EXCELing with Mathematical Modeling

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Week – 11

Lecture – 52 (Smoking model)

Hello, welcome to the course EXCELing with Mathematical Modelling.

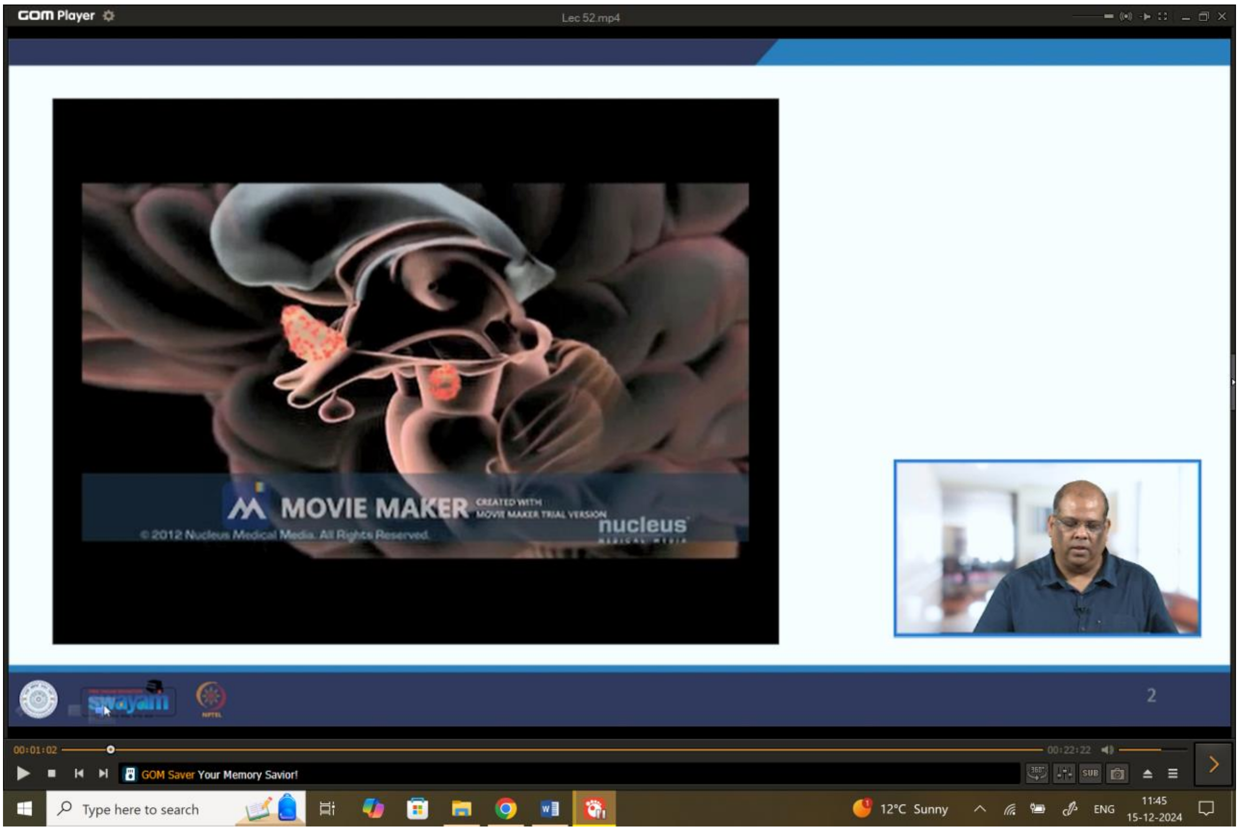
Today we will be talking about an interesting model, smoking model.

So when somebody inhales a cigarette, as you can see in the video, lots of toxic gases that get inside.

One of them, which we call nicotine, affects the brain.

So what does this nicotine does is, as we'll see that it highlights a part of the brain which actually give you a very pleasurable feeling.

So as those parts gets highlighted, that means you get addicted to the smoking.



So as we all know that the smoking affects the lungs due to a lot of toxic gases, a lot of toxic things or amounts that get inside the body and one of them is called cadmium.

So, this cadmium they get inside the lungs and they damage the lungs.

So, we will be talking about this smoking model where we model this amount of cadmium in the body.

So, some information is that each year a person smokes a single pack of cigarettes a day will absorb about 2.7 milligram of cadmium which is an extremely dangerous heavy metal pollutant and it is very toxic as you inhale the cigarette.

So, for simplicity in the model we assume that this cadmium it is absorbed at the end of the year and the elimination of cadmium is at the rate of 8 percent that is some people eliminate this cadmium from their bodies each year at the rate of 8%.

It could be 6%, 7%, even 9%.

It depends from person to person.

So with this information, mostly the elimination of cadmium at the rate of 8% and 2.7 mg of cadmium every year is added

Let us model this smoking problem.

So, if we take C_n that is the amount of cadmium in the body at the end of n years and r is the removal rate of cadmium and A is the yearly absorption of cadmium.

So, if we now model this, it is going to be

$$C_n = C_{n-1} - r\, C_{n-1} + A$$

So, this is the amount at the end of year n, this is the amount at the end of year $(n-1)$and removal is at a rate r and addition is A.

So, if you want to now find the equilibrium point, we put

$$C_n = C_{n-1} = C^*$$

$$\Rightarrow \; C^* = C^* - rC^* + A \Rightarrow C^* = \frac{A}{r}$$

So, this is the point of equilibrium. If you want to see the stability, about the equilibrium point $C^* = \frac{A}{r}$, we put

$$f(C) = C - rC + A \Rightarrow f'(C) = 1 - r$$

So, condition for stability is

$$|f'(C)| = |1 - r| < 1$$

$$\Rightarrow 0 < \; r < 2.$$

Now, before we check this numerically, let us derive generalized formula for this C_n. So, we have

$$C_n = C_{n-1} - rC_{n-1} + A$$

Let us assume that initially there is no cadmium inside the body.

So, your $C_0 = 0$,

$$n = 1, \qquad C_1 = C_0 - rC_0 + A = A$$

$$n = 2, \qquad C_2 = C_1 - rC_1 + A$$

$$= (1 - r)A + A$$

$$= (1 + 1 - r)A$$

$$n = 3, \qquad C_3 = C_2 - rC_2 + A$$

$$= (1 - r)C_2 + A$$

$$= (1 - r)(1 + 1 - r)A + A$$

$$\Rightarrow C_3 = [1 + (1 - r) + (1 - r)^2]A$$

$$n = 4, \qquad C_4 = [1 + (1 - r) + (1 - r)^2 + (1 - r)^3]A$$

In a similar manner,

$$C_n = [1 + (1 - r) + (1 - r)^2 + \cdots.. + (1 - r)^{n-1}]A = A\left(\frac{1 - (1 - r)^n}{1 - 1 + r}\right)$$

$$= A\left(\frac{1 - (1 - r)^n}{r}\right)$$

So, if I know the value of r and I want to find what is the amount of cadmium at certain time, I can just substitute, where your A value is given that is 2.7 and r is 8 percent, which is 0.08.

Now let us check this solution numerically.

So we will be using Microsoft Excel, I already have it open.

The value of r is 0.08 and the value of A is 2.7.

So, I just okay, so this is the n, this is the number of years, this is equal to 0 plus 1 and let me drag to few values, say, 50 of them, okay.

Now, the formula is or the model is

$$C_n = C_{n-1} - rC_{n-1} + A$$

So, C_n is the amount of cadmium at year n, this is the amount of cadmium in year $(n - 1)$ and there is a reduction of 8 percent, so, r is 0.08 of the amount in year $(n - 1)$ plus a constant input of 2.7 every year and initial value is taken to be 0.

We assume there is no cadmium and then this is equal to C_{n-1} , which is this minus r times which is a constant multiplied by C_{n-1} which is this plus the constant value of A, which is 2.7, and I make this a constant by putting dollars and enter.

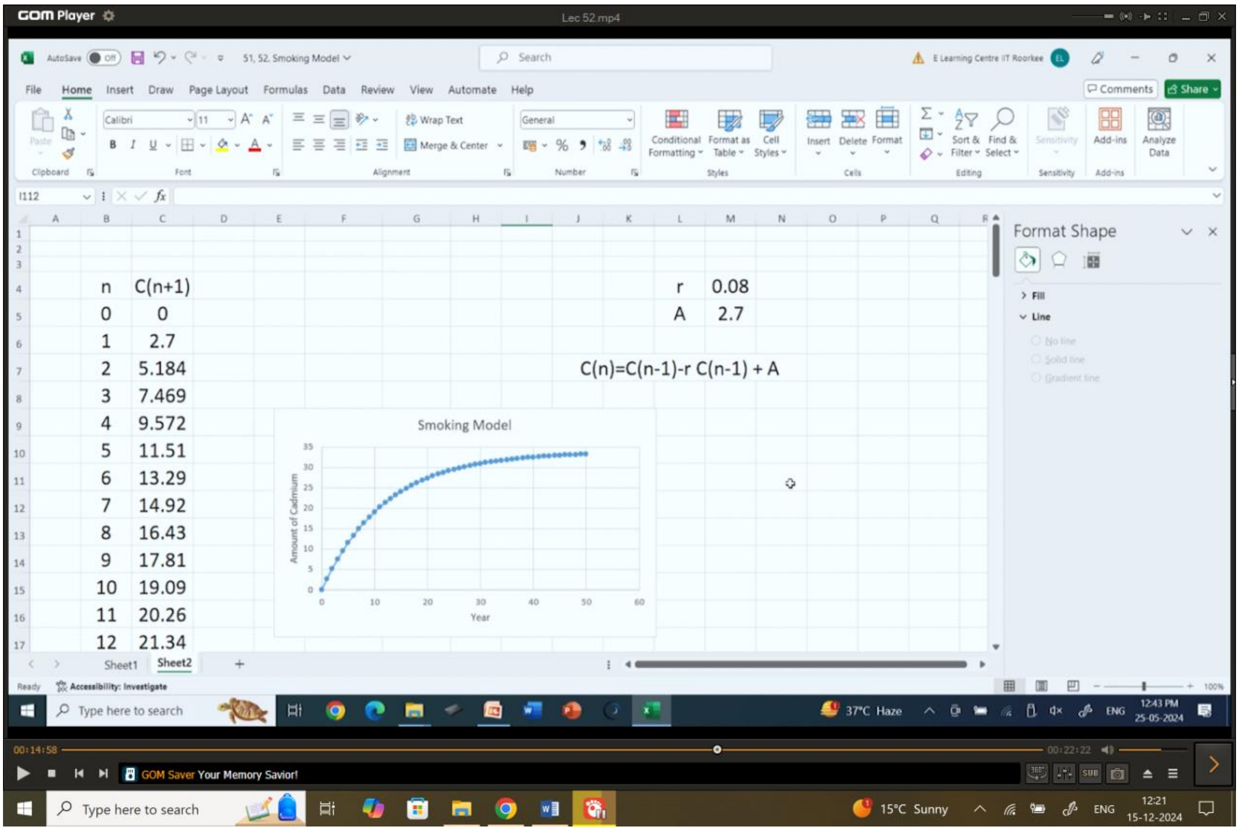
So, if I drag the values, let us see how much, okay.

So, I highlight this, go to insert, click this value and this one.

So, I can write this as smoking model, I want to enter the axis title, so this is year and this is the amount of cadmium.

So, this reaches some steady state value which I can see as 33 point something a little more.

So, if I drag these two a little more value, so it reaching 33.75, which is the equilibrium value and since this model is stable because you remember you get mod of 1 minus r and r is positive quantity 0.08 so 1 minus 0.08 will the modulus of that value will be less than 1 and hence this model is stable with these parameter values and you get the graph like this.



So, as you have noticed that the condition for stability is mod 1 minus r and if you check with this particular value, you can see that mod 1 minus r is going to be 1 minus 0.08 modulus of that 0.92 and which is less than 1. and hence your system is stable about the equilibrium point A star by r and if you substitute this value 2.7 by 0.08, you will see that this is something close to 33.75 and in the graph you have seen that this particular value starts from 0 and reaches the value 33.75.

Now if you want, what will be the amount of cadmium after say 20 years?

So, all you have to do is that you have to write the formula

$$C_n = \frac{A}{r}[1-(1-r)^n]$$

$$A = 2.7, \qquad r = 0.08, \qquad n = 20$$

So you want to calculate what is the amount of cadmium after 20 years.

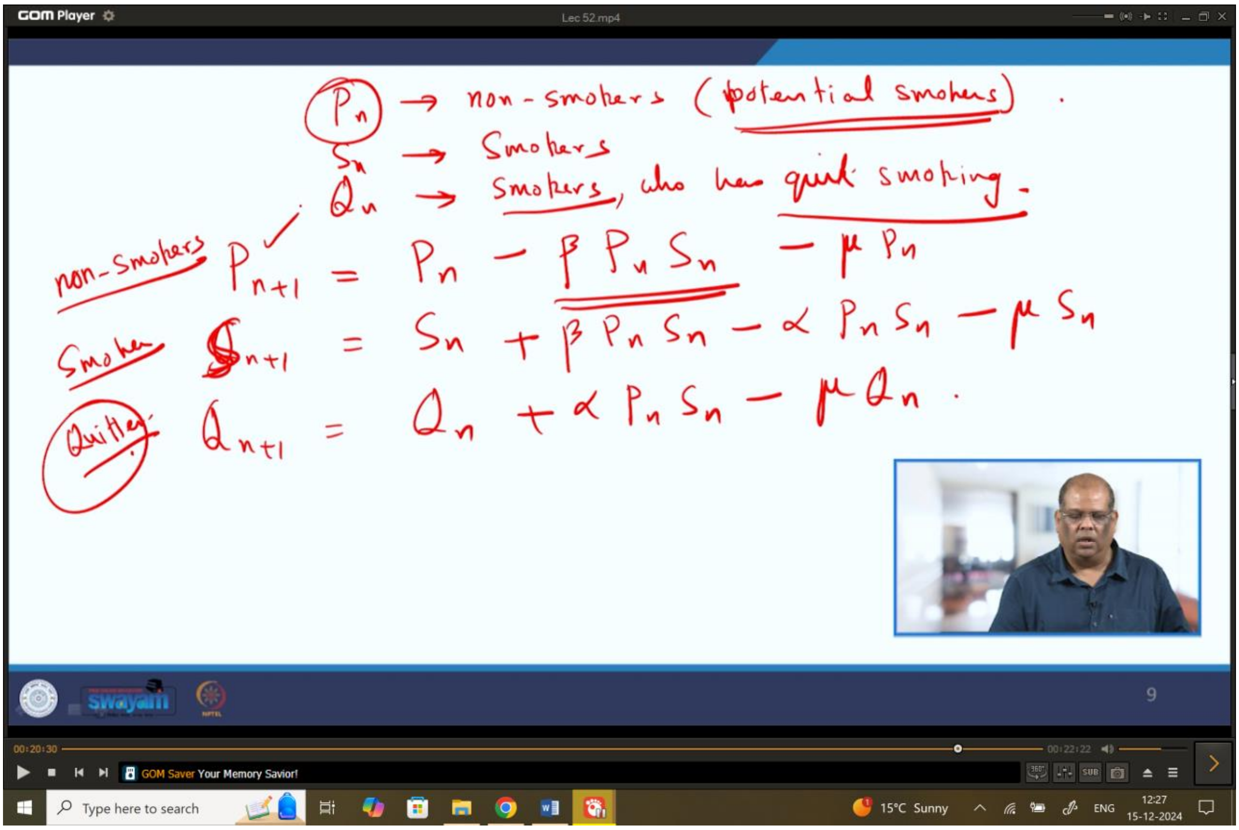
So you just plug in these values and you get

$$C_{20} = \frac{2.7}{0.08}[1-(0.92)^{10}] = 27.37\, mg$$

We next move to another interesting model for the smoking case.

Suppose we have P_n, we tell them as non-smokers but they are potential smokers and S_n to be the smokers and Q_n to be the smokers who has quit smoking.

So, this model is about smokers, the non-smokers and the person who used to be smokers but then they quit smoking.



So, we first write this one.

$$P_{n+1} = P_n - \beta\, P_n\, S_n - \mu P_n$$

$$S_{n+1} = S_n + \beta\, P_n\, S_n - \alpha\, P_n\, S_n - \mu S_n$$

$$Q_{n+1} = Q_n + \alpha\, P_n\, S_n - \mu\, Q_n$$

So, this gives you model among the non-smokers, the smokers and the person who has quitted smoking we call them quitters.

So, one more time, so this is the non-smoker, the non-smokers quitters, who comes across with the person who is smoking and then from this non-smoking class, they go to the smoking class and from the smoking class, there will be always some person who want to quit smoking which is happening at a rate alpha and then come to the quitter class and then these are the natural death of the persons.

Now, I leave this model to you, I give you some numerical values, say, $\beta = 0.2$, $\mu = 0.04$ and $\alpha = 0.12$.

You take the initial conditions and use these parameter values to get different dynamics of this model and use Microsoft Excel.

We have done so much examples.

I am sure you will be able to solve this particular smoking problem.

So, with this we come to an end of this interesting session where we have discussed this smoking model.

In my next lecture, we will be talking about this price and demand model.

Till then, bye-bye.