EXCELing with Mathematical Modeling

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Week – 05

Lecture – 22 (Drug Distribution Model)

Hello, welcome to the course EXCELing with Mathematical Modelling.

Today we will be discussing about the drug distribution inside our body that kind of model we will be discussing.

So, to start with this whole process that is the study of the movement of drugs in the body is called pharmacokinetics.

So, in this pharmacokinetics people use this mathematical equation and utilize them to describe the movement of the drugs through the body.

So how we model that?

So before going into the model let us see that how actually this drug works.

So if you see that once this drug enters inside your body it dissolves and it pass through certain membranes or sometimes especially the stomach or otherwise if it does not dissolve it go directly to the intestine.

So, in the intestine actually they start this dissolving they pass through the intestinal wall and then ultimately they come to this liver.

And from there, the drugs are distributed.

And if it is some sort of through the veins, then you can see they directly enter the bloodstream.

And if it is say muscular, the drugs, then again, they pass through the muscles.

And ultimately, they pass through these capillary walls and go to their targeted area where they are supposed to work.

So, this is how your drug distribution works inside the body.

So, while doing this mathematical modelling, so we follow that this drug that is present in the system, they follow certain laws and what are the laws?

So, in this particular case, so when the drug enters into your body obviously it will decrease and till it passes I mean till it finds the targeted region, and that is proportional to the amount present in the body.

So, this is one assumption there can be more assumption it can be say it is proportional to the square of the amount that is present to the body and so on.

So, in this particular problem if we take that the rate of decrease is directly proportional to the amount present in the body. So,

$$\frac{dc(t)}{dt} = -kc(t)$$

And let us assume that the initial drug that was given to the body is

$$c(0) = D$$

So we have

$$\frac{dc}{dt} = -kc, \qquad c(0) = D$$

So simple differential equation we just differentiate them sorry integrate them by separating the variables and we get

$$\int \frac{dc}{c} = -k \int dt \ \Rightarrow \ln c = -k\,t + \ln A$$

$$\Rightarrow \ln c - \ln A = -k\,t$$

$$\Rightarrow \ln \frac{c}{A} = -k\,t$$

$$\Rightarrow \frac{c}{A} = e^{-kt}$$

$$\Rightarrow c(t) = Ae^{-kt}$$

$$At \ \ t = 0, \qquad c(0) = D$$

$$c(0) = A = D$$

$$\Rightarrow c(t) = De^{-kt}$$

So, this is the concentration behaviour of the concentration of the drug at any time t. Now, let us see that sometimes it is required to give some doses of the drug.

So, let us assume, say, at time t = T, an equal dose, say, the same dose D of the drug or medicine is added.

So, what happens is that at time t = T, you already have

$$c(t) = De^{-kt}$$

So, if I want what is the drug at time $t = T$, this will be $c(T) = De^{-kt}$.

So, at the time t = T, another dose of drug is added, which is an equal dose D. So, this is the concentration of the drug from the previous dose and then you add another dose D. So, at time t = T,

$$c(T) = D + De^{-kT},$$

is the amount of drug now present in your body.

So, if you want to find what is happening after you have pushed another drug inside the body, then your initial condition changed. So, your equation was

$$c(t) = Ae^{-kt}, \qquad c(0) = D$$

But now you have to use the initial condition $c(T) = D + De^{-kT}$

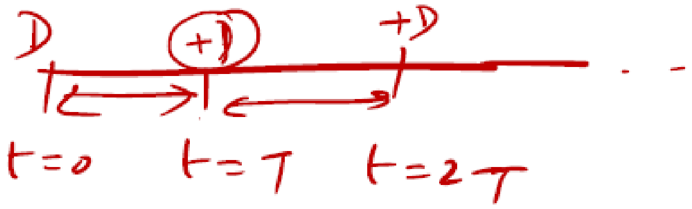
$$c(T) = Ae^{-kT} = D + De^{-kT} \Rightarrow A = De^{kT} + D$$

So, this new constant A will now be substituted here

$$c(t) = (D + De^{kT})e^{-kt}$$

Now as your T goes to 2T, then what will happen here?

So you have this interval, first it is at time t = 0, then at time t = T, now time t = 2T and so on.



So at time t = 0, you saw that your concentration starts with some D and here also +D is pushed, here also +D is pushed.

Now from here to here, the behaviour follows this particular law.

$$c(t) = De^{-kt}$$

But then again from here to here since another +D has been added now it follows this.

$$c(t) = (D + De^{kT})e^{-kt}$$

So as your

$$t \rightarrow 2T, \qquad c(2T) = (D + De^{kT})e^{-2kT}$$

So, once it reaches this 2T so the concentration of the medicine of the drug at time t = 2T,

$$D + (D + De^{kT})e^{-2kT}$$

This is coming from this particular equation, which says that as t tends to 2T, your concentration tends to this particular value.

So at exactly at the time 2T, this is the drug which is present there plus another D is added to it and hence you get this particular thing plus a D and this is your new initial condition, and the process goes on.

So if I continue this then concentration of the medicine or the drug after time = nT,

So, as you can see from here if I multiply this, this is going to

$$D + De^{-2kT} + De^{-kt} = D(1 + e^{-kT} + e^{-2kT})$$

So, this can be simplified to this. This is for t = 2T. So, the concentration of the drug after time T equal to nT can be written

$$D(1 + e^{-kT} + e^{-2kT} + \cdots \dots \dots + e^{-nkT}).$$

So, if it is t = 2T, here it is minus k times 2T. So, if it is t = nT, it is minus n times kT.

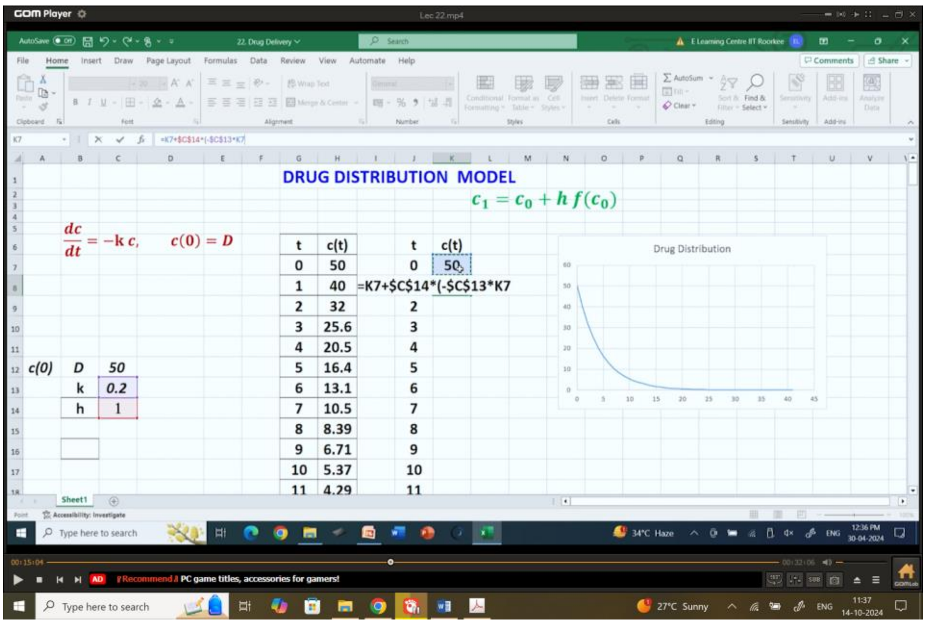
And this comes in a GP series. So, if I want to find the sum, with common ratioe^{-kT}, So,

$$c(nT) = D\left[\frac{1 - e^{-(n+1)kT}}{1 - e^{-kT}}\right].$$

So, this is the concentration of drug after time t = nT and as your n becomes large your c(nT) will tend to a steady value that this part goes to 0, and

$$c(nT) \to \frac{D}{1 - e^{-kT}}.$$

Let us now see the numerical solution of this particular model for which I will be using this Microsoft Excel.



So, as you can see that I already have this equation

$$\frac{dc}{dt} = -kc, \qquad c(0) = D$$

the numerical value I have chosen D to be some 50, this K equal to some 0.2 and h = 1.

And we will be using this equation, this is the Euler's method to solve this differential equation.

Already I have done it but to show you again this is say T and this is C T

Let me increase the font size 20 make it a bit bold in the middle. So, let me plot about say 40 points. So, first this is 0, and this value is 0 plus 1 and then I just drag to the next 40 points and up to 40.

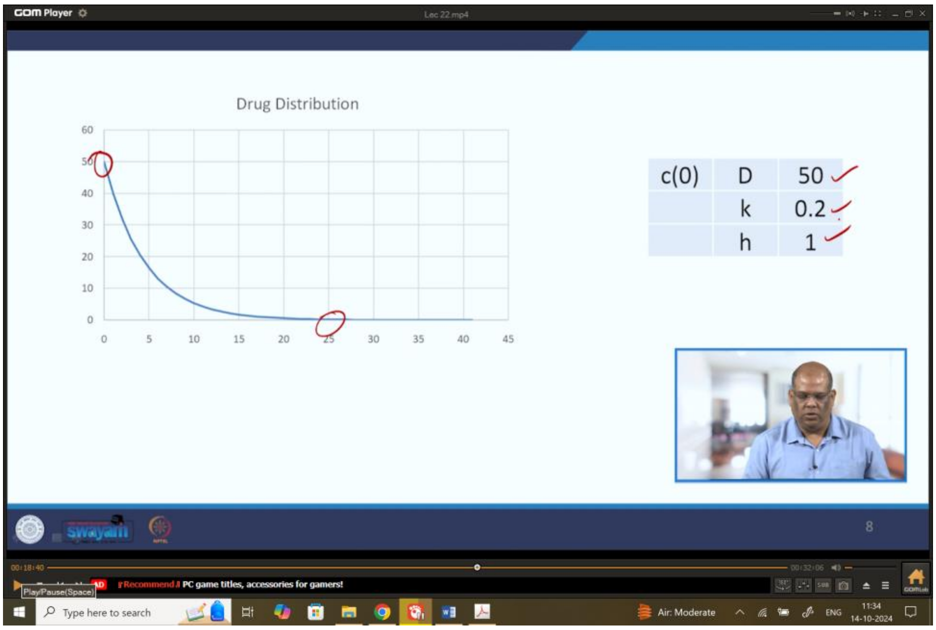
Next we calculate this sorry this value is 50 as mentioned here so I will calculate this is equal to so this is the initial concentration of the drug.

And this is equal to C0 plus H which is a constant so I put a dollar sign multiplied by minus KC so minus K is 0.2 again it is a constant multiplied by C that is the initial value close it and enter so this I drag the next 40 values and I get the values.

Now I want to plot it, so I highlight this, I go to insert, I go to this scattered diagram and I choose this. So if I click this plus sign, so I see I don't want the grid lines, I want the axis, I want axis title and if you want you can have the legend.

So if I want to change the chart title, I click here, and you just say drug distribution in a body, This is your time. This is the concentration of the drugs, and if you want to change this the title here the series one if you want you go to this drug design you can like select data and here is the series one you click Edit and you just write concentration and you click okay and okay.

So you see that here the concentration has come. So this is how you generate the figure.



So I do that again and again so that you just you know get accustomed to it that how to get the numerical solution of a particular problem. So we come back to our original problem.

So now you saw that our solution is going to be in this type that you start from some initial value which is 50 and then you go like this. So I think I have the curve here like this.
So you start with the initial value which is 50 and then your drug concentration ultimately goes down to 0 at some point here and these are the initial conditions which you have used.

So with this we come to a particular problem where the drug concentration is directly proportional to the amount of drug present in the body.
if you want to vary the problem you can say that okay I can take

$$\frac{dc}{dt} = -kc^2, \qquad c(0) = D$$

or you can again change the law by saying okay let me put a bit more complicated

$$\frac{dc}{dt} = -ke^c, \qquad c(0) = D$$

So that is how by changing the law you can give a different kind of solution and but the curve will sort of the dynamics of the curve will remain the same because it will start with certain value and slowly the drug is used up inside your body.

So this is going to decline and come to 0.

Now the declination may be like this or declination can be like this whether depending on the law which you are choosing so that the concentration comes down at a slower rate or sometimes at the higher rate.

So, clearly when this is exponential it is going to come down at a higher rate.

We now look into the numerical solution of these models and let us check whether what I have just told whether it matches with the solution.

So, the first equation is

$$\frac{dc}{dt} = -kc^2, \qquad c(0) = D$$

So, our initial condition is 50, K is 0.01 and his 1.

So, I have my time T and my concentration as C. So, I change the fonts to be 20.

Center them, make a bit bold. And let's make the font size same as 20 and center them.

So the first thing is this value is 0.

The next value is 0 plus 1, and let me drag this to the next 40 values.

This is equal to, we will use the Euler's method, this initial value is 50 and this is equal to your C0 plus h, which is a constant.

So, I put a dollar sign and this is multiplied by, I open a bracket, minus K C0 square.

So, minus then K which is again a constant multiplied by this is C0 square bracket closed and end and let me drag this.

So, if I now plot this I will choose these two, using the cursor I go down up to 40 values and I click insert, I choose the charts and I take this smooth diagram.

So, name of the chart.

Drug distribution, I remove the grid lines, I want axis title and this will be time and this will be the concentration of the drug.

So, this is the figure which is the graph which you get from the equation

$$\frac{dc}{dt} = -kc^2$$

where the law has changed from minus kc to minus kc square.

Let us now look into the other equation where you have dc dt equal to minus k times e to the power c.

$$\frac{dc}{dt} = -k\,e^c, \qquad c(0) = D$$

The initial concentration is d that is 50.

However, the value of k we have taken very very small otherwise this because of the function e to the power c the concentration reaches zero very, very fast.

So that's why such a small k we have taken.

So again, we have this C, the time, the concentration, we increase their font size, make it bold and this is 0 and this is 50.

Just quickly make them 20.

So, this first we increase them by 1 plus 1 and then drag them to the 40 values and this is equal to, again we use the Euler's formula for solving this first order differential equation.

So, this is equal to C0 plus h, which is a constant multiplied by minus K. It is quite a small value but again a constant multiplied by e to the power c, so exponential c which is c0 in this case.

This is closed, again closed by another bracket because this exponential is in this first bracket and then this bracket has to be closed by the last bracket and I get the value.

So sometimes if it is coming in exponential, you can just go to the number and either 2 or 3 or 4 decimal places.

So that will give you in terms of decimal values.

Now you have to plot this to see the dynamics.

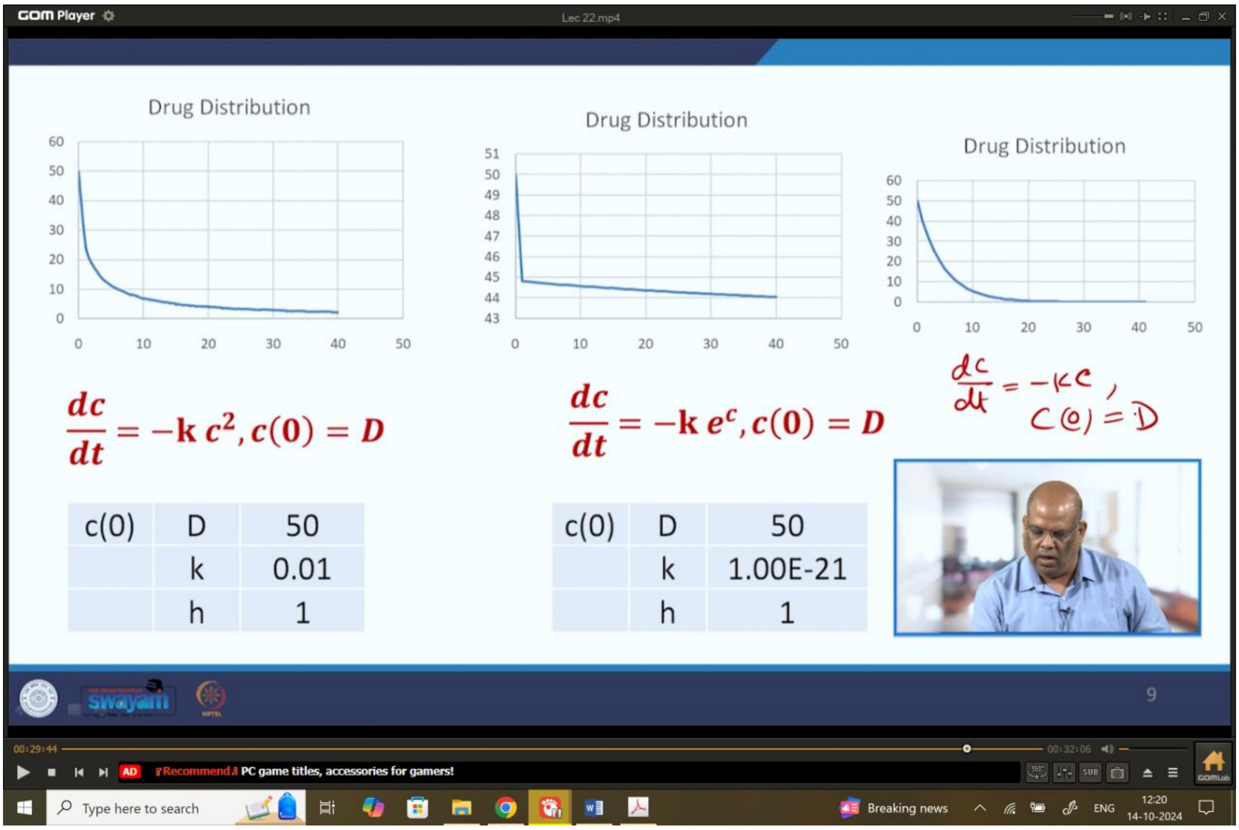
So I choose the 40, T and the concentration values, go to insert, go to chart and then click.

So this is the graph. I format them in a little, say drug distribution, see what is my name drug distribution same and I choose this I remove the grid lines I choose the axis title and this is the time and this is the concentration of the drug

Okay, so you can see there's a sharp fall and then it's going to some steady value.

So let's go back to the slides again and compare.

So as you can see that here we have three graphs.



This one is

$$\frac{dc}{dt} = -kc, \qquad c(0) = D$$

and the value of K also same here it is 0.01.

So if you see this particular diagram the plots with the law minus Kc you can see that it starts with the concentration 50 and gradually goes down to 0.

Then we come to this law which is

$$\frac{dc}{dt} = -kc^2, \qquad c(0) = D$$

we have the same initial condition 50 The value of K is 0.01 and then it starts from 50 but because it is minus K c square you can see that this decline is sharp 1 than this decline.

So with this minus K c square the concentration of the drug will be used up rapidly with a sharp decline and then it is going to some steady values. When it is

$$\frac{dc}{dt} = -k\, e^{c}, \qquad c(0) = D$$

here the decline is much, much sharper, straight down.

However, the value of k has to be extremely small here because otherwise this will just go to 0 in just 4-5 steps.

The concentration of the drug is kept as 50.

So, sharp decline and then it is going to some steady state value.

So, as we have told before that the concentration of the drug its dynamics will depend on what kind of law you are choosing and depending on a particular drug we choose a suitable law.

So, in our next lecture I will be taking up some growth and decay model of the electric circuit and till then bye-bye.