EXCELing with Mathematical Modeling

Prof. Sandip Banerjee
Department of Mathematics
Indian Institute of Technology Roorkee (IITR)

Week – 09

Lecture – 42 (Bank Account Models)

Hello, welcome to the course EXCELing with Mathematical Modelling.

We all have bank accounts, we are familiar with the loans, with the mortgages, with the credit card bills and so on.

So, today we will be looking up into some bank account problems that we usually face in our day-to-day life.

So, let us consider that you have a savings account which gives you 4 percent interest compounded yearly.

Generally, it does not happen in a savings account but somehow the banks offer this kind of offer which gives you okay you open a savings account with us we will give you 4% interest compounded yearly.

So, you make an initial deposit of say rupees 10,000. And you say that at the end of each year, you will deposit rupees 5000. So, the bank says okay. So, the interest will be added.

And, then at the end of next year, whatever the amount is, we will again give interest on that particular thing because it is compounded yearly.

So, if you want to model this kind of situation, so you take, say, a_n that is, the amount at the end of year n. So, what will be your a_1?

If I take, say, r to be my rate of interest, then I have my initial deposit a_0, the rate of interest is r multiplied by the initial deposit a_0 and I get $(1+r)a_0$, that is,

$$a_1 = a_0 + ra_0 = (1+r)a_0$$

And, please note that this rate of interest this is usually not given in percentage but if it is given in percentage you have to divide it by 100 and then that value will become the r.

So, in the second year I have my new principle, which is

.

$$a_2 = a_1 + ra_1 = (1+r)a_1$$

$$- - - - - - - - - - - - -$$

$$a_{n+1} = a_n + ra_n = (1+r)a_n$$

But now you said that, okay, every year I will be adding 5000 rupees which is a constant thing. Then the model become

$$a_{n+1} = (1+r)a_n + 5000.$$

For n = 0, $a_1 = (1+r)a_0 + 5000.$

So if you now want to calculate what is your a_1, your rate of interest r is 4% and as I told you, you have to divide it by 100 to make it 0.04. Then,

$$a_1 = 1.04\, a_0 + 5000 = 1.04 \times 10000 + 5000 = 10400 + 5000 = 15400$$

Similarly,

$$a_2 = 15400 \times 1.04 + 5000 = 21016,\ a_3 = 1.04 \times 21016 + 5000 = 26857,$$

and so on.

So, we can get what is the amount at the end of whatever year you want.

Now, let us consider a different scenario. So, in this particular scenario, no deposits are made.

So your model was an plus 1 equal to 1 plus r into an initially and then we say that every month we add this 5000 rupees to it so you have added 5000. But now this 5000 will not happen no deposits are made instead rupees 2000 is withdrawn at the end of each year.

We keep the rate of interest at 4%. So, if we calculate

$$r = 4\% = \frac{4}{100} = 0.04$$

So, what do you want to know is that how much initial deposit you must make such that your balance is never zero.

So, to find that there are two assumptions we just take.

First is the money is withdrawn after the interest from the previous year is added. So, you can only withdraw the money once that interest is deposited in the account. And, number two, no penalty for withdrawing money.

See the bank has given a special offer. The offer is that it is a savings account but then your interest gets compounded.

In such a case the bank can put some clause that okay if you have an early withdrawal then we will put a penalty but in such case in this case there is no penalty on withdrawing the money and money will be withdrawn after you have deposited after the bank deposited the interest.

So with these two assumptions we need to find that what will be the initial deposit such that it never run out of cash.

So, you have to deposit the initial amount in such a clever way that with this 0.04% interest you will still have and your constant withdrawal of rupees 2000 you will still have money in the bank.

So, if you want to model this, we get

$$a_{n+1} = (1+r)a_n - 2000$$

$$\Longrightarrow\ a_{n+1} = 1.04a_n - 2000$$

So, this is the model which you get for this particular scenario.

Now to get the initial deposit what you have to do is so you are withdrawing 2000 every year and If there is no change from one generation to another generation that is from one year to another year then what you get is the equilibrium solution.

So, basically you have to get that equilibrium solution such that your account never run out of cash. So, you put

$$a_{n+1} = a_n = a^*$$

$$\Rightarrow a^* = 1.04a^* - 2000 \Rightarrow a^* = \frac{2000}{0.04} = 50000.$$

So, if you make an initial deposit a_0 to be 50000, you will see that you never run out of cash. So, for example,

$$a_1 = 1.04a_0 - 2000 = 50000$$

Let us now see the numerical solution of this with the help of Microsoft Excel to see that whether our analytical result matches with the numerical one.

So, this is my n. So, for a1, a2, and a3.

So I have three solutions here or rather you can put it as a1 because you have taken the equation in the form of a but I guess you understand that.

So why these three values?

That is because we will start with initial amount of 50000 and we will see what happens if it is less than the chosen initial amount and greater than the chosen initial amount.

So, this value is equal to this plus 1. So, I just give an increment of 1 and let us calculate up to 40 of them.This value is equal to so I will use this equation.

So, this is n equal to 0. So, this is A1 and this is value is A0. So, A1 is equal to 1.04 multiplied by A0 minus 2000.

So, you see you get this 50000 back here also equal to 1 that is your initial value A0 multiplied by 1.04 minus 2000 and here equal to 1.04 multiplied by the initial value of 60000 minus 2000

So, let me calculate the next 40 values, just drag them, let me increase their font size 20, make them bold, make them in the center.

So, now let us plot and let us see what you get.

So, go to insert, go to scattered diagram and choose this one.

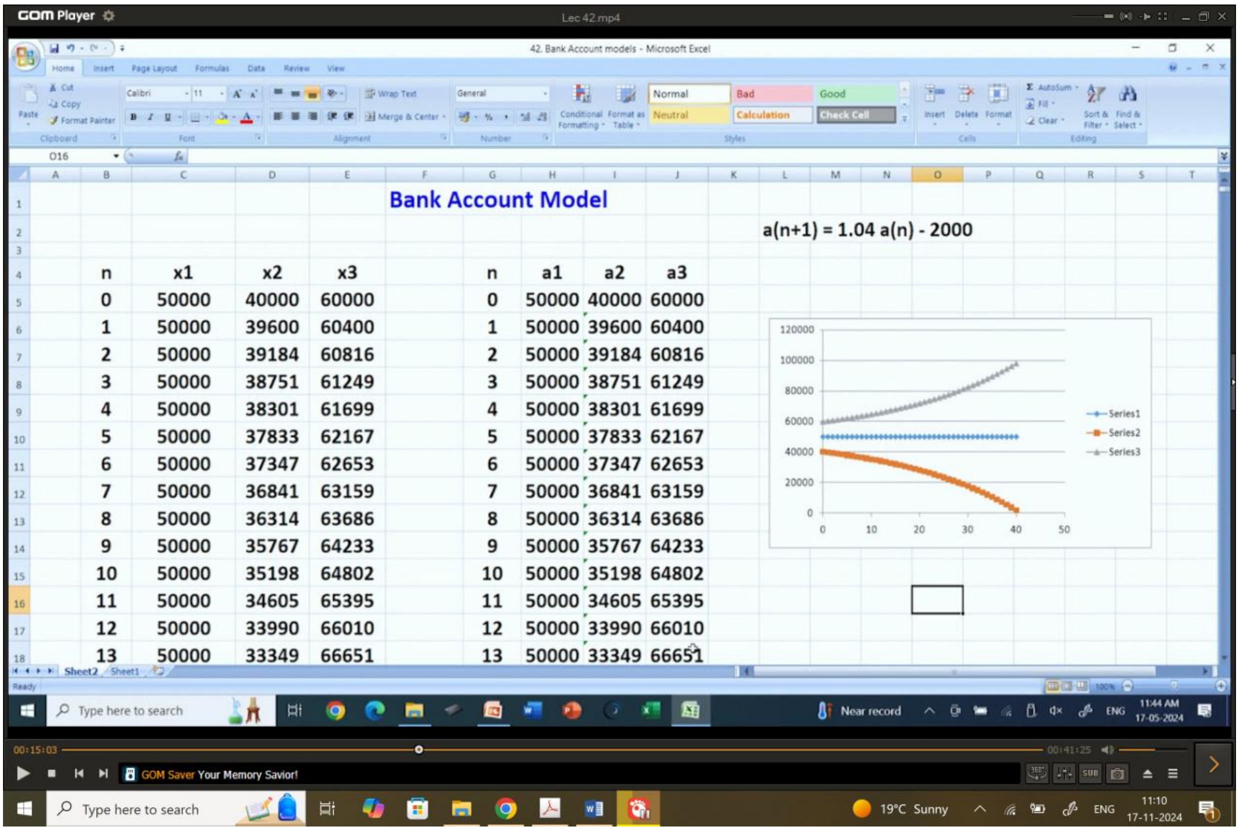
So, you get something like this okay so let me now explain it from the slides so we have this kind of chart which you can see in the slide so when this is 50,000 you get a constant one so if you make your initial deposit to be 50,000 as shown analytically your account will maintain 50000 throughout and even if you deposit 2000 from here this is going to maintain 50000.

So, the cash will never run off.

So, that is the problem which we are facing and we have solved it.

But obviously you will be curious what will happen if A is less than 50000 then you can see that slowly your amount is decreasing and ultimately goes to 0 and if it is more than is greater than if it is greater than 60000 sorry 50000 then your amount will increase and it becomes an unbounded solution.

So, basically these two solutions are needed for a real life scenario, but again if it is more than 60000 your money will keep growing till there is any change in the rate of interest or in the initial amount.



n	x1	x2	x3	n	a1	a2	a3
0	50000	40000	60000	0	50000	40000	60000
1	50000	39600	60400	1	50000	39600	60400
2	50000	39184	60816	2	50000	39184	60816
3	50000	38751	61249	3	50000	38751	61249
4	50000	38301	61699	4	50000	38301	61699
5	50000	37833	62167	5	50000	37833	62167
6	50000	37347	62653	6	50000	37347	62653
7	50000	36841	63159	7	50000	36841	63159
8	50000	36314	63686	8	50000	36314	63686
9	50000	35767	64233	9	50000	35767	64233
10	50000	35198	64802	10	50000	35198	64802
11	50000	34605	65395	11	50000	34605	65395
12	50000	33990	66010	12	50000	33990	66010
13	50000	33349	66651	13	50000	33349	66651

Now, let us change it into a loan problem with the same setting say a person wants to take a loan.

So, in that particular case also you have the payment which will depend on the rate of interest and then you make a monthly instalment of d.

So, let us assume that the bank charge 12% interest annually.

The monthly income of the person, say, 96,000 and the bank put a clause that monthly instalment cannot exceed one-fourth of the monthly income.

So, your maximum repayment of the loan will be 8000 divided by 4 which is 2000.

So, this is, say, yearly income.

So, this divided by 12 will give you 8000 and one fourth of the loan is 2000.

Your monthly rate is 12%, you take the percentage 0.12 and you divide it by 12, so 0.01.

So, if you now put everything here and this

$$P_{n+1} = P_n + rP_n - d$$

And we know the solution of this is

$$P_{n+1} = P_0(1+r)^{n+1} - d\left[\frac{(1+r)^{n+1} - 1}{1+r-1}\right].$$

Now, say, he took the loan for 25 years, which is 300 months. So, if you sum up this problem that this is now a loan which has been taken from the bank.

So, how it works?

This is the amount for $(n+1)^{\text{th}}$ year and this is the amount for the n^{th} year. Bank will charge an interest r on the principal that is there and minus the monthly instalment.

So, this is the monthly amount that has been accumulated and this is the instalment you pay, your rate of interest is r.

So if your yearly income is 96,000, you divide it by 12 which comes to 8,000 and bank put a clause that you cannot pay more than one-fourth of the monthly income which is 2,000.

Your monthly rate will be 12 is your annual rate divided by 12 and then divide by 100 and you get it 0.01.

This will have a solution in this form, you substitute everything here and you say that in 300 months my loan amount will be zero.

So, you put this equal to zero, your initial amount you want to calculate. So, what will be your P_0?

$$P_{300} = 0 = P_0(1.01)^{299+1} - 2000\frac{(1+0.01)^{299+1} - 1}{0.01} = 0$$

$$\Rightarrow P_0 \times 19.79 = \frac{2000}{0.01}(19.79 - 1)$$

$$\Rightarrow P_0 = 189893.886 \approx Rs.189894$$

So, if you take this much amount of loan that is the maximum amount this much then you will be able to repay back in 300 months.

We now look into another interesting model we name it as retirement investment or account model.

So, often you visit bank, this personal, they suggest you different schemes where you can invest.

So, this retirement investment or retirement account is one such scheme.

So, suppose a person who is doing a high profile job he went to the bank and the personal suggest that you must have a retirement account.

So, what they are going to offer?

So they say that say in this account he will earn 0.75% per month also with his high paying he will be able to invest rupees 20,000 per month and to take care of the inflation and of course there are pay raises.

It is suggested to increase this amount by this amount means this 20,000 by 0.5% per month.

Now to see what will be the accumulation of the fund say after 20 years.

So, if we want to find that what will be the accumulation of his account or fund after 20 years which is 240 months.

So, we are going to model this situation where a person went to the bank and he have been asked that you please invest in this retirement scheme and in the scheme they said that you will earn 0.75 percent every month which is 9% per year and if you invest every month 20,000 that is the initial investment and let us find out what will be the amount after 240 months and it is suggested that every month he should increase this amount by 0.5% to keep into account the inflation and the pay rises.

So, if we want to model it, so let us say this an that is the monthly amount after n months in the retirement account and say R be the inflation rate.

$$A_n = A_{n-1} + rA_{r-1} = (1+r)A_{n-1}$$

Then,

$$A_1 = (1+r)A_0$$

$$A_2 = (1+r)A_1 = (1+r)(1+r)A_0 = (1+r)^2\, A_0$$

$$A_3 = (1+r)^3\, A_0$$

$$-----------$$

$$A_n = (1+r)^n\, A_0$$

So, this is one part. Let us take another variable B_n, which gives the balance in the retirement account after n months. Let your *i* be the interest rate and B_0 is your initial value, then

$$B_n = B_{n-1} + iB_{n-1} + A_{n-1} = (1+i)B_{n-1} + A_{n-1}$$

$$\Longrightarrow\ B_1 = (1+i)B_0 + A_0 = A_0$$

Now initially $B_0 = 0$, as there is no balance because the initial investment start from 20,000. Similarly,

$$B_2 = (1+i)B_1 + A_1 = (1+i)A_0 + (1+r)A_0$$

$$B_3 = (1+i)B_2 + A_2 = (1+i)[(1+i)A_0 + (1+r)A_0] + (1+r)^2\,A_0$$

$$= [(1+i)^2 + (1+i)(1+r) + (1+r)^2]\,A_0$$

$$= \left[\frac{(1+i)^3 - (1+r)^3}{(1+i) - (1+r)}\right] A_0$$

$$B_4 = (1+i)B_3 + A_3$$

$$= (1+i)[(1+i)^2 + (1+i)(1+r) + (1+r)^2]\,A_0 + (1+r)^3\,A_0$$

$$= [(1+i)^3 + (1+i)^2(1+r) + (1+i)(1+r)^2 + (1+r)^3]\,A_0$$

$$= \left[\frac{(1+i)^4 - (1+r)^4}{(1+i) - (1+r)}\right] A_0$$

So, if I want to get the formula for B_n, so I have to just follow the generalization, this will be

$$B_n = \left[\frac{(1+i)^n - (1+r)^n}{(1+i) - (1+r)}\right] A_0$$

So, this is basically what you get after the simplification of the model. So, let us now find what will be his funding after 240 months.

So, according to the problem

$$A_0 = 20000, i = 0.75\% = 0.0075,$$

$$r = 0.005, n = 20 \text{ years} = \ 240 \text{ months}$$

So, if I substitute Bn this will be equal

$$B_n = 20000\left[\frac{(1+0.0075)^{240} - (1+0.005)^{240}}{0.0075 - 0.005}\right]$$

$$= 8\times 10^6[(1.0075)^{240} - (1.005)^{240}] = 21591576.39$$

Let us now check this result numerically. So, I already have the model here in the form of basic equations which is

$$A_n = A_{n-1} + rA_{r-1} = (1+r)A_{n-1}, \qquad B_n = (1+i)B_{n-1} + A_{n-1}$$

So the initial condition is there is no balance which is zero.

This value is 20000 with which he starts.

Your value of r is 0.005 and the value of i is 0.0075 just change the font size and make it bold and set.

So, let us first calculate this A_n so this is equal to 1 plus r 1 plus so though it looks like 0.01.

Let us now calculate this value of A_n which is equal to 1 plus r which is a constant multiplied by a0 which is this value and you get the value and this is equal to 1 plus i. So, 1 plus i which is again a constant multiplied by B_0 initial value is 0 plus A_0 which is this.

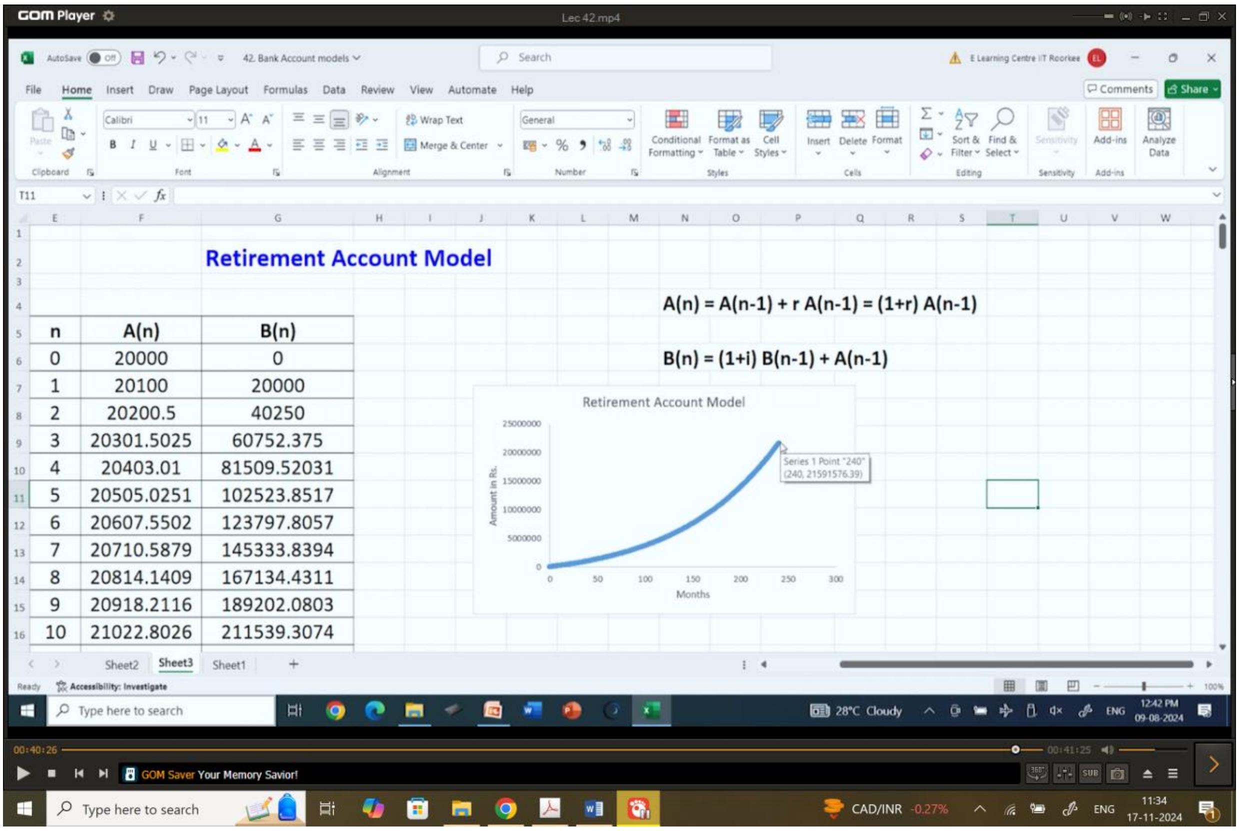
Let me now drag this to the next 240 values.

So if we want to plot this as I want to see what is the balance basically I want to see this Bn so I will highlight this first up to 240 values and then I will take the third column so I press control I click this then press shift and the down cursor.

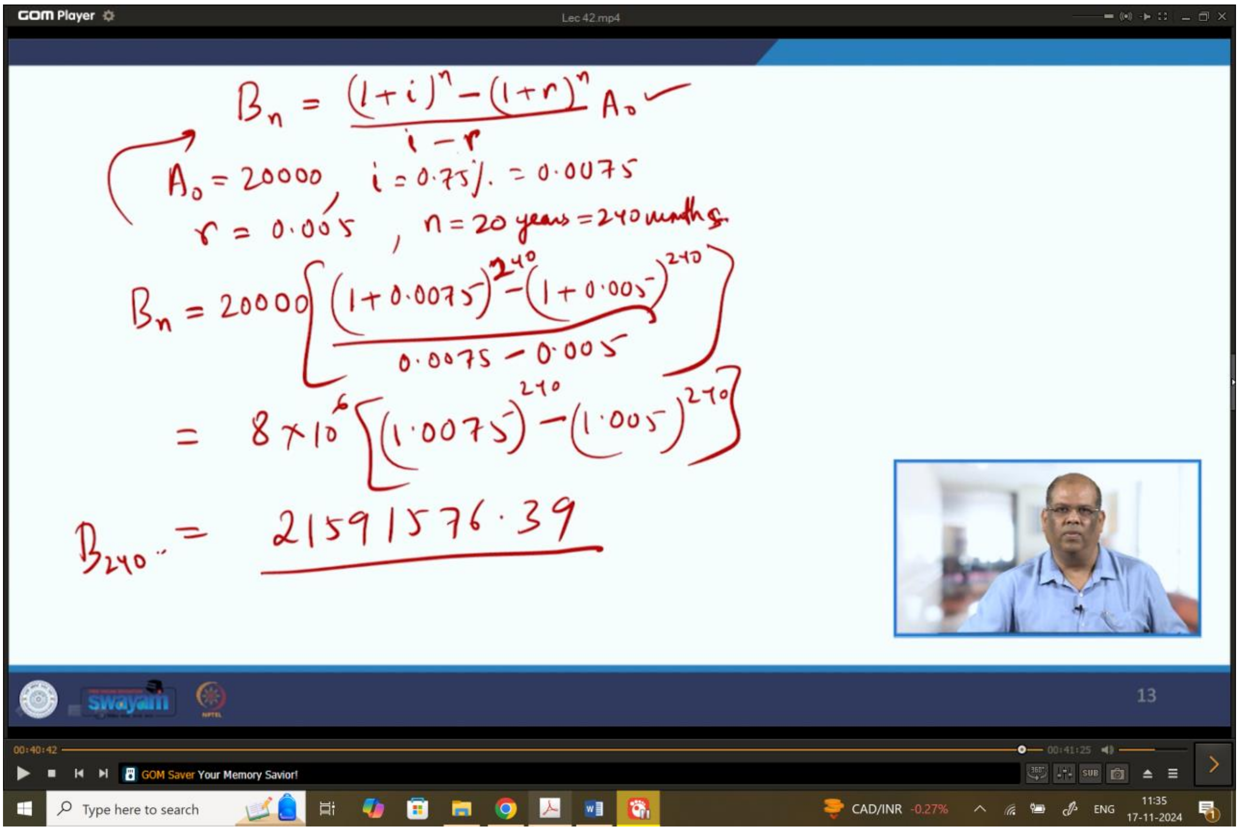
So, it will choose all the 240 values.

I go to insert, I go to chart and I plot this.

So, I can write this as retirement account model remove the grid lines if I want the axis title this is the month and this is the amount in rupees.



So, if I want to see what is the amount after 240, so I can just bring the cursor here and as you can see it is written that 0.240 and the amount is 21591576.39 which matches exactly with the analytical solution this 21591576.39.



So with this, we come to the end of this lecture where we have taken few examples that is related to the bank and its accounts.

So in my next lecture, we will be talking about some economic models, discrete economics models.

Till then, bye-bye.