EXCELing with Mathematical Modeling

Prof. Sandip Banerjee
Department of Mathematics
Indian Institute of Technology Roorkee (IITR)

Week – 06

Lecture – 30 (Rumor Model)

Hello, Welcome to the course EXCELing with Mathematical Modelling.

Today, we will be talking about mathematical models of rumour.

So, what is a rumour?

So, I am sure you all know what the rumour is, but the definition goes like this, that it is a form of social communication and can shape the public opinion and affects the belief of an individual, which can lead to the changes of individual's attitude towards economic, political and social aspects.

So, a rumor is an unconfirmed truth and has negative impact on society though an intervention can be implemented to minimize the negative aspects or negative impacts.

So, we will use here mathematical model to understand this spread of rumor and to figure out some strategies that how to reduce its negative impact on society.

So to start with, let us divide the individuals into three categories.

The very one first is ignorant. So, ignorant are the ones who does not know about the rumours.

Then comes the spreaders. So obviously the spreaders are the ones who spread the rumours and the third category we name it as Stifler.

So Stiflers are those guys who already know the humour but somehow they lost interest in spreading them.

So with these three categories we now form our differential equation.

So, this is the rate of change for the ignorance who does not know about the rumour, this is the rate of change of the spreaders and this is the rate of change for stiflers.

So, we assume that there is a constant input of this ignorant, call it λ and when ignorant are coming in contact with the spreaders, so, they came to know the about the rumors. So, $-\beta IS$, because when the ignorance are coming in contact with the spreaders, they get the rumor and then they come to the category of spreaders. So, here it will be βIS and then we put a natural death μI of the ignorant.

Similarly, μS natural death of the spreaders and a part of the spreader who has lost the interest they do not spread it anymore they become the stiflers, let us put it, say, αS and they come to the Stifler group and μF is the natural death for the Stiflers.

So, we have taken the same natural death for simplicity for all these three categories.

So, we have a model for the spread of rumour. So let us now get into the analysis of the model.

So as explained before, so this is the ignorant, this is the spreaders and this is the stiflers.

$$\frac{dI}{dt} = \lambda - \beta IS - \mu I = f \ (say)$$

$$\frac{dS}{dt} = \beta IS - \ \alpha S - \mu S = g \ \ (say)$$

$$\frac{dF}{dt} = \alpha S - \mu F = h \ \ (say)$$

So the very first thing is we find the equilibrium points. So we put

$$\frac{dI}{dt} = \lambda - \beta IS - \mu I = 0$$

$$\frac{dS}{dt} = \beta IS - \ \alpha S - \mu S = 0$$

$$\frac{dF}{dt} = \alpha IS - \mu F = 0$$

And if you solve them, you will get the equilibrium points to be

$$\left(\frac{\lambda}{\mu}, 0,0\right), \left[\frac{\alpha + \mu}{\beta}, \frac{\beta\lambda - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}, \frac{\alpha}{\mu}\frac{\beta\lambda - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}\right].$$

So we have two equilibrium points and now we will be doing a stability check.

So, this can be easily shown you just put

$$\lambda - \beta IS - \mu I = 0, \ \ \beta IS - \ \alpha S - \mu S = 0, \alpha IS - \mu F = 0$$

and solve them. Now, we start our stability analysis with this equilibrium point.

So our Jacobian matrix A is going to be

$$A = \begin{pmatrix} \frac{\partial f}{\partial I} & \frac{\partial f}{\partial S} & \frac{\partial f}{\partial F} \\ \frac{\partial g}{\partial I} & \frac{\partial g}{\partial S} & \frac{\partial g}{\partial F} \\ \frac{\partial h}{\partial I} & \frac{\partial h}{\partial S} & \frac{\partial h}{\partial F} \end{pmatrix} = \begin{pmatrix} -\beta S^* - \mu & -\beta I^* & 0 \\ \beta S^* & \beta I^* - \alpha - \mu & 0 \\ 0 & \alpha & -\mu \end{pmatrix}$$

and they need to be calculated at the equilibrium point (I^*, S^*, F^*)

So, I replace this by the equilibrium point. So, now we look for the stability analysis at each of these equilibrium points. So, the first point is

$$\left(\frac{\lambda}{\mu}, 0,0\right)$$

And if we substitute it in this matrix, we will get

$$A = \begin{pmatrix} -\mu - \lambda & -\frac{\beta\lambda}{\mu} & 0 \\ 0 & \frac{\beta\lambda}{\mu} - \alpha - \mu & 0 \\ 0 & \alpha & -\mu \end{pmatrix}$$

And I have to find its eigenvalues which will be given by

$$\det(A - xI) = 0 \Longrightarrow \begin{vmatrix} -\mu - x & -\frac{\beta\lambda}{\mu} & 0 \\ 0 & \frac{\beta\lambda}{\mu} - \alpha - \mu - x & 0 \\ 0 & \alpha & -\mu - x \end{vmatrix} = 0$$

$$\Longrightarrow (-\mu - x)\left(\frac{\beta\lambda}{\mu} - \alpha - \mu - x\right)(-\mu - x) = 0$$

$$\Longrightarrow x = -\mu, \quad -\mu, \quad \frac{\beta\lambda}{\mu} - \alpha - \mu.$$

Now, we can see $(-\mu)$ is negative, and if the system has to be stable at the point $\left(\frac{\lambda}{\mu}, 0,0\right)$, we must have

$$\frac{\beta\lambda}{\mu} < \alpha - \mu,$$

and unstable if

$$\frac{\beta\lambda}{\mu} > \alpha - \mu.$$

So, this is the condition which need to be satisfied for the system to be stable and for the system to be unstable.

Let us move on to the next equilibrium point which is

$$\left[\frac{\alpha + \mu}{\beta}, \frac{\beta\lambda - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}, \frac{\alpha}{\mu}\frac{\beta\lambda - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}\right]$$

Now please notice that this is a positive quantity and since they represent the population this has to be positive and this has to be positive and condition for that is

$$\beta\lambda - \mu(\alpha + \mu) > 0$$

So, for the existence of the non-zero equilibrium point,

$$\beta\lambda - \mu(\alpha + \mu) > 0,$$

this condition must be satisfied, otherwise, the population will be negative which does not make any sense.

So, now we do the stability analysis.

And if we calculate the Jacobian matrix A, this will give me

$$A = \begin{pmatrix} -\mu - \beta\lambda - \mu(\alpha+\mu) & -\alpha - \mu & 0 \\ \dfrac{\beta\lambda - \mu(\alpha+\mu)}{\alpha+\mu} & 0 & 0 \\ 0 & \alpha & -\mu \end{pmatrix}$$

So, it is advisable that when you want to calculate the eigenvalues, and it gives you some complicated expression, then go for this characteristic equation.

The characteristic equation will be given by

$$|A - xI| = 0.$$

So, the equation will be of the form

$$(x+\mu)\big[(\alpha+\mu)x^2 + \beta\lambda x + (\alpha+\mu)\big(\beta\lambda - \mu(\alpha+\mu)\big)\big] = 0$$

So, from here we can clearly see one of the root is $-\mu$, and we get a quadratic equation

$$(\alpha+\mu)x^2 + \beta\lambda x + (\alpha+\mu)\big(\beta\lambda - \mu(\alpha+\mu)\big) = 0$$

Now if you recall the stability criteria for

$$x^2 + a_1 x + a_2 = 0,$$

Then, the Routh Hurwitz condition or criteria, says that an equation of this form will be stable if

$$a_1 > 0, \qquad a_2 > 0$$

So we rewrite this equation

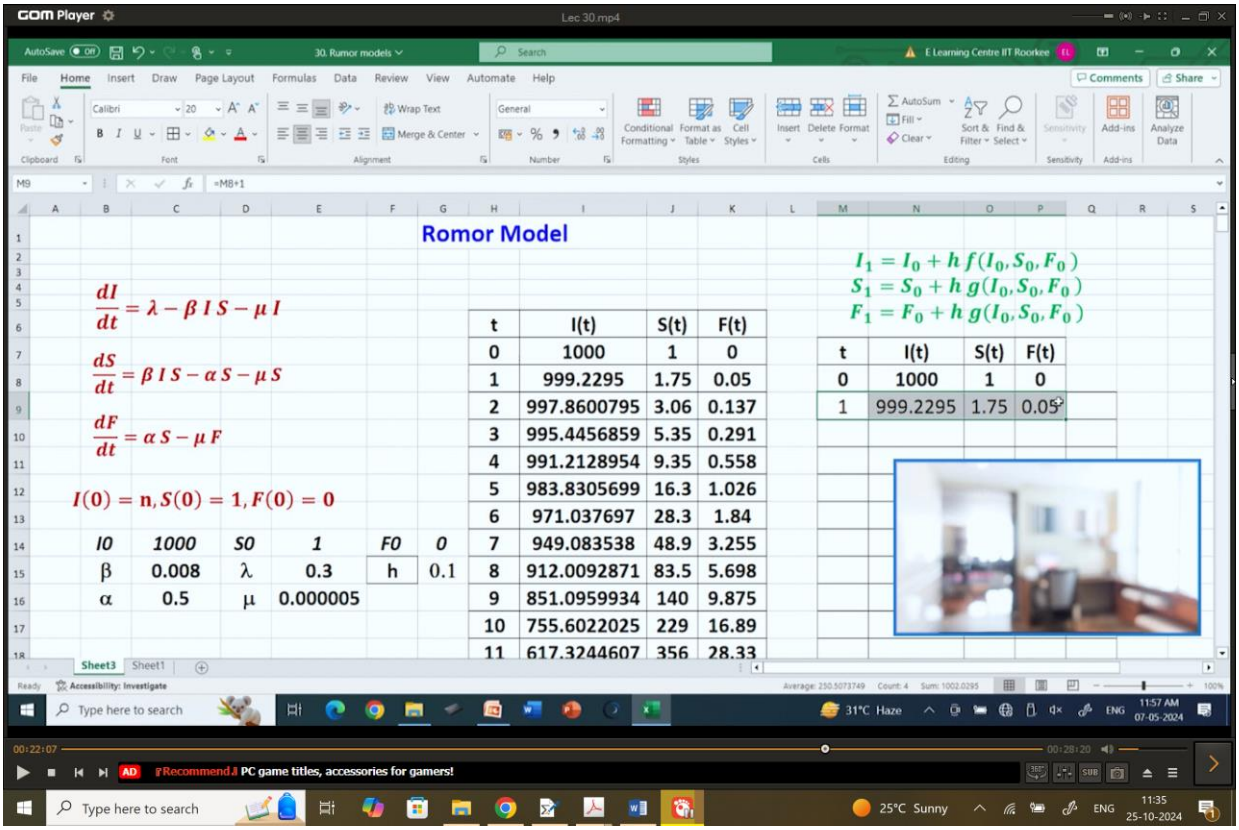
$$x^2 + \frac{\beta\lambda}{\alpha+\mu}x + \big(\beta\lambda - \mu(\alpha+\mu)\big) = 0$$

So this quantity is positive as all the parameter values are positive, and $\beta\lambda - \mu(\alpha + \mu)$ is also greater than zero, due to the existence of the equilibrium point which we have already shown here.

So, this equation satisfies the Routh-Hurwitz criteria and the system is always asymptotically stable about the equilibrium point, we call it the endemic equilibrium point, or you can say the non-zero equilibrium point in this particular case, because endemic we generally use when we consider a disease or disease model.

So, since it is a rumor models, we say that non-zero equilibrium point.

Now, let us see about the numerical solution of this particular rumor model for which we will be using the Microsoft Excel.



I already have it opened. So as you can see this is the first equation of the ignorance.

These are the spreaders and this is the stiflers.

Now initial condition though I have put n here the value I have chosen to be 1000.

Say there is only one spreader and no stiflers.

So here is the initial values at time t equal to 0, the ignorance were 1000, only one spreader and no stiflers.

So, this let us calculate this will be 0 plus 1, this is going to be this initial value plus H is 0.1 and it is a constant. So, I put a dollar multiplied by open a bracket this whole thing.

So, lambda which is 0.3 and again a constant minus beta which is 0.008 multiplied by I which is 1000 multiplied by S which is 1 minus mu times which is a small value multiplied by I, close it and end. So, the next value is again S0 plus h times which is a constant multiplied by this whole thing.

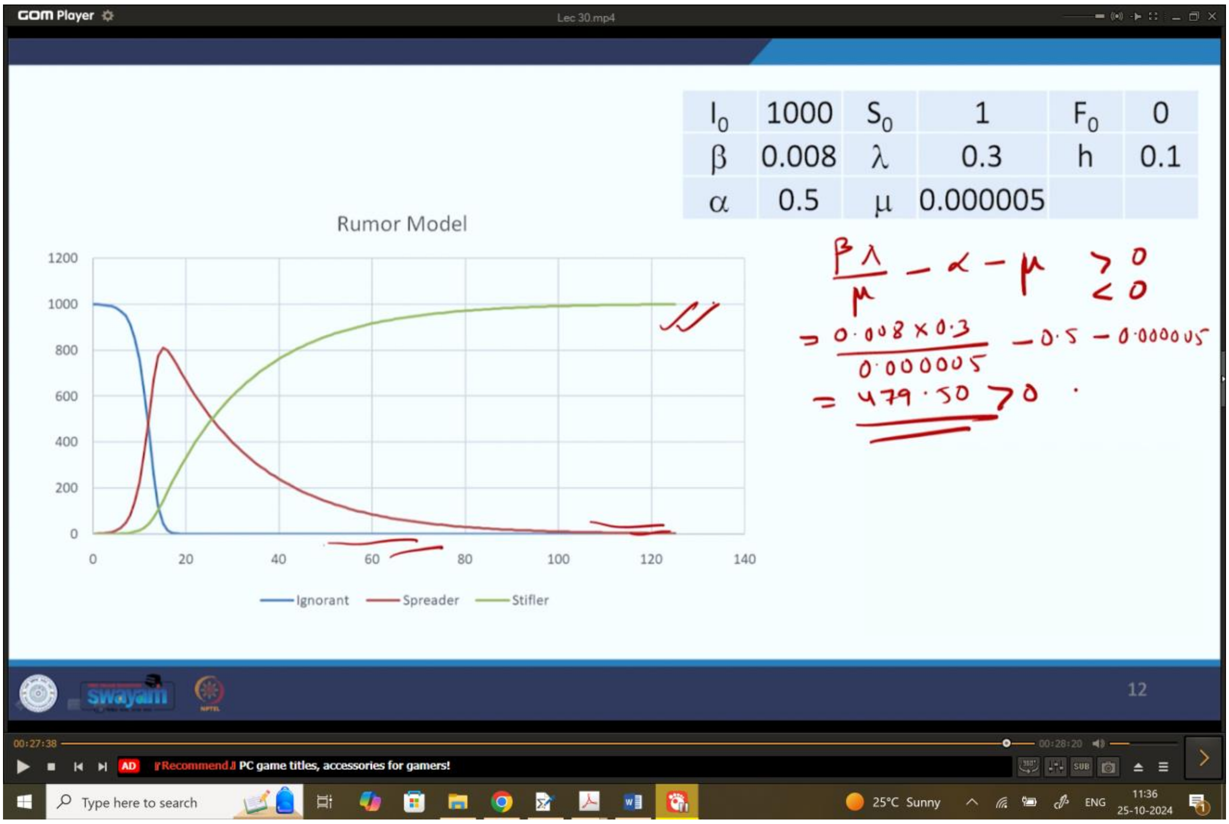
So, beta times I into S minus alpha s alpha into s minus mu into S And the stiffness is equal to the initial value is 0 plus h multiplied by this whole thing alpha times S sorry dollar alpha times S minus mu times F and we drag all this generate series of values and we take it up to 150 and

So, we get the values and now we have to plot them.

So, I highlight these values by pressing the shift key and the cursor, go to insert the scatter diagram and this one.

So, the chart title is going to be click that it is rumor model. The series, select data, click the series, go to ignorant, go to edit and this is ignorance, next one edit, spreaders and the last one edit, Stiflers, and you get the numericals.

If you want to remove the grid lines, go to plus and remove the grid lines. So, this is how you generate the numerical figures. Let us move back to the explanation of this graph.



So, I have already copied and pasted the graph here and the initial values that so the ignorance are 1000, the spreader is 1, the Stifler is 0, these are the parameters of alpha, beta and H. So, what this graph tells?

So, the graphs tells you that the ignorance started with 1000. So, total population is 1001.

So, they ultimately goes to 0 that means all the ignorance they come across with the spreaders and then came to know about the rumors and then comes the spreader.

So, they start spreading spreading spreading and then slowly they also goes to zero and because most of once you get the news you spread it becomes old and then slowly you lose the interest it is just like the your whatsapp thing you get a message the moment you get the message you start sending it to other groups and friends and at one point once you see that the message is quite old, you lose the interest and that is where you form the stiflers and go to an equilibrium point.

So, at the end all the spreaders ultimately become the Stiflers But there is a peak as you can see that where the message or the rumors are being shared and that is the peak of the spreaders but slowly it also goes down to zero.

If you now look into the stability condition, we have

$$\frac{\beta\lambda}{\mu} - \alpha - \mu$$

and we have shown that if this is positive the system is stable about the non-zero equilibrium point and if this is negative then the system is unstable about the non-zero equilibrium point.

Now if we calculate this value I substitute 0.008 multiplied by 0.3 divided by mu which is 0.00005 minus alpha minus mu.

If I calculate this value, I will be getting 479.50 is positive, this system is stable about the non-zero equilibrium point but the question is as you can see that these are approaching zeros.

So, the thing is the values of the parameter are chosen in such a way that this goes to a very small amount but they are never zeros they are going to some very small value compared to this 1000 and hence your Stifler's they reach this value 1000, where the other values are coming to a negligible value which in the graph compared to this 1000 looks like 0.

So, with this we come to an end of this very interesting rumor model.

In the next lecture, we will be taking some more interesting model.

Till then, bye-bye.