EXCELing with Mathematical Modeling

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Week – 11

Lecture – 54 (Paper Towel Model and Burning Calorie Model)

Today we will be discussing two models.

One is the paper towel model and another one is the burning calorie model.

So, to start with let us go into this paper towel model.

These days we many of us use this paper towel mostly in the kitchen or the dining table rather than not much in the toilet. So, the problem involves is what is the that length of that paper and including the radius the thickness of the paper.

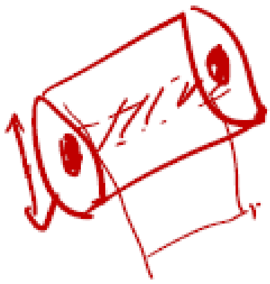
So, let us dig into that.

Paper Towel Model

So, suppose you have a roll of paper as paper towel.

So, we are formulating the problem first.

Let the radius of the cardboard core be 0.5 inch. So, if you look into this paper towel, it will look something like this, and then a little bit of paper holding out and this is rolled in some cardboard tube, which is termed here as cardboard core and suppose the thickness of the paper be 0.002 inch.



So, we define the variables R_n as the radius of the paper wrapped n times and L_n with the length of the paper used for wrapping till n times.

So basically you have to get a relation between the radius of the paper and the length.

So as you roll the paper this radius starts increasing and so is the length of the paper.

So we want to find a relation between the radius and the length of the paper.

So if

$$R_n = R_{n-1} + t,$$

where t is the thickness of the paper.

So, you will have

$$R_1 = R_0 + t$$

$$R_2 = R_1 + t = R_0 + t + t = R_0 + 2t$$

Similarly,

$$R_3 = R_2 + t = R_0 + 2t + t = R_0 + 3t$$

So, if you notice the pattern you can see that

$$R_1 = R_0 + 1.t$$
$$R_2 = R_0 + 2.t$$
$$R_3 = R_0 + 3.t$$

$$R_n = R_0 + n.t$$

So, this gives your R_n in terms of the initial R_0, the number of times it has been wrapped and the thickness of the paper.

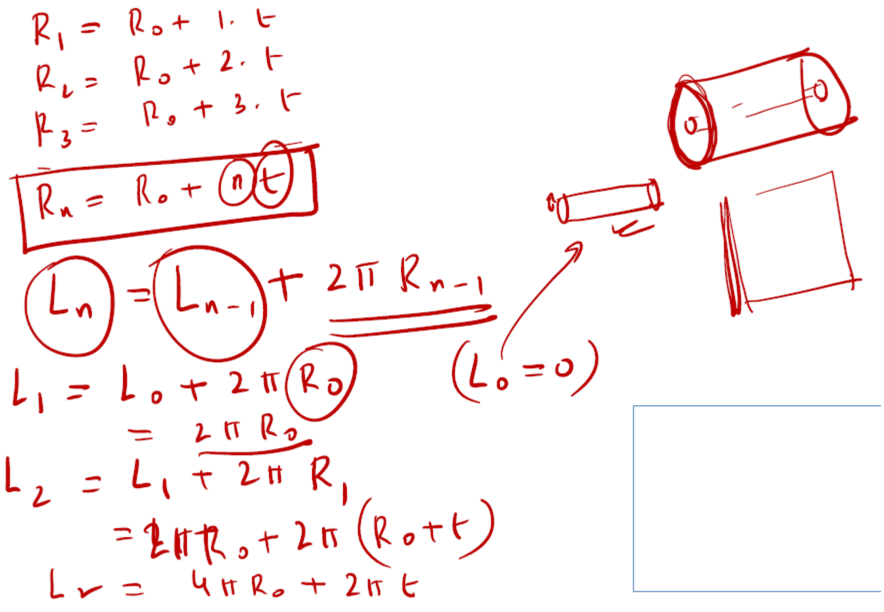
Let us look into the length.

So, if I open it once, so it basically it is going to take this length.

So, what you will get is

$$L_n = L_{n-1} + 2\pi R_{n-1}$$

So, this is the current length, this is the previous length and if you add one revolution which is this circumference of this circle, you get $2\pi \times R_{n-1}$.



So, similar manner I find

$$L_1 = L_0 + 2\pi R_0$$

and initially there is no length. So, your $L_0 = 0$. So, basically

$$L_1 = 2\pi R_0$$

Similarly,

$$\begin{aligned} L_2 &= L_1 + 2\pi R_1 \\ &= 2\pi R_0 + 2\pi(R_0 + t) \\ &= 4\pi R_0 + 2\pi t \\ &= 2.2\pi R_0 + 2(2-1)\pi t \end{aligned}$$

$$\begin{aligned} L_3 &= L_2 + 2\pi R_2 \\ &= 4\pi R_0 + 2\pi t + 2\pi(R_0 + 2t) \\ &= 6\pi R_0 + 6\pi t \\ &= 3.\,2\pi R_0 + 3(3-1)\pi t \end{aligned}$$

$$\begin{aligned} L_4 &= L_3 + 2\pi R_3 \\ &= 6\pi R_0 + 6\pi t + 2\pi(R_0 + 3t) \\ &= 8\pi R_0 + 12\pi t \\ &= 4.2\pi R_0 + 4(4-1)\pi t \end{aligned}$$

So now you get the pattern and now you can write your

$$R_n = R_0 + nt$$

$$L_n = 2\pi n R_0 + n(n-1)\pi t$$

So, these are the two equations that represents the length and the radius.

Let us now answer the question.

So, what will be the length of the paper when the radius of the roll is 2 inch.

So, either you know the length, you find the radius or you know the radius and you can find the length. So, you have your

$$R_n = 0.5 + 0.002\, n$$

If you quickly recall, this is the length the thickness of the paper and this is the radius of the cardboard core.

So, we got this equation for R_n. Next, it is given that $R_n = 2$ inch. So, if you substitute it here you will get

$$2 = 0.5 + 0.002 \times n$$

$$\Rightarrow n = \frac{2 - 0.5}{0.002} = 750.$$

Now, you substitute this value of n here, this value of t here, you get the

$$L = 2\pi \times 750 \times 0.5 + 750(750 - 1)\pi \times 0.002 \approx 5888.10 \text{ inch}$$

So, we took paper towel problem where we have the radius of the wrap of the paper and the length in terms of the variables.

If you want to find the length relation between the length and the , this is how it is done and you can answer the question that what is the length if the radius is either 2 inch or 2.5 inch or 1.5 inch.

So, we have discussed a particular paper towel problem, let us now move to the burning calorie problem.

<u>Burning Calorie Model</u>

We all exercise to burn the extra few calories that we take so that we can keep ourselves fit, we look good, we look trim and slim.

So the burning calorie model exactly deals like this.

So suppose your weight is 169 pounds and you consume x pounds worth of calories each week. Assume that your body burns off the equivalent of 3% of its weight to the weight of the body each week through normal metabolism.

And this 3% varies from person to person, becomes slow with the age.

In addition, you burn off one fourth pound through daily exercise.

Now with this information, you try to form a model and then answer the question that what is that x you must take the calories each week such that your weight lie between 144 pounds to 146 pounds.

So if you want to formulate this model, let us define W_n that is the weight after n weeks.

The calories you consume is x and W_0 is your initial weight. So, you have

$$W_n = W_{n-1} - \frac{3}{100} W_{n-1} - 0.25 + x.$$

So, this gives a model for your weight and you want to find that what the range of this x, such that your weight lies between 144 and 146 pounds.

So, let us simplify this model. So, you have

$$W_n = W_{n-1} - 0.03W_{n-1} - 0.25 + x = 0.97W_{n-1} + (x - 0.25)$$

$$W_1 = 0.97W_0 + (x - 0.25)$$

$$\begin{aligned} W_2 &= 0.97W_1 + (x - 0.25) \\ &= 0.97[0.97W_0 + (x - 0.25)] + (x - 0.25) \\ &= 0.97^2 W_0 + (x - 0.25)(1 + 0.97) \end{aligned}$$

Similarly,

$$W_3 = 0.97^3 W_0 + (x - 0.25)(1 + 0.97 + 0.97^2)$$

$$W_4 = 0.97^4 W_0 + (x - 0.25)(1 + 0.97 + 0.97^2 + 0.97^3)$$

Ultimately, we can get

$$\begin{aligned} W_n &= 0.97^n W_0 + (x - 0.25)(1 + 0.97 + 0.97^2 + \cdots + 0.97^{n-1} \\ &= 0.97^n W_0 + (x - 0.25)\frac{1 - 0.97^n}{1 - 0.97} \\ &= (0.97)^n W_0 + \tfrac{x - 0.25}{0.03}[1 - (0.97)^n] \end{aligned}$$

So, the question was that how much calorie you must take so that you can keep the weight between 144 and 146 pounds in one year, which is 52 weeks.

So, here $n = 52$. I have to find the value of x such that the weight in 52 weeks must lie between 144 and 146 pounds and the initial value of the weight $W_0 = 169$ pounds.

So, all you have to do is to substitute these values to get

$$144 < W_n < 146$$

$$\Rightarrow\ 144 < (0.97)^{52} W_0 + \frac{x - 0.25}{0.03}[1 - (0.97)^{52}] < 146$$

And if you simplify this, you will get

$$4.375 < x < 4.45,$$

that is, x lies between 4.375 to 4.45.

So, if your calorie intake lies between 4.375 and 4.45, you will be able to bring down your weight between 144 and 146 from 169.

So with this, we have an idea that how burning of calories can be calculated or can be achieved by this simple mathematical model.

In my next lecture, we will be talking about kidney function model.

Till then, bye-bye.