EXCELing with Mathematical Modeling
Prof. Sandip Banerjee
Department of Mathematics
Indian Institute of Technology Roorkee (IITR)
Week – 02
Lecture – 06 (Basics of EXCEL II)

Hello.

In my previous lecture, we have learned the basics of this Microsoft Excel.

Continuing with that, today we will be learning how to solve a system of equations in Microsoft Excel.

It can be linear, it can be non-linear.

So, let us start with the system of linear equations.

So, if your equation is of the form

A X = B, where your A is a matrix.

So, we will solve them in the matrix form, where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},\ X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ and } B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Then, we know that if your inverse exists you multiply it by A^{-1} inverse, the both sides, and your solution is

$$X = A^{-1} B.$$

So, we will be using this technique to find the solution of a system of linear equations.

So, let us open our excel sheet and here I again highlight, I make the font size a bit bigger, make it 20 and it make it justified.

So, the first system of equations which we will be solving is

$$2x + 3y = 4$$
$$3x + 5y = 12$$

So the first thing is you write them in the matrix form as

$$\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$$

where, $A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix},\ x = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$

The first thing is I need to find what is my A^{-1}. So, A^{-1}, I give the symbol like this.

Since this is a 2 cross 2 matrix, so what you have to do is, you highlight 4 cells 2 cross 2, and in this you just put an " = " .

Now you need to find the inverse.

So, the command for that is you click this, you write M, matrix inverse, did not get anything.

So, again you retry m inverse and you get this MINVERSE, because it is familiar to me you may try with matrix, matrix inverse but ultimately you will see that m inverse is the command, you give okay.

So, MINVERSE of the array. So, array you will choose these four, and click OK but then you see it is only this 5 which is giving this all are missing.

So, what you have to do is, I repeat this process you take this as highlighted this 4 cells, put is equal to then your MINVERSE, it will come, then you highlight this cell, close it.

Now, to get the answer for all these four cells you have to shift control shift and then enter.

So please remember you have to shift, you have to press the keys the control the shift at the same time and then press enter and then you get the A^{-1} of this particular matrix.

So once you get A^{-1}, then I have to multiply it by B. So I rewrite my B here.

So B is let me put it here. It does not matter. So, I put $\begin{pmatrix} 4 \\ 12 \end{pmatrix}$.

So, this is my x and y. So, it will be A^{-1} B.

So, I have to now do the matrix multiplication.

Again, you highlight this 2 because your answer will be 2 cross 1.

You put " = " , and then for multiplication, the command is, M means matrix, already it came here MMULT. So, MMULT, if you double click that, it will come, Matrix multiplication.

So, you choose this array and shift and highlight all this. So, one of this matrix has been chosen. We call it array.

You give a comma and then again come to the second matrix choose it and then close it.

Again, if you want the output, you have to press control shift and then press enter, and you get your answer to be $\begin{pmatrix} -16 \\ 12 \end{pmatrix}$ minus 16 and 12.

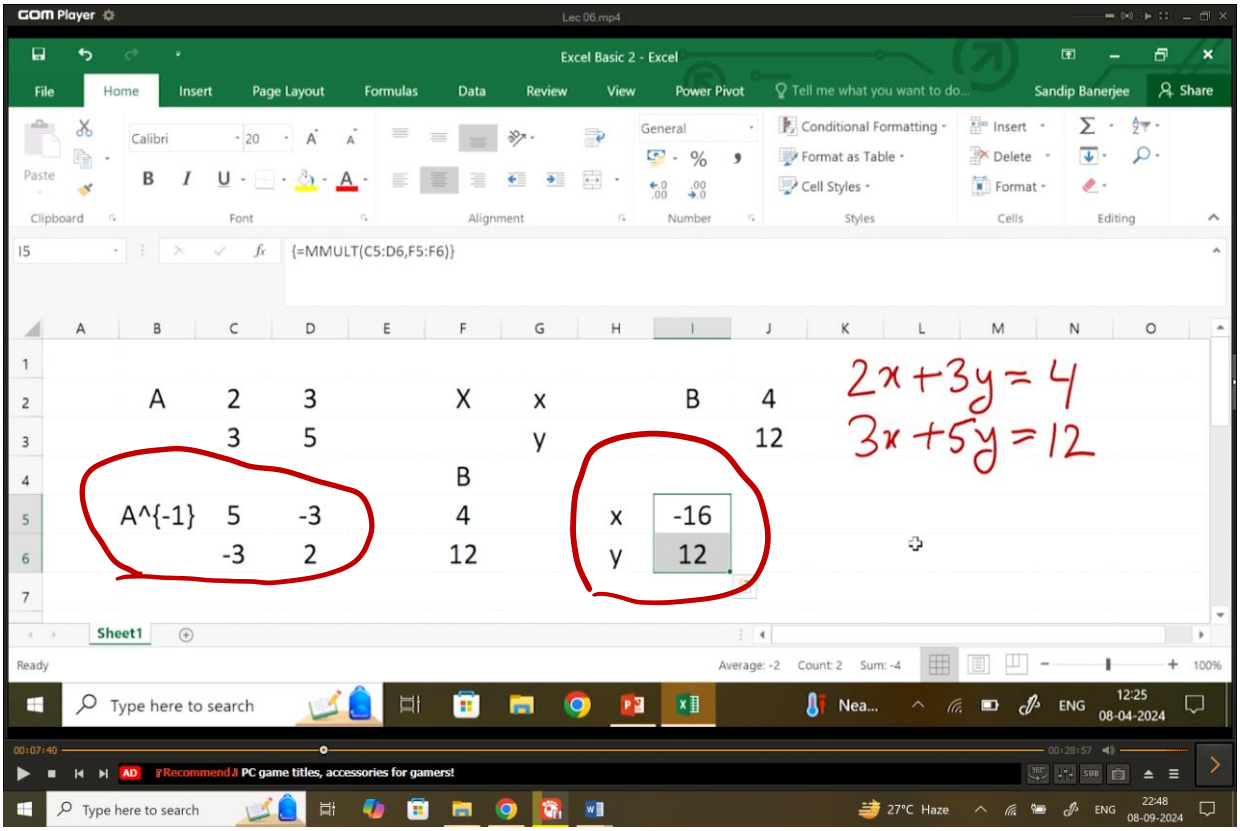
So you have a linear equation

$$2x + 3y = 4$$
$$3x + 5y = 12$$

and you use the matrix method

$$X = A^{-1} B$$

where your A^{-1} exists, and this is how your Microsoft Excel solve the equation.



Let us take another one, say I increase the variable, say,

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y - z &= -6 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

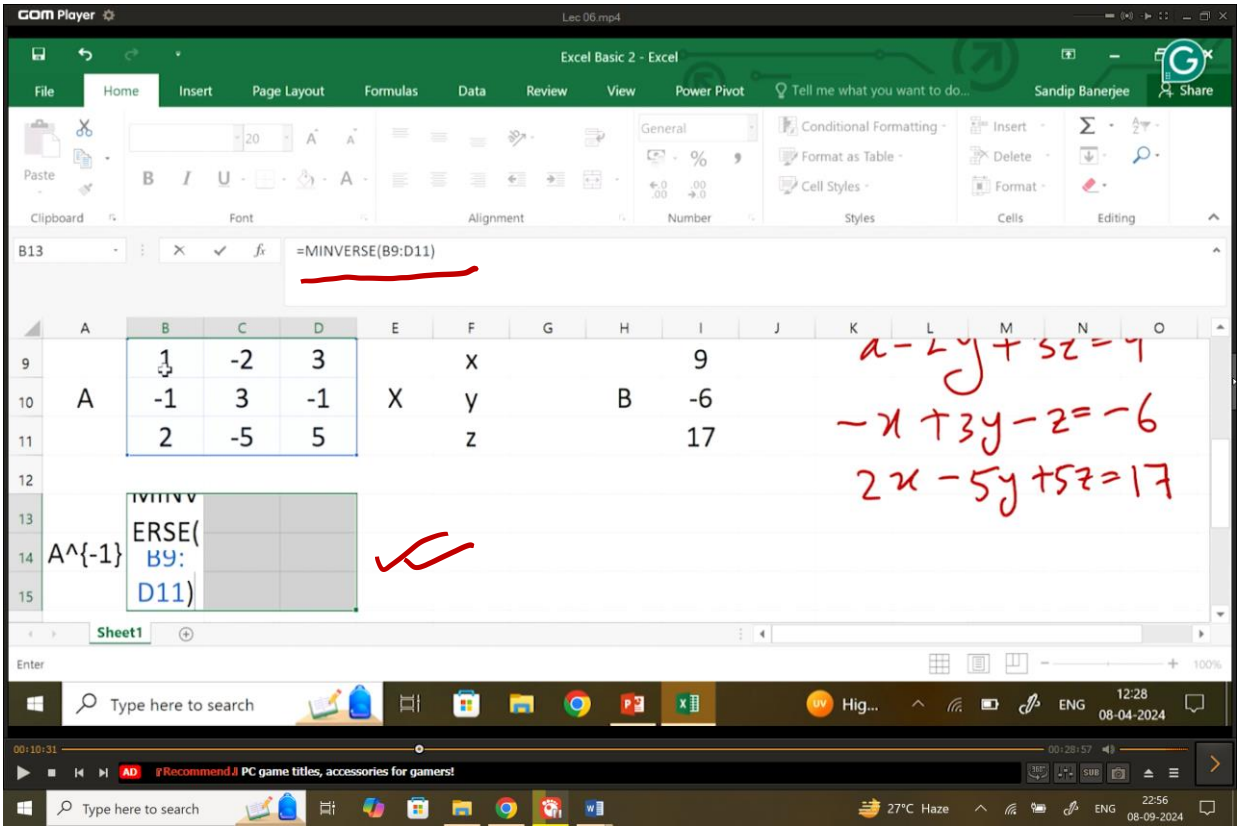
So, the same technique, let me put the value A. So,

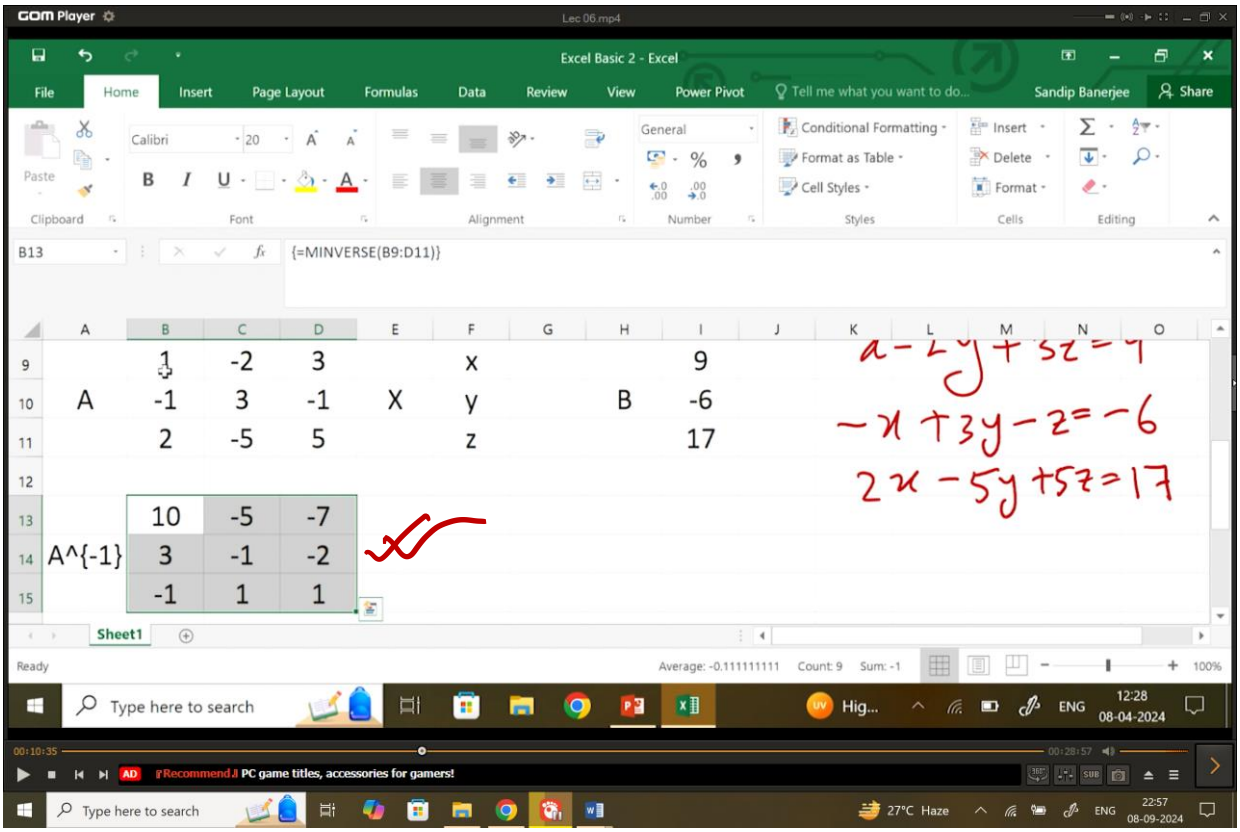
$$A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } B = \begin{pmatrix} 9 \\ -6 \\ 17 \end{pmatrix}$$

So, as before, let us find the A^{-1}.

So, I will highlight this time, 3 cross 3, I put an " = " to, then type MINVERSE, here it has come already, I choose this matrix, so you click, one cell is highlighted, press the shift, move the cursor to choose the whole array.

Close it with the parenthesis now to get the answer, control-shift pressed at the same time, and then press enter, and you get the value of A^{-1}.





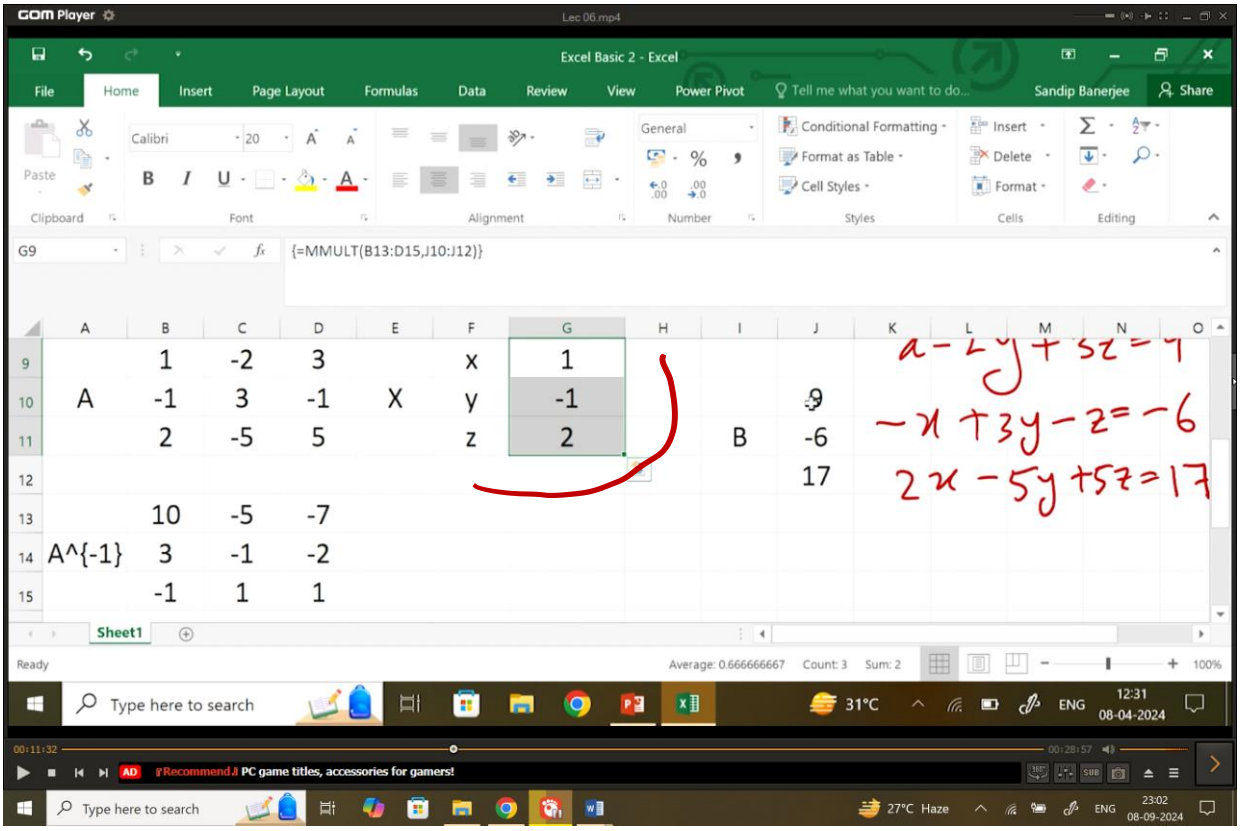
Okay, so once I get this A^{-1}, now I have to find the values of this x, y, z, which will be

$$X = A^{-1} B$$

So, here I highlight the cell, I put an equality, then matrix multiplication, so MMULT, this has already come, then I have to choose A^{-1}, so I highlight this array, shift and choose this array, give a comma, come to B, choose the array, close.

Now to get the answer, control, shift and enter.

So, you get the value to be $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ 17 \end{pmatrix}$.



So, this is the way how you can solve a system of linear equations, whether it is 2 variable, 3 variable, 4 variable, whatever it can be and you get the solution.

Next, let us move to a system of non-linear equations.

In this system of non-linear equation, we will be using the solver.

So, let us take an example, say,

$$x^2 - y + 2x - 1 = 0$$
$$x + y = 1$$

Suppose, this non-linear equation we want to solve. I name this expression as f1 and this expression as f2, so I will bring this to this side, minus 1 equal to 0.

$$f_1 = x^2 - y + 2x - 1$$
$$f_2 = x = y - 1$$

Now, so what we do first is I need some initial values of x and y, so here is an x.

So, I format the cells. So, I make this 20 and justify.

So, this is the x value, this is the y value and I put f_1 here and I put f_2 here.

So, I have to choose an initial value of x and y. So, I will choose say x equal to 1, y equal to 1.

So, if x equal to 1 and y equal to 1 is the root of this system of non-linear equation, then if I substitute those values here, f_1 will be 0 and f_2 will be 0.

So, the idea is to, exactly like the similar one, you first find the value of f1 with x equal to 1 and y equal to 1.

Similarly, for f_2 x equal to 2, f_2 x equal to 1 and y equal to 1.

So, for f_1 I calculate the value x square.

So, x square minus y plus 2 times x minus 1. So, the value you got is 1.

For f_2, it is x plus y minus 1.

So both of them becomes 1, 1 and you can easily find out that if you put x equal to 1 and y equal to 1 here, this comes to 1, 1 minus 1, 0, 2 minus 1, 1 and, hence 1 and 1.

Now if whatever value of x and y you choose to be the root, the aim is then they are f_1 and f_2 is going to satisfy and they will be 0.

So, we go to the solver, and your objective is you set this value f_1, it will go to 0, by changing the variables here, x and y.

So, C4 and C5, this two.

Now, subject to constraint, so here the constraint is, that my f_2 is also has to be 0, for whatever changing values of x and y. So, you click this add.

So, the moment you click this add, now you have the cell reference.

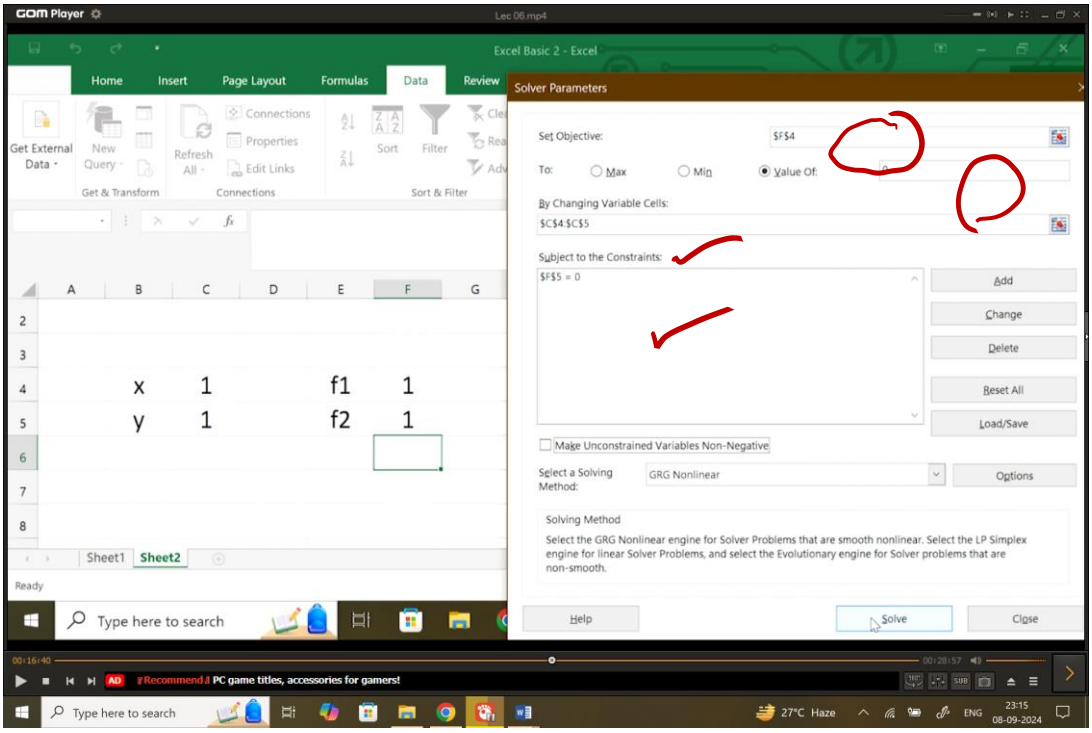
So, this is my f_2, this has to be equal to 0, not less or equal to 0.

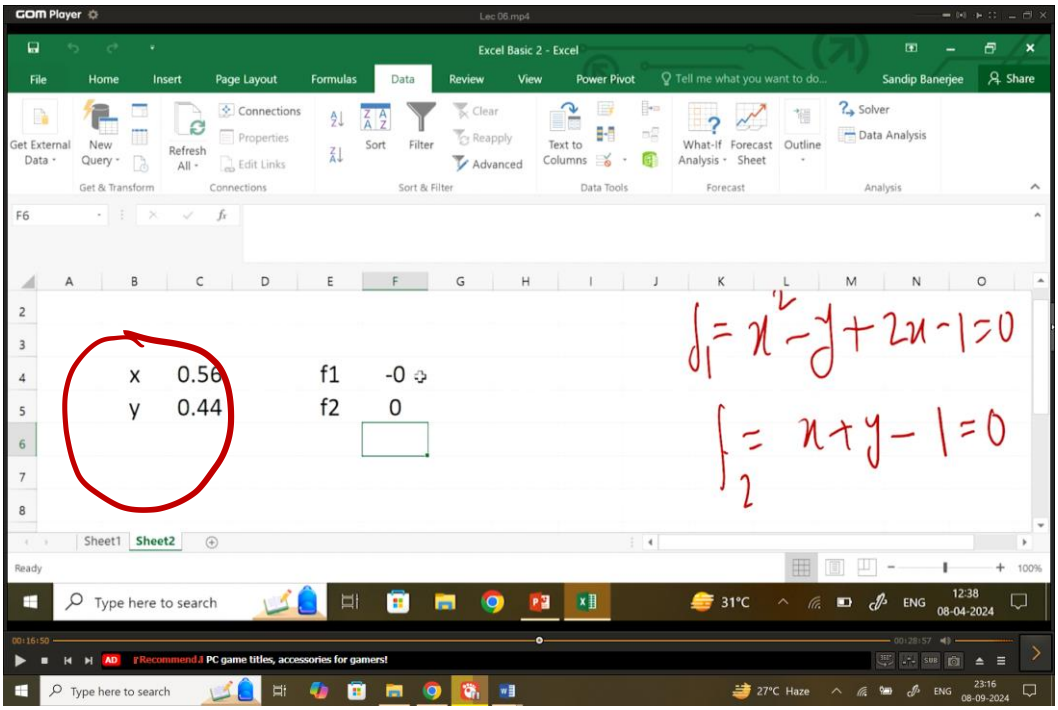
So, I put an " = " , and I put a 0 and I put okay.

So, now you get the same window again with the objective F4, that is, this one, whose value will be 0 by changing the variables this two x equal to 1, y equal to 1 and subject to the constraint f_2 is equal to, equal to 0.

I remove this make unconstraint variable non-negative keep this as GRG non-linear and solve.

Keep the solver solution and your solution is 0.56 and 0.44.





These two are zeros and as I told you, if you increase the cell size, you get exactly like this.

If you want to see them in decimals, right click, go to the format cells, go to numbers again numbers increase the decimal point to 4 or 5 and you get this.

If you want the same format in this next cell, you have to go to home, you highlight this cell, this will copy whatever format in that cell and if you press here, this has pasted what has format both of them give to be 0 and you get your answer as x equal to 0.56 and y equal to 0.44.

Now how to choose this approximate root again I have to do it graphically so you take, say, this x value. So, I can start with x = –5. So, before that, I have to rewrite the equation as

$$y = x^2 + 2x - 1, \quad y = -x + 1.$$

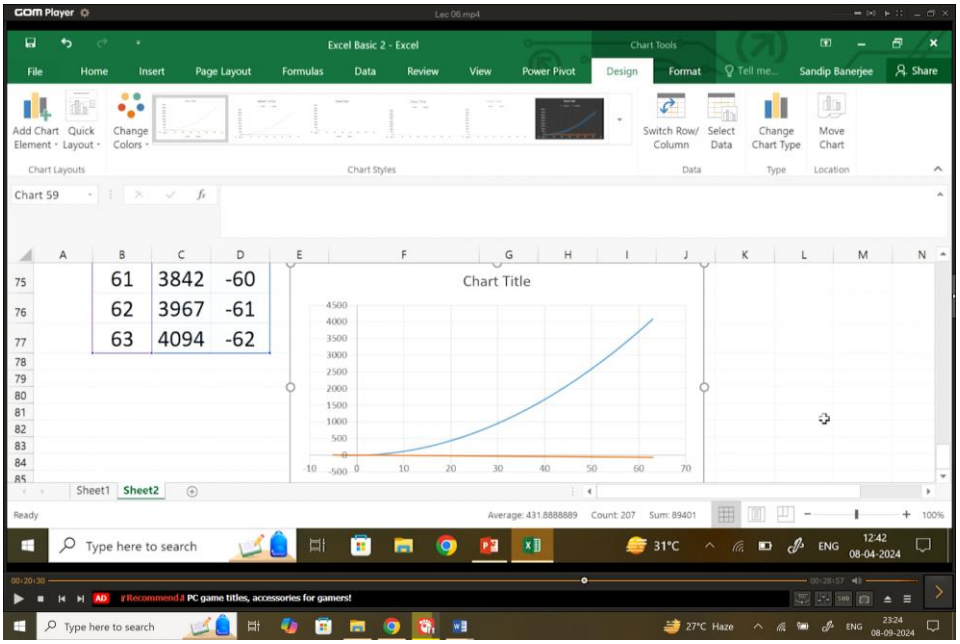
So, those two equations are rewritten like this as a function of y.

I put x = –5, and this cell, I will just add this, plus 1 and I will drag this a little, say up to, 10-20 values. Now my first y, I put it y_1.

$$y_1 = x^2 + 2x - 1, \text{ which is V9.}$$

So, sometimes if you cannot find that the cell can be highlighted you just know that that is v9. So, just substitute V9 minus 1 and you get the value.

So, you just drag it, and your y_2. So, $y_2 = -x + 1$, okay.



Now if I want to plot them, I highlight all the cells. I go to insert, I go to this curve and I draw this curve.

I can remove because the point of intersection is somewhere here.

So, that is why it is desirable that you choose some value between 0 and 1, to get an approximate root.

Let us take another example.

We are solving the same thing, that is, a system of non-linear equations with a different example.

So, now let us take

$$x^2 - x + y = 0.5, \quad y + 5 \times y - x^2 = 0.$$

Now before using the technique, let us see that how you choose the initial values of x and y. The very first one can be written as

$$y = 0.5 - x^2 + x,$$

and the second one,

$$y = \frac{x^2}{1 + 5x}.$$

So, let us quickly plot these two graphs.

So, I have my x values, I have my y_1 and I have my y_2.

Say, I put, say, minus 5, and the next value is this, I give an increment of 1 and I drag few of them, say 25.

So, y_1 is going to be, so I will be using this formula, $0.5 - x^2 + x$, which is A2 and add.

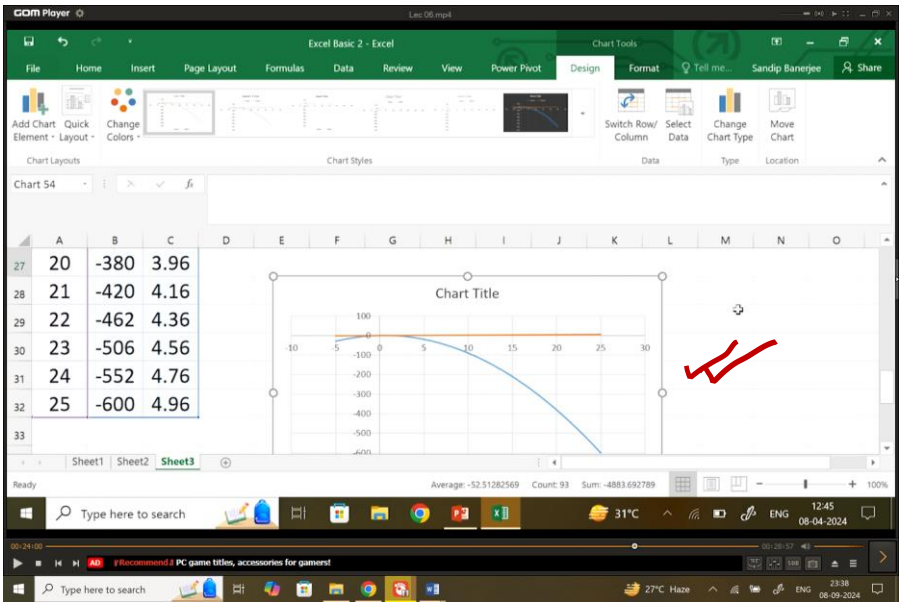
So, you get this to be minus 30, you drag them get all the values of y_1.

Then,

$$y_2 = \frac{x^2}{1 + 5x},$$

enter, and you get this value of y_2, you drag like this.

Let us visualize this. Go to insert, go to this curve, and you see.



So, again you have an idea that they lie somewhere between 0 and 1, the point of intersection is somewhere here.

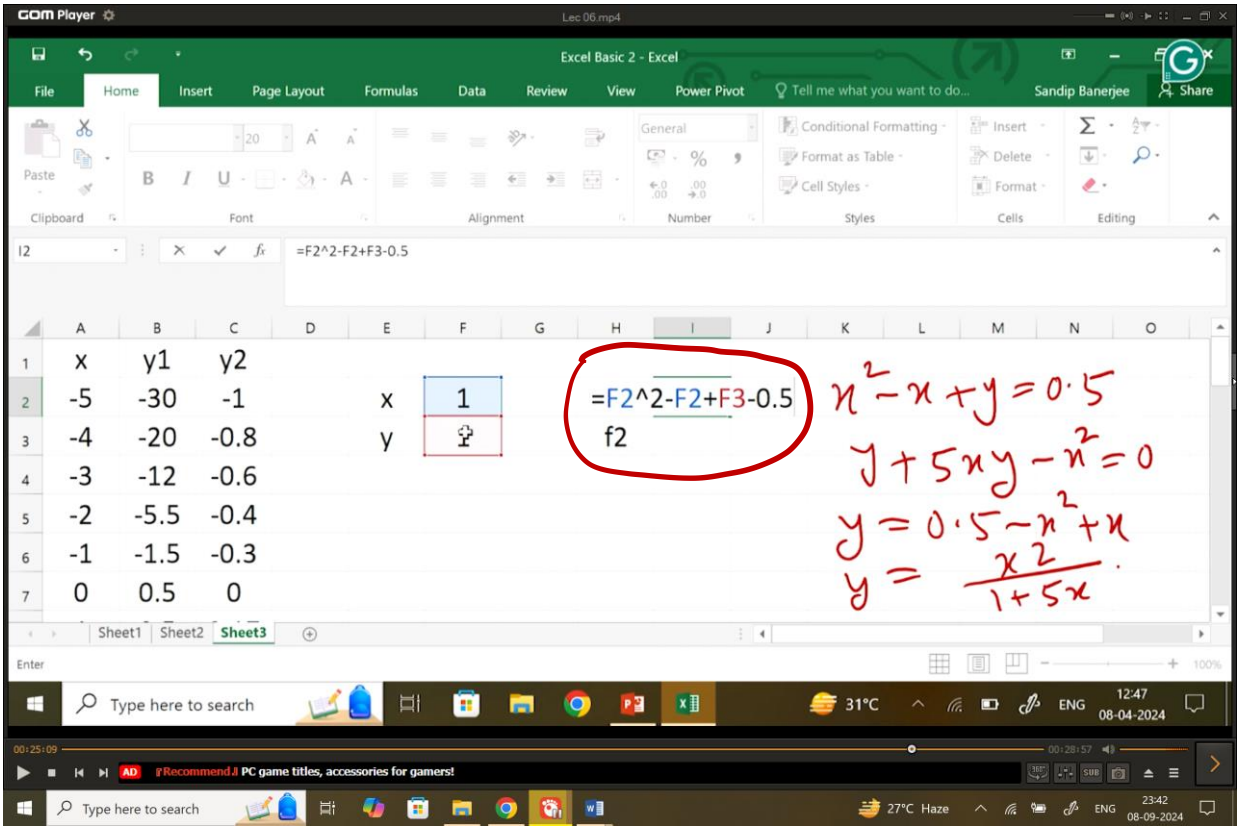
So, that is why, you choose your initial value.

So, let us x and y, let us again choose that to be $\begin{pmatrix} 4 \\ 12 \end{pmatrix}$.

I have my f_1 and I have my f_2. So this whole expression will be f_1 by taking this to this side and this whole expression is f_2.

So, the idea is again the same that I, if $x = x_0$ and $y = y_0$ are the roots of this, then this value f_1 and f_2 must be 0, because they must satisfy the equation.

So, this is equal to x square, so, this square minus x again this plus y minus 0.5



and enter, so, you get the value, and for f_2 equal to y plus 5 times x multiplied by y minus x square and that.

So, we get these two values and now we use the solver, we go to the data, we go to the solver.

So, if I highlight this now, this is your I2.

I have to make the value to be 0, by changing the variables x =1 and y = 1.

Now, subject to the constraint that I have to add a constraint.

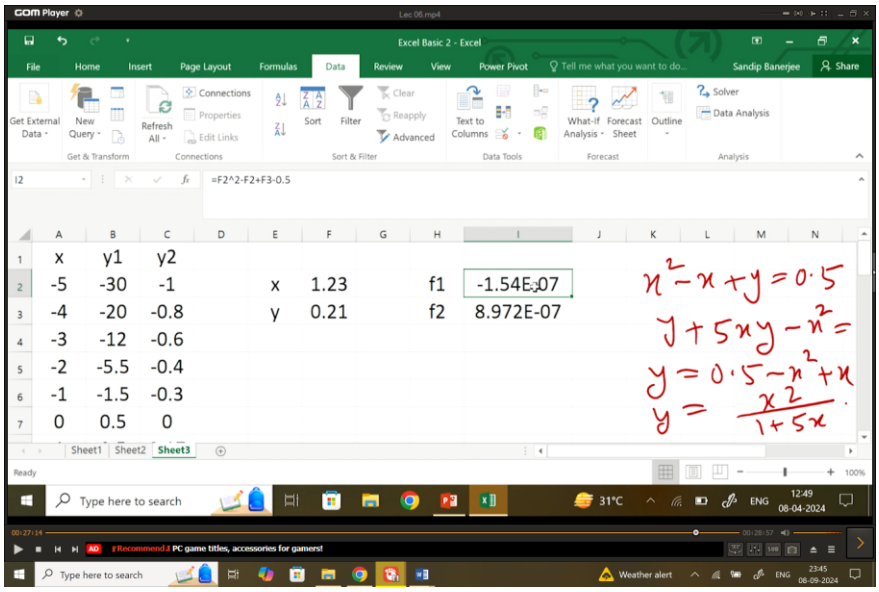
So, I have to add the constraint is this f_2, this is also equal to 0.

I click okay, I get this back.

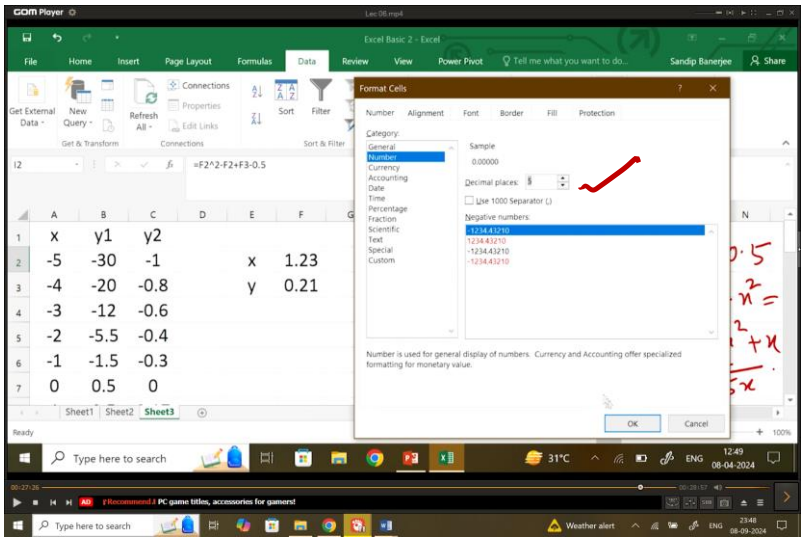
So, now the objective is, this f_1 value should be 0 by changing the variable x =1, y = 1 subject to the constraint f_2 is also equal to 0.

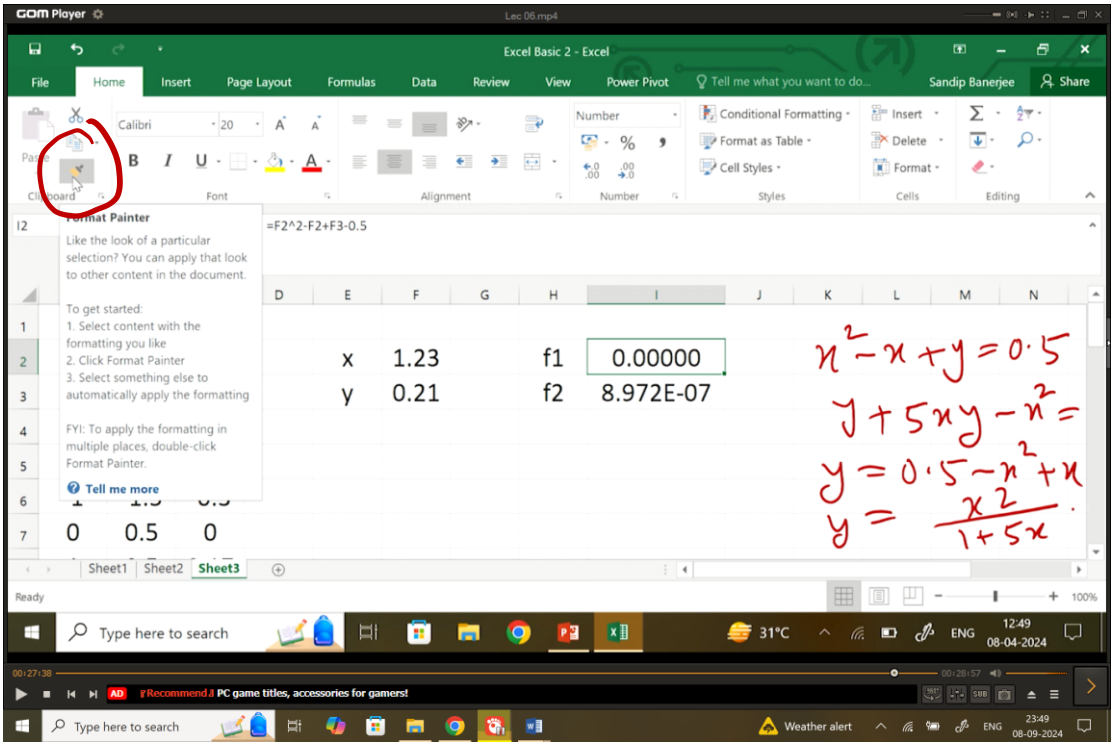
I remove this, so that it is non-negative also are taken into account and you solve.

Keep the solver, so you see the answer is 1.23 and 0.21 and as usual if I expand this size of this cell you get that this is minus 1.54 into 10 to the power minus 7, and 8.97 to 10 to the power minus 7.



If you want to see them in decimals, you go to right click, then format cells, numbers, again numbers and change them, say to 5, okay, you can repeat the process here but otherwise if you want the same format, go to home, highlight this cell, click this brush, which is the format and click it here. So the same format is transferred here.





So up to 5 places of decimal, your answer, so this is correct.

You can drag this a little to see okay.

This also gives 6 places of decimal.

So, 4 places of decimal can you can always take that your root is correct.

So that is how you use this Microsoft Excel to solve a system of linear equations or nonlinear equations.

In my next lecture, I will continue this and take it to the differential equations, that how to solve a single differential equation and a system of differential equations with the help of Microsoft Excel.

Till then bye-bye.