Rocket Propulsion

Prof. K. Ramamurthi

Department of Mechanical Engineering

Indian Institute of Technology, Madras

Lecture No. # 05

Theory of Rocket Propulsion

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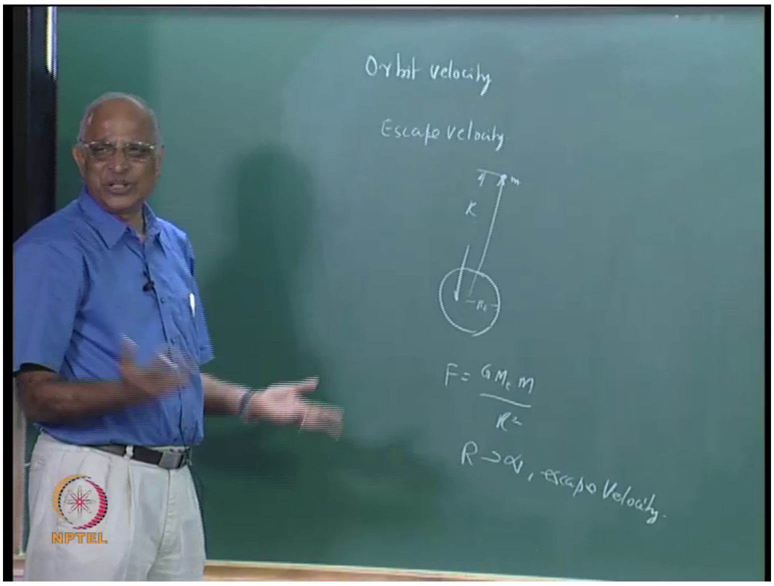

In the class today, we will look at Orbit velocities and illustrate it with some examples. Since you mention something about Escape Velocities, we will see that it is the velocity required to escape from let say the Earth or from some other planet. Let us discuss escape velocities first. I have the Earth here and I want to escape from this Earth. That means, I want the force with which the Earth is attracting me to vanish; that means I escape. In other words, the gravitational force is given by GMM_E/R^2 in which M_E is the mass of Earth, m is the mass of the body and is divided by R square; This means that R must become very very large or rather infinity for which this force would vanish.

In other words, I am looking for a body to escape from the surface of Earth i.e., radius R_E; I want to go to infinity to escape from the Earth and I have to determine the corresponding velocities; and that velocity required to be provided and becomes the Escape velocity. Therefore, escape velocity is the velocity required such that I escape from the attractive force or gravitational field of the Earth. Therefore, what must be the value of R to escape? I want the force to be zero. I want to escape from the attraction; therefore, R must be infinity in order to have the Escape velocity. How do I do this problem?

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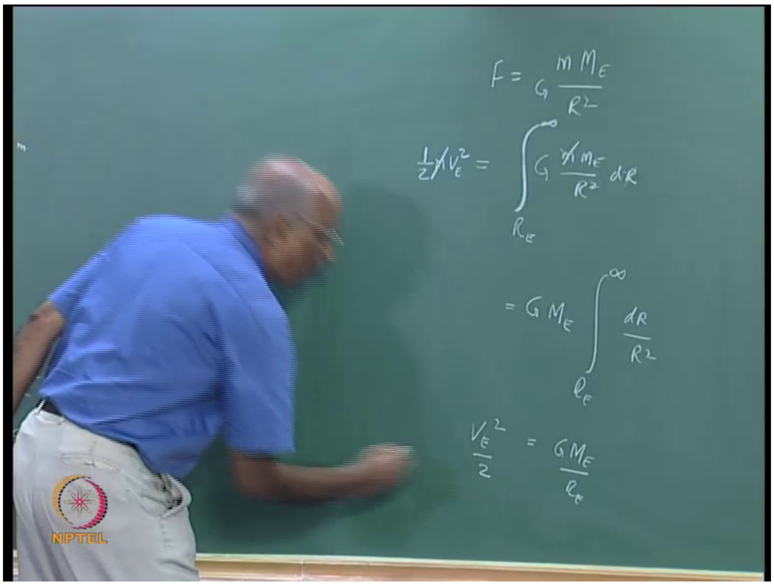

I am looking at the force, which is equal to the mass of the body m, mass of the earth M_E divided by R square into G, and what is the work which I must do to take it from to infinity. The work, which I must do when I travel a small distance dR for a radius R is equal to GM $M_E/R^2 \times dR$, is the small amount of work when the distance traveled is dR. And now I want to escape from let us say from the surface of the earth having radius R_E; I want to go to infinity and therefore, this must be the work that is done. And how do I do this work? I give the kinetic energy to the body therefore, I give ½ m ×V escape square; and it is this velocity which is the escape velocity.

And let us find out what this is? We again the find that mass of the body cancels out; and when we say GM_E/ R^2 square, which is again equal to GM_E and here I write the value of distance going from R_E to infinity. You see this small increment in the radius dR. Integrating, we get minus of one over R_E and this become equal to GM_E by R_E. And what is the value I get for $(V \text{ escape})^2$ divided by 2?

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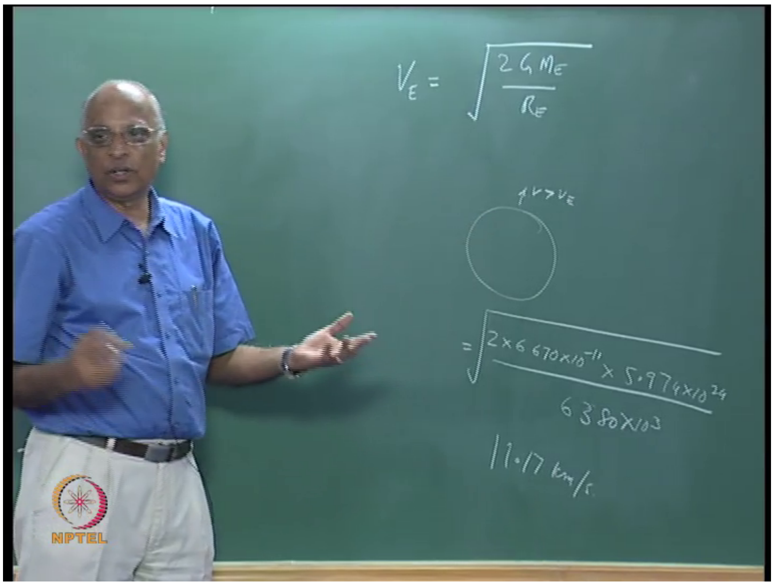

We get the Escape velocity V_E^2 is equal to 2 G M_E by the value of the radius of the Earth R_E. In fact it is very interesting to note that when the Earth was born, we had lot of hydrogen, which was available around the earth. Hydrogen is the very light gas and when the gas is light we shall see later on that a lighter gas provides much higher velocities for a given value of energy when we get into theory of Rocket propulsion a little later. The velocity of hydrogen is greater when it is hot and the Earth was hot when it was formed. Since the hot hydrogen moves at high velocities, which is greater than the escape velocity, we lost the hydrogen. Anything which travels at a velocity greater than the escape velocity escapes from the surface of the earth or more correctly from the gravitational field of the Earth.

Let us calculate the value of this Escape Velocity on the surface of Earth. Escape velocity from the surface of earth is equal to $\sqrt{2}$ ×gravitational constant 6.670×10^{-11} Newton meter square by kilogram square × mass of the earth 5.974×10^{24} kilogram ÷ the radius of the earth 6380 into 10^3 meters. And this is the Escape velocity from the surface of the earth. You calculate it to be something like 11.17 kilometers per second. Supposing you want to go to the moon you have to escape from the Earth's gravitational field; that means, you need to have the Escape velocity to get out of the Earth's gravitational field, then we get into the gravitational field of the moon. And supposing I want to come back, I need to be provided with the Escape velocity to leave the gravitational field of the moon and enter the gravitational field of Earth.

And this is how we work out the total orbital requirements or the total velocity requirements to put a spacecraft or any body to go to different planets.

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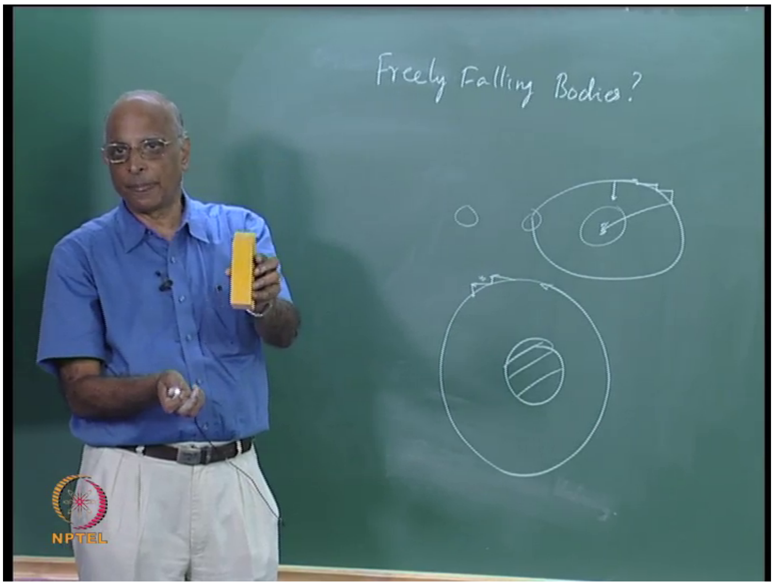

Let me take one or two small examples. We will do these one or two small problems such that we are very clear. However, before doing that, I also want to tell something about freely falling bodies. What do we mean by freely falling bodies? What did we say earlier? We have the eight planets which are going around the Sun in elliptical orbits. We told that all these planets are just falling freely on to the surface of the Sun, falling towards the Sun, but why is it not just crashing; because by the time its falls it goes through some distance corresponding to the orbital velocity and again it goes through some distance as it falls, it is always falling toward the center of the Sun but never reaches the sun. Therefore, all planets and all of us are freely falling on to the Sun as it is.

And how does a spacecraft orbit the Earth? I say this is the Earth; let us for ease consider something like a circular orbit. As the spacecraft orbits, it is going at a constant linear speed; however, it tends to fall towards the center of the Earth as it goes horizontally because of the orbital velocity. It therefore goes horizontally as shown but it falls in the process. Therefore, all the bodies which are in orbit are freely falling bodies. It is as good as I drop a stone and it falls freely. So, also all the bodies, which are in orbit are freely falling objects, and what is the function or what is the thing, which we understood by freely falling bodies.

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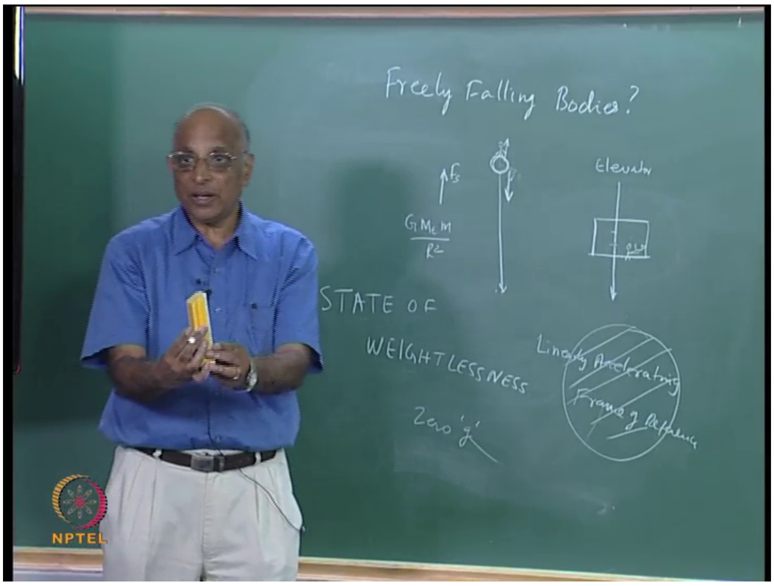

Let us take two examples: in one a stone falls freely to the surface of Earth; it falls freely, but let us assume that there is no resistance due to air, because we are talking of space. Therefore, it just keeps falling freely. I also take an example of an elevator or lift, and let us say the elevator falls freely. Let us take an example, I am in the elevator which is falling freely, I am going down, and I am holding in my hand a cup of tea; we all would have noted this. Now, what happens to the teacup which I am holding? What sensation do I have?

This stone is freely falling. The frame of reference of the stone is not in an inertial frame of reference, because it is picking up acceleration. Therefore, it is something like a linearly accelerating frame of reference; It is different from the rotational frame of reference which we considered while deriving the orbital velocities.

Now if I am sitting on the stone or if I am standing the lift with a tea cup in my hand what is the force which I will experience or which the tea cup will experience? Can somebody tell me? I have the Earth attracting me and therefore I have to move. But I am on the stone or I am in the elevator and I cannot move with respect to the stone or with respect to the elevator. And therefore, I have to correct my motion if I were to refer my motion with respect to the stone or with reference to the elevator, because it is not in an inertial frame of reference. I have to put some corrective force. May be the stone is freely falling; I have to put some force here because I am not moving at all with respect to the stone. Therefore, I have to put a pseudo force opposite to the attractive force of the Earth to describe my state of motion correctly with respect to the stone. Attractive force of the earth is GM_Em by R^2, and I have to put this pseudo force here, which is equal to the above force. With this correction I do not move in the given frame of reference.

And the moment I put a pseudo force over here my motion is taken care of I am able to describe my motion correctively because I am not moving with respect to the stone. I am not moving with respect to this lift which is correct; but when I put a force equal and opposite to the force with which I am getting attracted, the net force on me become zero. And when the force on me becomes zero, I am weightless, or I am in a state of weightlessness. Why is it? It is not that I have lost my mass. I have my mass, but to be able to correctively define my motion with respect to the stone or elevator , because I am dropping along with them, I am sitting on the stone; the stone is coming down, but I have to correct my motion because I am not moving with respect to the frame of reference of this stone. Therefore, I need to put the pseudo force, vertical and opposite to the gravitational force of the Earth so that I am not moving with respect to the stone on which I am sitting.

Therefore, the moment I put the pseudo force, I do not have any force or weight, I am in a state of weightlessness when I consider my reference to be the stone or the elevator. Some people call it as zero "g"; actually it is not zero g; g is the gravitational field; the gravitational field is always there, but a body in orbit which is also a freely falling body, is in a state of weightlessness.

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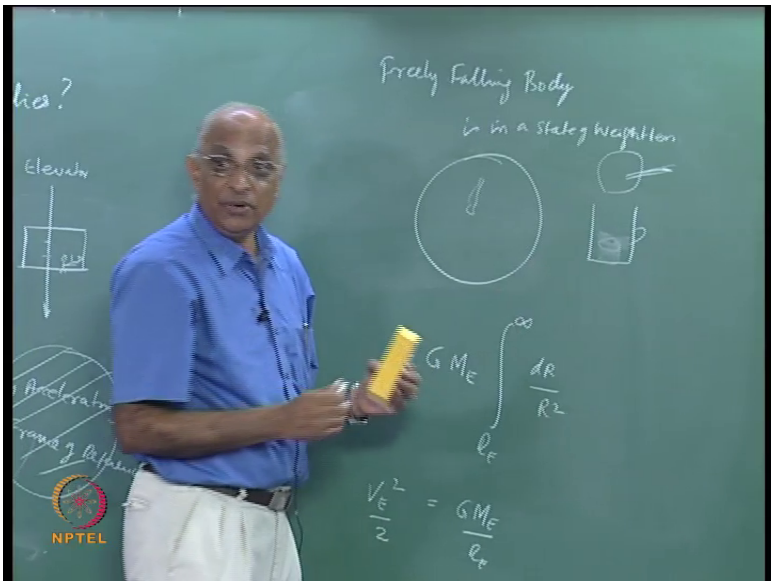

That is a freely falling body is in a state of weightlessness. It does not seem to have any weight. And so also if in the case of the lift or elevator coming down I hold a tea cup and come down, you will feel it is not heavy at all, it is as if it is very light in the confinement of the elevator as it is descending. I have to correct my motion using a pseudo force, and that is why whenever you see the picture of astronauts in space, you see that they are all floating around with respect to the space capsule they are in. It is because you have to correct their motion with respect to the space capsule by a pseudo force which makes them appear weightless. And you know to be able to drink a cup of water while I am orbiting up in space, supposing I were to go there and I am very thirsty, is going to be difficult. The water does not settle to the bottom of the cup or tumbler.

Therefore, it will be freely floating; that means, I have water, but it will just be floating, therefore what has to be done is to arrest it somewhere, then may be put a straw and suck it through then only even drinking little bit of water in space while orbiting is possible. This is what we call as a state of weightlessness or some people call it as zero “g”.

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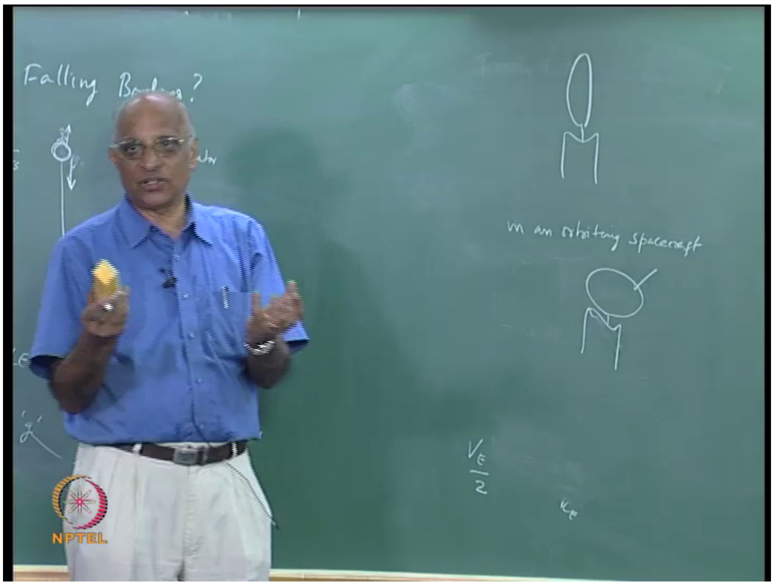

But let us not confuse: g is always there; in fact since, many of you are from mechanical engineering and working in combustion, if you look at a candle flame; the flame rises because of convection and it is like this. There are some experiments done in a space capsule while in orbit. What does the flame look in an environment of weightlessness? Any guesses on what should be the shape of the flame?

The rising flame is because the gas becomes lighter on being heated and the lighter gas rises and you have the candle flame like this on the ground. If I were to look at it in a spacecraft in which the objects are in a state of weightlessness, there is nothing to rise up and the candle flame must be a pure sphere. It does not have any light weight or strong weights rising up or coming down; it just is a perfect sphere and that is what my equations, in the absence of the gravity term, give. I can solve for it in a spherical frame of reference.

I can match my solution for the diffusion and chemical reaction equations and for the energy release equation in the state of weightlessness by neglecting the gravitational field. I am able to find out what is the mechanism of diffusion flame which is different from an experiment on the ground where gravity influences the shape and properties. And, may be later on, I will show some pictures to show how may be an astronaut drink water in space, how does a plant and how does a flower look like when grown in an orbiting spacecraft up in space or how it will be different from that on Earth.

Therefore, what is it we have done so far? Let us quickly summarize and do one or two small problems, which will make sure that we have understood this subject.

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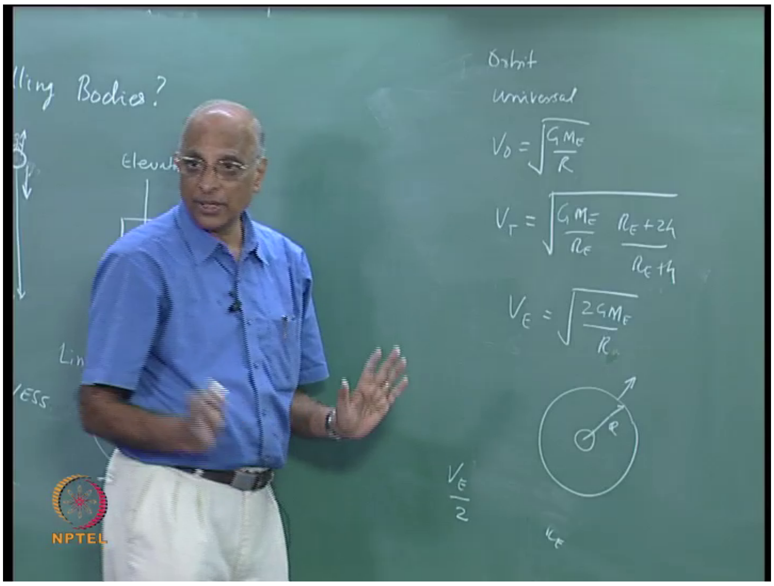

We talked in terms of orbits of the different planet in our solar system. We went ahead, we formulated the universal law for gravitation as determine by Newton. We told that all planets are freely falling just as an apple is falling from the tree to the ground. A heavier body attracts a lighter body and you have the universal gravitational law. We used the gravitational law and we found out the orbit velocity V_0 is equal to √the square root of GM_E/ R, where R is the orbit radius. We also found out that the total velocity required to orbit for a circular orbit is equal to $\sqrt{GM_E/R_E} \times (R_E + 2\,h) \div (R_E + h)$.

We also talked of geo-synchronous orbit, polar orbit which is used for remote sensing, so that I can see entire Earth as the spacecraft rotates; for communication and weather prediction may be geo-synchronous is better suited. I can also have low Earth orbit around the Earth and we found out the total velocity requirements for orbits at different heights. We also talked in terms of the escape velocity, we said it is equal to $\sqrt{2GM_E/R_E}$ from the surface of the Earth. If we have a spacecraft which is orbiting at its distance R from the center of the earth and I want to push it to infinity then in this case R has to be substituted in place of R_E; and this is all what we have done so far.

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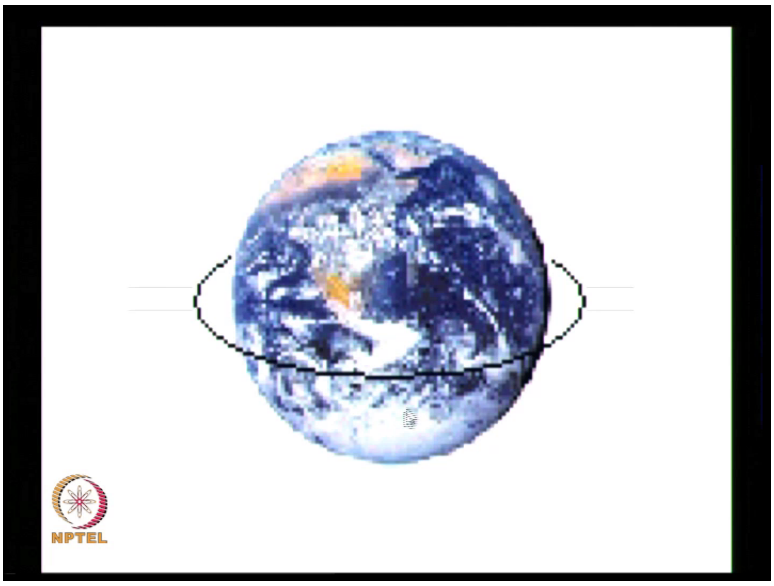

Let us first take a look at this power point presentation. Here, we see the low Earth orbit i.e., a body going around the Earth as it circles around it.

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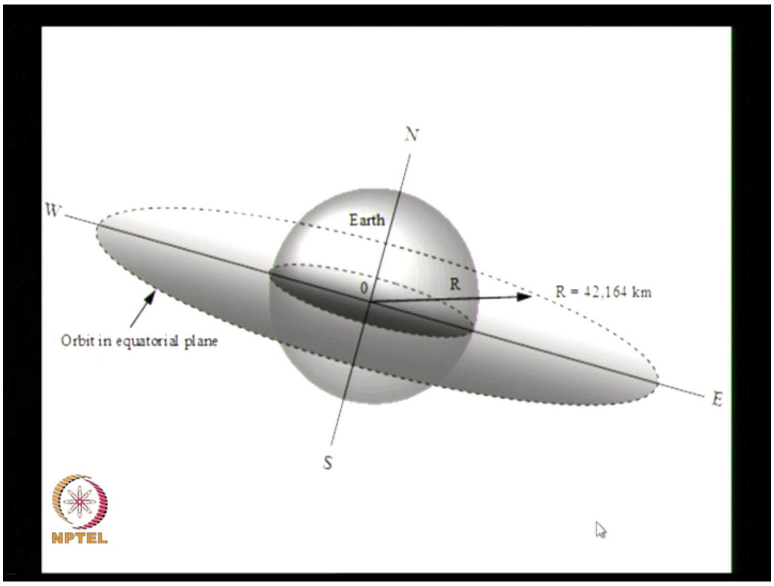

This shows the geostationary orbit, you have the orbit in the equatorial plane of the Earth i.e. along the East to West along the equator and you find that the spacecraft is going around at a radius of 42,164 kilometers; this is the Earth and this dotted line is the orbit. You subtract the radius of the earth from the radius of the orbit and that is the height of the stationary orbit.

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Geostationary, this again shows the satellite in equatorial plane going around east to west.

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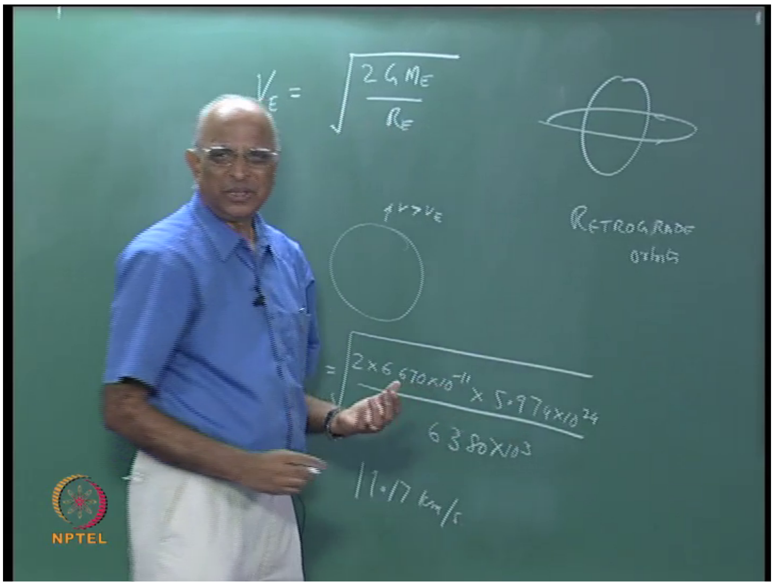

And supposing we have an orbit let us say instead of going from east to west, I go from west to east. I am going against the rotation of the earth. The orbit is no longer synchronous and such orbits are known as retrograde orbits. It is not useful at all because why should I go against the rotation and not get any benefit at all.

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Let us go to the next one; this show Syncom 2, the first geostationary satellite, which was launched by US. Syncom 1 was not successful and the second one was successful. The launch was on 26 July 1963, and Syncom 2 was used to relay the Tokyo Olympics.

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This shows the polar orbit; the orbit is in north – south direction. The inclination to the equatorial plane is not exactly 90^o , but little more than 90^o.

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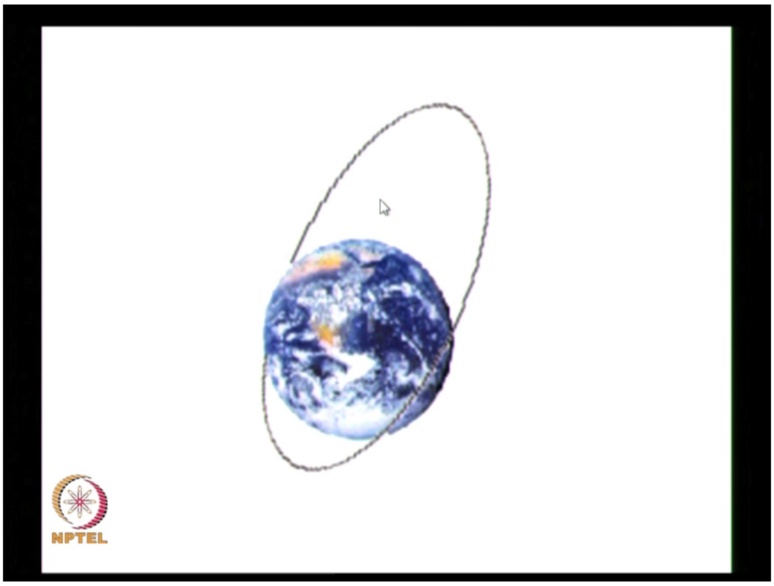

This shows the highly elliptical orbits; this is the perigee, this is the apogee. This distance we set for the apogee is something like 42000 km; the perigee is of the order of 6000 km, and this is an elliptical orbit.

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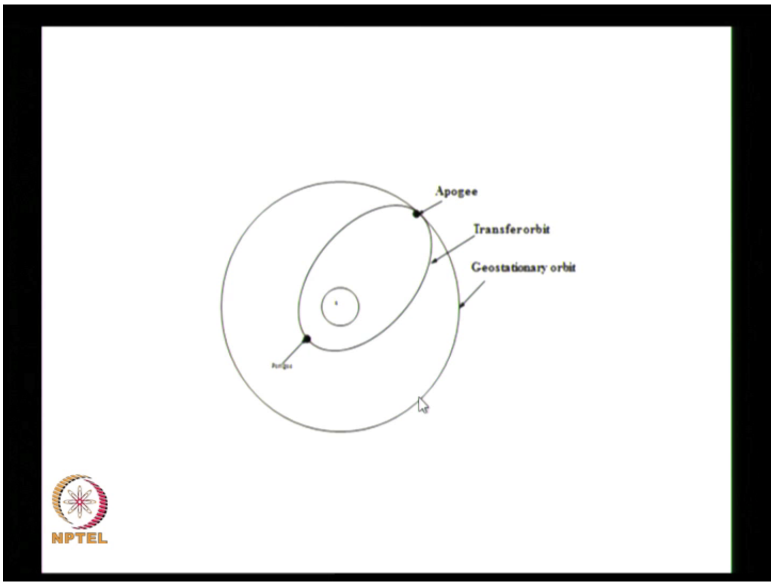

And now that you know about orbits and we want to launch satellites in a given orbit, say into geostationary orbit, we first take off from the ground, we put the spacecraft in an elliptical orbit; we put the apogee equal to something like 40,164 km highly elliptical orbit and then it comes to the apogee; we make sure we fire a rocket and circularize it and make sure it goes along the geostationary orbit. We call the initial elliptical orbit as a transfer orbit.

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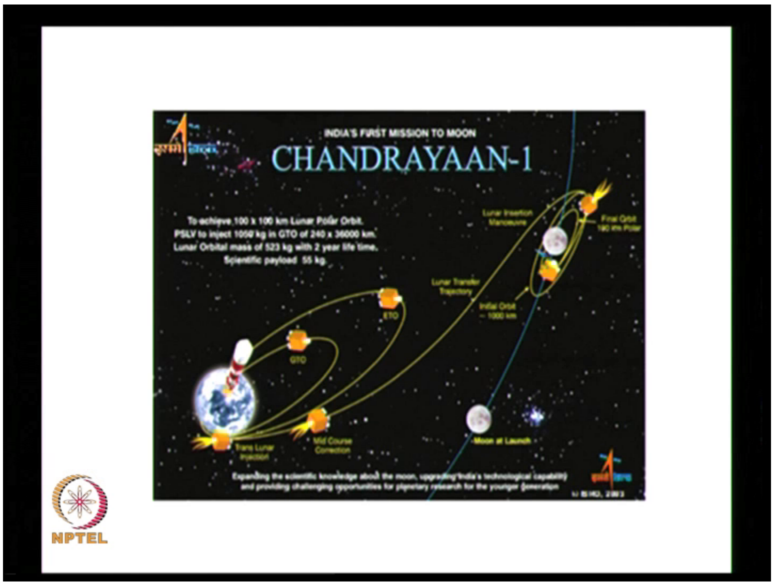

Suppose, we are interested in a mission to the moon; something like Chandrayaan-1 of ISRO which orbited around the moon. From the Earth, we keep going through a series of elliptical orbits, we escape from the Earth i.e., escape the gravitational field of the Earth, and get inserted on to the moon's gravitational field and thereafter orbit around the moon. If we want to come back to Earth from the moon, we again come out of the moon's gravity, we escape from the moon and reenter Earth's gravitational field.

In the previous slide, we had the transfer orbit; that means we do not take the satellite directly to the geostationary orbit. But to be able to go to this, we first put it in a transfer orbit, and then when the apogee is the radius of the geosynchronous orbit, we circularize it. Well these are all about the different orbits and it is about time for us to do a problem or two.

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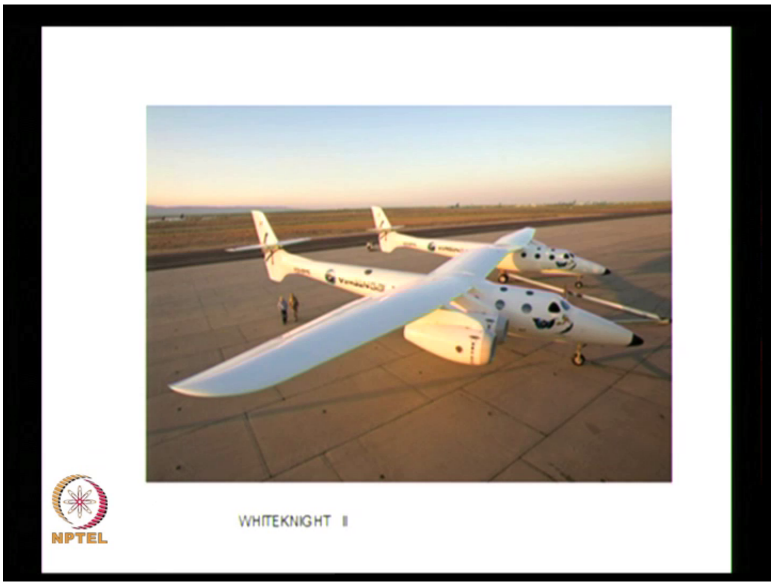

And I take a problem, which is something related to a recent rocket. An innovator by name Sir Richard Branson wants to ferry tourists to space.

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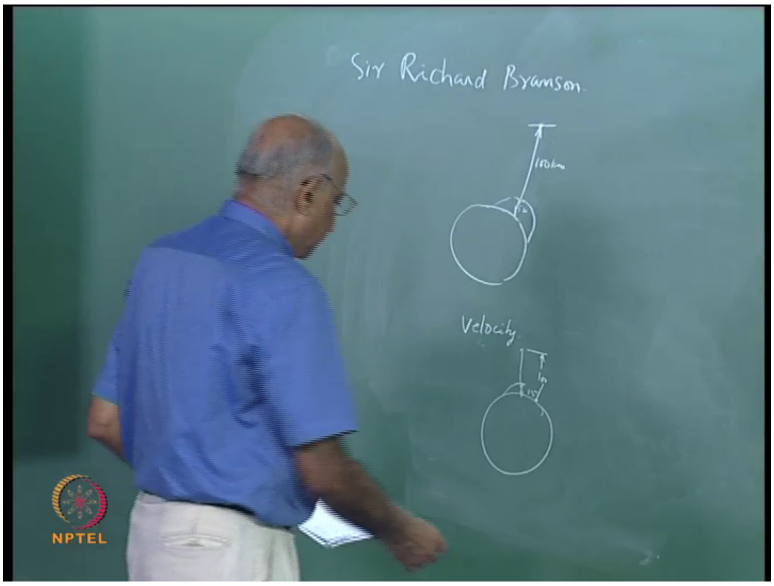

He take off from the surface of the Earth into deep space so that the tourists could go above the Earth and see the Earth as it where from space; how it looks and it seems to be very fascinating. Therefore, what he did is that he starts with an aircraft from the ground; this aircraft is known as White Knight. It carries a rocket and a space capsule. The aircraft takes off from the surface of the earth goes to a height to a something like 15 kilometers, and it returns. At 15 kilometers, you fire a rocket, separate it from the aircraft and it takes you to a distance of something like a 100 kilometers.

Therefore, the problem that I pose to you is for the rocket to traverse from 15 kilometer above the surface of the earth to 100 kilometers, what is the value of velocity to be provided by this rocket? Because, the velocity required from the surface of the earth to 15 km is given by the aircraft. The rocket goes from 15 kilometers to a distance of 100 kilometers; that means, I am looking at this up to 15 kilometers the aircraft White Knight 2 is used. And from here to a height of may be 100 kilometers the rocket is used, I want to know what is the velocity, which must be provided by the rocket.

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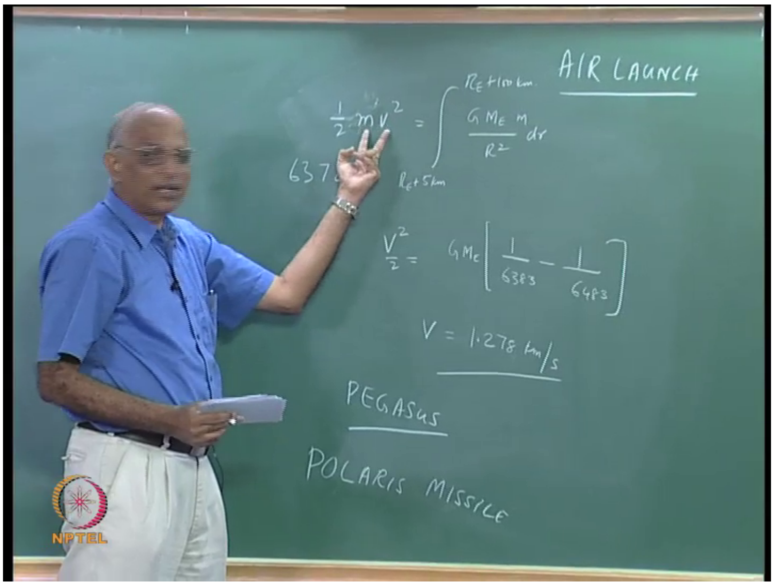

Let us calculate it. We know, that ½ m into V square is the kinetic energy what we are giving. This must be equal to the work done or the energy required to start from 15 kilometers above the surface of the earth; that means, R_E + 15 kilometers, I go a distance of R_E + 100 kilometers. And what I do? I give this kinetic energy, which will give me the work done in traversing from 15 km to 100 km; and what is the work done? It is $GM_Em/R^2 \times dR$ for a small distance dR. The total work is the integral from R_E +15 km to R_E + 100 km.

And now how to solve this; I have to integrate this equation and find out. What is the value? How do I do it? I find that m and m gets cancelled, GM_E/R^2 gives me minus 1/R. Therefore, we get $V^2/2$ is therefore equal to $GM_E \times 1$ over radius of the earth R_E plus 15 kilometers is 6383 minus 1 over R_E plus 100 km; that is 6468. We substitute the value of $G = 6.670 \times 10^{-11}$, the mass of the Earth and we get the velocity. And this velocity will come out to be something like 1.278 kilometers per second. And this is the velocity, which is required.

Now the question is if we had started from the surface of the earth, which is we say 6368 km. I find that the difference velocity is going to be very small. We readily do not see any advantage in launching from an airplane and going up. But there is something which we seem to forget. When an aircraft flies, it also gives a horizontal component namely an orbital velocity component; and that what make it advantageous. You know we have some rockets that are air launched and one of the rockets is known as Pegasus rocket. What is done in this rocket is you take the rocket and the space craft in an aircraft to height of something like 10 to 15 kilometers till atmosphere is available and then launch the rocket and that way the rocket need not be very powerful, but it can much smaller to do job.

We call such rockets launched from the air as air-launched rockets. It is not mandatory that rockets are launched from the ground. It might as well be launched from under the sea; we have sea launch. We have a missile known as the Polaris Missile, which is launched from the submarine from under the water: it comes up to the surface of water and it propels through air. So, wherever we want a launch, we need the value of the velocity that is to be provided such as the orbital velocity, total velocity, etc.

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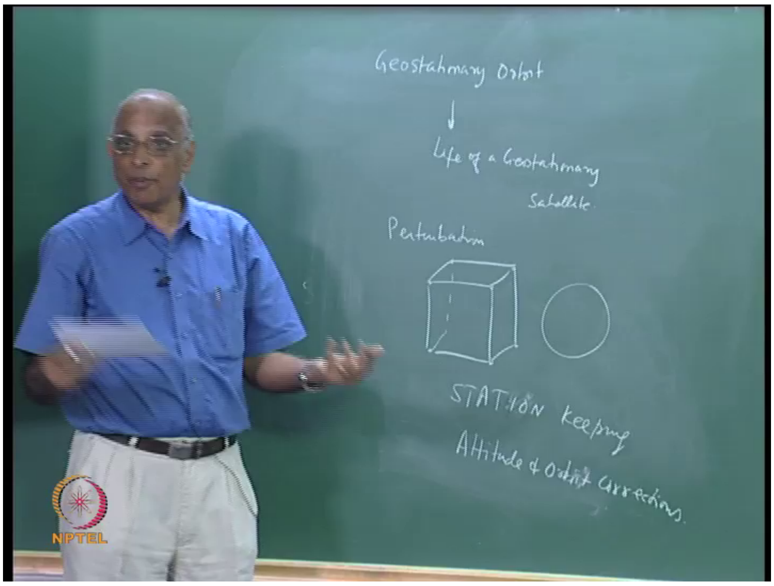

May be I will take a next example which is again very illustrative. We talked in terms of geostationary orbits. And you know nowadays, we find many countries wanting to launch spacecraft into geostationary orbit, because this is very useful for communication purposes. As far as India is concerned, the satellite in geostationary orbit at the given altitude points towards Nagpur, which is the centre of the India; India gets covered by the spacecraft and TV programs among others are relayed by the spacecraft. The program is beamed to the satellite from a given place and it beams it back throughout the country.

Let us consider the Indian National Satellite INSAT. Many INSAT satellites have been launched. Can we keep on going continuously using a satellite or is there a life for a satellite? And if there is a life to a satellite, why should it have a life? This is because electronics can continue to function for 100's of years therefore why should there is be a life? What is your opinion? People say the satellite has a life time of 15 years, some say it has 20 years, some say it is only 5 years.

What decides the life time of the satellite? Because I keep telling everything is freely falling; everything is vacuum, everything is going round and round in perfect orbits. Why should there be something like a life of a geostationary satellite?

You are telling may be the satellite may deviating from its path or orbit? Why should it deviate? You are partially correct, but then why it should it deviate?

There are many other forces like for instance, you have the Sun's gravitational field, you have solar flares, the gravitational field of the Sun is changing, may be we have the moon somewhere near at a geosynchronous radius of something like 43000 kilometers; there is the moon's attraction that is also a variable. Therefore, it is quite possible that there are perturbations or changes in the forces on the spacecraft as it orbits. How do you take care of these perturbations? We have the satellite in the form of a box and in it we have will have something like 16 rockets placed at the corners or at some other locations. And whenever you find something is changing have to fire these rockets and generate a force or momentum which can overcome the disturbance. That means I have to do something like what we call as station keeping to keep the satellite in its orbit. And make sure it as always pointed as required, If there is a drift, I have to correct it. This means that attitude, position, and orbit need corrections. I need energy for the corrections that are required.

Therefore, for all these things, we fire rockets and therefore, I have to keep in the satellite and use it as and when required. And once my fuel is over, the life of the satellite is over; and that is the reason for the life of a spacecraft. We keep talking in terms of the exotic propulsion like a electrical propulsion, which may not have so much requirement of a fuel, which can be there for much longer time and therefore we will cover this things as we go along.

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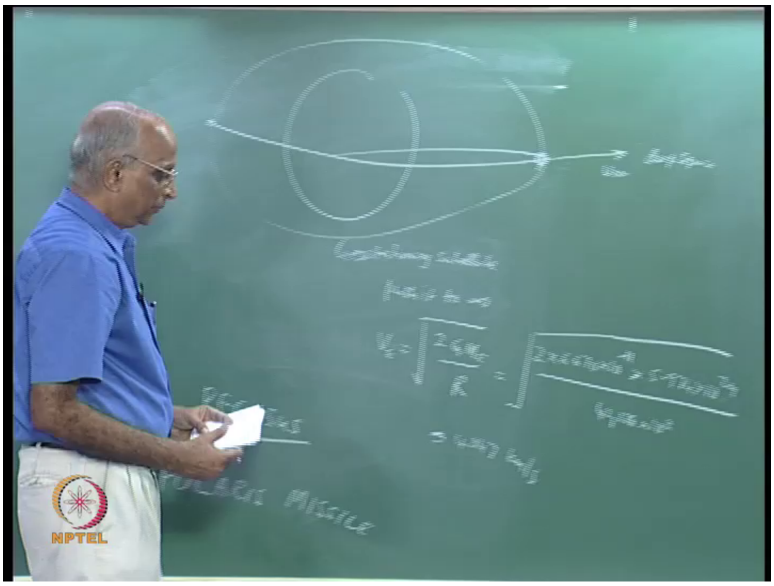

Let us consider a satellite in geosynchronous orbit at a radius of about 43,000 km. And let us say that the lifetime of the geostationary spacecraft is over. If it is going to be left to orbit it may be pose a danger to other geosynchronous satellites as it may collide with it. If we have old non-functional satellites it may be hazard for the others.me. This means that there is the problem even in space even though the space is so large.

Therefore, it is necessary that once the life time of a satellite is over, to push it out with escape velocity such that it goes into deep space and I have no such problems. How I do it? In other words, before the INSAT satellite life is over, I should make sure that with the remaining fuel, I must push it out of the geostationary orbit.

Therefore, let us do this problem. All what I am saying is I have a geostationary satellite; and now I want to push it out of the orbit; that means, I have to push it to infinity.

So what is the velocity required to push a satellite out of the geostationary orbit into deep space? I have to escape from the orbit; let as forget about the pull of the moon, other planets and all that and let as assume only Earth is attracting. Therefore, I want to find out what is the escape velocity? Escape velocity is $\sqrt{2GM_E/R}$. What is the R now? I want to escape from this orbit, and what is that R? R is equal to what we said was something like 42,000 kilometer. I put the value of R, substitute the value of G and Mass of Earth M_E.

And we find that we still require to push it out with a velocity of something like 4.347 kilometer per second. Let us puts the numbers: $2\times6.670\times10^{-11}\times$ mass of the earth 5.974×10^{24} divided R which equal to 42178×10^{3} meters. And this comes out to the 4.347 km/s. That means, I must keep some fuel reserve such that with this fuel, I will be able to push it out; and to keep this amount of fuel reserve is mandatory.

Well, this is all about orbits. I think be have covered it to some extent. Why do we need rockets? We now go back and ask how to push in space?

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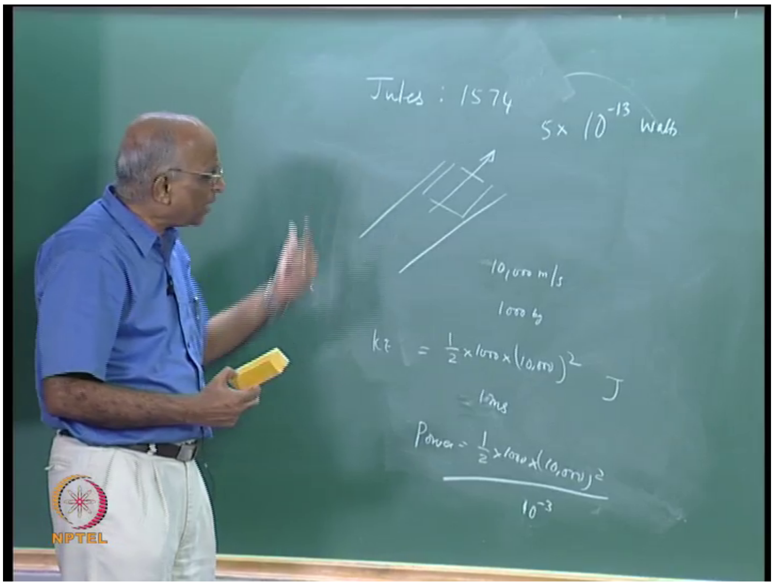

Joules Verne in his Science Fiction book "From Earth to Moon" suggested we have a canon; in the cannon, you put a spacecraft you push it out with extremely high velocities such that it gets into the orbit. What is the type of typical orbital velocities for geostationary orbit? It is around 13 kilometers per second or let us say that we need an orbital velocity of around 10 kilometers per second that is 10000 meters per second. Supposing the mass of the body, which I want to orbit is around we say 1000 kilogram, because 1000 kilogram is the least required, wherein I can account for some equipment, may be one or two people can be there; we have something for life support and all that and 1000 kilogram seems reasonable.

Therefore, what is the energy I must have? The kinetic energy is ½ × 1000 kilogram × 10000^2 so much Joules. This is the energy, what I have to give to the body as kinetic energy. And what is this value? Supposing, I have to launch it instantly using a canon in something like in 0.1 milli second or let us say 1 milli second, because it has to get out of the canon fast. Therefore, the power required is equal to ½ × 1000 × $10{,}000^2 \div 10^{-3}$. And what is the number we are now talking of? We talking of 500 × 10^8 divided by 10^{-3}. We are talking of the huge numbers something like 10^{13} watts. If you take the entire electricity which is generated in a super thermal power plant, it is very much lower than this; therefore, we cannot use such a canon for launching the space capsule.

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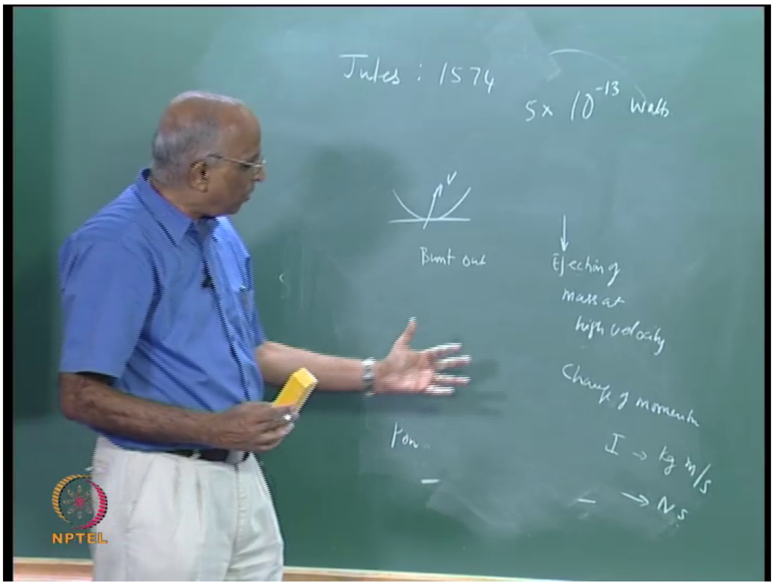

And let us take a look at what others were suggested in very early times. Imagine a ship is sailing on the sea; there is the giant storm and you get a high velocity waves over the sea. The ship gets launched by the high velocity wave motion into the atmosphere. Even if by chance, I get a high velocity, when the body with the high velocity is traveling through atmosphere, it will get burnt out, because you have frictional resistance of the air; therefore, we need some other type of launching. When we imply rocket propulsion all what we mean is, you have continuous ejection of mass from the rocket at high velocities. What it does is that it provides momentum or rather some change of momentum. And what do we mean by change of momentum? We call it as impulse (I). What is the unit of a momentum: momentum is kilogram meter per second. Therefore, impulse has the same unit kilogram meter per second, but Newton is equal to kilogram meter per second square; therefore, impulse also has units of Newton second. Please be careful about units. Therefore, when we launch a body by a rocket, we give some impulse continuously to the body. During the process the mass of the rocket keeps decreasing as it ejects mass out and the velocity of the body increases rapidly.

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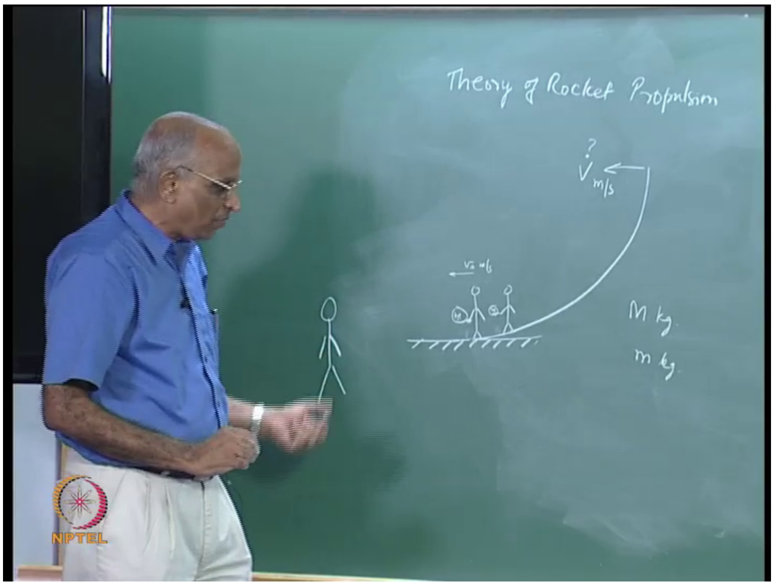

This brings us to the theory of rocket propulsion. All what are telling is that we must give some momentum to the body; and how do I give it a change of momentum? I throw some mass out of the body at sufficient velocity and cause a momentum change in the body. Therefore, let us take an example; we will start with this example; it is a very fascinating example. I borrow this example from my teacher who taught mechanics to the first level students. Supposing we have something like a rigid sled; what is the sled? Sled is something on which you slide down the slope of a mountain. And this sled, let us say, is on level ground and we presume that there is no gravitational field. There are no external forces present on the sled.

On this sled we have two-boys; the sled is stationary and these two boys want to move the sled; the sled is on a level ground let say the ground is so slippery that there is no friction. We idealize this situation. This sled is stationary, these two boys find that there is all ice all around - there is no friction between the ground and the sled; they do not want to get out; but they want to move this sled. Therefore, they say let us provide some impulse or let us provide some change of momentum to the sled.

Therefore, in the example, we consider each of the boys carry a stone of mass m. Let the mass of the stones, the two boys and the sled be M kg; lets the mass of each stone be m kg. Now, the boys want to move, how do they move? They say let both of us throw the stone out simultaneously at a velocity v_0, so many meters per second. Therefore, both the boys simultaneously through this mass with a velocity v_0 meters per second.

Now, I want to find out whether this sled will move or not. How do I solve? I go back to my inertial frame of reference. What I do is stand outside the sled; and I am in the inertial frame of reference, because I am moving at constant velocity and therefore, I describe the motion of the body. I am watching these things happen. Now, I want to know the velocity at which the sled moves. The two boys throw the stone in a given direction with the velocity v_0; let me assume that the sled also moves in this same direction at a velocity V meters per second. I want to determine the value of V. I am looking at it from the inertial frame of reference and therefore, what will be the equation for change of momentum?

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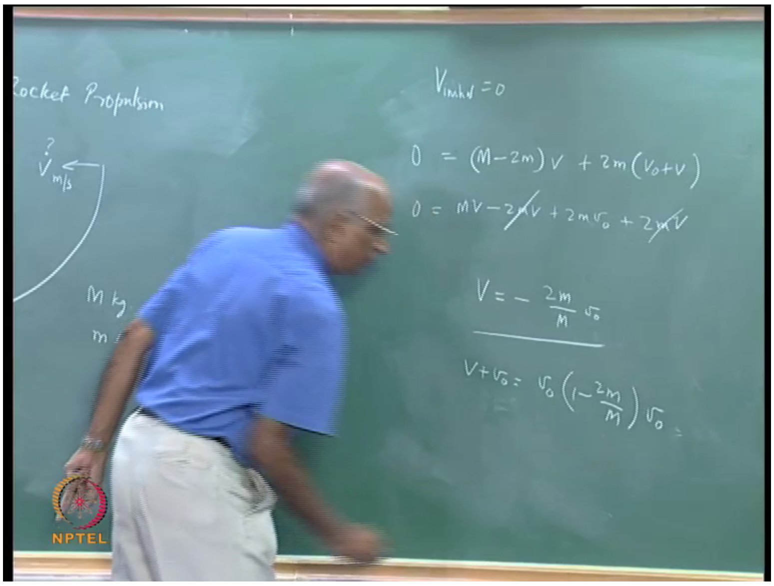

The initial momentum of the sled, the boys and the stone put together, they are all at rest and the velocity V is equal to 0 initially. And therefore, the value of the initial momentum is 0. What is the final momentum? The momentum is conserved in the inertial frame of reference. Therefore, what is the final momentum? Let us calculate it. Now the final mass of the sled is M − 2m since the two stones have left. The sled has the velocity V, the stones are hurled with a velocity v_0. What is the velocity of the stone as I am watching it from the inertial frame of reference? The sled is moving with the velocity V, stones are moving with the velocity v_0 and therefore, the velocity as seen by the observer in the inertial plane of reference is $V + v_0$. The momentum is therefore $2m \times (V + v_0)$ in the inertial frame of reference. Momentum is conserved; initially it 0 and final must be equal to this, because I am talking of the inertial frame of reference. And therefore the initial momentum is 0 and is equal to the final value of $(M - 2m) \times (V + v_0)$. 2mV gets cancelled. We want to find out the final velocity of this sled, which is capital V? What is the value that we get? V is equal to minus 2m/M × the velocity v_0 with which the stones are thrown. What does the negative sign in this equation tell us? If the stones are thrown out in a given direction, the velocity will be in the opposite direction.

Therefore, just through the action of these two boys throwing the stone, they are able to move this sled at this velocity. Now, I ask the second question. What is the relative velocity of the stone? $V + V_0$. Therefore, I take v_0 outside into $(1 - 2m/M) \times v_0$.

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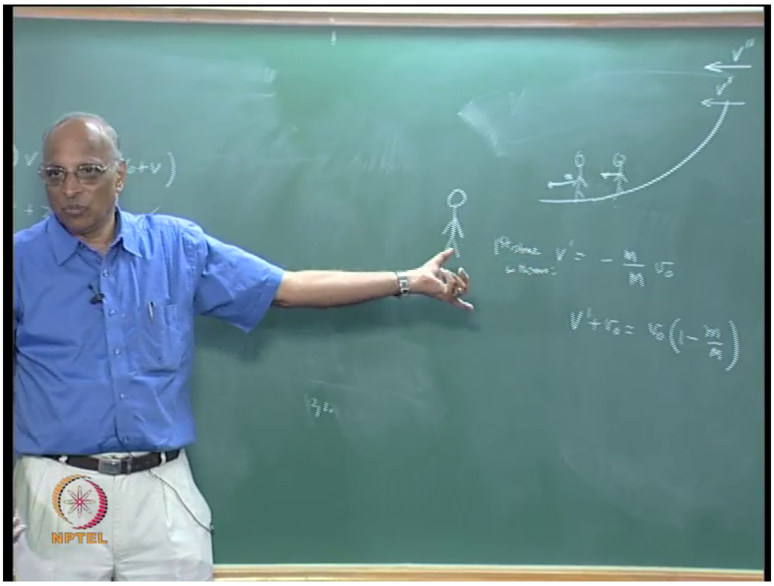

The next question we ask is if these two boys, given the same set of stones, and throwing the stones at the same velocity can they move the sled even faster. The first boy throws the stone with velocity V_0 and this is followed after some time by the second boy now throwing the second stone with the same velocity V_0. In fact, they do not throw the stones simultaneously that is 2m mass of stone thrown together, but rather one stone after the other. If the stones are thrown one after the other, what will be the final velocity of the sled? Let us call it as V''. What should be the value of V''? Let us write the equation of motion again.

Again I am in the inertial frame of reference. I stand over here; watch the fun in the inertial frame of reference. I find that the first boy throws the stone and let the value of V' be the velocity of the sled after the first stone is thrown. And what is the relative velocity? The relative velocity is equal to V' + V_0. Following from the last example the value of V' is – m/M× v_0. The relative velocity of the stone which was thrown is therefore V' + v_0 which is equal to v_0 (1- m/M). At this point in time the second boy throws the stone and therefore, what is the final velocity of the sled? When the first stone is thrown you have V' as the velocity of the sled, when the second stone is thrown, you get V''. Let us balance the momentum in the inertial frame of reference. We get:

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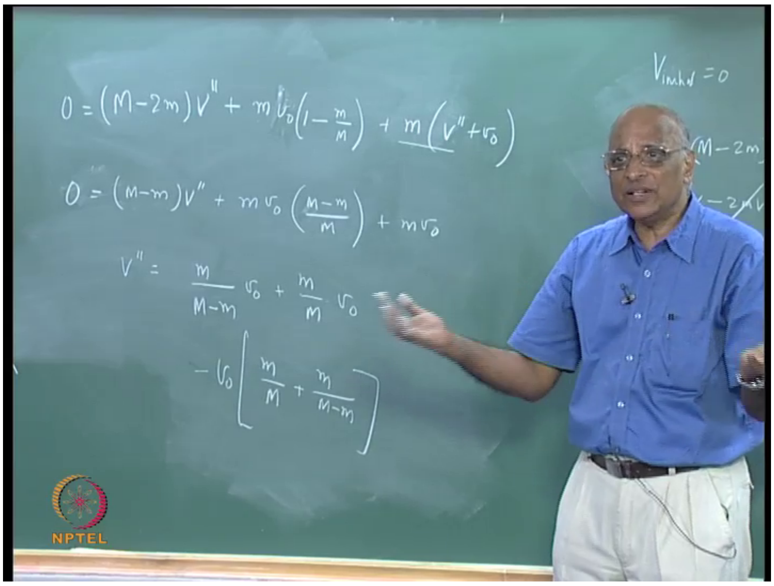

Initially the momentum is zero. The final momentum is (M−2m) × V''. The first stone goes with a velocity of relative velocity in the inertial frame of reference, which is equal v_0 into 1− m/M. And the second stone goes with a momentum of m into the velocity equal to V''+ v_0 relative velocity; This becomes my equation for momentum balance. If I solve this, what do I get? I want to find out what is the final velocity V''? Minus 2m×V'' + m ×V'' gives −m× V''. The next term is simplified as m into v_0 × M − m by M and the third term is left with m × v_0.

What do we get for the value of V''? $V'' = m/(M-m) \times v_0 + (m/M) \times v_0$ or rather we get it as equal to $v_0 \times [\, m/M + m/(M-m)]$. Now, we find that when one stone is thrown after the other, I get this velocity whereas when both the stones are thrown together, we had the velocity as $(2m/M) \times v_0$; in both cases we should have had the negative sign since the direction of velocity of the sled is opposite to the direction in which the stone is thrown

And therefore, what is the comparison? We have m/M plus m/M when both the stones are thrown together. We have $m/M + m/(M-m)$ when one stone follows the other. M minus m is smaller than M. Therefore, in the second case the velocity of the sled will be greater; therefore, the throwing one stone after the other gives a higher velocity than when both the stones are thrown together. Now we can generalize, instead of having two stones, I keep on throwing one stone after the other, what is going to happen? I will get the velocity, which is much better than the spontaneous throwing of all these stones together. And this is the basis of the rocket propulsion.

What we do in a rocket is keep on ejecting mass till we achieve the required velocity. I will continue with this; in the next class. We will derive Tsiolkovsky's equation, which is known as the rocket equation following this analogy. But this is basically the principle using which we must be able to design new forms of rockets.