EXCELing with Mathematical Modeling
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Week – 08
Lecture – 38 (Basics of Excel 4)

Hello welcome to the course EXCELing with Mathematical Modeling.

Today we will be learning about how to solve these difference equations with the help of Microsoft Excel.

In my previous lecture, we have come across the solution of the difference equation in the form

$$u_n = ck^n,$$

where c is an arbitrary constant.

And we have seen that for various values of k, how the solution behaves.

For example, if it is $k > 1$, we get the solution that as n increases, the solution becomes unbounded.

When $0 < k < 1$, as your n increases, the solution converges to zero or the solution decreases.

If $-1 < k < 0$, then there is a oscillatory behaviour but the damping oscillation and ultimately the solution decreases, and at the end if $k < -1$ then we have the same damping oscillations but with increasing magnitude.

So now what we will do is we will replicate these solutions with the help of Microsoft Excel.

So our solution is of the form

$$u_n = ck^n.$$

Now I have taken the initial value of $c = 10$.

For $k > 1$,so I put $k = 1.5$, for $0 < k < 1$, I put $0 < k = 0.5$, for $-1 < k < 0$, I put $k = -0.5$ and for $k < -1$, I put $k = -1.5$.

So, we will be solving and visualize the solution for this different values of k. To start with I put this value to be zero, and this is equal to I give an increment of 1 and calculate this values.

Now the first u_n that is when $k > 1$ so in that particular case this is equal to so if I put $n = 0$ here, I get $u_0 = ck^0 = c$.

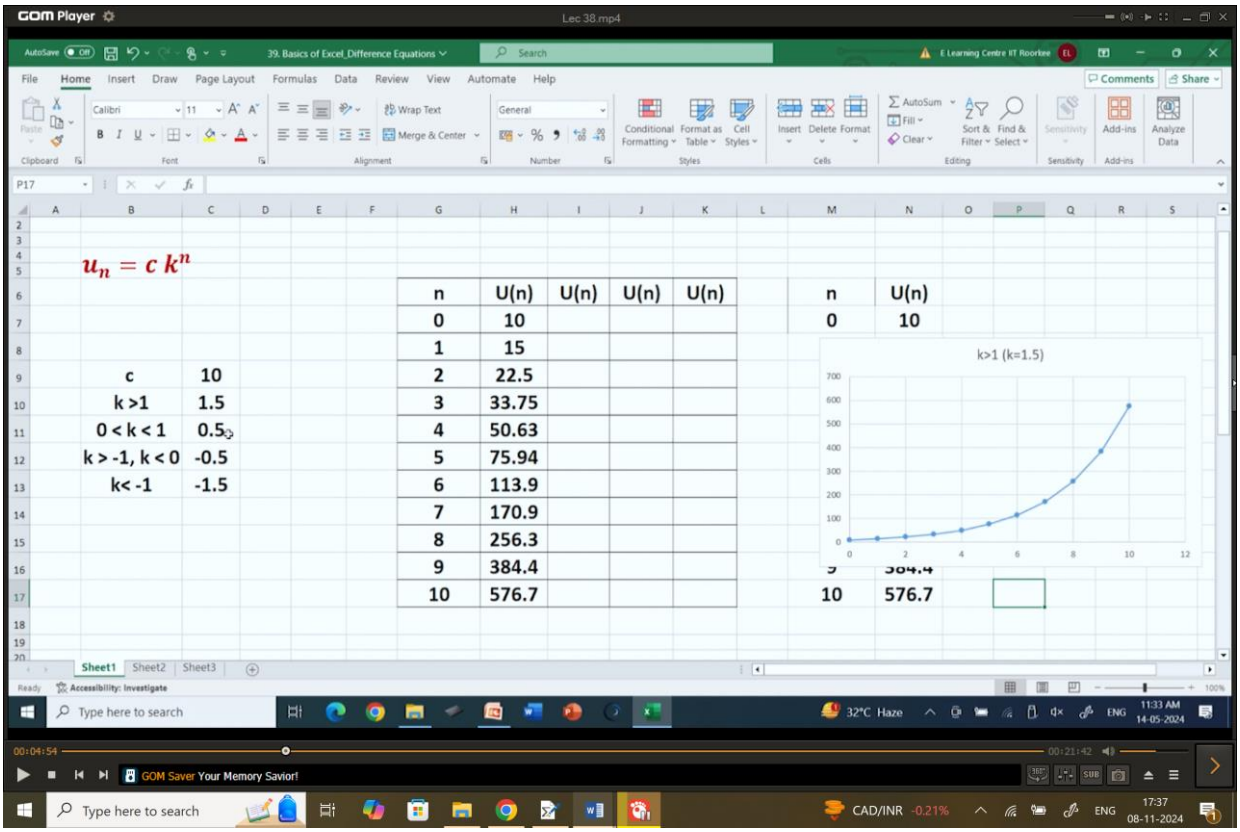
So, I can write $u_0 = ck^0$

Now, c is a constant value so I put a dollar value of k again a constant so I put another $ here and G7.

So, I get this value and I drag the next 10 values.

So, if I want to plot them now, I will highlight this by pressing shift and the cursor key go to insert go to this scattered diagram and instead of choosing this continuous line I will choose this one.

So, that you get an idea that these are the points the dots and we have joined those points because they are discrete case and we have get a curve like this.



If you want to change the title you can just type this is $k < 1 (k = 1.5)$.

Now let us move when your $0 < k < 1$ $(k = 0.5)$.

So, again this

$$u_0 = ck^0 \Longrightarrow u_0 = c(0.5)^0 = c$$

and the value of k is also taken to be constant which is $k = 0.5$ and I drag them.

Now to plot this particular column and this particular column what you have to do is you have to first highlight this by pressing the shift key and the cursor then go there.

Press the control button choose this one press shift and then the down cursor and you will this will be highlighted.

Then you go to insert again choose this and this particular figure.

So, when your $0 < k < 1$, you have a decreasing value of the curve. Now, let us draw this one.

This is the value of $k = -0.5$ here.

So, this is equal to again

$$u_0 = c(-0.5)^0 = c$$

This c is constant and the value of k is also constant.

So, while dragging you make sure that your plus sign changes from this to a smaller one with black and you drag them and you get all these values again to choose this and this you have to first highlight this press shift choose these 11 values press control go here highlight this and then press the shift key and the down arrow and you will get these two columns highlighted.

Go to insert, click this chart and again you get this.

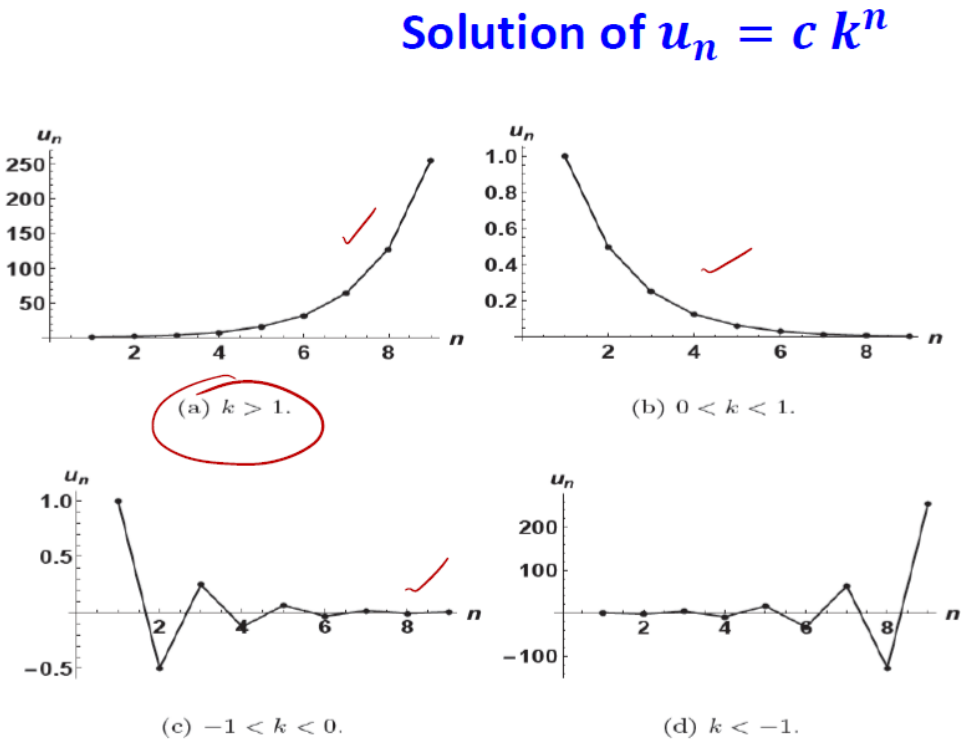
So, you get an oscillatory behaviour but decreasing a damping one which ultimately goes to 0.

And the final one is again

$$u_0 = c(k)^0 \Longrightarrow u_0 = 10(-1.5)^0 = c$$

and this 13 is also a constant -1.5 you get this value you drag them.

So again if I highlight this one by pressing shift and the down arrow key this is highlighted press control go to this cell click highlighted press the shift and the down arrow.



So this will be both the columns have been chosen go to insert go to this chart and again click this, the values where the oscillation is increasing for the value of $k < 1$.

I already have these charts here.

So, the first one tells you that okay this is when $k > 1$, the value increases unboundedly.

When $0 < k < 1$ then the value decreases and ultimately goes decreasing in this case it goes to zero.

When $-1 < k < 0$, this is the oscillatory behavior, but a damping oscillation where the oscillation slowly decreases and dies out.

And when $k < -1$, then again it is an oscillatory behavior, but in this particular case, the oscillation increases as n increases.

So, these are the four dynamics which we have already seen here.

These are the four dynamics and we now we have just solved this equation with the help of Microsoft Excel to get a replica of these cars.

Now, let us move to the second kind of equation.

So, we have a differential difference equation like two difference equation and mostly you will be solving difference equation like this maybe some non-linear terms but the process remains the same this is

$$x_{n+1} = 1.3x_n + 0.4y_n$$

$$y_{n+1} = -0.6x_n + 1.05y_n$$

So, if I just want to calculate, so I put this value is 0 and the next value is 0 plus 1 and let me drag them to 10 values.

Let me change the font to 20 make them bold and center.

Now the initial value I have chosen this to be

$$x(0) = 100 \text{ and } y(0) = 8000.$$

Give a little gap. Now, I calculate x_{n+1}. So, if I put $n = 0$, here, this is

$$x_1 = 1.3x_0 + 0.4y_0$$

$$y_1 = -0.6x_0 + 1.05y_0$$

So, I will type here

$$x_1 = 1.3 \times 100 + 0.4 \times 8000$$

$$y_1 = -0.6 \times 100 + 1.05 \times 8000$$

So, I highlight these two values go here and calculate up to 10 values.

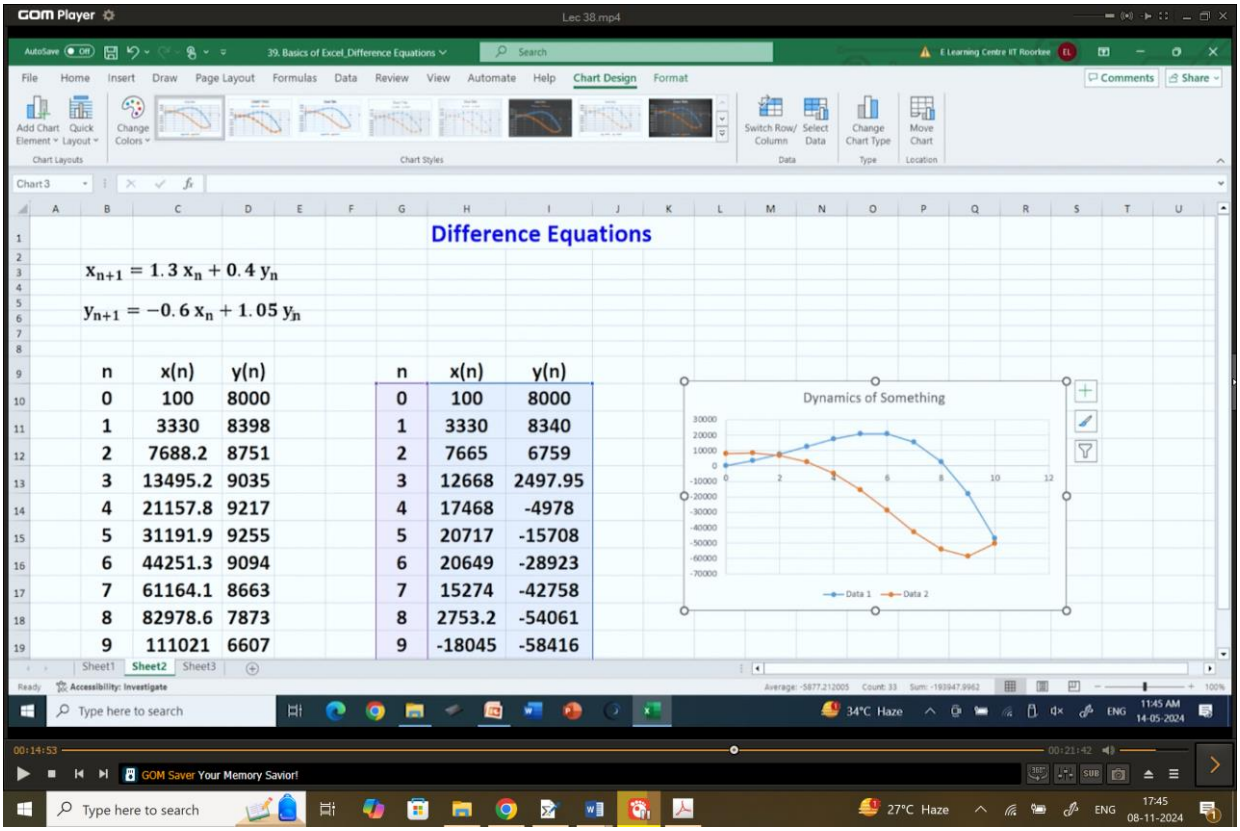
Now, if I want to plot them I highlight these values go insert and plot this.

If I want to change this title, I just type this is dynamics of something.

So, you replace something by whatever problem you are dealing with.

And if you want to change the series, it remains the same, you go to this chart design.

select this data, select series 1, edit and I name it, it is a data 1 and series 2, go to edit and data 2 and you get some this kind of timeline.



n	x(n)	y(n)	n	x(n)	y(n)
0	100	8000	0	100	8000
1	3330	8398	1	3330	8340
2	7688.2	8751	2	7665	6759
3	13495.2	9035	3	12668	2497.95
4	21157.8	9217	4	17468	-4978
5	31191.9	9255	5	20717	-15708
6	44251.3	9094	6	20649	-28923
7	61164.1	8663	7	15274	-42758
8	82978.6	7873	8	2753.2	-54061
9	111021	6607	9	-18045	-58416

So, that is how you solve a difference equation consists of two variables it could have happened that this is ${y_n}^2$ particular case you just multiplied $y_n \times y_n$.

So, that is the only difference when you get the non-linear terms of suppose this is $x_n \times y_n$

So, instead of only multiplying this y_n you have to multiply the component x_n also.

There is one small difference which you have noticed that this 1.3

So, instead of taking some $A = 1.3$ and multiplying by this value and then putting $ sign, you could have directly multiplied by the constant which is 1.3 and 0.4.

So, that also gives you the same result.

But sometimes in a long calculation where these constants also need to be specified there we choose a cell and then you multiply it by that particular value with a $ sign like in this particular example I have kept changing the value of this k. So, if I put 1.5 here in the equation then I have to change that equation again and again.

So, instead I put this and I put a $ sign.

So both are correct and you can use both in getting the solutions.

We now solve a non-linear difference equation with the help of Microsoft Excel.

So as you can see the equation is

$$x_{n+1} = x_n + 2.5y_n - 0.1{x_n}^2 - 1,$$
$$y_{n+1} = y_n + \frac{5}{x_n} - 1$$

Initially, we have taken that at $n = 10$, the value of $x(0) = 3, y(0) = 110.$

So, we first write this with an increment of 1 and let me drag this 68,

be a little more, okay up to this much.

Then this, so this is

$$x_{n+1} = x_n + 2.5y_n - 0.1{x_n}^2 - 1,$$

So I get this value.

Let me drag.

This is

$$y_{n+1} = y_n + \frac{5}{x_n} - 1$$

So let us take up to 20 values.

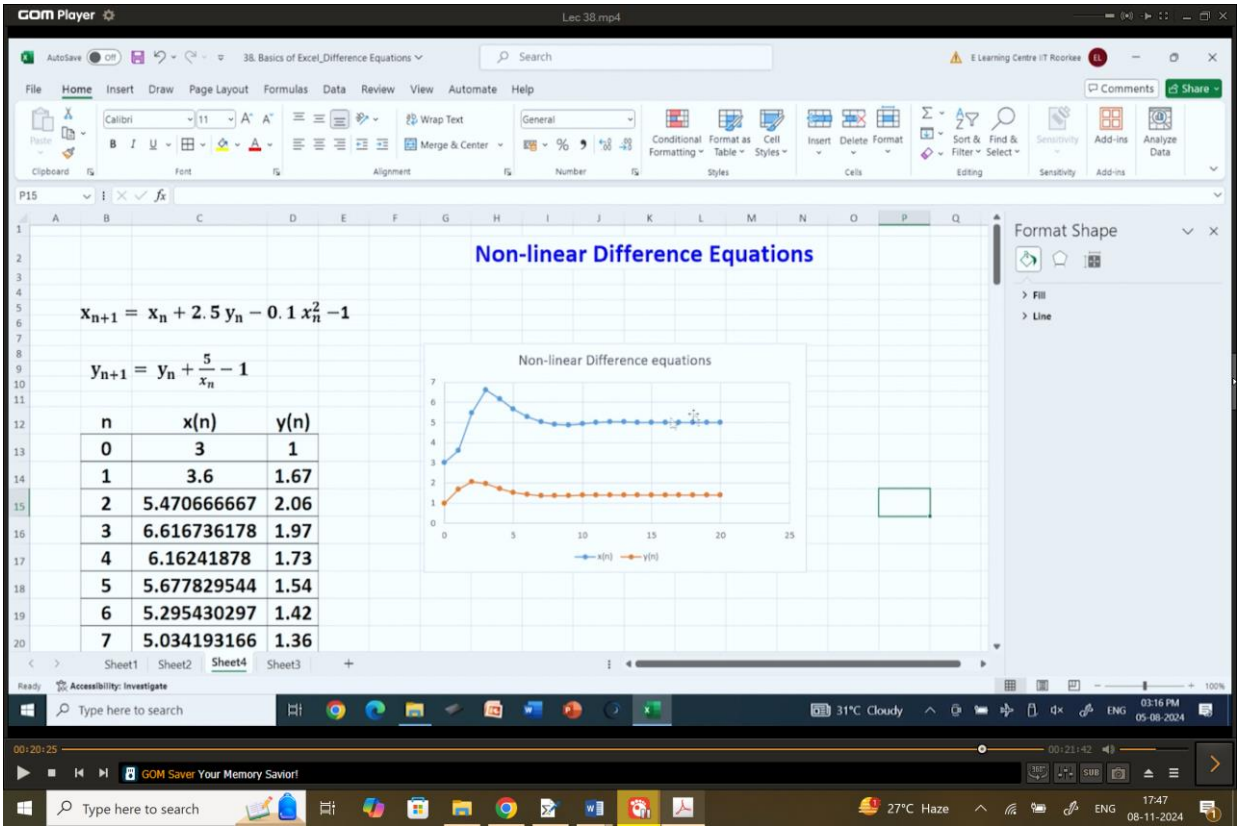
So now we plot, we select these numbers.

We go to insert, go to charts and select this.

If you want to write this, write non-linear difference equations for the series, go to chart design, you select data.

You edit and you write this as x and , similarly series 2, go to edit and write this as y and so you just solved a non-linear difference equation and get these two curves.

Later on you will be solving the model which will represent this non-linear difference equation and from there you will get this curve from which you have to interpret.



n	x(n)	y(n)
0	3	1
1	3.6	1.67
2	5.470666667	2.06
3	6.616736178	1.97
4	6.16241878	1.73
5	5.677829544	1.54
6	5.295430297	1.42
7	5.034193166	1.36

So, with this we come to the end of this particular lecture where you have learned that how you can solve a difference equation with the help of Microsoft Excel.

So, as you go along you will come across the models which will be represented by the difference equation and you will be using this Microsoft Excel to get the solution and its corresponding visualization.

In my next lecture, we will be talking about the stability analysis and the equilibrium points of the difference equation.

Till then, bye bye.