EXCELing with Mathematical Modeling
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Week – 12
Lecture – 59 (Estimation of Parameters-III)

Hello, welcome to the course EXCELing with Mathematical Modelling.

In this lecture we continue with our study for this estimation of parameters and today we will be learning about how to estimate the parameters with difference equations in discrete cases.

So, what you will be given is again a set of data because it is a discrete case so it is $n = 0,1,2,3$ ... and some variable x_n and this data will be given say x_1, x_2, x_3 and so on, and you will be having discrete equation of the

$$x_{n+1} = f(x_n),$$

which contains the parameters.

We will be using the method of list squares to estimate the value of the unknown parameters.

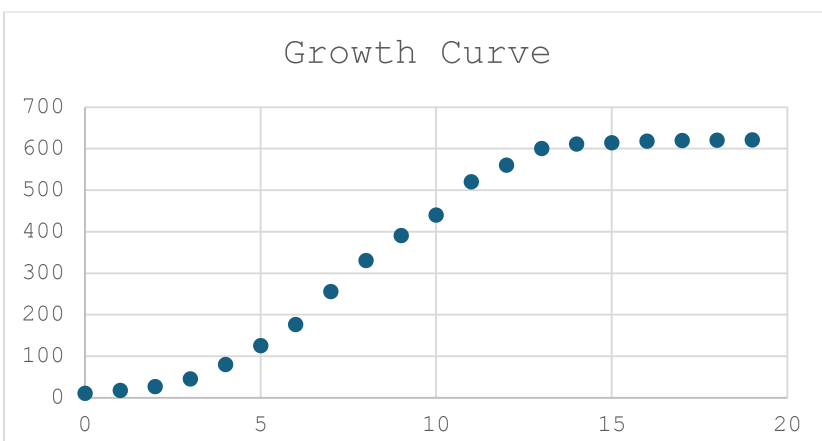
So, straight forward we go to the excel spreadsheet.

But before that let me tell you what equation we will be doing. So I will be taking the logistic growth equation in discrete case and the equation looks like

$$a_{n+1} = a_n + b(c - a_n)a_n,$$

where b and c are unknown parameters. This b is the intrinsic growth rate and c is the carrying capacity.

So, let us go to the data. So, this is the measured value of some growth data. So, let us see that how this graph looks. So, this is the graph. So I just see this as a growth curve.



So the equation which I have is the logistic equation and this will look like this that

$$a(n+1) = a(n) + b*(c - a(n))*a(n)$$

and I just make this want 20.

So, this is the equation b and c is the unknown parameters.

Now, as I can see that this curve is going towards the value 600 and above and here I can see it is 621. So, one way is that I can take this value directly to be 621. So, I know my carrying capacity from the graph as it is going to 621.

Then the only thing is that what will be the value of this c.

So the predicted a_n, so this is the predicted a_n, the initial value, say, I coincide with say I put it as 10, so this will be equal to, if I put n= 0 here, so

$$a(1) = a(0) + b*(c - a(0))*a(0)$$

Now, this is equal to a(0), which is this plus b times, so I have to put a value here, say 0.1, this is equal to

$$a(1) = a(0) + b*(c - a(0))*a(0)$$

So, I already took the value of c to be 621 minus a(0), which is this, and multiplied by again a(0).

So, I get this value.

Okay, let us now plot these values first.

So I have to come here, I have to select data and I have to add data.

This is a predicted, so the values of n, enter and the values of predicted.

So let's make this continuous line.

So change the chart type, the predicted or the continuous line.

Okay.

Open the legend, remove the grid lines, and the series 1, I rename it as data.

Edited.

Okay.

So, now you can play with this value a little so that this curves comes here.

So, 0.001.

Okay.

Oh, wow.

Okay.

So, if you put some value here, so as I was started playing with, I see that this becomes a quite a good match here.

But then, now you have to check using the method of least squares.

So, first you calculate residuals.

So, the residuals is this value minus this value and then the residual square. So this square.

So once you get the residual square, you have to calculate the sum.

So, which is equal to the sum of the square of the residuals.

So, I got the sum.

So since this is quite a good match, this value is not going to change much.

So that was just a guess and it sort of matches very well.

But then the technique remains the same.

You go to, once you get the sum of the square of the residuals, you have to minimize this, go to data, go to solver and then in this objective,

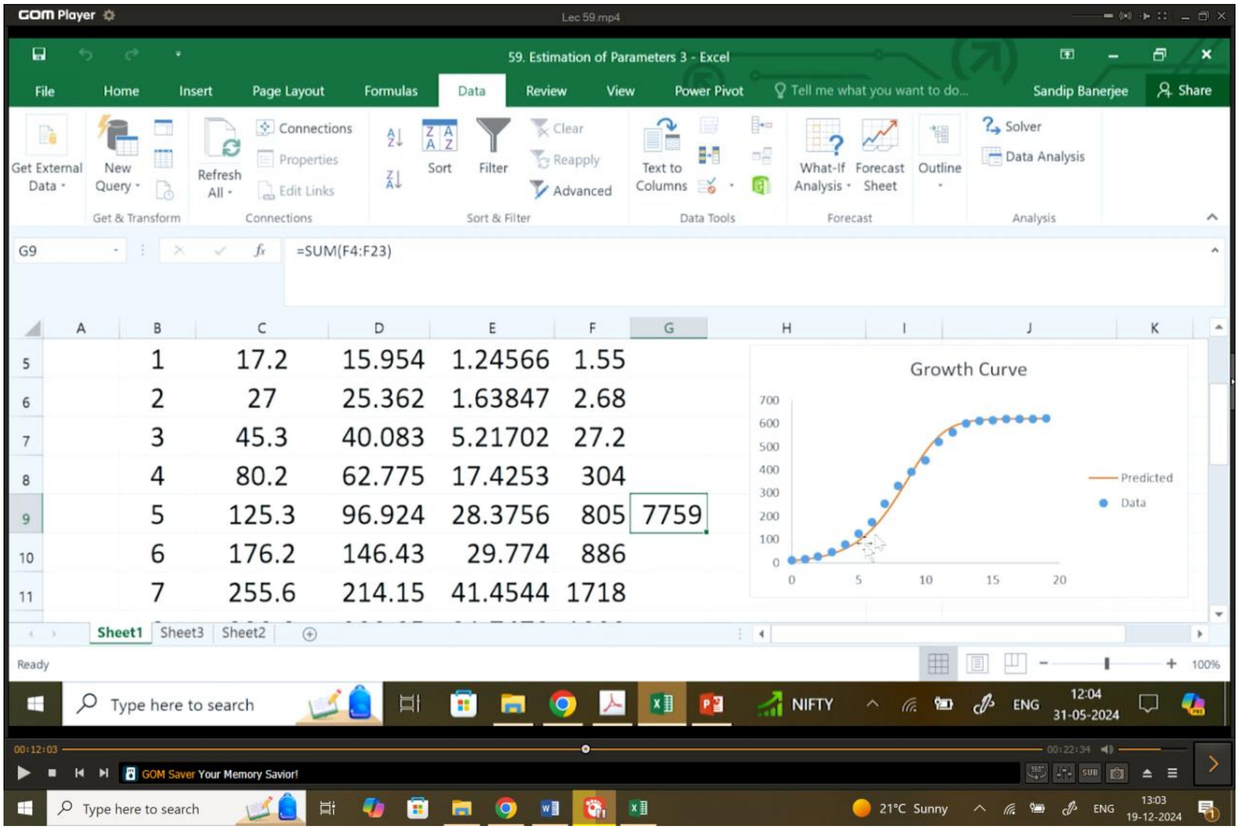
I choose this one which is this cell I have to minimize and by changing the variable cell which is b here and I will click solve.

So a very slight change and you see the value is 0.00097.

So that is why it was a good match because I just put 0.001.

So that is why it becomes a good match and from here you can see that it is a quite good match with this particular set of data and the estimated value for b because I have taken the c value to be 621.

So, the estimated value of b is 0.00097452.



Let us quickly take another example where I have this E. coli data and I am going to, the one which we used for the continuous case.

So, only I make the time this is n now and this is some growth data a(n). I make them 20.

So in this particular case also I will, since it is a growth data, I will be using logistic equation but now here I will estimate both b and c. So I just put it, say, some b = 0.1 and c = 0.1.

So, the reason of choosing such a less value, you see the data and see where it is approaching. So, they are all 0 point something. So, that is why instead of choosing 1, 2 or something, I choose some value which lies somewhere in this range. And I will know that the logistic value that it will ultimately go to its carrying capacity.

So, let us see. So, it is approaching to some value as you can see 0.7 something. So, I could have taken it as 0.7 also, but then let us estimate it.

So, the first thing is this is the measured value a(n). So, you got the growth data from some experiment and now the predicted a_n.

I take this value to be 001 and then this value is

$$a(1) = a(0) + b*(c - a(0))*a(0)$$

So, this will be D4. So, I get some value. So, I drag them.

So, let us now plot this, however I should have plotted the measured value first, but since we are doing it for previous examples.

So, let us now draw all three of them at the same time, I go to insert, I go to chart, and say I have this chart name it again as growth curve so this one let's change it to series type

So this one has a continuous line. And let me remove the grid lines. The legends are here.

So I go to design, select data, series 1, edit, name it as data and series 2 edit image as predicted.

So this is the data, this is the predicted, not a very good match but you have to play with.

the values, so you have b and c. So, let us take the value of this 2.5.

Okay, so once I change these values to 2.5 and 5, I see this curve is going up.

Let me remove this one so that you can have a look here.

So 0.6, so again moving up again 0.6.

So slowly you can see that if I adjust it, I can bring it somewhere here.

But then let us see the sum of the residuals.

So this is equal to the measured value minus the predicted value and I square them.

So sum, this is the square of the residuals.

So sometimes if suddenly this comes, do not panic, what happens is that you just click double thing and you get the rest of the values here.

So if they are quite small actually what happens is the values are approximate to certain decimal places.

Now I have to get the sum. So this is equal to the sum of the squares.

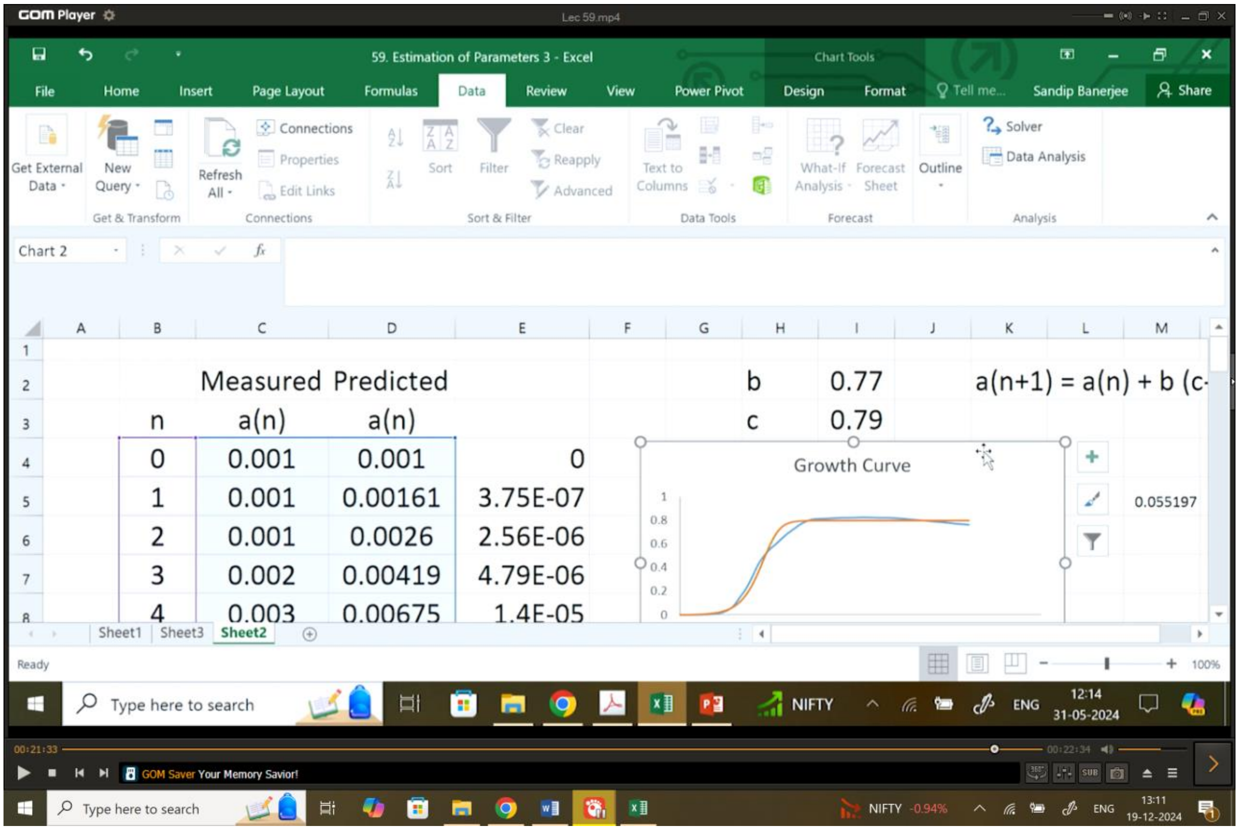
So now this is the data, this is the predicted value, I have to estimate.

So using solver, I go to data then click solver and I have to minimize the sum of the square of the residuals.

So here this is the value sum of the square of the residuals, I have to minimize the value by changing the variables of b and c, rest remains same, I click solve, okay.

So you can see it's a quite a pretty good match because this is embedded here, you cannot see it but let me see if I can.

So if I just change it from dots, you can see that this is the blue one is the data and the orange one is the predicted, so pretty good match and the values are 0.77 and 0.79 of b and c. b is the intrinsic growth rate and c is the carrying capacity.



So, using this Excel spreadsheet, you can calculate or you can estimate the value of the parameters of your model equations, whether it is differential equations or whether it is a difference equation and you can move on with the modeling part.

So with this, we come to the end of this lecture of estimation of parameters.

In my next lecture, we will be looking into something different.

It is called the simulation modeling.

Till then, bye-bye.