670×10^{-11}. We have the mass of the earth as 5.974×10^{24} kg and therefore, if now I find the value of R_G, it works out to be something like 42164 kilometres. To summarise, we had $2\pi R$ as the distance, we had $m\omega^2 R$ which is equal to the psuedo force and we said it is balanced by the by the gravitational force of attraction due to the Earth and that is how we got this expression. And therefore, we got the angular rotation of the orbit is this much and the angular rotation of the Earth which was equal to it goes to 360 degrees or 2 pi radians in the same time and therefore, we get the distance at which the period of rotation of the object and the period of rotation of the Earth are same as equal to 42164 km.

Now the height of the geosynchronous orbit is therefore, equal to R_G minus R_E (the radius of the Earth). R_E we said is 6378 kilometres. Rather the height at which a space craft appears stationary is equal to 42164 minus 6378 which is equal to 35786 kilometres.

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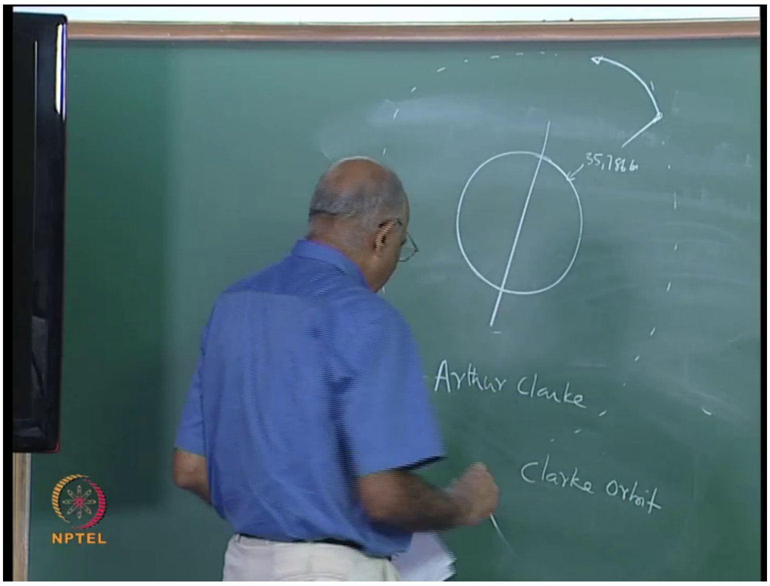

Let us draw the Earth and look at the rotation of the Earth on its axis and the rotation of the spacecraft. The earth is rotating once in 24 hours the angular velocity of rotation is 2π divided by 24 into 60 into 60 radians per second. And may be the period at which if we are at a height above the Earth which is 35800 km, the period of rotation of the spacecraft and the period of rotation of the Earth are the same. If further, the orbit is in the equatorial plane, the spacecraft will appear to be stationary.

This concept was not developed by rocket engineers or people working in space missions, but was told by the famous science fiction author Arthur Clark. Arthur Clark has been writing a lot of science fiction books. In fact, he settled down in Srilanka. He passed away several years ago, He was a very prolific writer. And he proposed this orbit in the year around 1945 and therefore, the geostationary orbit is also referred to as Clark orbit after him. Therefore, let me just repeat this yet again, once again. All what we are telling is that we have the earth rotating east to west once in 24 hours.

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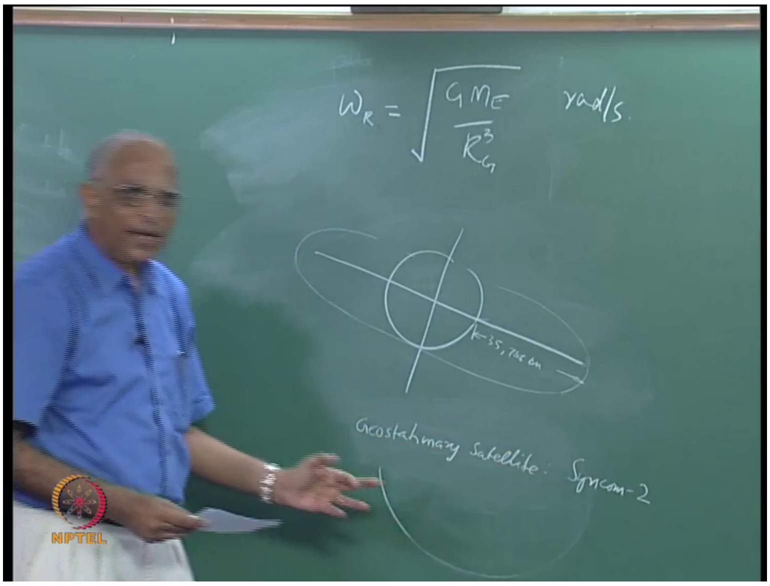

And now if I have a plane which is on the equator, equatorial plane and in this plane we have an orbit - a circular orbit- in which a spacecraft rotates once in 24 hours, the height that we are talking is something like 35786 kilometres. And at this point the rotating object will appears stationary. In fact, this was recognised and it has been the effort to have such orbits for communication satellites. The first geostationary satellite; that was developed was known as Sincom 2. It was launched by US in 1963; I think July 26, 1963. Wee had this geosynchronous satellite there at that height may be looking at the Earth always there and that was the year when Tokyo Olympics were held and it was the first time we had live TV communication from Tokyo and people could watch it live in the US and may be in some other countries also. Therefore, the geostationary satellite is one for which the period of a single rotation is same as the period of the rotation of the earth on its axis and the orbit is in the equatorial plane. Therefore, what we have done so far is we started with the velocity of the orbit. Let us just again put down the equations clearly such that we are very clear.

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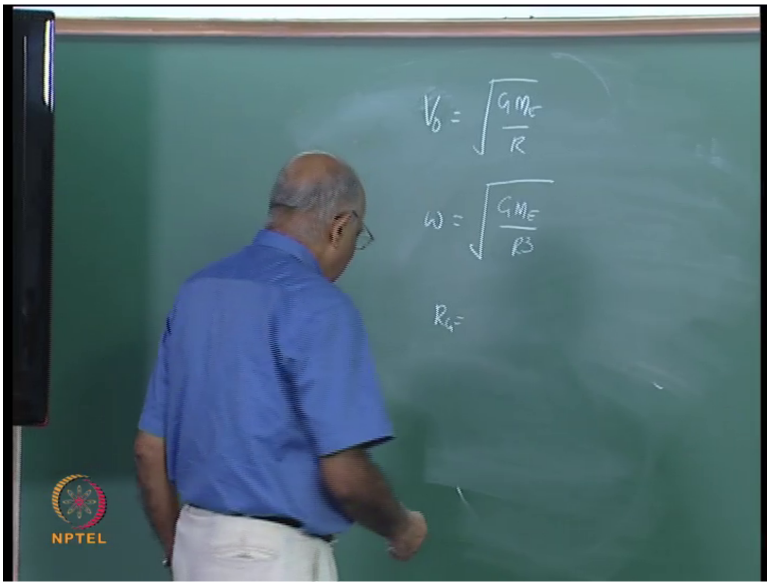

The velocity of the orbit V_0 is equal to $\sqrt{GM_E/R}$. The angular rotation of a satellite in orbit is equal to $\sqrt{GM_E/R^3}$. When we are looking at the period of rotation, when the period of rotation of the Earth on its axis and this value is the same, we get the value of R_G as equal to 42000 or the height above the earth as equal to something like 35800 km. And this was postulated by the science fiction author Arthur Clark; it is also known as the Clark orbit. Now, many countries have their geostationary satellites. In India we have INSAT satellites. The first one was from US viz., Syncom in the year 1963. Having seen the geostationary orbits let us go to some other orbits

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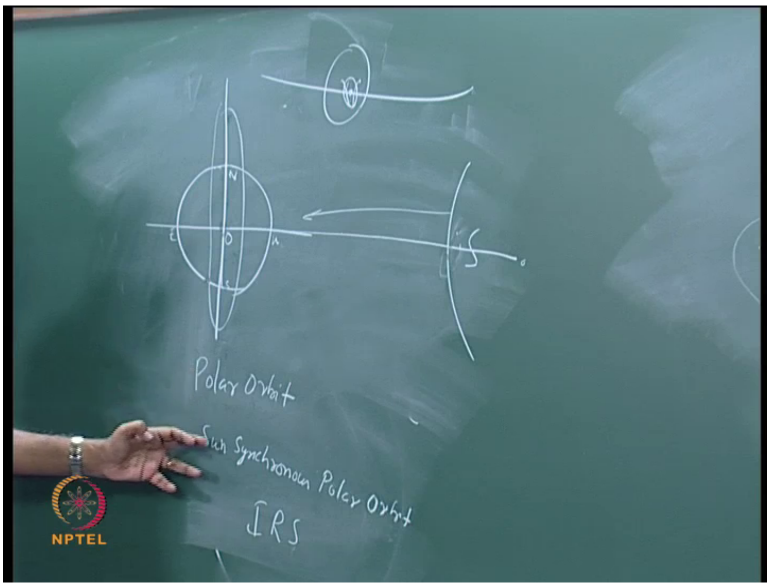

Let us see if we can have a satellite which goes from north to south; that means, this is the Earth, with the north pole, the south pole, the satellite to go round from north to south. And there is one advantage, if a satellite can go round like this in a circular orbit between the poles. We are still talking of circular orbits; the Earth, this is the east and west, earth is rotating like this. If we have the satellite which goes round and round between the poles, as the earth rotates this particular satellite can see all parts of the earth. This orbit going from pole to pole that is between the north pole to south pole, is known as polar orbit. The equations are exactly identical. Supposing, I want to put it at a distance R from the centre of the earth, I calculate the value of ω and I find out the period or I calculate the value of the velocity of the orbit.

Therefore, we have polar orbit. But then, the polar orbit is really not 90^o to the equatorial plane because the Earth is little chubby not really a sphere. Supposing you want the sunrays to come; that means, we are talking of the imaginary line between the centre of the earth to the centre of the sun, this is the Earth to Sun line or axis and we have the orbital plane over here. If the angle between the orbital plane and the imaginary centre line of between the Sun and Earth is kept constant we call the orbit it as sun synchronous polar orbit. Why do we need this? You know if the line joining the centre of the Earth and Sun here and the polar orbit makes the same angle, the intensity of the sunlight which the satellite sees on the Earth will be the same. And therefore, I can compare the reading of what I take today and may be some other day and this is known as sun synchronous polar orbit. And we have in our country, Indian Remote Sensing satellites (IRS) which keep sensing or keep looking at the Earth.

Now let us divert our attention a little bit more and ask ourselves, why all these different orbits? You know if I say geosynchronous orbit that means, I have the Earth as it were traveling from east west and the object or satellite at a height of 35800 km. You know the satellite is always appearing stationary with respect to the Earth if it is in the equatorial plane. If on a clear day, we go out at night and we can see the satellite: we can see INSAT geostationary satellite. And when I look at it, I will see the stars and all that you know we said the stars are in a state of continuous motion. We will see the stars and other heavenly objects drifting across, but this satellite will be dead stationary. That is because both are rotating at the same speed. And because it is stationary may be I have the INSAT satellite pointing may be towards the centre of India may be at Nagpur and it is able to cover the entire Indian sub-continent and it is able to provide communication, telephony may be TV programs and other services.

When I talk in terms of polar orbits and talk in terms of sensing the earth why do we need this? You know, supposing, we say some crop is grown in some part of Andhra Pradesh and I want to find out if the crop is healthy or not. I can think in terms of crop is not healthy then, it withers. The frequency what I see, or the colour what I see is going to be different. I can find out from this particular satellite the nature of the crop and I can warn the people, look here, your crop is not doing well or may be if somebody wants to catch fish; fish always prefer to be in the ocean when the temperatures are little higher may be I monitor the temperature of the ocean and give a message to the to the fisherman to go to the warm waters and catch fish. And that is how you use the remote sensing polar orbit and which is again if it is sun synchronous I will get the same illumination, I will be able to compare the data obtained on different days and we talk therefore, in terms of may be polar orbit and a geostationary orbit – these are the two major orbits.

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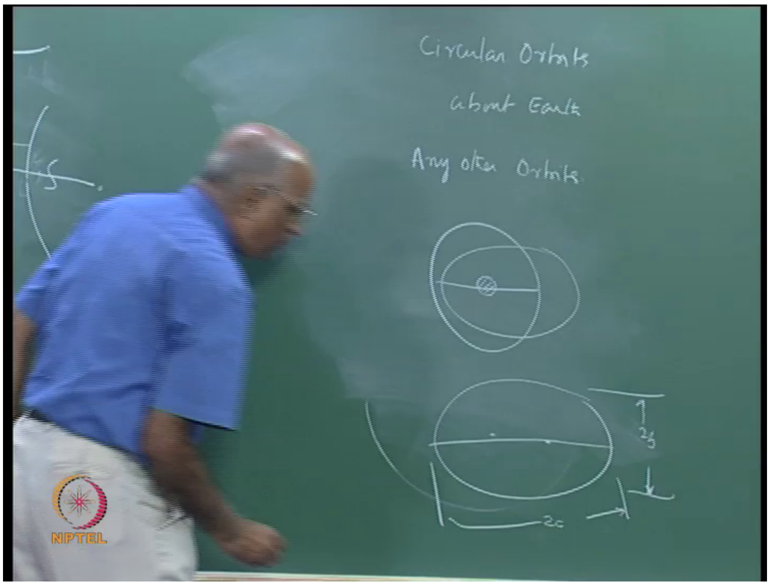

Let us go one step further. We could have a low earth orbit around the Earth we could have something like medium Earth orbit. In other words, I have the Earth may be I go to low altitudes. The low earth orbits are normally used for scientific studies. You all would have heard of scientific studies being conducted and what are the scientific studies about? May be we talk in terms of troposphere, we talk in terms of stratosphere. In this stratosphere we have the wind, you would like to find out the wind velocity. We were also told there are some charged particles which are available in the stratosphere. I would like to measure some of these things and that is the way lower orbit is used. But there is a limit to the height of the lower orbit. Typically, it has to be greater than 200 to 300 kilometres otherwise, the air or atmosphere which is there will cause drag on the satellite.

Therefore, can I assume that we are fairly clear at this point in time relating to circular orbits above the earth or for that matter why should it be Earth alone? If I go and have a stationary satellite about Jupiter, it should be on the same lines. I take the mass of Jupiter, I take the radius of Jupiter and I can find out at what height I must have. Therefore, I think at this point let us ask ourselves, are there any other orbits other than circular orbits? You will recall when we looked at the orbital velocities of the planets around the Sun, we said all orbits are elliptical. What is the difference between a circular orbit and an elliptical orbit? In other words, instead of the earth being here, let us say smaller, if I say its circular, if I say it is elliptical, may be I am talking in terms of elliptical, something like this. In other words, how do we define an ellipse? We define something like a foci 1 and 2 and you have major axis which is equal to 2a and a minor axis which is equal to 2b. Therefore, we have, may be Sun at a focal point. We told that the Sun is at the foci and Earth is rotating around it. Similarly, if I have the Earth here and the satellite is in elliptical orbit, I have Earth as the foci and the satellite going around this elliptical orbit.

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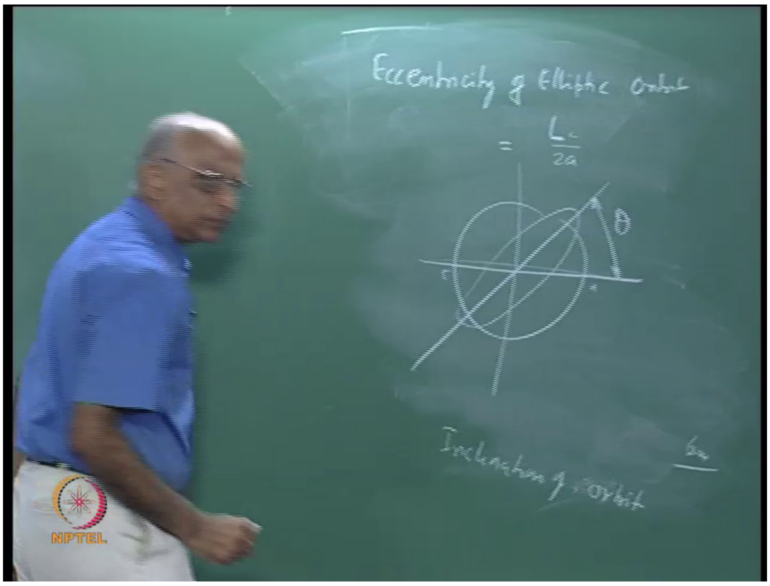

Therefore, you define two terminologies; eccentricity of elliptic orbit, this is the distance between the 2 foci Lc. That means, this is the Lc divided by the major axis 2a. And you find for a circular orbit, Lc is 0 because I have a centre here and the eccentricity becomes equal to 0 for a circular orbit.

There is another point which we must keep in mind and that is the orbit need not always be east to west, or need not always be polar; we could have in between. May be an orbit could be like this; may be at an inclination this is the orbital plane, this is the east west, I say this is the inclination θ; that means, we say the angle between the orbital plane and the equatorial plane is what we call the inclination of the orbit.

Let me take you through an example:

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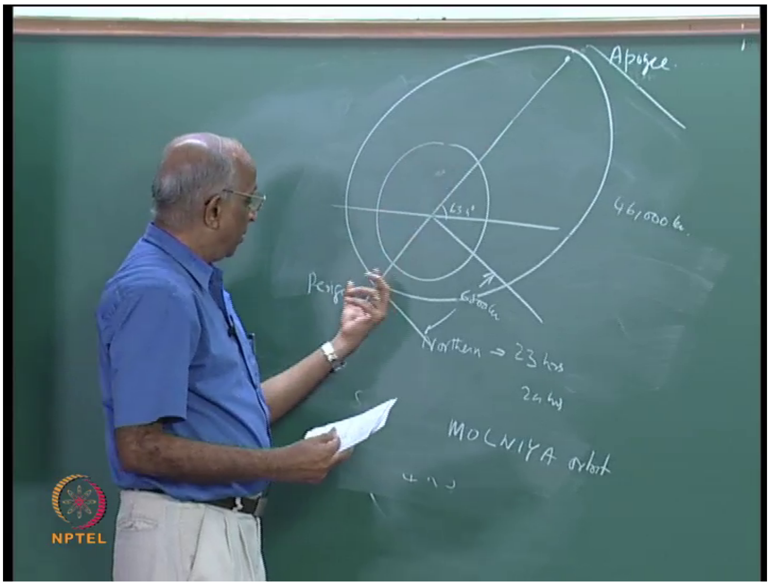

We take a country like Russia. And this particular country is in the northern hemisphere and if I were to have something like an equatorial orbit; they are not able to see the satellite distinctly over a sufficient time. And supposing if I have something like an elliptical orbit at an angle of something like may be like 63.4 degrees. I will come back to this particular inclination a little later. And then I have an orbit which has a longer travel distance here at the northern end and a much smaller distance here at the southern end that is, the orbit is something like elliptical orbit. I find that the space craft or the object which is rotating, spends much more time in the northern latitude and correspondingly smaller time in the southern latitude. And what was the second law of Joahanes Kepler? Equal area swept in the same time. And therefore, it spends much more time in the northern hemisphere; the satellite can spend something like 23 hours out of 24 hours. And this particular type of highly elliptical orbits is what the Russians call as Molniya orbit.

It has an inclination of something like 63.4^{o}. The distance to the top most point from the centre is 46000 kilometres and the distance from the centre to the nearest point on the orbit is something like only 6800. That is, you see the distance between the centre of the Earth and the furthest point from the Earth which is over here, and this is the furthest point away from the Earth over here. The distance between the centre of Earth and the nearest point is what we call as perigee; that means, this is the point which is nearest to the Earth. The point furthest away from the Earth is known as apogee.

Therefore, in an elliptical orbit we also define something known as perigee and apogee. Perigee is something which is the orbital distance nearest to the centre of the Earth and apogee is the farthest distance from the Earth. These are for the case of orbits which are elliptical.

To summarize; we have to put a psuedo force to balance the attractive or universal law for attraction of the object to the centre of the planet and then figure out the radius of the orbit. But something which we have really not done is about orbital velocity. What is this orbital velocity?

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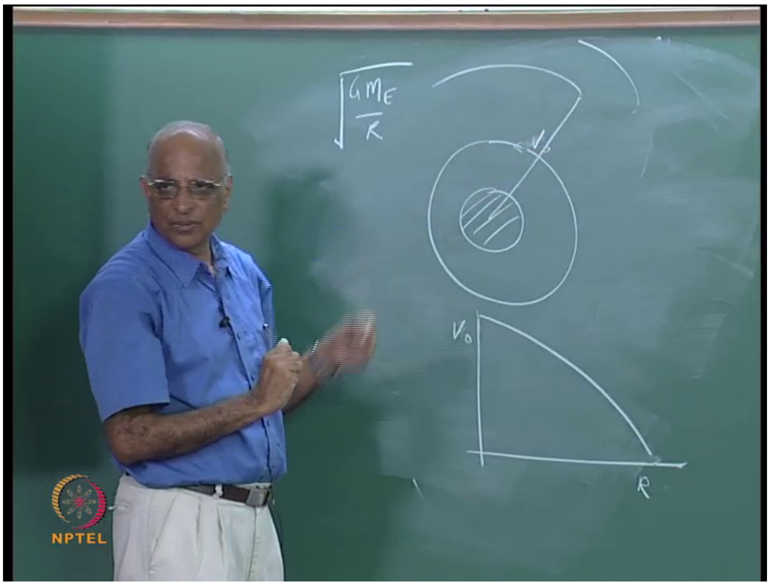

We found that if we plotted orbital velocity V_0 (mind you we did it in the last class) and we had the orbital velocity V_0 in metres per second equal to $\sqrt{GM_E/R}$. We find, if the radius is infinite the orbital velocity is 0. If the distance from the centre of the Earth increases, the orbital velocity keeps falling and reaches 0 as the distance goes to infinity. Why should the velocity of the orbit goes to 0 at infinity? How would you explain? Is there any suggestion ? Why should it be 0? Any thinking on it.?

We did tell that, supposing the spacecraft were to leave the Earth i.e., escape from the Earth; well it has to go to infinity. Therefore, when I talk of infinity you have no attractive force due to the Earth and therefore, it is not in orbit anymore and therefore, you find that the orbital velocity keeps decreasing and ultimately becomes zero.

Therefore, the orbital velocity V_0 keeps decreasing as I increase the height because as I increase the height above the Earth the attraction by the Earth keeps decreasing. Now the next question is, from the surface of the earth you have to go to this height which we have not yet considered. That means, I require a certain velocity or potential energy to be given to reach this point or I have to do some work in taking a mass from the surface of the earth to go to the particular orbit at a distance let us say R or I have to increase the height from the surface of the earth by h. How do I include this? See in other words, so far I have talked only of the orbital velocity V_0: we have not considered how much velocity is required to start from the surface of the Earth and go to this height and then provide the necessary orbital velocity.

Therefore, let us now find out what is the total velocity required for orbiting viz., taking the space craft from the surface of the Earth or some other planet to the particular radius of the orbit and then injecting it with the given orbital velocity. How do I do it? If we have understood what is discussed so far, we must be able to do it very readily.

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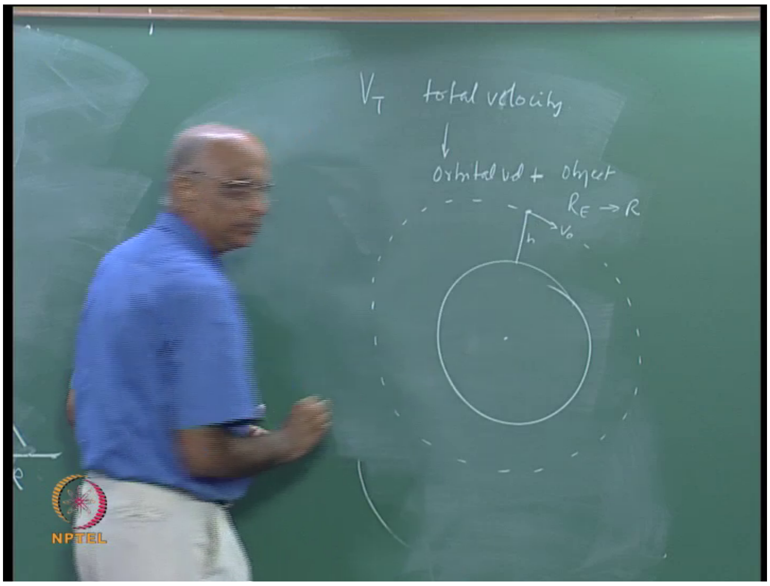

Let us try to do it. We want the total velocity which includes the orbital velocity plus the velocity required to take the object from the surface of the Earth let us say at R_E to the particular orbit at R. How do I determine it? I have the Earth here; I want to take the spacecraft above the earth to a height h. And I have to insert into orbit for which I have give an orbital velocity and then it will continuously rotate. Therefore, what is the total velocity which I must give? How do I determine it using the same set of equations?

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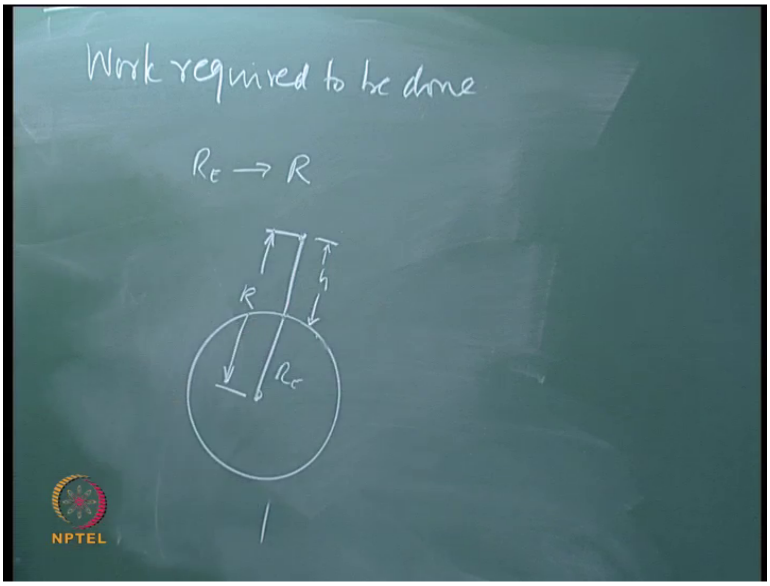

We talk in terms of the universal law of gravitational forces. We ask ourselves what is the work required to be done to increase the height of the spacecraft or to take the spacecraft or the object from R_E to R. Let us consider the Earth and what I am asking is I have the earth here, I have R_E here, I want to take the spacecraft of mass m from R_E to R_E + height h. Let us say this final distance is R and this height is h how do I do this? What is the work required to be done. What is the potential energy required to take this spacecraft from the surface of the earth to a height h. How do I do it? Any suggestions on how will you solve this problem? I want the work which is required and work is equal to force into distance. What is the force? Yes. The force to be overcome is the gravitational force of attraction due to the Earth.

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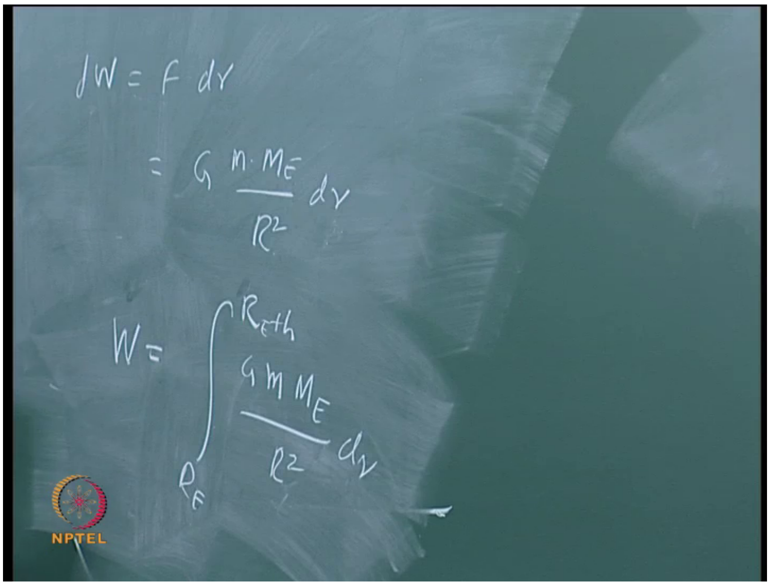

Therefore, let us put it together. We find that the force is equal to G × mass of the earth M_E × mass of the body m divided by at any radius R^2. What is the value of the work done, when it moves through a small distance let us say dr. We assume that it goes from a height R to a height R plus dr. What is the small work which is required to be done? We call this small work as dW and is equal to the force into the small displacement dr and therefore, dW must be equal to $GmM_E/R^2 \times dr$. What is the work required as I go from may be the surface of the earth having radius R_E to a height R? I just have to integrate out or the total work required as equal to integral from the surface of the earth to the radius R. But mind you; let me qualify again that we are illustrating with respect to the earth. I could have all these things for any planet or body. Supposing somebody wants to go from the surface of the moon to some height. I just have to take the mass of the moon the radius of the moon plus the particular height is what is to be considered. Therefore, I have $GmM_E/R^2 \times dr$ and integrate it from the value R_E to R as I go through the height h. And this work must be equal to the potential energy available at the height h because you have increasing the height by h, Therefore, I have higher value of potential energy. But then, we also know that we need to provide orbital velocity V_0; that means, I have to give some kinetic energy and what is the total energy of an orbiting satellite at a height h? I have, this potential energy plus I have the kinetic energy.

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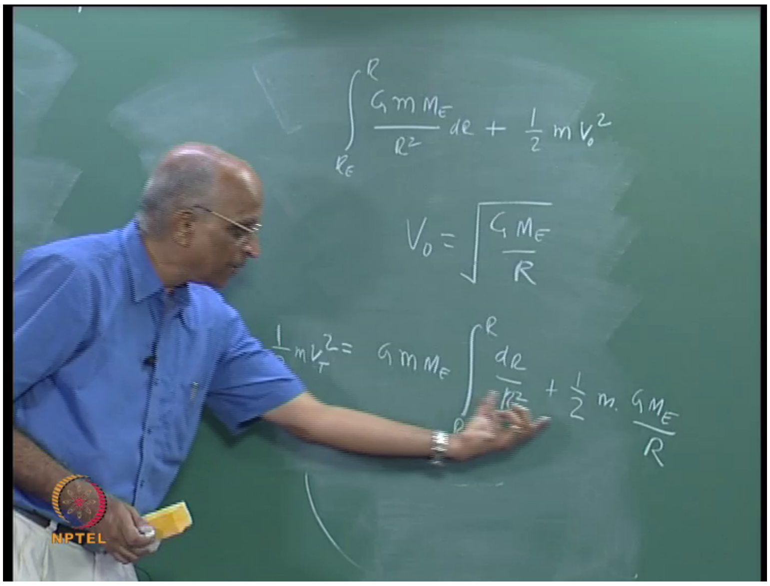

And therefore, I can say that total energy of a rotating spacecraft in orbit must be equal to integral R_E to R of GmM_E/R^2 ×d r plus the kinetic energy of a rotating body equal to ½ mV_0^2 , where V_0 is the orbital velocity. And what is V_0 square? We have derived it as $\sqrt{GM_E/R}$. And therefore, we can say that the total energy in orbit is therefore equal to: let us integrate this out, mass of the object is constant, mass of the earth is constant therefore, we have G gravitational constant m M_E into we have integral R_E to the value of R of dr by R^2 plus I have ½ m into V_0^2 viz., GM_E by the particular orbital radius R. Is it all right?

And if I get started by giving a kinetic energy that is I supply some kinetic energy to this satellite and I call it as the total velocity V_T: this must be equal to ½ m into V_T square, that is the total velocity I give to the spacecraft. This total kinetic energy must equal potential energy required to increase the height plus the kinetic energy due to orbit. And therefore, the total velocity what I give to the spacecraft can be derived. Let us do it. m cancels out over here.

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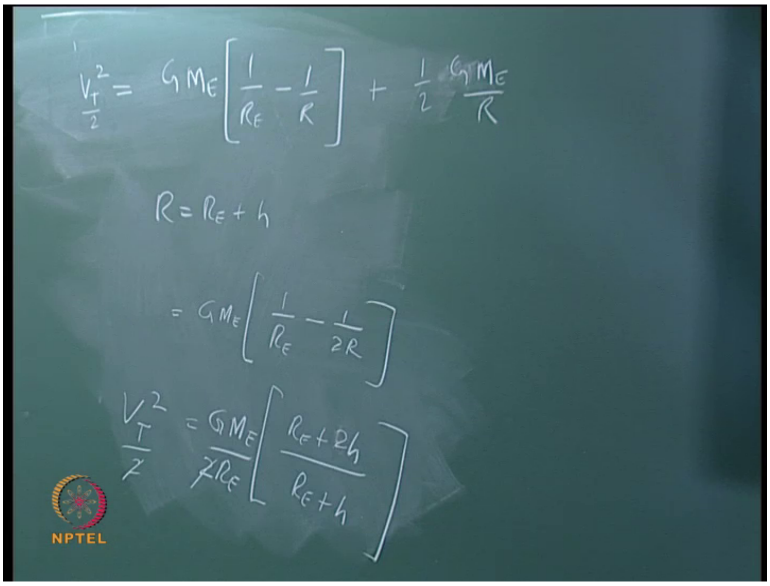

And therefore, we get V_T square is equal to lets us get this here; half V_T square by 2 is equal to GM_E into 1 by R_E minus 1 by R plus half of GM_E by R. Therefore, this become with R equal to R_E + h and add the two terms to give one over R_E minus 1 over two R. We substitute R as equal to the radius of the earth R_E + height h. Now, I have taking $2R_E$ outside, the term within brackets as 1 minus R_E by 2R. With R equal to R_E plus h, we get R_E plus h in the denominator and R_E plus 2 h in the numerator. Therefore the total velocity V_T square divided by 2, 2 and 2 gets cancelled and therefore, the total velocity $V_T^2 = GM_E/R_E \times [(R_E + 2h)/(R_E + h))$.

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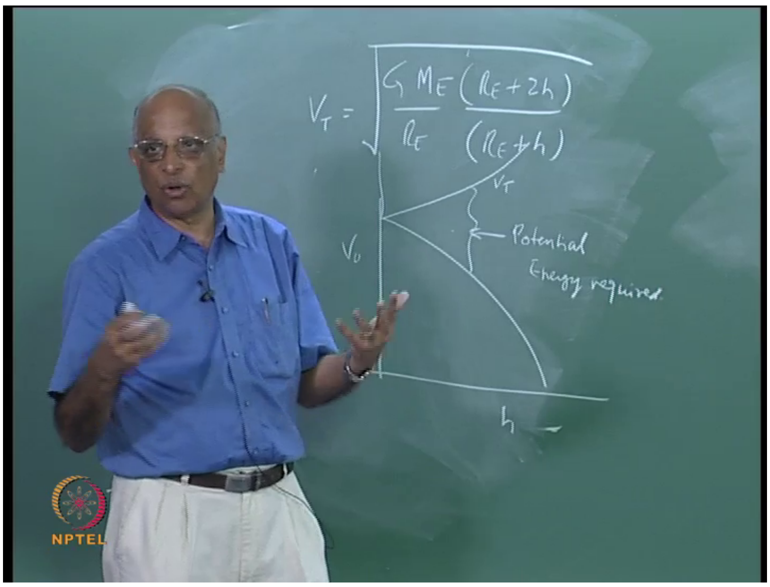

When the spacecraft is orbiting at a height h above, the total velocity required to be provided is given by V_T equal to $GM_E/R_E \times (R_E +2\ h) \div (R_E + h)$. Now, what is it we find? While the orbital velocity keeps decreasing as the height above the earth increases, we find the total velocity is added by 2h in the numerator and added by h in the denominator.

The numerator being multiplied by 2h, implies that the value in the numerator increases faster as compared to the denominator as h increases and the total velocity therefore increases as h increases. That means, as I go higher and higher I need to give higher values of total velocities to the spacecraft which is going to be much higher than the orbital velocity and this difference is what constitutes the potential energy required. Is it alright?

In the next class we will continue with some problems involving the potential energy and kinetic energies and solve for different orbits.