Mass Transfer Operations - I
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Diffusion Mass Transfer - II
Lecture - 05
Diffusion through variable cross-sectional area

Welcome to the 5th lecture on Mass Transfer Operation -I. We are discussing a diffusion mass transfer. Before going through this discussion, let us have recap on our previous lecture.

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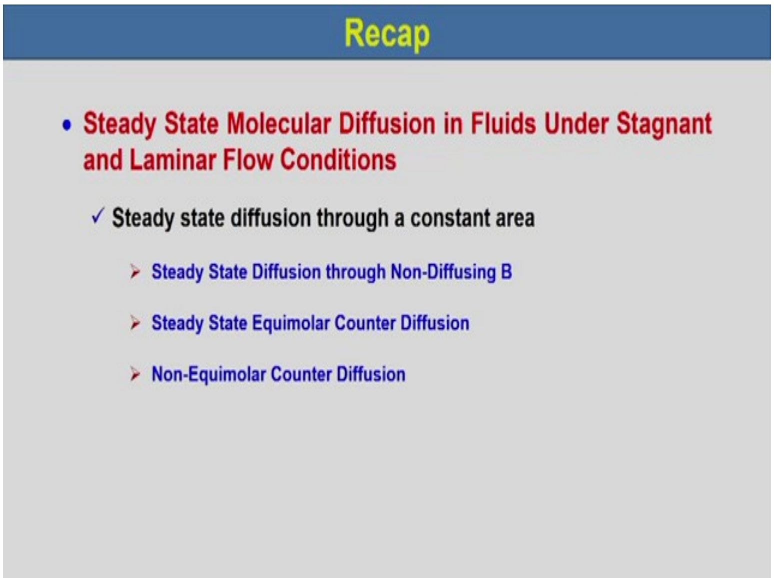

We have considered Steady State Molecular Diffusion in fluids under stagnant and laminar flow conditions. Under this the first case we have considered Steady state molecular diffusion through a constant area. Under constant area we have considered the first case is Steady State Diffusion through Non-Diffusing B, Steady State Equimolar Counter Diffusion and the 3rd case we have considered Non-Equimolar Counter Diffusion.

In this lecture we will consider Steady State diffusion through a variable area.

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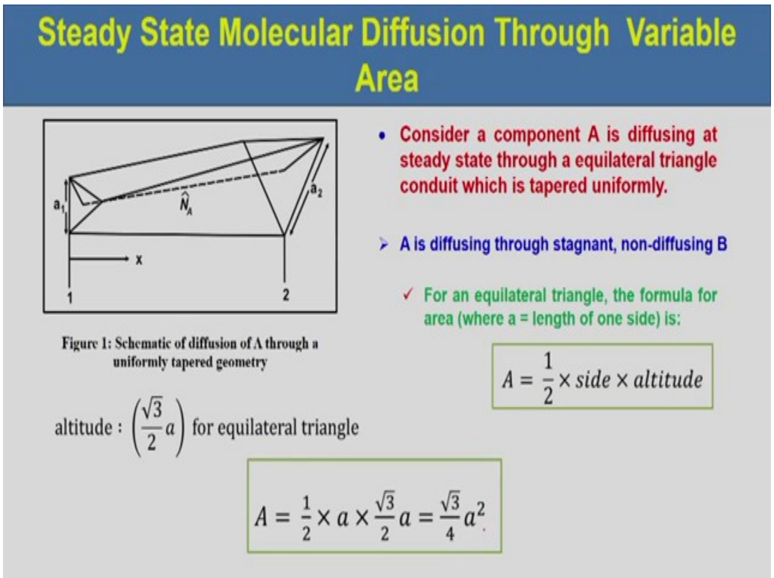

The left hand side of this figure shows the schematic of diffusion of a through a uniformly tapered geometry. So, as you can see the area on the right hand side of the figure of a triangular triangle whose side is a 2 to which is tapered towards the left side of this figure which is that the triangular shape no face which is area of a 1. Obviously, a 1 is at point 1 and a 2 is at point 2 and a 1 is less than a 2. So, consider a component a is diffusing at steady state through a equimolar triangle conduit which is tapered uniformly.

Let us consider A is diffusing through stagnant non-diffusing B component. Now, for an equilateral triangle the formula for area where a is the length of one side can be written as A is equal to half into side into altitude. The altitude of triangle, equilateral triangle is root 3 by 2 into a. A is the side of the triangle. Now, the area A would be half into side is a into the altitude which is root 3 by 2 a which is equal to root 3 by 4 a square.

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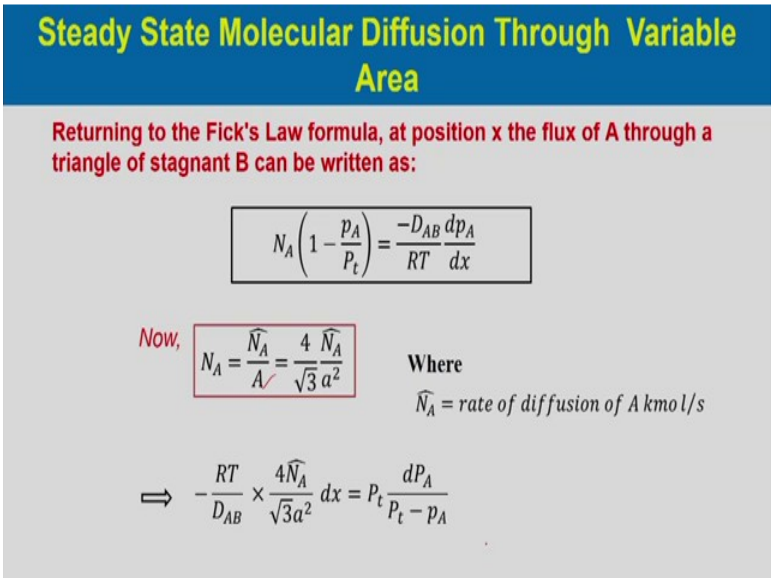

So, now returning to the Fick's law formula at position x of A through a triangle of stagnant B can be written as N A into 1 minus p A by P t is equal to minus D AB by RTdp A dx.

N A is the flux, molar flux and p A is the partial pressure of a component, P t is the total pressure D AB is the binary diffusion coefficient of component A into the non-diffusing B, R is the universal gas constant, T is the temperature and dp A by d x is the partial pressure gradient with respect to x. Now, if we define N A is equal to N A cap divided by A where N A cap is the molar flow rate which would be equal to if we equal to 4 by root 3 N A cap by a square.

If we substitute the area in this equations as we have calculated before, so N A as we said N A is the rate of diffusion of A in kilomole per second. So, the above equations can be written as minus RT by D AB into 4 NA cap by root of our 3 a square d x which would be equal to P t into d PA by P t minus p A.

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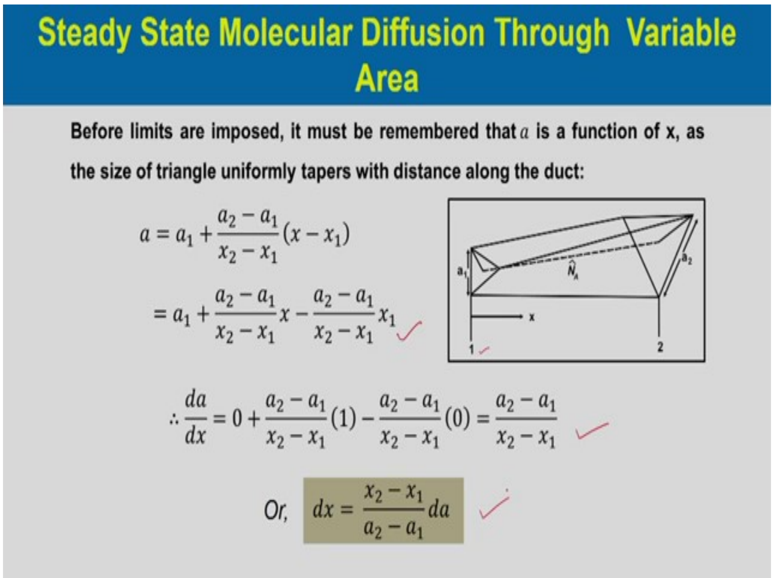

Now, before limits are imposed it must be remembered that a is a function of x. That means, a is varying with respect to the distance x as the size of the triangle uniformly tapered with distance along the duct. So, we need to have a relation between a and x. Now, a would be equal to a 1 which is at point 1 in this figure plus a 2 minus a 1 divided by x 2 minus x 1 into x minus x 1.

So, this is the relation between the a and the distance. if we just simplify, it would be equal to a 1 plus a 2 minus a 1 by x 2 minus x 1 into x minus a 2 minus a 1 divided by x 2 minus x 1 into x 1. So, if we just differentiate with respect to x this equation, then it would be d a by d x would be equal to 0 plus a 2 minus a 1 by x 2 minus x 1 into 1 minus a 2 minus a 1 divided by x 2 minus x 1 into 0, so which will be simplified to a 2 minus a 1 by x 2 minus x 1. Therefore, d x would be equal to x 2 minus x 1 divided by a 2 minus a 1 into d a. So, we have a relation between d x and d a.

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Steady State Molecular Diffusion Through Variable Area

Substituting x for a, and integrating with limits of p_{A1} at triangle of side a_1 and p_{A2} when the triangle is of side a_2:

$$\frac{-RT\widehat{N_A}}{D_{AB}}\frac{x_2-x_1}{a_2-a_1}\int_{a_1}^{a_2}\frac{4}{\sqrt{3}a^2}da = P_t\int_{p_{A_1}}^{p_{A_2}}\frac{dp_A}{P_t-p_A}$$

$$\Rightarrow \frac{-4RT\widehat{N_A}}{\sqrt{3}D_{AB}}\frac{x_2-x_1}{a_2-a_1}\left[\frac{-1}{a}\right]_{a_1}^{a_2} = -[\ln(P_t-p_A)]_{p_{A_1}}^{p_{A_2}}$$

$$\Rightarrow \frac{4RT\widehat{N_A}}{\sqrt{3}D_{AB}}\frac{x_2-x_1}{a_2-a_1}\left(\frac{1}{a_1}-\frac{1}{a_2}\right) = \ln\left(\frac{P_t-p_{A_2}}{P_t-p_{A_1}}\right)$$

$$\Rightarrow \widehat{N_A}\left(\frac{a_1-a_2}{a_1a_2}\right) = \frac{\sqrt{3}D_{AB}P_t}{4RT}\left(\frac{a_2-a_1}{x_2-x_1}\right)\ln\left(\frac{P_t-p_{A_2}}{P_t-p_{A_1}}\right)$$

Now, if we substitute x for a and integrating with limits of partial pressure of component a at 1 at triangle of side a 1 and partial pressure of component a at point 2, that is p A2 when the triangle is at is of side a 2, we can write minus RT NA cap divided by D AB into x 2 minus x 1 by a 2 minus a 1 integral a 1 to a 2 4 divided by root 3 a square d a which would be equal to p t integral p A1 to p A2 d pA divided by P t minus p A.

So, this we can simplify we can integrate this and we will obtain minus 4 RT NA cap divided by root 3 D AB into x 2 minus x 1 divided by a 2 minus a 1 into minus 1 to minus 1 divided by a with a limit a 1 to a 2. So, this would be equal to minus ln Pt minus PA with a limit pA1 to p2 and hence, the left hand side can be written as 4 RT NA cap divided by root 3 D AB into x 2 minus x 1 divided by a 2 minus a 1 into 1 by a 1 minus 1 by a 2 if we put the limit in the earlier equation which would be equal to ln Pt minus p A2 divided by p T minus p A1.

If we simplify this it would be equal to NA cap a1 minus a2 divided by a1 into a2 which would be equal to root 3 D AB P t by 4 RT into a2 minus a1 divided by x2 minus x1 ln Pt minus p A2 by p T minus p A1.

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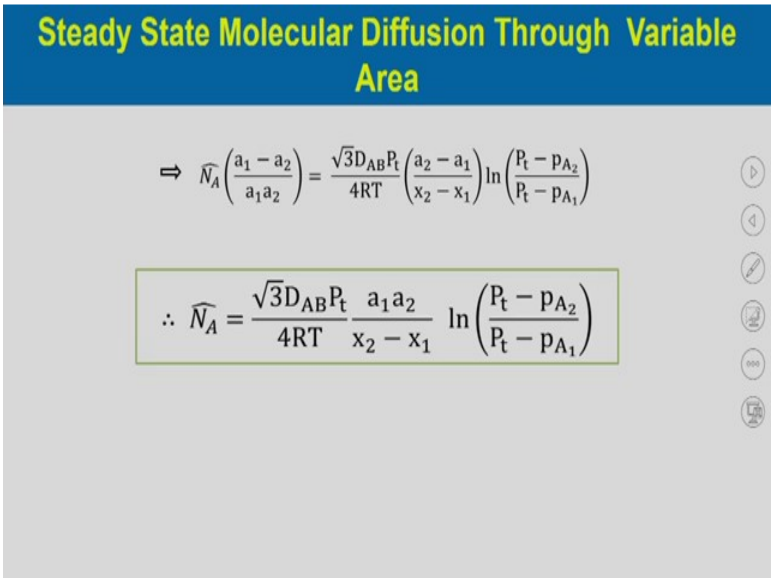

So, now this equation we can just rearrange and simplify from where we can write NA cap would be equal to root 3 D AB by 4 RT into P t into a 1 into a 2 divided by x 2 minus x 1 into ln Pt minus p A2 by P t minus p A1. So, this is the final one equation for Steady State Molecular Diffusion Through Variable Area.

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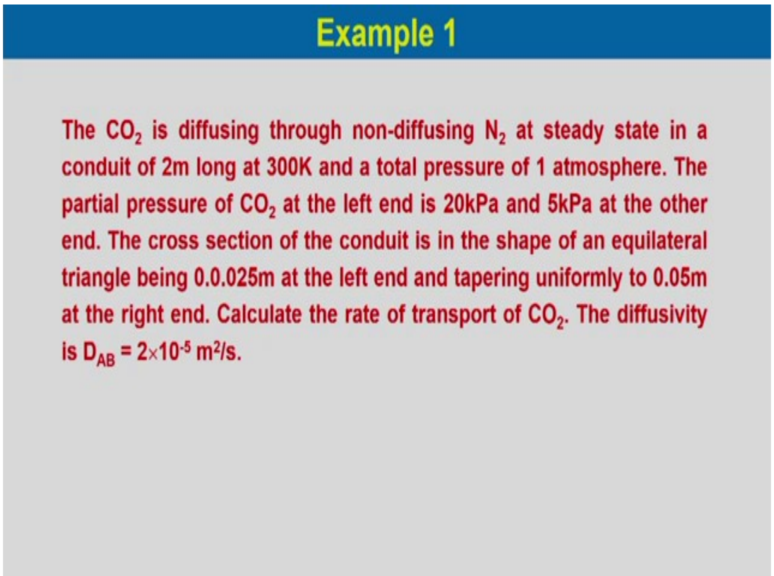

Now, let us consider one example. The carbon dioxide is diffusing through non-diffusing nitrogen at steady state in a conduit of 2 metre long at 300 Kelvin and a total pressure of 1 atmosphere. The partial pressure of carbon dioxide at the left end is 20 kilopascal and 5 kilopascal at the other end. The cross section of the conduit is in the shape of an equilateral triangle being 0.025 metre at the left and tapering uniformly to 0.05 metre at the right end.

Now, we need to calculate the rate of transport of carbon dioxide. The diffusivity is given for D AB equal to 2 into 10 to the power minus 5 metre square per second. So, the value which are given diffusion coefficient which is equal to 2 into 10 to the power minus 5 metre square per second.

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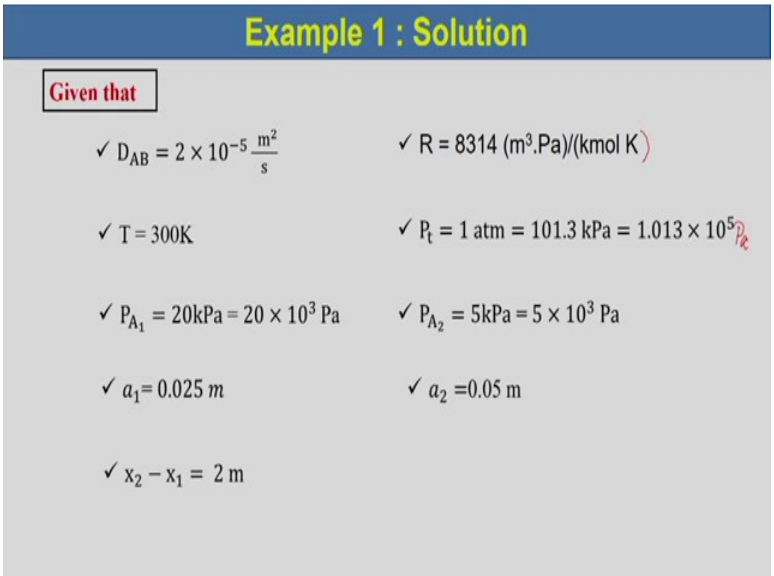

R is known to us 8314 metre cube pascal per kilomole kelvin, T is the temperature which is given at 300 kelvin total pressure Pt is 1 atmosphere. So, it is 101.3 kilopascal which is equal to 101.3 into 10 to the power 5 pascal, P A1 is equal to 20 kilopascal which is equal to 20 into 10 to the power 3 pascal, P A2 is 5 kilopascal which is 5 into 10 to the power 3 pascal, a1 is equal to 0.025 metre, a2 is equal to 0.05 metre, x 2 minus x 1 is 2 metre the distance between the two end.

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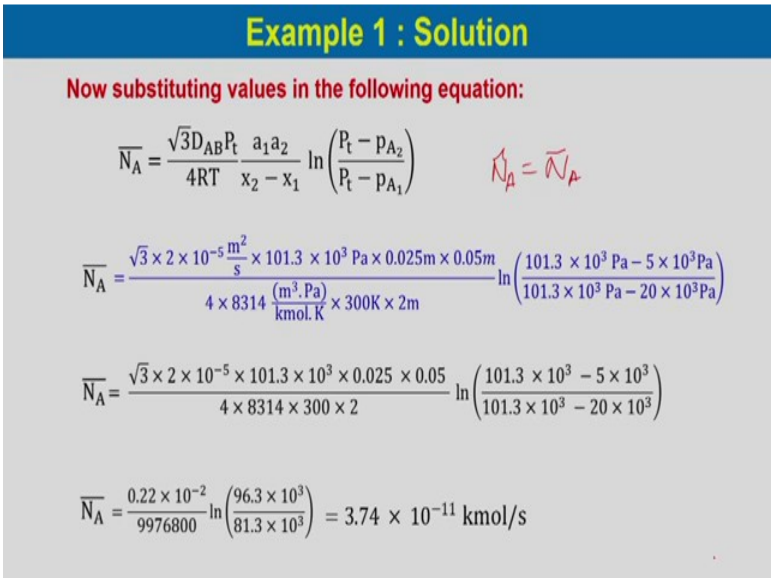

Now, substituting the values given in the following equations NA bar would be equal to root 3D AB Pt divided by 4RT into a 1 a 2 into a 1 into a 2 divided by x 2 minus x 1 into ln Pt minus p A2 divided by P t minus p A1. So, this NA cap is equal to NA bar here. Now, if we substitute the values root 3 into 2 into 10 to the power minus 5 metre square per second for diffusion coefficient, total pressure multiplied by the total pressure 101.3 into 10 to the power 3 pascal into a 1.

The length of side at point 1 of the triangle which is 0.025 metre and then a 2 which is that side of the triangle at point 2, that is 0.05 metre divided by 4 into r 8314 metre cube pascal by k mole into Kelvin into temperature is 300 Kelvin, the distance x 2 minus x 1 is 2 metre into ln Pt is given 101.3 into 10 to the power 3 Pascal minus P a 2 is 5 into 10 to the power 3 Pascal divided by P t which is 101.3 into 10 to the power 3 Pascal minus 20 into 10 to the power 3 Pascal.

So, if we just calculate, it would be equal to 3.74 into 10 to the power minus 11 kilo mole per second. So, this is the molar flow rate which is given in the problem to calculate.

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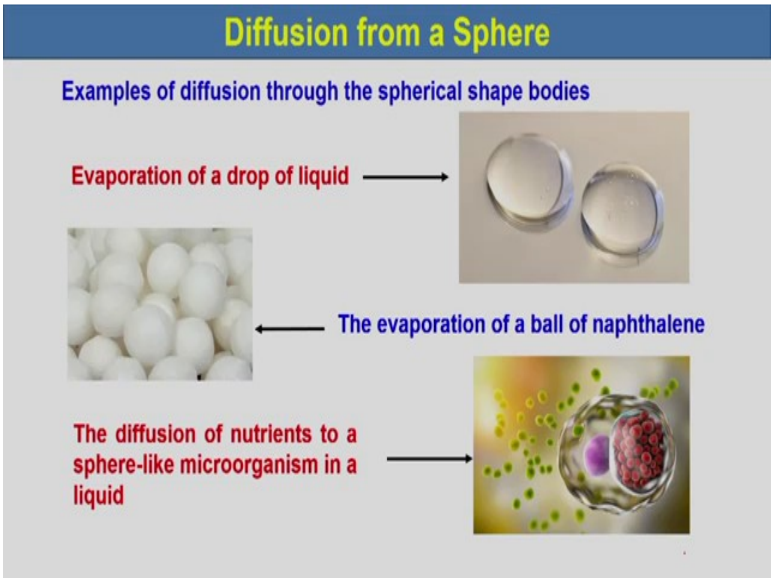

Now, let us consider diffusion from a sphere where the area will vary. Examples of diffusion through the spherical shape bodies one of them is the evaporation of a drop of liquid. So, if we take a drop of liquid and just put on a plane surface, the liquid droplet will slowly evaporate and its diameter will reduce slowly and hence, the area for the diffusions will also vary.

Another example is the evaporations of a ball of naphthalene if we just keep spherical naphthalene ball on to the air, then it will diffuse and slowly its diameter will decrease because the naphthalene will diffuse to the air. This is another example and then, the third examples is the diffusion of nutrient to a sphere like microorganism in a liquid.

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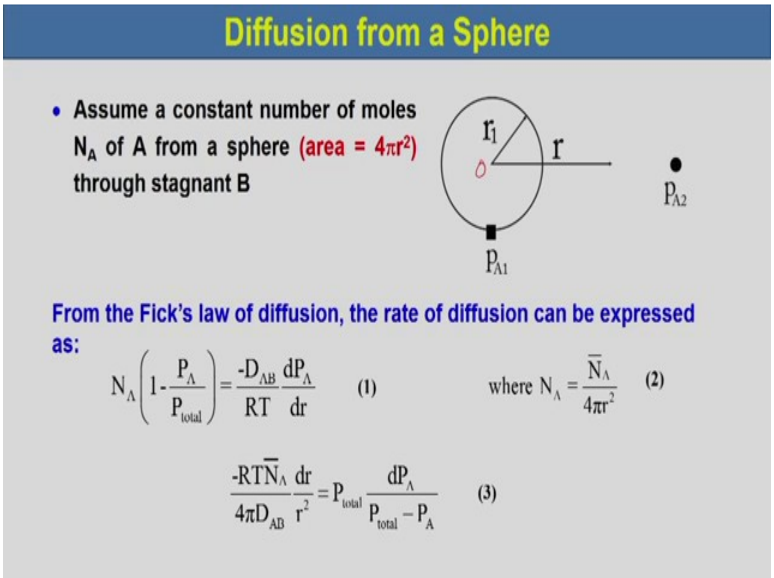

So, let us consider a sphere whose centre is O and the radius is r 1 and then, the area radius is varying with time and the partial pressure at the bulk is P A2 and at the surface of the sphere is P A1.

Assume a constant number of moles NA of A from a sphere whose area is equal to 4 pi r square through stagnant B. Now, if we consider Fick's law of diffusion, the rate of diffusion can be expressed as NA into 1 minus PA by P total PA is the partial pressure of component A which is equal to minus D AB by RT into dp A dr where NA is equal to NA bar by 4 pi r square. So, we can just substitute this NA is equal to NA bar 4 pi r square. The equation 1 would be equal to minus RT NA bar divided by 4 pi D AB into dr by r square which would be equal to P total into dP A by P total minus PA.

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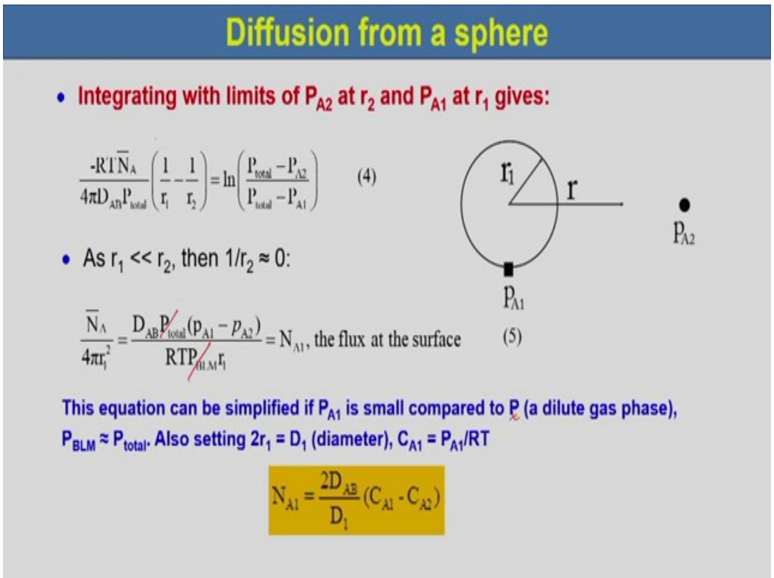

Now, integrating with limits of P A2 at r 2 and P A1 at r 1, it gives minus RTN A bar divided by 4 pi D AB P total into 1 by r 1 minus 1 by r 2 is equal to ln P t or P total minus p A2 divided by P total minus p A1. Now, as r 1 is very very less than r 2 which is at far away, then 1 by r 2 would be approximately equal to 0. So, then the above equation 4 would be reduced to NA bar by 4 pi r 1 square into D AB. The above equations would be reduced to NA bar divided by 4 pi r 1 square equal to D AB P total into p A1 minus p A2 divided by RTP B ln r 1 which would be equal to NA1 that is the flux at the surface.

P BLM is the log mean partial pressure difference as we have discussed before. Now, these equations can be simplified if p A1 is small compared to p that is total pressure P t since a dilute gas phase, then P BLM would be approximately equal to P total. Now, also if we set 2 r 1 is equal to D 1 that is diameter and then, C A1 would be equal to p A1 by RT the above equation , this equation. P BLM and P total will be cancelled out and P A1 by RT would be C A1 which is the concentration at point 1 of component A and P A2 by r 2 would be equal to C A2 that is at concentration of component A at point 2. So, the above equations will reduce to NA1 would be equal to twice D AB by D 1 into C A1 minus C A2.

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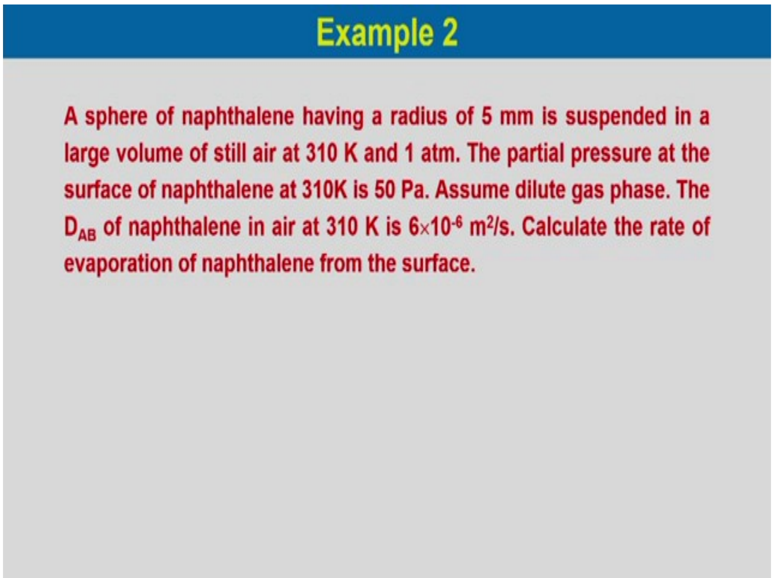

Now, let us consider one example. A sphere of naphthalene having a radius of 5 millimetre is suspended in a large volume of still air at 310 Kelvin and 1 atmospheric pressure. The partial pressure at the surface of naphthalene at 310 Kelvin is 50 Pascal. Assume dilute gas phase. The diffusion coefficient of component that is D AB of naphthalene in air is at 310 Kelvin is given as 6 into 10 to the power minus 6 metre square per second.

Now, we need to calculate the rate of evaporation of naphthalene from the surface.

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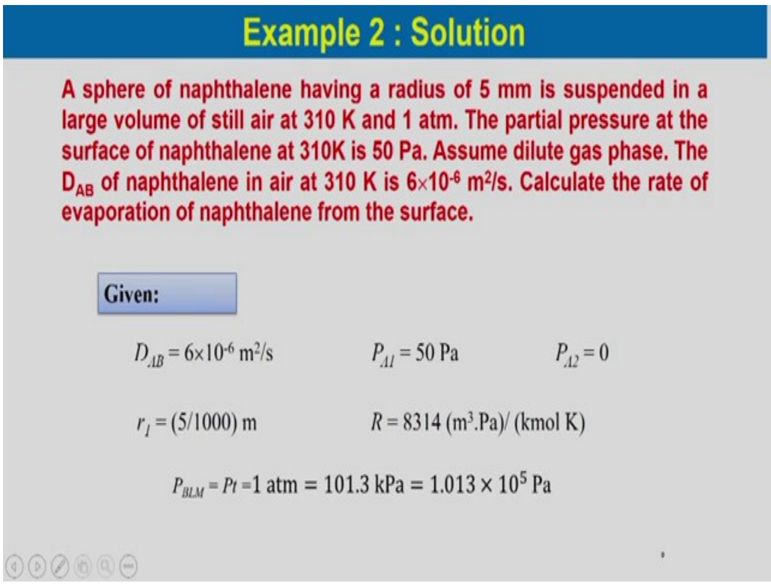

So, what are the parameters given? Let us note them down. D AB the diffusion coefficient of naphthalene into air. Air is component B which is equal to 6 into 10 to the power minus 6 metre square per second. Partial pressure of component A at point 1 is 50 Pascal and at P A2 is equal to 0 because it is dilute gas phase, r 1 is equal to 5 by 1000 metre since it is 5 millimeter, Capital R which is universal gas constant is known to us 8314 metre cube Pascal divided by kilo mole Kelvin. P BLM since it is dilute gas P BLM would be equal to the total pressure P t which is equal to 1 atmosphere we can write this is 101.3 kilopascal, which is equal to 1.013 into 10 to the power 5 Pascal.

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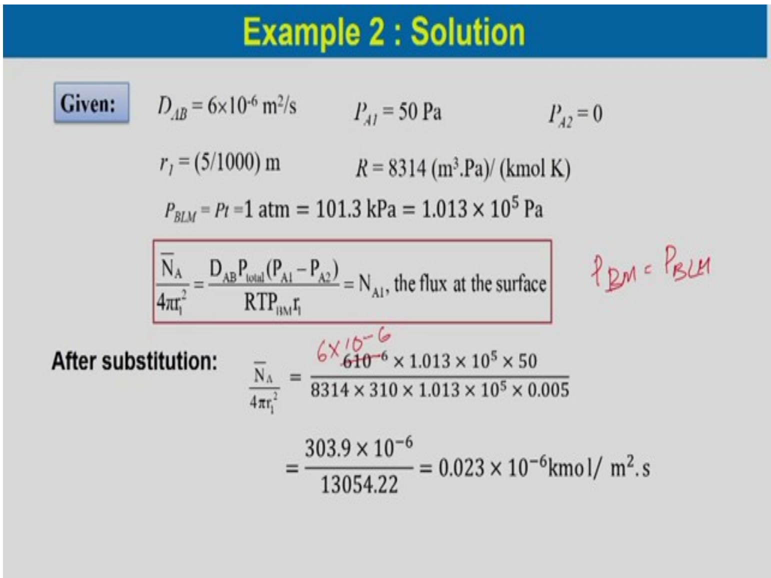

Now, these are the parameters known to us and this is the relations of the flux at the surface. We know for the spherical surface NA cap or NA bar divided by 4 pi r 1 square is equal to D AB P total P A1 minus P A2 divided by RTP BM r 1. Here P BM is P BLM that is log mean partial pressure difference which is equal to N A1, the flux at the surface. So, if you substitute the values which are given over there, it would be 6 into 10 to the power minus 6 into 1.013 into 10 to the power 5 into 50 divided by 8314 into 31 0 into 1.013 into 10 to the power 5 into 0.005.

So, this would be equal to 303.9 into 10 to the power minus 6 divided by 13054.22 which is equal to 0.023 into 10 to the power minus 6 kilomole per metre square second. So, this is the flux for the ball at the naphthalene at the surface.

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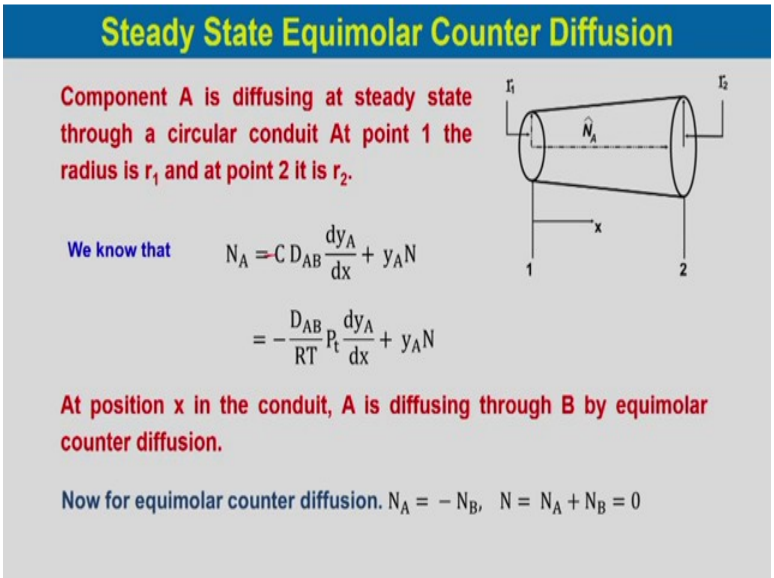

Now, let us consider steady state equimolar counter diffusion. So, this is a cylindrical geometry uniformly tapered. So, component A is diffusing at steady state through a circular conduit. At point 1 the radius is r 1 and at point 2 it is r 2.

So, we know that N A is equal to minus C into D AB dy A dx plus y A N. Now, it would be minus D AB by RT into P t dy A dx plus ya N. At position x in the conduit, A is diffusing through B by equimolar counter diffusion. Now, for equimolar counter diffusion we know that N A would be equal to minus N B. So, the total molar flux N would be equal to N A plus N B which would be equal to 0.

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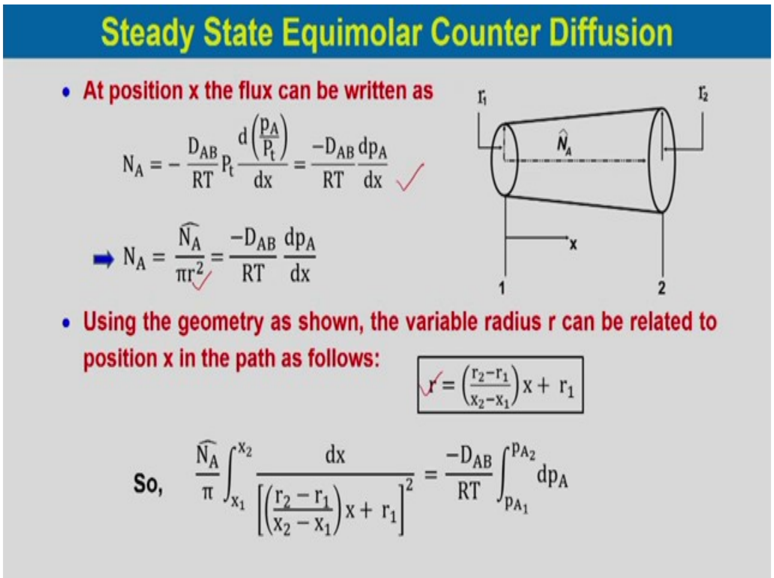

At position x the flux can be written as N A would be equal to minus D AB by RT into P t into dp A by p t divided by d x is equal to minus D AB by RT into dp A dx. Now, from this we can write N A would be equal to N A cap by pi r square which is equal to minus D AB by RT into dp A dx. Now, using the geometry as shown the variable radius r can be related to position x in the path as follows r would be equal to r 2 minus r 1 divided by x 2 minus x 1 into x plus r 1. Now, if we substitute this value of r into over here, the above equations will lead to N A k f (cap) by pi integral x 1 to x 2 d x divided by r 2 minus r 1 by x 2 minus x 1 into x plus r 1 whole square which would be equal to minus D AB by RT integral p A1 to p A2 into dp A.

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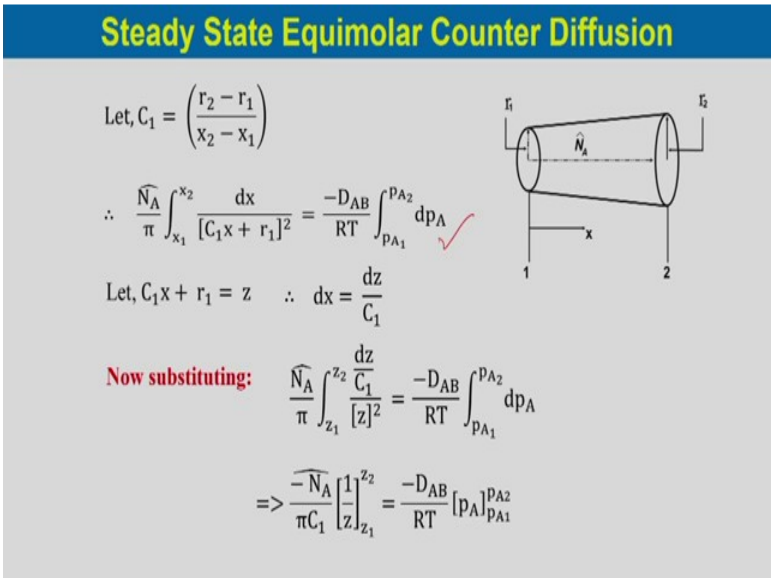

So, now let us consider C 1 is equal to r 2 minus r 1 divided by x 2 minus x 1. So, we can write the above equation as N A cap by pi integral x 1 to x 2 d x divided by C 1 x plus 1 whole square which would be equal to minus D AB by RT integral p A1 to p A2 dp A.

Let C1 x plus r1 is equal to z. So, if we just differentiate, then it would be dx would be equal to dz by C1. Now, if we substitute the above equations, this equation would be NA cap by pi integral z 1 by z 2 d z by C1 whole divided by z square would be equal to minus D AB by RT integral p A1 to p A2 dp A. So, if we integrate it would be minus N A cap divided by pi C 1 into 1 by z with a limit z 1 to z 2 which would be equal to minus D AB by RT into p A with a limit p A1 to p A2.

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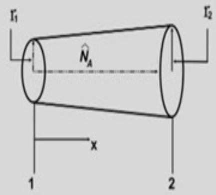
Steady State Equimolar Counter Diffusion

$$\Rightarrow \frac{\hat{N}_A}{\pi C_1}\left[\frac{1}{z_1}-\frac{1}{z_2}\right]=\frac{D_{AB}}{RT}\left[p_{A_1}-p_{A_2}\right]$$

$$\Rightarrow \frac{\hat{N}_A}{\pi C_1}\left[\frac{1}{C_1x_1+r_1}-\frac{1}{C_1x_2+r_1}\right]=\frac{D_{AB}}{RT}\left[p_{A_1}-p_{A_2}\right]$$

$$\Rightarrow \frac{\hat{N}_A}{\pi\left(\frac{r_2-r_1}{x_2-x_1}\right)}\left[\frac{1}{\left(\frac{r_2-r_1}{x_2-x_1}\right)x_1+r_1}-\frac{1}{\left(\frac{r_2-r_1}{x_2-x_1}\right)x_2+r_1}\right]=\frac{D_{AB}}{RT}\left[p_{A_1}-p_{A_2}\right]$$

$$\Rightarrow \hat{N}_A=\frac{D_{AB}}{RT}\frac{\pi\left(\frac{r_2-r_1}{x_2-x_1}\right)\left[p_{A_1}-p_{A_2}\right]}{\left[\frac{1}{\left(\frac{r_2-r_1}{x_2-x_1}\right)x_2+r_1}-\frac{1}{\left(\frac{r_2-r_1}{x_2-x_1}\right)x_1+r_1}\right]}$$



So, this would be written as N A cap by pi into C 1 whole into 1 by z 1 minus 1 by z 2 which is equal to D AB by RT into p A1 minus p A2.

So, from there if we substitute z 1 and z 2, then it would be N A cap by pi C 1 into 1 by C 1 x 1 plus r 1 minus 1 by C1 x 2 plus r 1 which would be equal to D AB by RT p A1 minus p A2. So, now if we substitute C A1, it would be N A cap divided by pi r 2 minus r 1 divided by x 2 minus x 1 whole into 1 by r 2 minus r 1 divided by x 2 minus x 1 into x 1 plus r 1 minus 1 by r 2 minus r 1 divided by x 2 minus x 1 into x 2 plus r 1 which would be equal to D AB by RT p A1 minus p A2.

So, finally if we rearrange the molar flow rate NA cap would be equal to D AB by RT into pi r 2 minus r 1 by x 2 minus x 1 into the partial pressure gradient p A1 minus p A2 whole divided by 1 by r 2 minus r 1 divided by x 2 minus x 1 into x 2 plus r 1 minus 1 by r 2 minus r 1 by x 2 minus x 1 into x 1 plus r 1. So, we would obtain this relation.

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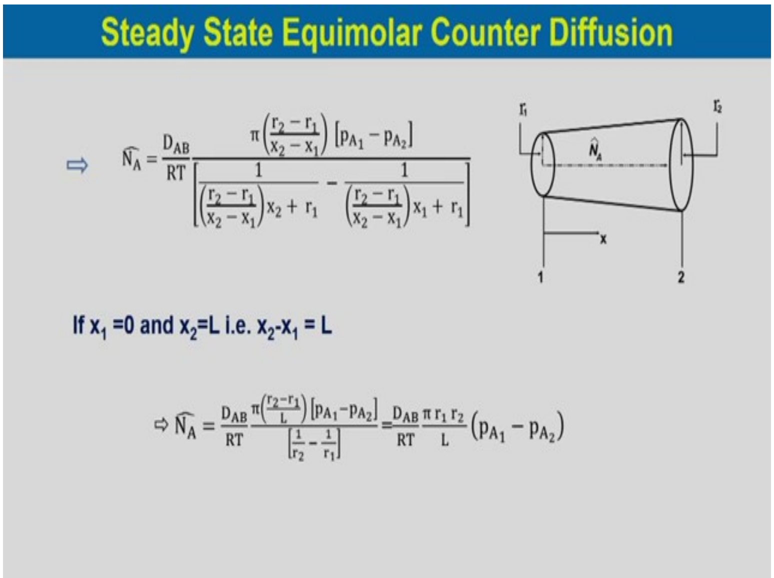

Now, after now if we put x 1 is equal to 0 and x 2 is equal to L and x, that means x 2 minus x 1 is L. The total distance from point 1 to point 2 the above equations, we can write N A cap would be equal to D AB by RT into pi r 2 minus r 1 by L into p A1 minus p A2 divided by 1 by r 2 minus 1 by r 1 which would be equal to D AB by RT into pi r 1 into r 2 by L into the partial pressure gradient p A1 minus p A2.

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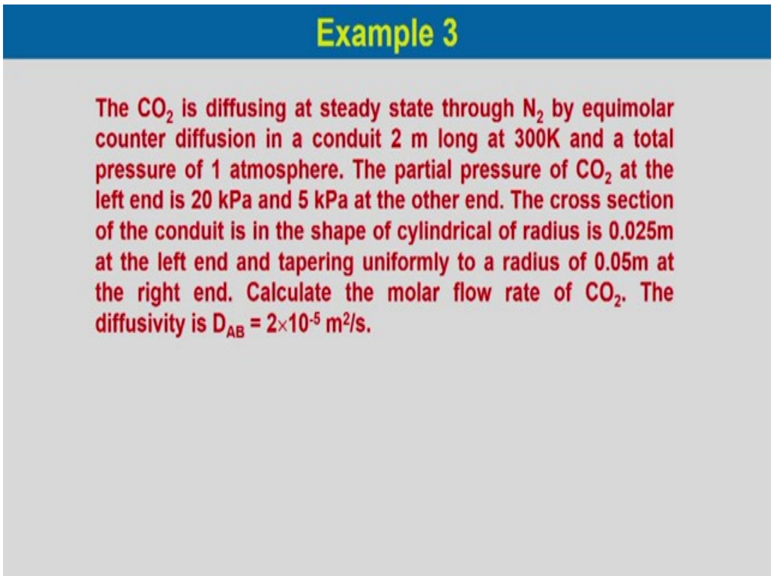

Now, let us consider one example. The carbon dioxide is diffusing at study state through nitrogen by equimolar counter diffusion in a conduit of 2 meter length at 300 Kelvin and a total pressure of 1 atmosphere. The partial pressure of carbon dioxide at the left end is 20 kilo pascal and 5 kilo pascal at the other end. The cross section of the conduit is in the shape of cylindrical of radius 0.025 metre at the left end and tapering uniformly to a radius of 0.05 meter at the right end. We need to calculate the molar flow rate of carbon dioxide. The diffusivity of the component is D AB is equal to 2 into 10 to the power minus 5 meter square per second.

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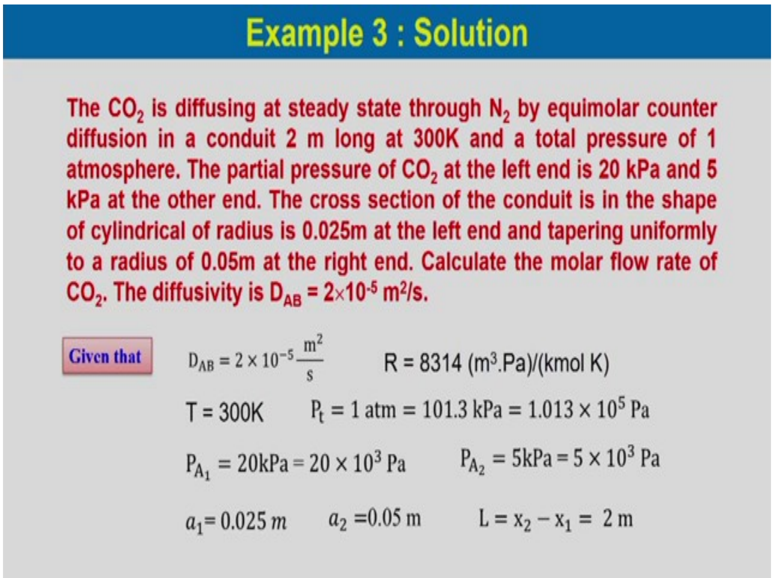

Now, the data which are given can be noted D AB is equal to 2 into 10 to the power minus 5 meter square per second, R is 8314 meter cube Pascal per kilo mol Kelvin, T is the temperature which is 300 Kelvin, P t is the total pressure which is at atmospheric pressure which is equal to 101.3 kilo Pascal which equal to 1.013 into 10 to the power 5 Pascal , P A1 is 20 kilo Pascal which is equal to 20 into 10 to the power 3 Pascal and P A2 is 5 kilo Pascal which is 5 into 10 to the power 3 Pascal, a 1 is 0.025 meter and a 2 is 0.05 meter, the distance between the two end or two point x 1 and x 2 point which is L is equal to x 2 minus x 1 which is 2 meter.

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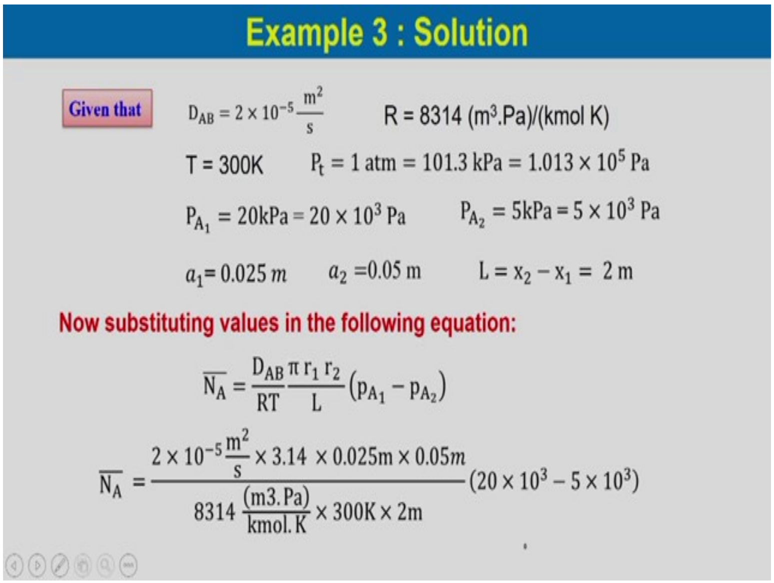

Now, if we substitute the parameters which are given D AB RT Pt and partial pressure of component a at 1 and partial pressure of component two component a at 2 and then, the other values a 1 a 2 and the length.

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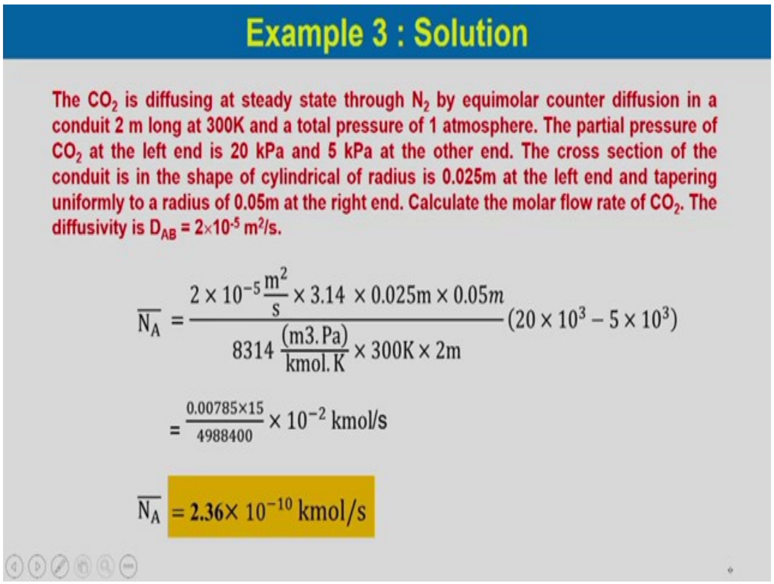

So, if you substitute these values in the following equation, you would obtain N A bar would be equal to this which would be equal to 0.00785 into 15 divided by 4988400 into 10 to the power minus 2 kilo mole per second which is equal to N A bar is equal to 2.36 into 10 to the power minus 10 kilo mole per second.

Thank you very much for hearing this lecture.