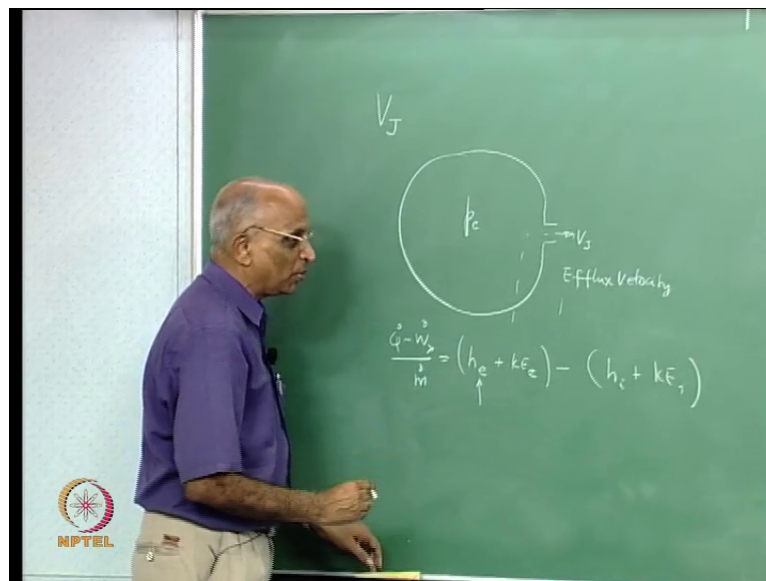


**Rocket propulsion**  
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**Lecture No. # 09**  
**Theory of Nozzles**

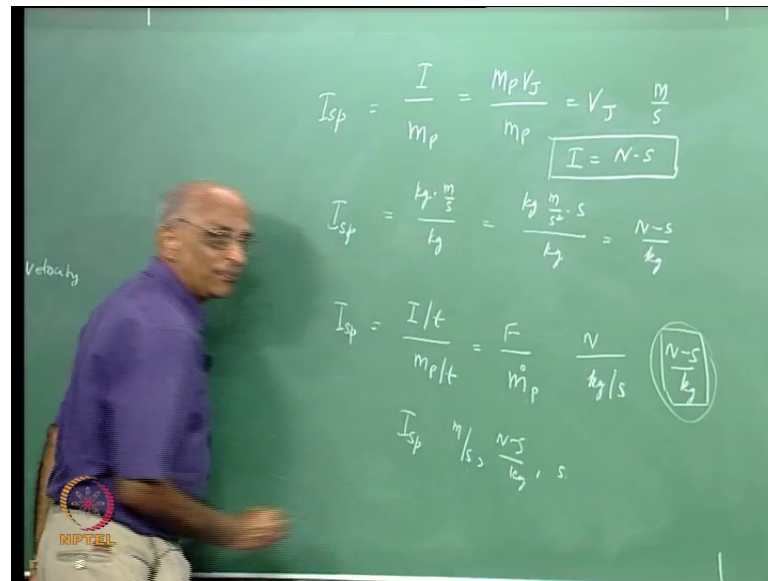
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Good morning. We will develop the equation for  $V_j$ . We have a high pressure container. It contains a gas at chamber pressure  $P_c$ . We have something like a hole or a vent through which the gas squirts out at velocity  $V_j$ .

We wish to find out the efflux velocity  $V_j$ , which is called as jet velocity or we call it as efflux velocity  $V_j$ . However, before I do this since there was a question of specific impulse, and unit of specific impulse let us just spend some 2 or 3 minutes on this issue.

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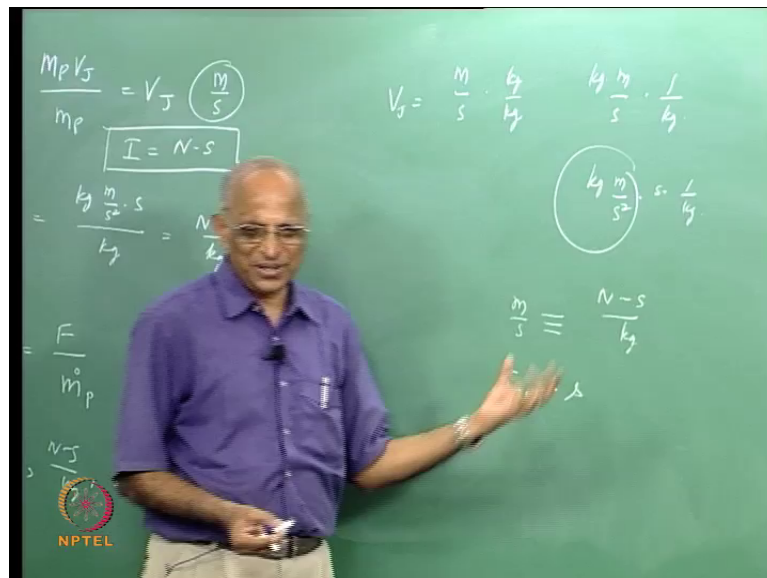


What was specific impulse? We told that specific impulse is equal to impulse  $I$  divided by the mass of the propellant, that is the impulse generated from unit  $M_p$  mass of propellant. Impulse is change of momentum and therefore is equal to  $M_p$  into the velocity  $V_j$  divided by  $M_p$  which is equal to  $V_j$ . As per this logic, the unit of specific impulse should be meter per second, which is same as the efflux velocity  $V_j$ . But how did we define specific impulse? It was defined it as impulse per unit mass of propellant or equivalently thrust per unit mass flow rate of propellant. In other words impulse has unit of momentum, change momentum viz., kilogram  $\times$  meter per second  $\div$  kilogram and when we say specific impulse we get back m/s. From force considerations, we get force into a given time; that means, we can write the change of momentum as equal to kilogram, meter per second square into second divided by kilogram and this is Newton second by kilogram. Therefore I also see that unit of specific impulse can be expressed in Newton second by kilogram which gets reduced to m/s.

So far so good. Let us derive the units of specific impulse by expressing specific impulse as equal to impulse per unit time (Force) divided by mass flow rate of propellant per unit time. And impulse per unit time is force, force into time is impulse, in other words I have force divided by  $m^\circ$ . In other words force as unit of Newton, mass flow rate of unit kilogram per second, therefore the unit of impulse specific impulse comes out to be Newton second by kilogram which again reduces to m/s.

Therefore, whether I express specific impulse as impulse per propellant mass or specific impulse as force per unit mass flow rate we get the same unit as Newton second by kilogram, therefore the unit for specific impulse is Newton second by kilogram. The unit for impulse should have been Newton  $\times$  second, that is what gives the value of impulse kilogram meter per second as Newton second. Therefore let us keep ourselves very clear; impulse has unit of force into second: Newton second, specific impulse has unit of Newton second by kilogram, but well there are many text books which specify the specific impulse not in meter per second, not in Newton second by kilogram, but as second.

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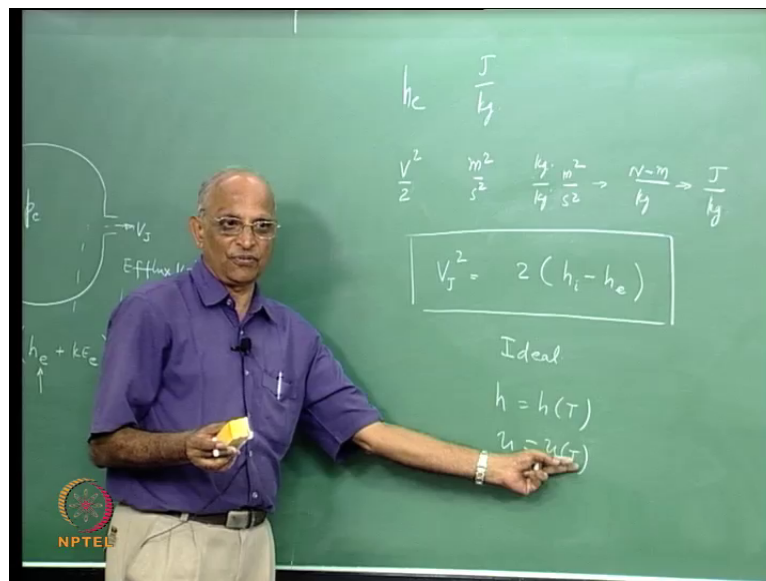
See when we say the specific impulse is so many meters per second, we could have multiplied both numerator and denominator by kilogram and kilogram, and what is it we get; we get the unit of  $V_j$  as equal to kilogram meter per second, divided by kilogram. Now we again multiply the numerator and denominator by second to gives us kilogram meter per second square  $\times$  second per unit kilogram. This gives us Newton second by kilogram. Therefore meter per second is actually identical to Newton second by kilogram. Either of the units is ok,

Why is it some people use the unit as seconds. They specify the force in kilogram or pounds, therefore pounds and pounds gets cancelled and second is left. Therefore whenever somebody gives the units in second it is our duty on Earth to multiply by the

gravitational field  $g_c$ , and then use it in Newton second by kilogram. May be you all should go through it, but in this class we will always address specific impulse in Newton second by kilogram or in terms of meter per second both of which you see have the same identical units.

Units are very important in engineering. With wrong units we will be talking of something but getting of some other numbers. Let us get back to our problem on  $V_J$ . We want to find the  $V_J$ . We derived the equation for control volume we said  $Q$  minus the work done by the particular vent  $W_x$  divided by let say  $m^\circ$  is equal to the enthalpy which is leaving (let say  $h_e$ ) + the kinetic energy per unit mass which is leaving - ( $h_i$  + the kinetic energy per unit mass that is entering the vent. We told that the change potential energy between the exit and the entry is zero and therefore we can neglect it.

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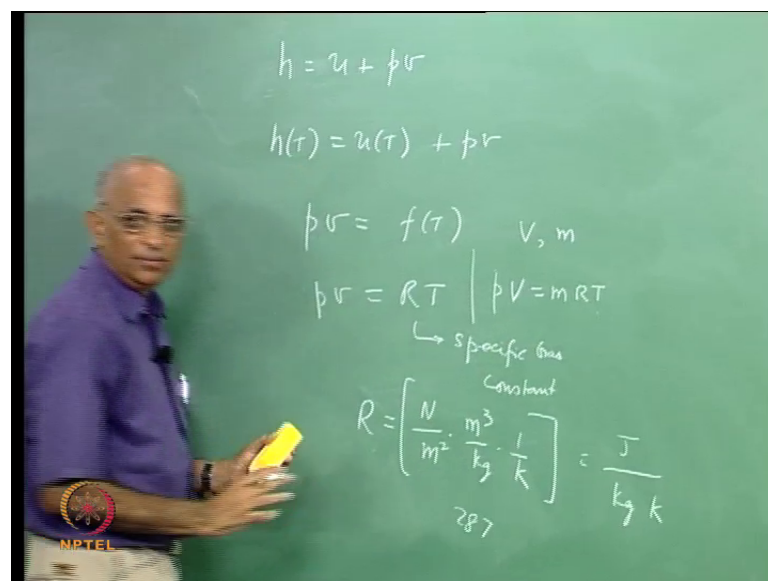


And we told that we are to looking at  $h_i$  per unit mass, this is enthalpy  $h_i$  for unit mass and let us put the units clearly; the unit of  $h_e$  is therefore joule per kilogram, the kinetic energy is equal to  $V^2$  divided by 2 and that is equal to meter squared per second squared. We multiply numerator and dominator as usual by kilogram, meter square by second square, this is equal to kilogram meter square per second square. This gives Newton meter per kilogram, which is equal to joule per kilogram. Therefore the kinetic energy  $V^2/2$  has units of joule per kilogram; that means we are taking per unit mass of the gas which is moving and determinig its kinetic energy.

Therefore let us put things together. We told that the process of expansion is adiabatic in the vent, it is not something which can dilate the nozzle and work can be done by it. If you were to apply the same problem to our heart valve, see heart also pumps our blood, but the valve is also something like flexible when we write the same equation for the control volume for the flow by blood through one of the arteries and valves in our body for which  $W_x$  is not 0. Maybe that is what makes modelling of the heart more difficult compared to a vent over here. Therefore now let us put things together: we have  $V_j^2$  which is equal to the exit velocity squared and that is equal to 2 of the enthalpy which is entering minus enthalpy with exiting the nozzle i.e.,  $V_j^2 = 2(h_i - h_e)$ .

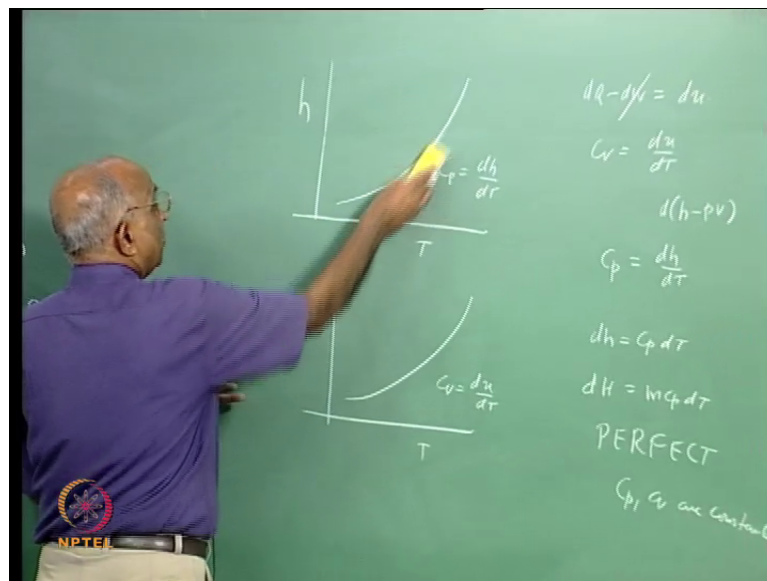
Now, we want to solve this equation and we want to make sure that we solve it in terms of the properties of a particular gas. What are the gas properties? They could be the temperature, could be the molecular mass, could be some other property which we need to consider. Now to be able to solve this we have to make further assumptions. Let us assume that the gas is ideal and what do you mean by the gas is ideal? A gas is ideal when the enthalpy per unit mass or the specific enthalpy and specific internal energy are only functions of temperature. I think this definition is important. Let me just briefly go through the definition of the enthalpy is only a function of temperature and the internal energy is only a function of temperature for which we say that the gas is ideal.

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And what is the consequence of this, we have the definition that  $h$  minus  $u$  are rather we define enthalpy as equal to  $u + p \times$  specific volume  $v$ , and therefore we find for an ideal gas if  $h$  is function of temperature, internal energy is the function of temperature plus  $p v$ , are rather we get  $p$  into the specific volume is only a function of temperature, and therefore we write it as  $R T$ . Therefore an ideal gas for which enthalpy and internal energy are only the function of temperature has an equation of state which specifies  $p v$  is equal to  $R T$ , and this  $R$  is what we call as specific gas constant. Let us again just repeat what we said.

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We told that for an ideal gas may be  $h$  is only a function of temperature, similarly  $u$  is function of temperature and now if we take the slope anywhere we get the value of specific heat  $C_p$  is equal to  $dh$  by  $dT$  and we get  $C_v$  is equal to  $du$  by  $dT$ . How did this come for any system? We have  $\delta Q$  minus  $\delta W$  is equal to  $du$  and for a constant volume system work done is 0, therefore, heat required per unit temperature change per the unit mass is therefore the value  $C_v$  is equal to  $du$  by  $dT$ . That means for a constant volume system work done is zero and I get this value. For a constant pressure system what did we do we defined  $dh$  as instead of  $du$ ?

I write now  $du$  as  $dh - d(pv)$  and the last term gives  $p dv$  and  $v dp$ . I have  $p dv$  over here it cancels with  $p dv$  of  $du$  and I get  $C_p$  for a constant pressure process as equal to  $dh$  by  $dT$ . We are still considering an ideal gas with specific gas constant  $R$  and what is a unit

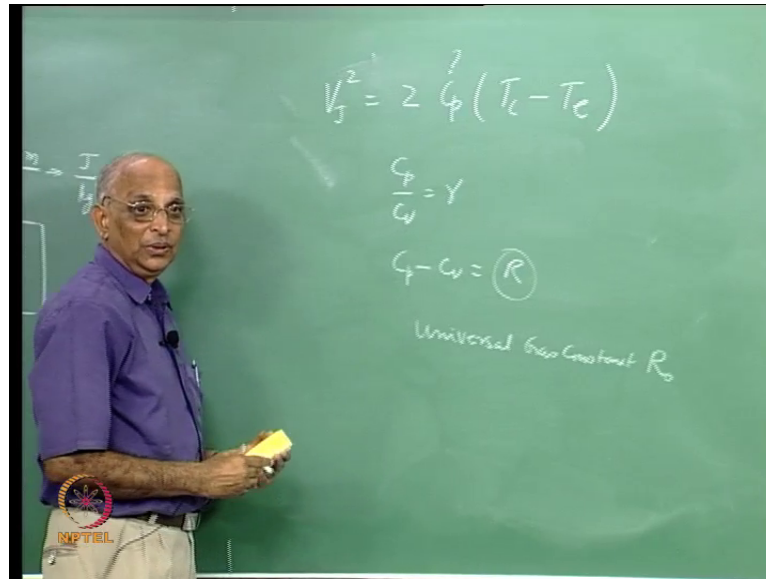
for R, let us put it down pressure, Newton/ meter<sup>2</sup> × volume, meter cube by kilogram by its specific volume ÷ Kelvin giving Newton meter viz., joule per kilogram per Kelvin; that means we have joule per kilogram Kelvin, which for air is about 287. It is a value specific to the gas, air has value 287 and may be CO<sub>2</sub> will have a lower value, may be helium may have a higher value, this is R depends on the type of gas which we use. Therefore we say well I have the equation of state of this gas given p v is equal to R T, or if I consider the volume of a gas V which has a certain mass m, we can write the same equation pV = m R T, and this is what we have been studying in thermodynamics.

Let us now go forward. We find h is the function of temperature, u as a function of temperature, the curve is varying, Cp is also a function of temperature and Cv keeps varying with temperature. If I have to write from this equation, the value of dh, we get as equal to Cp dT or rather the enthalpy of the mass of gas m is given by H = m Cp T. But Cp changes with temperature now my problem is going to become more complicated because I have to consider the Cp variation with temperature.

Therefore, I put another idealisation and in this idealisation I say that the gas is even better than ideal; I call it as a perfect gas. A perfect gas is one for which Cp and Cv are constant in addition to the value of h and u being as a function of temperature. This implies one more assumption and we find that the functional dependence of enthalpy on temperature is not like this but the functional dependence is straight line. Cp and Cv are constants for a perfect gas.

Therefore, we will solve this equation assuming let say a perfect gas. If the gas pressure is p<sub>C</sub> and its temperature is T<sub>C</sub> and if the exit temperature at this plane be say T<sub>e</sub>, my job is clear and I need to derive the value of V<sub>J</sub>.

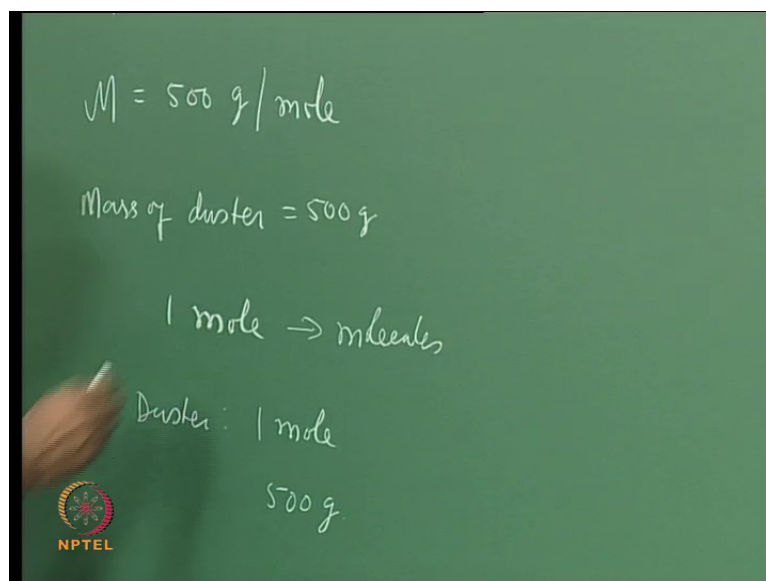
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We now write  $V_j^2 = 2 (h_i - h_e)$ ; with  $C_p$  is a constant  $V_j^2 = 2 C_p \times$  the change in temperature. The value  $h_i - h_e$ ,  $h_i$  corresponds to the temperature is  $T_c$ , while  $h_e$  corresponds to temperature  $T_e$ . Now we want find out what is the value of  $C_p$  and we want to know the properties of  $C_p$  of gas in terms let us say the gas constants may be in terms of the a molecular mass of the gas. Therefore we again go through the relation  $C_p$  by  $C_v$  is equal to  $\gamma$  viz., the specific heat ratio. Also from equation  $h - u$  is equal to  $p v$  is equal to  $R T$ , we get the expression  $C_p$  minus  $C_v$  is equal to  $R$ . How did this come?  $h - u = p v = R T$ . I take differential  $dh$  by  $dT$ ,  $du$  by  $dT$  is equal to  $R$  and therefore  $C_p - C_v =$  specific gas constant  $R$ . Now we find  $R$ , the specific gas constant, keeps changing with type of gas and supposing we keep changing the gas here and I have to change the value  $R$ . Why not I express the  $R$  in terms of the universal gas constant, which is same for all gases namely  $R_0$ .



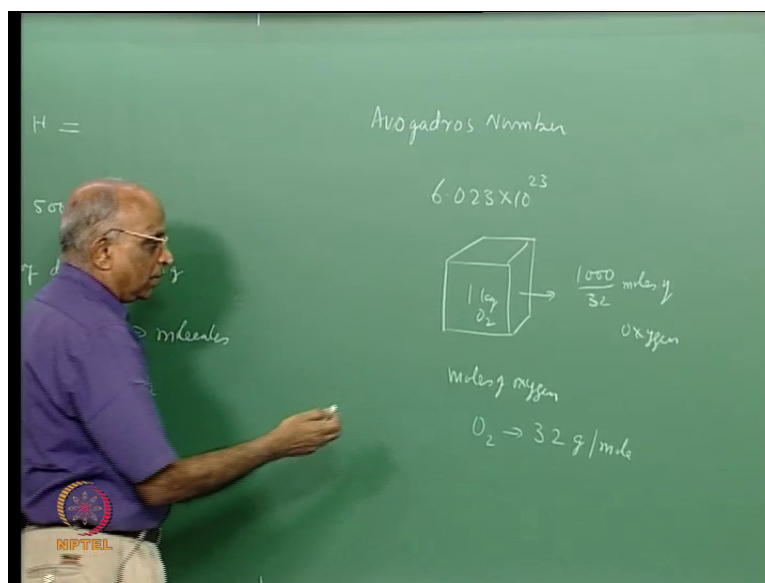
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And now how do I define  $R_0$  with respect to  $R$ ? Let us again talk in terms of little more basics over here. What did we talk about it when we started this course. We told that the quantity of matter could be expressed in terms of kg, I have a given amount of matter and how do I say kg - well somebody maybe in 1827 or 1830 he kept some mass in some lab in France which said the mass to be 1 kg. The quantity of matter is what I express in kg; but why it should be kg? It could also be another unit. Instead of saying that the mass of the duster is let us say 500 grams why not express it in some other unit, let us say the molecular mass of wood. If the molecular mass of wood is something like 500 grams per mole for this particular duster, I can as well say that this duster contain one mole of the wood. Instead of defining the mass of this duster as 500 grams, I can say the duster consists of 1 mole of the substance wood. Therefore I can also define the quantity of this duster in terms of a mole just as we define it in terms of mass of 500 grams.

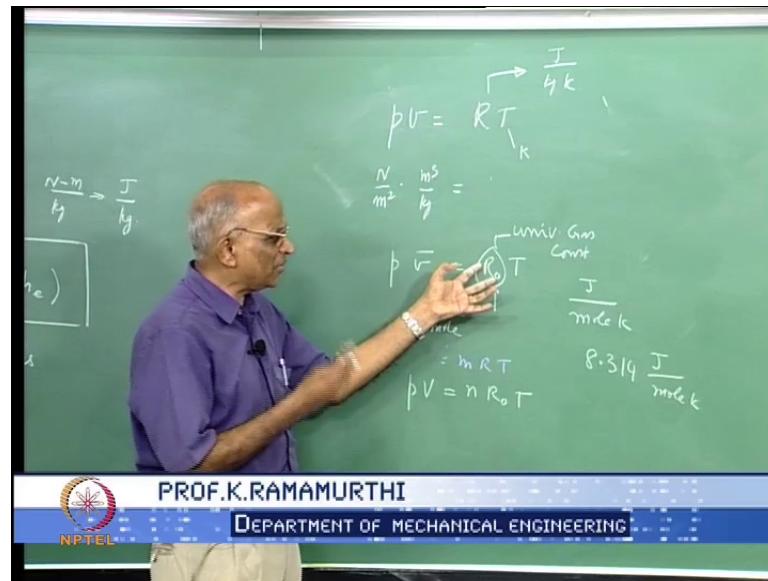
Getting back to some more details we must remember that this mole is different from the number of molecules. The number of molecules in 1 mole is what we call as Avogadro's number.

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And the number of molecules in 1 mole is  $6.023 \times 10^{23}$ . In other words 1 mole of any substance has something like  $6.023 \times 10^{23}$  molecules. Now we must be a little more clear and let me take one more example. Supposing I consider let us say a box; an empty box into which I introduce a mass of 1 kg of oxygen. Now instead of saying 1 kg of oxygen, I did rather describe this quantity oxygen in terms of moles of oxygen. We all know that the oxygen  $O_2$  has a molecular mass of 32 grams per mole, therefore instead of saying 1 kg of oxygen, I can as well say I have 1000 divided by 32 so much moles of oxygen. Therefore to state the amount of matter which is there or amount of matter which is available, we can express it either in mass or in terms of moles.

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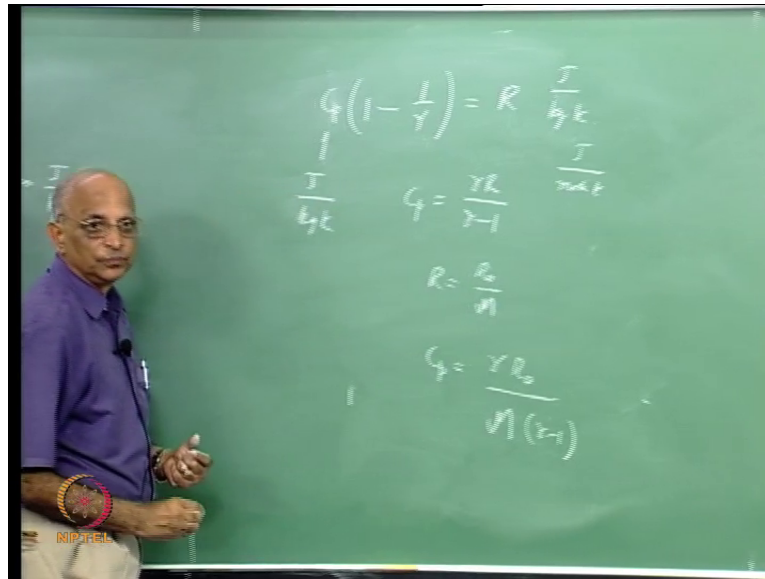


We saw the equation  $p \times$  the specific volume  $v$ ;  $p$  is a pressure with units Newton/meter<sup>2</sup>, specific volume has units of meter<sup>3</sup>/kilogram, as equal to  $R T$ , where  $R$  is the specific gas constant.  $R$  has units of joule per kilogram Kelvin since temperature has a unit of Kelvin.

Now instead of expressing this specific volume in meter<sup>3</sup>/kilogram, we can also write it as meter<sup>3</sup>/mole. With this unit my right hand side becomes  $R_0 T$  where  $R_0$  for all gases is the same. This is known as the universal gas constant. In other words we write  $pV = nR_0T$  instead of  $pV = mRT$ . If we consider  $m$  in kilograms I have  $RT$  if I consider this term in mole I get it as  $R_0T$  with the same value of  $R_0$  for all gases which now become universal. We say  $R_0$  is universal gas constant which should have the units as joule per mole Kelvin and the value is 8.314 joule per mole Kelvin. Therefore we just refreshed ourselves with a little bit of thermodynamics.

We say we are talking in terms of a perfect gas for which  $C_v$ ,  $C_p$  are constants we also learn to distinguish between  $R$  and  $R_0$  which is very primary but which is very essential. We can now go back to this particular equation  $= 2 C_p (T_c - T_e)$ . Note that we said  $C_p - C_v = R$  and  $C_p/C_v = \gamma$ .

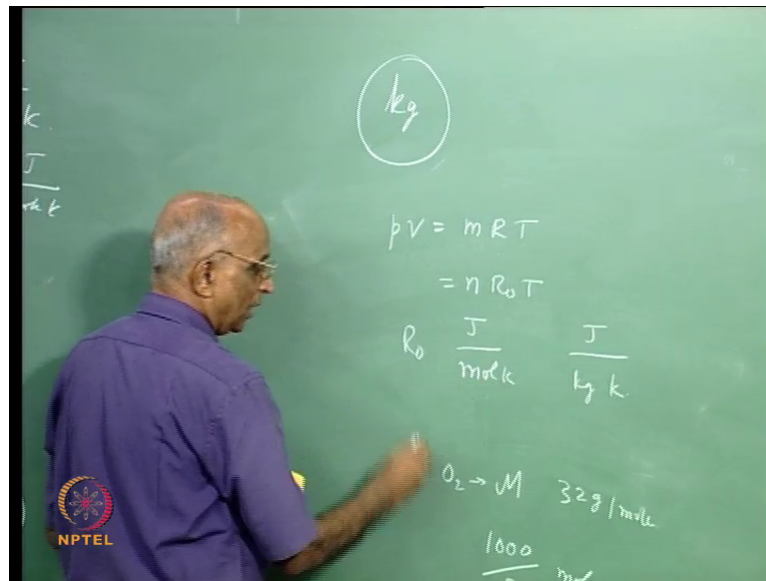
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And therefore we could write that same equation as  $C_p$  into 1 minus  $C_v$  by  $C_p$ ,  $C_p$  by  $C_v$  is gamma and therefore  $C_p(1 - 1/\gamma)$  is equal to specific gas constant  $R$ . Let us keep track of the units.  $R$  has the unit of joule per kilogram Kelvin,  $C_p$  has units of heat required per unit mass per unit Kelvin, and hence  $C_p$  and  $R$  have same units.

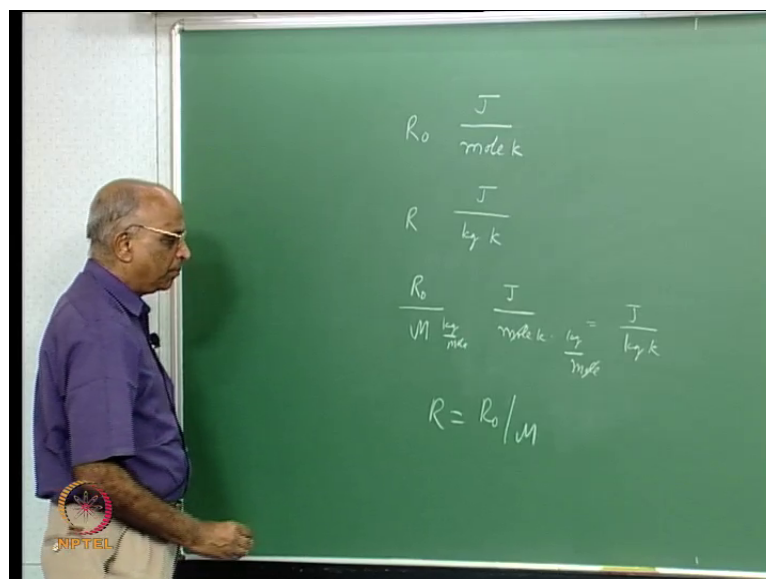
We can write the value of  $C_p$  as equal to  $\gamma R / (\gamma - 1)$ . Supposing we want to write it in terms of moles instead of mass, all that we do is instead of writing  $R$  in joule per kilogram we have to write  $R$  in terms of joule per mole Kelvin. Therefore we say  $R$  by comparing  $pV$  is equal to  $mRT$ ,  $pV$  is equal to  $nR_0T$ , we get  $R$  is equal to  $R_0$  by the molecular mass  $M$ , where  $M$  is the molecular mass in kilogram per mole. Rather if we now simplify the equation, we get the value of  $C_p$  in joules per kilogram Kelvin as equal to  $\gamma R_0 / M(\gamma - 1)$ .

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I just swallowed 1 or 2 small steps. What are the steps that I did not show? We could write  $p \times V$  volume in meter cube is equal to  $m R T$  are rather this could also be  $= nR_0T$ , where  $R_0$  is in joule per mole Kelvin.  $R$  in joule per kilo gram Kelvin to convert to  $R_0$ , we have to multiply  $R$  by  $\text{kg/mole } M$ . That means  $R_0$  divided by molecular mass is equal to  $R$ , and this  $R_0$  has the unit of joule per mole Kelvin.

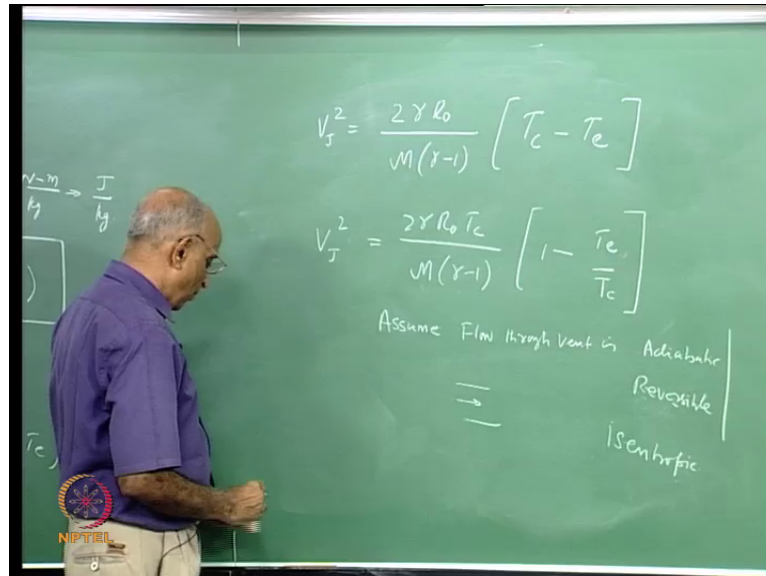
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And if I have to convert joule per kilogram Kelvin into joule per mole Kelvin, multiply it by the molecular mass in kg per mole.

Therefore we have  $R$  is equal to  $R_0$  divided by  $M$ , the the molecular mass of the particular gas.

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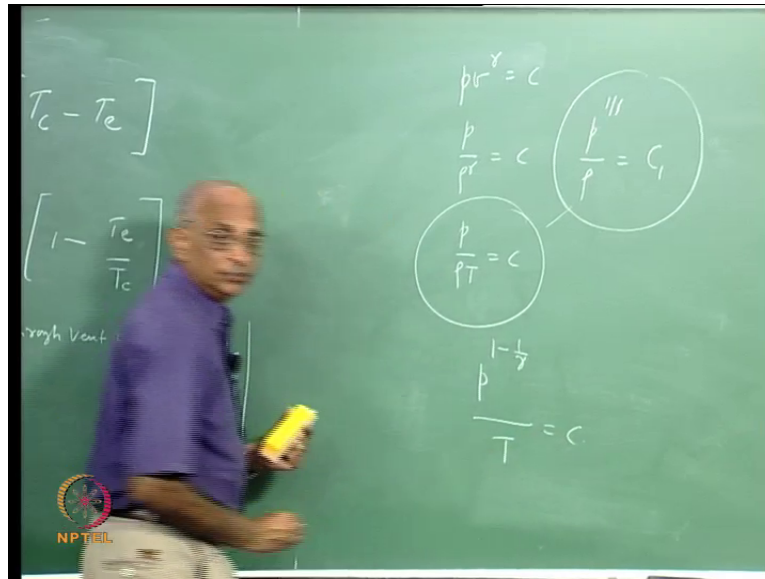


Therefore, let us now substitute this in the expression here for  $V_J^2/2 = \gamma R_0/[(\gamma-1)M] (T_c - T_e)$ . Taking  $T_c$  outside, we have  $V_J^2 = [2\gamma R_0/(\gamma-1)M]T_c(1-T_e/T_c)$ . This is the value of the efflux velocity squared or jet velocity squared.

So far we have made no assumptions of the nature of the expansion process. Since we do not know what is the exit temperature but we know the exit pressure, we would like to convert  $T_e$  by  $T_c$  in terms of pressures. Let us assume the next assumption as flow through the vent or flow through the hole is adiabatic. We have already assumed it earlier and said there is no heat transfer. Let us make one more small assumption.

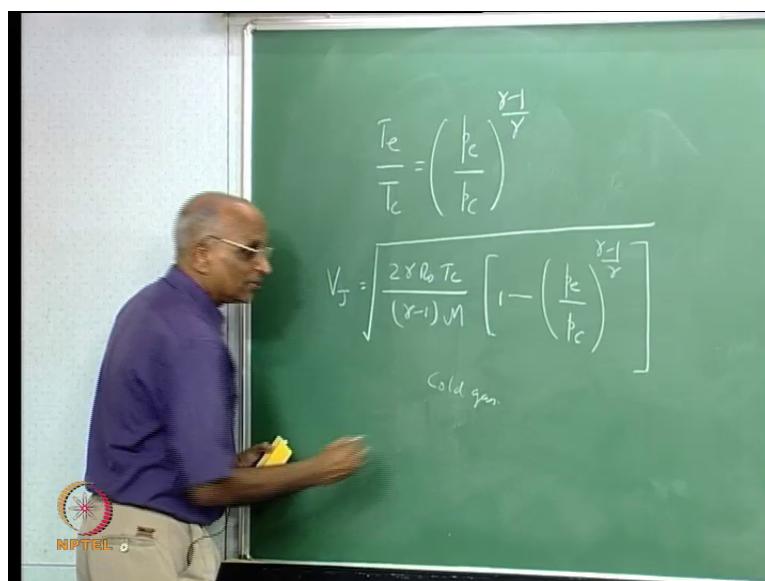
Let us assume that the flow through the particular vent is quite slow. What do I mean by slow? I mean its slow enough such that the flow is reversible. What do you mean the flow is reversible; in other words the flow is not so fast that it cannot retrace back. It goes through the series of equilibrium states and therefore I say that the flow through the vent is adiabatic and reversible or rather the expansion process is isentropic.

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And if we have the isentropic flow in the vent, the equation for the process involving this isentropic process is going to be  $p v^\gamma = \text{constant}$ . With  $v$  being the specific volume i.e.,  $1/\text{density } \rho$ , the equation becomes  $p/\rho^\gamma = \text{constant}$ . But we know that the gas is ideal gas or rather perfect. This is already assumed. Based on this, we can write the equation of state as  $p/(\rho \times T)$  is a constant. We take to the power of gamma of this equation viz.,  $p^\gamma/(\rho^\gamma T^\gamma) = \text{constant}$  and compare this equation along with the isentropic equation. I divide one by the other and get  $p^{1-1/\gamma}/T^\gamma$  is a constant.

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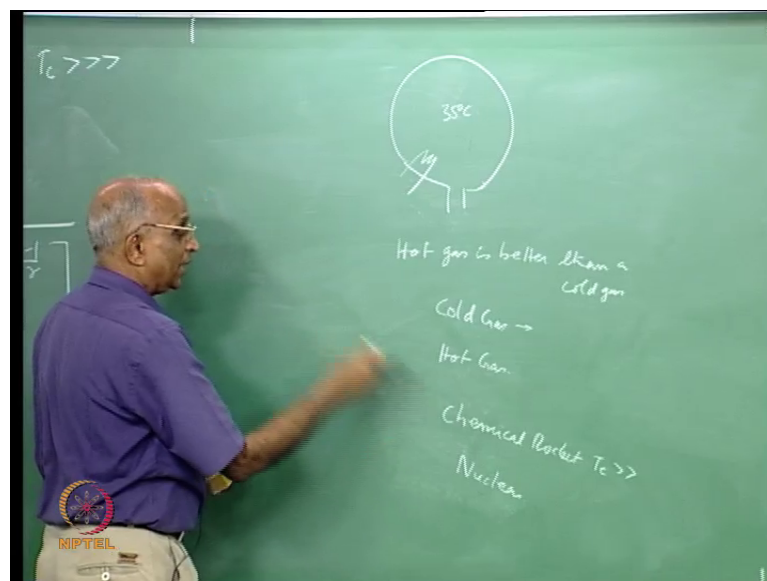


And therefore we can write the equation as the value of  $T_e/T_c$  is equal to  $(p_e/p_c)^{\gamma-1/\gamma}$ .

Therefore the expression for jet velocity  $V_J = \sqrt{2\gamma R_0 T_c / (\gamma - 1) M \{1 - (p_e/p_c)^{(\gamma-1)/\gamma}\}}$ , so many meters per second and this is the expression for the efflux jet velocity. That means we are able to derive an expression based on the assumption for perfect gas, the flow through the vent is adiabatic and it is reversible i.e., it is a slow flow. It gives us the jet velocity as so many meters per second. If this expression is clear, maybe we can draw some conclusions based on this equation and that is what we will be doing in the next couple of minutes. Is it clear how we get the jet velocity. The only assumption which we made is flow through the vent is adiabatic and also reversible because we set it as a slow process.

We will keep these assumptions in mind. Let us discuss the equation. When will we get a high value of  $V_J$ ? When the temperature  $T_c$  is very high. And what does this tell me? It tells me if I use something like a cold gas such something like

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let us say I take the same balloon and use it as a rocket, my balloon over here, I have this particular vent over here, I have the balloon which was initially filled with air at 35 degree centigrade; that means if my temperature is small, I do not get a very high value of  $V_J$ . If I can increase this temperature in some way I can get a much higher  $V_J$ , and that is why a hot gas is better than a cold gas. I could have a cold gas rocket, I could have a hot gas rocket but hot gas is definitely better. Therefore, first thing we say is  $T_c$  should



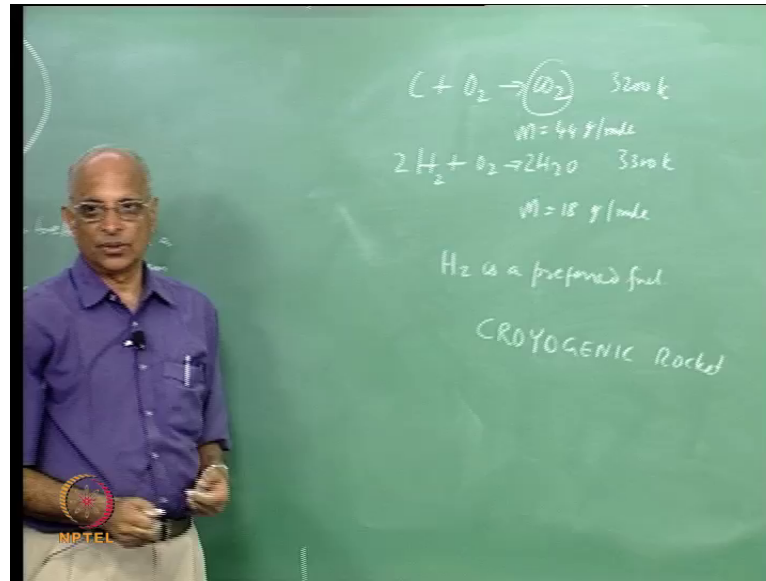
have the highest possible. But is there some limitation? The material must withstand the high temperature and therefore there is a limit to this temperature. We normally use temperatures of 3000 to 4000 Kelvin. And how do we generate it? We burn fuel and oxygen which we call as propellants; that means we use chemical reactions to generate high temperature or rather get a high value of  $T_c$ .

Therefore, a rocket could be a cold gas rocket, could be a hot gas rocket. In a chemical rocket you generate high temperature with chemical reactions, or we could introduce resistance wire to electrically heat the gases to a high temperature  $T_c$ . Cold gas rockets are also used wherein we need small jet velocities. We allow gas in a chamber and allow the gas to expand in a vent to give thrust. You have chemical rockets. You could also have nuclear reactions, and in nuclear reactions we can get an even higher temperature and can get a higher jet velocity. We have classification of rocket as cold gas rocket, hot gas rocket, chemical reaction rocket, electrical rocket, nuclear rocket and we say that temperature is one of the major parameters that contribute to jet velocity.

Let us examine the other parameters. Consider the exit pressure. If exit pressure is very small, well the fraction  $p_e$  by  $p_c$  becomes small. Therefore, we would like  $p_e$  must be small, or if my exit pressure cannot be small I can set  $p_c$  must be large. That means if we can store gas at very high pressure such that we can get a higher value of the pressure ratio; in other words the ratio  $p_e$  by  $p_c$  must be small are rather the chamber pressure must be high and the exit pressure must be small. This is the second conclusion from this equation for the jet velocity  $V_j$ .

Let us take a look at the molecular mass of the gases which are exhausted out through the vent. We find  $V_j$  is equal to  $1/\sqrt{M}$  i.e., one over under root of molecular mass; that means, if I can have a gas which is very light i.e., small molecular mass like hydrogen or let say or helium well my  $V_j$  will be higher, but how do I get a light gas. Let us take an example.

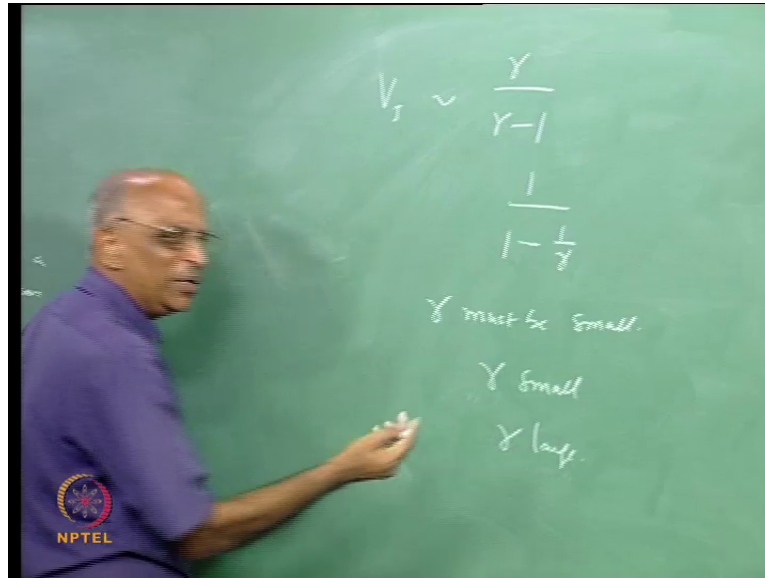
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Supposing I have a chemical rocket in which I take carbon, I burn it with oxygen and I get product combustion of carbon dioxide,  $C + O_2 = CO_2$ . We assume complete combustion. I have another rocket, I have Hydrogen  $H_2 + \frac{1}{2} O_2 = H_2O$ ; or  $2 H_2 + O_2 = 2 H_2O$ . What is the difference in these two rockets, as far as  $V_J$  is concerned? The first rocket gives a temperature  $T_c$  around 3200 Kelvin while the second one gives around 3300 Kelvin at the same pressure conditions. The temperatures are not very much different. We look at the molecular mass of carbon dioxide; the molecular mass is equal to 12 plus 32 = 44 gram per mole, while for water the molecular mass is 16 plus 2 = 18 g/mole. In other words if we burn hydrogen and oxygen we get a very low molecular mass and the  $V_J$  which is directly proportional to the under root of molecular mass will therefore be large and that is why we find hydrogen is a preferred fuel. Even if we have a solid propellant; we would like the solid propellant to contain as much hydrogen as possible. Since to carry hydrogen gas in bulk is very difficult, we liquefy hydrogen at low temperatures and carry it as liquid hydrogen. This is what we mean by cryogenic liquid propellant. Cryogenic propellant rockets have better performance. For the cryogenic propellant rockets, the specific impulse is about 4600 Newton second by kilogram whereas for an ordinary fuel it is of the order of 3500 Newton second by kilogram. You get immense benefit out of the smaller value of molecular mass.

Therefore we infer the parameters 1. Temperature, 2.  $P_e/p_c$  and 3. Molecular mass which must be as low as possible. Can we say what must gamma be like? Should gamma be small or large and if so why?

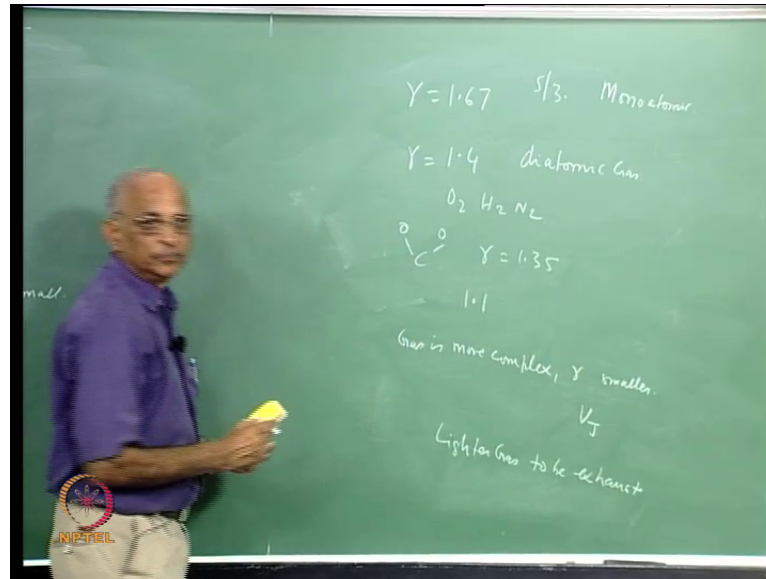
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We find  $V_J$  goes as  $\sqrt{\gamma/(\gamma-1)}$ . Therefore would we like a small value of gamma or large value of  $\gamma$ ? Let us divide numerator and denominator by  $\gamma$  to give  $1/(1 - 1/\gamma)$  within the underroot sign. We find if we have the smaller value of gamma, we subtract a larger number in the denominator and this gives a higher value for  $V_J$ . A small value for  $\gamma$  is preferable. Therefore gamma must be small. Because if gamma is small, we subtract a larger quantity and my denominator comes down and for the same numerator  $V_J$  is larger.

Therefore we also tell that  $\gamma$  must be small. What is the sensitivity of gamma? It is not inversely proportional as molecular mass because it is  $\gamma/(\gamma-1)$ ;  $\gamma$  is not very much influential. Let us take an example; how do I make gamma small, how do I make gamma large and how does gamma depend on the gas?

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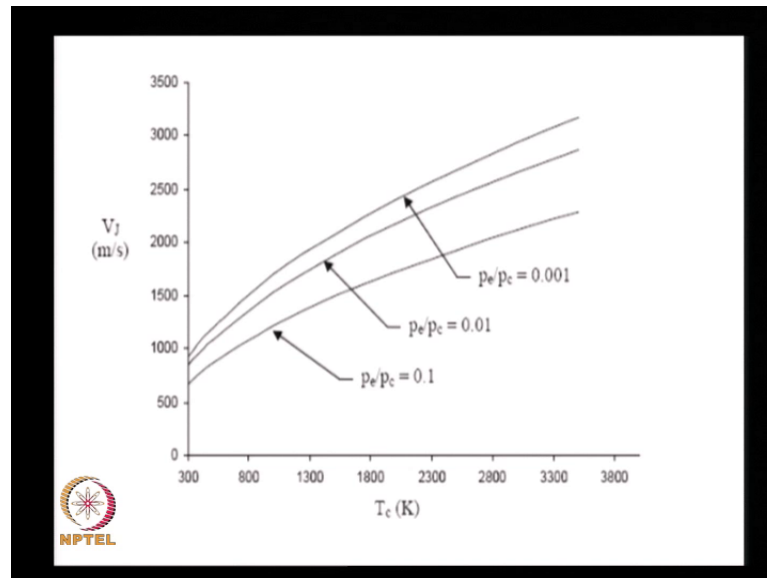
Let us consider helium; this is a mono atomic gas has a value of  $\gamma$  is equal to 1.67 or rather is equal to 5 by 3. If we take air  $\gamma = 1.4$ . If I take nitrogen or oxygen  $\gamma = 1.4$ . If we consider  $\text{CO}_2$ ,  $\gamma \approx 1.35$ . If the gas molecule is more complicated like we have Freon gas, used as refrigerant,  $\gamma \approx 1.1$ ; that means as the molecular mass of the gas increases the gamma becomes smaller. If the molecule of the gas is more complex, gamma is smaller, and therefore the influence on  $V_j$  will be better.

But when we looked at the molecular mass; if the exhaust gases have lower molecular mass, we obtained higher  $V_j$ . This is contradictory from viewpoint of  $\gamma$ . The role of molecular mass and gamma is just the opposite, but it so happens the effect of gamma is much smaller than the molecular mass and therefore we would still like to have a lighter gas to be exhausted out.

I will just repeat the four salient conclusion which I draw from this particular jet velocity equation.

The temperature of the gases must be as high as possible; the ratio  $p_e/p_c$  must be small are rather the ratio of the chamber pressure to the exit pressure as must be large as possible; molecular mass of the gas which is coming out must be as small as possible and lastly we would like gamma to be small, but this gamma being small contradicts the requirements of the small molecular mass, and therefore we do not really pay much attention to gamma since it is not as influential as molecular mass.

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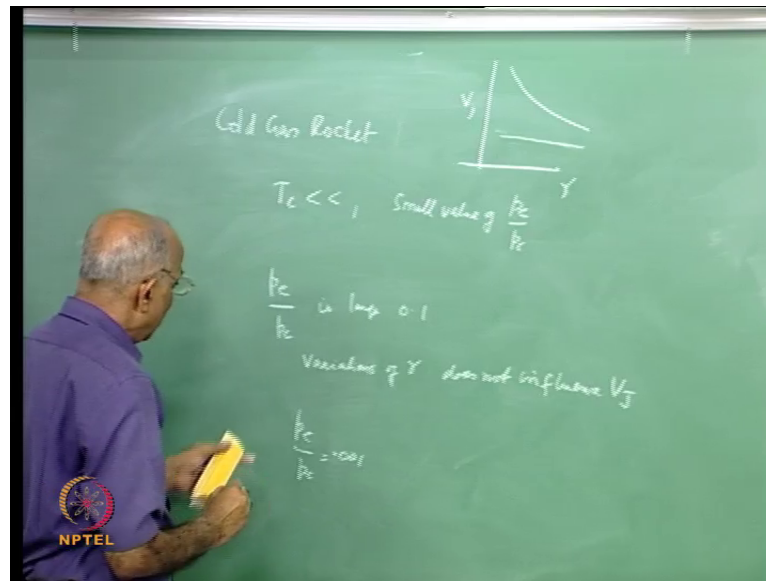


Let us quickly go through the conclusions on the above slide, because I plotted the equations for different values of temperature. In the first slide, I have plotted the value of  $V_j$  in meters per second as the function of temperature, the temperature varies from 300 to 3800 Kelvin. The range of  $T_c$  in chemical rockets is of the order of maybe something like 3000 to 4000 Kelvin, much lower than 4000 Kelvin, and therefore I restrict myself to 3800 here. I also plot for different values of  $p_e$  by  $p_c$ . You find as the value of  $p_e$  by  $p_c$  decreases I get the higher value of jet velocity, and this is the conclusion that we drew looking at the equation.

If I have say a gas at a low temperature such as used in a cold gas rocket and if I decrease the value of  $p_e$  by  $p_c$  from 0.1 to 0.001 I do not really get much benefit, whereas if I have a high temperature rocket;  $T_c$  is high and I get much larger benefit. Therefore if we were to design a cold gas rocket I can make a small rocket and I do not need to really expand it much. I can as well have a lower chamber pressure.

Therefore a conclusion that we could draw is maybe for a cold gas rocket, and let us put it down as an important conclusion: for a cold gas rocket for which  $T_c$  is quite small we do not really require a small value of  $p_e$  by  $p_c$ .

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Because you find when the temperature is small, the gain what we got in expanding the gas by a large amount is very small while if I have really a high temperature chemical rocket this gives me higher temperature and much better gain in  $V_j$ .

In the next slide, the value of the jet velocity in meter per second as a function of the molecular mass is given. I find when the molecular mass is small, we get the value of  $V_j$  which is higher, and of course the same trend continues. This is at a mean temperature of 3000 Kelvin. This tell me very clearly that as the molecular mass is smaller, we get a higher value of  $V_j$ .

In the last slide, I show the influence of  $\gamma$ ; we show the value of  $V_j$  as a function of gamma what we find is at a low value of  $p_c/p_c$  of 0.1, gamma really does not influence  $V_j$ . We find that the curve is quite flat and it is independent of gamma as it were. However, when the value of  $p_c/p_c$  is quite small of the order of 0.001, I find as gamma increases I get a smaller value of  $V_j$ . The conclusions which we drew that as gamma decreases, the value of  $V_j$  increases is seen to be more effective at smaller value  $p_c/p_c$ . Whereas when I have a higher value of  $p_c/p_c$  such as 0.1, the effect of gamma changes does not influence  $V_j$ .

If we have a rocket for which the expansion ratio is not very high like in a cold gas rocket, we can even use helium for which gamma is 1.67. Whether I use helium or we use air with gamma of 1.4, it really does not make things worse, and therefore a cold gas

rocket normally uses a light gas like helium, and helium has low molecular mass around 4 gram per mole, and therefore we get the benefit of the molecular mass and we do not lose any effect due to gamma.

Therefore what is it we have done thus far. We derived this particular equation for  $V_J$ , we looked at the effect of temperature, molecular mass, expansion ratio and also  $\gamma$  on  $V_J$ . We found for a cold gas rocket  $p_e/p_c$  need not be very high and the effect of gamma is not dominant. But if we were to look at the effect of temperature in a chemical rocket for which temperature is high, we can operate at a much lower value of  $p_e/p_c$  and gamma effects will also become significant. Therefore by now we must be very clear that if where to have a choice of propellants for my rocket, we must have propellants which will generate a high temperature and a low molecular mass gas. We do not have much control over  $\gamma$ ;  $\gamma$  is not very controlling provided the pressure ratio is not too low.

I think this is all about this jet velocity and we have to now relate it to choice of propellants viz., solid propellant rockets, liquid propellant rockets and other forms of rockets. In the next class what we do is address the shape of this vent which we give us a high value of velocity. In other words we move into the chapter of nozzle, shaping of nozzle and what are the problems with nozzles.