EXCELing with Mathematical Modeling
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Week – 12
Lecture – 60 (Simulation Modeling)

Hello, welcome to the course EXCELing with Mathematical Modeling.

Today is the last lecture of this course and we will be discussing something different from we are discussed before.

So, this is called simulation modelling.

Now, what do you mean by this simulation modelling?

In real life scenario, most of the cases are random, to be more precise, probabilistic.

So, we have two categories, one is the deterministic, another is the probabilistic.

The deterministic part which we have already done, that is, mostly using the difference equation or the differential equation to model the real life scenario, though most of them are a bit probabilistic.

So, in this particular modelling, what we will be doing is that we will create an environment of a particular problem using the technique of programming.

Now, do not get scared when I say this programming, it is just few commands that you need to know and kind of environment will be created to solve the problem.

So, as we have discussed or as you already know the deterministic system is determined once you have all the parameters that is set their constants and you get the solutions, whereas, the probabilistic system is determined by the random events and in this simulation modeling, the model uses the random numbers.

So, the one which we will be doing is called the Monte Carlo simulations and get their name they are often used to study the game of chance.

And this simulation consists of three basic steps.

That is, the first one ,that you construct the model using random numbers.

Then you run that model many times using different set of random numbers and once you get those results you statistically analyze those results.

So, whatever problem we take, we will be following these three basic steps, which is related to this Monte Carlo simulations.

Now, so we start with this problem of flipping a coin.

So, we all know that flipping a coin is a random experiment.

So, either you get a head or you can get a tail, but you cannot know when you will get a head or when you will get a tail.

So, the problem is, what is the probability that you have to calculate of getting 8 heads, when a coin is flipped 12 times.

So, you have to write an algorithm.

So, the first thing is that you simulate 12 random flips of a coin.

So, 12 times you will just toss the coin and you note the results.

That is one trial.

Then you repeat it for many as many as in this case we are doing repeating it for 200 trials and then calculate the number of heads rather at least 8 heads were obtained.

This number changes it can be 7 heads, 8 heads, 9 heads and this will be called a success and then you calculate the probability that is the total number of heads that is the number of success divided by the total number of trials, which in this case is 200.

Now the answer which you will get will not be exact if you calculate the value analytically that means I can calculate that this random experiment they generally follow the binomial distribution and if you recall the probability mass function of the binomial distribution is something like this.

So, this is for the rth trial, n is the total number of trials, p is the probability of success and q is the probability of failure which is nothing plus 1 minus p and this is your r and r goes from 0, 1, 2 till n.

Now, in this particular case, the probability of getting a head that is also half and the probability of getting a tail that is also half. So, this is your success, this is your failure.

And if I do it say 12 times, so this particular formula will be

$$^{12}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{12-r} = {}^{12}C_r \left(\frac{1}{2}\right)^{12}$$

So, we are calculating the number of trials with at least 8 heads. So, by at least 8 heads will be 8, 9, 10, 11 and 12.

So, I have to put the value of r to be starting from 8, then 9, then 10, then 11, then 12.

So, what will be the probability? It will be

$$\left({}^{12}C_8 + {}^{12}C_9 + {}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12} \right)\left(\frac{1}{2}\right)^{12}$$

and this value you can calculate this will be approximately equal to 0.19.

Now, the thing is that this I have calculated the probability as 0.19, whether you will get the same value from the simulation model or not.

Most probably you will not get the same value, you will get a value which is close to this value.

So, that is an approximate value that you get through this simulation.

So, let us now see how this is done. So, I will do a bit slowly here so that it is easy to understand.

So, I have opened a fresh EXCEL sheet here.

I have already done this problem, but then let me explain step by step.

So, let me first copy this.

So, I have is the trial number so I have to flip the coin 12 times and I have to note the number of heads here.

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	Total Heads

So this is the trial number this is the total heads and I will repeat, so this is first trial and first trial so I will do this 200 times so that is equal to this plus 1 and I will drag it to up to 200.

Okay, these are the extras.

So let me increase the font size to 20.

So, the next thing is that I have to use the random number.

So, either you are flipping the coin once. So, either it will be head or it will be tail.

If it is head, I name the success as 1. If it is a tail, I name the success as 0.

So, that is something which I have to put it here.

Now, how that is done?

So, if I see, I click it here. The command is the random number between 0, 1. So, the command is RAND between 0, 1. So, here is equal to I will write rand. So, already it came random number between 0, 1. I close it and enter.

So, I get it 0. So I just drag it up to this much and I get the number. So these are random numbers and they will keep changing.

So as you can see it suddenly change. So what you do is you click any cell I mean you just keep the cursor here, and you press F9. So, F9 and it keeps changing.

So, at the first flip of the coin, there is no tail, no head, no head, no head, no head, sorry, yes, only tail, sorry, no head and this is head, head, head, head, okay. And the total heads will be the sum of all this up to 12 and enter.

Okay, so this will be the random. So random between 0, 1.

Yeah, so the total heads is the sum of this select up to 12 close the bracket end.
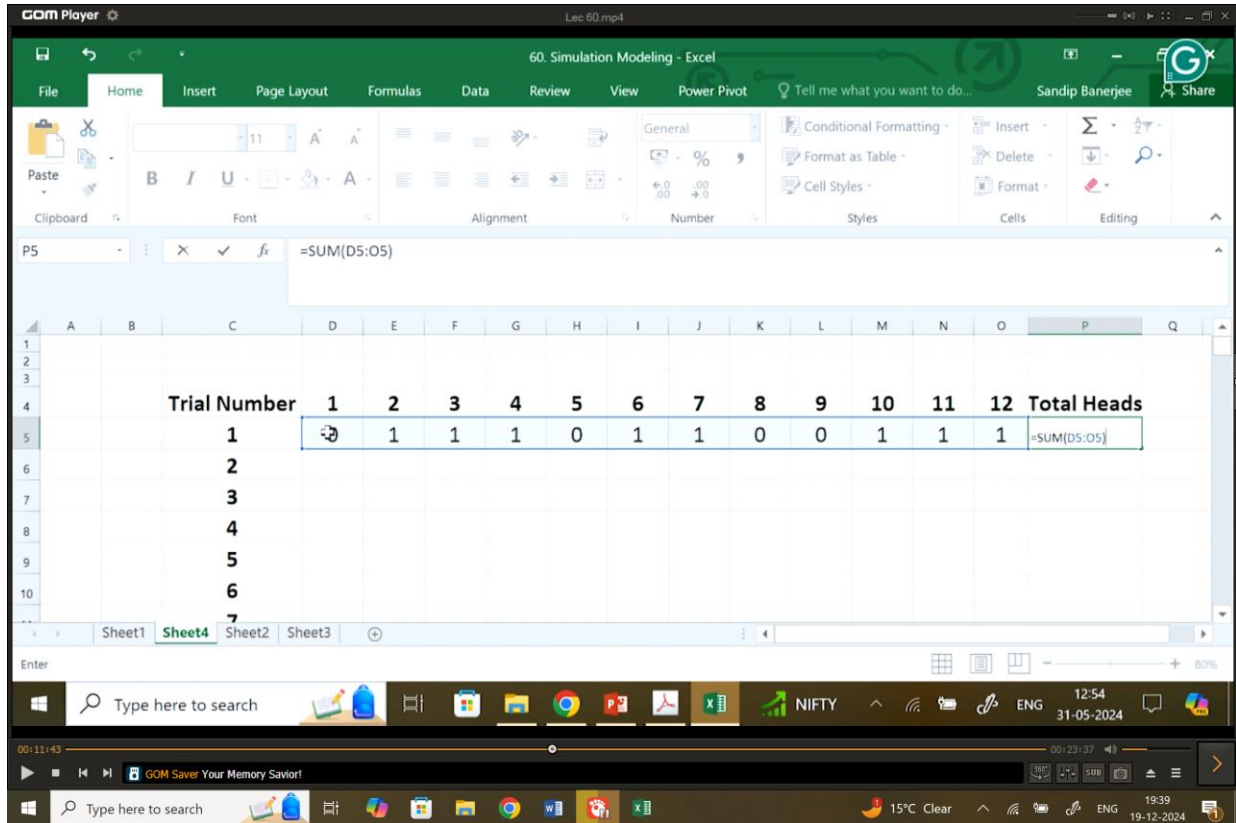
So you can now check, you click your cursor here or here or any of the cell and press F9.

So you can see there is only 4 heads and this is 4. Flip again.

So the moment you press F9, this keeps changing because they are random and they will be generated.

So now take this till 200 trials.

So this is trial 1.

Let us go for 200 trials.

So, you have 200 trials, this gives the total number of heads and my cursor is here or here and I press this F9 and you can see this keeps changing.
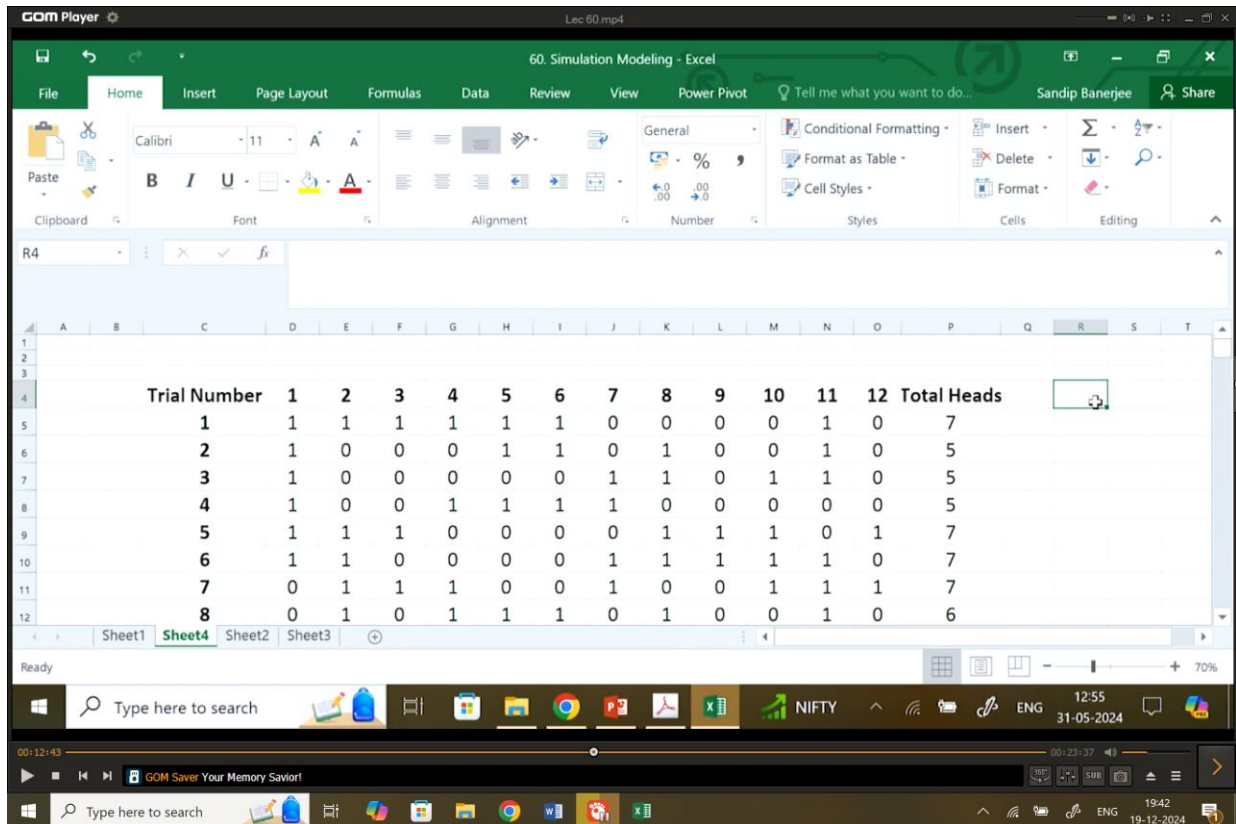
So what I got is that I have flipped the coin 12 times.

This is my trial 1 and this is the number of heads which I got.

1 represents the head and 0 represents no head or tail.

Then I calculate or I add this total 12 events and I see that 1, 2, 3, 4, 5 of them are heads.

So here it is head.

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	Total Heads
1	1	1	1	1	1	1	0	0	0	0	1	0	7
2	1	0	0	0	1	1	0	1	0	0	1	0	5
3	1	0	0	0	0	0	1	1	0	1	1	0	5
4	1	0	0	1	1	1	1	0	0	0	0	0	5
5	1	1	1	0	0	0	0	1	1	1	0	1	7
6	1	1	0	0	0	0	1	1	1	1	1	0	7
7	0	1	1	1	0	0	1	0	0	1	1	1	7
8	0	1	0	1	1	1	0	1	0	0	1	0	6

So, this is the success. So, the next thing is total success.

So, total success will be that how many of these trials this total head is greater than or equal to 8 like this one, this is greater than 8, this is greater than 8, this is greater or equal to 8, this is also greater or equal to 8.

So, now I have to calculate all from all these 200 trials that how many of the event is greater than 8.

So, how many of these trials have the head which is greater or equal to 8.

Now what to I mean how to do that?

So, this is your total successes and this is the probability.

Okay, we make them bold also, little change in color.

So, the total success if I look it here and I click it here now look it here.

So, what you are doing is you are counting this total thing.

So, you are counting that count if from cell 06 that is from here till your cell 205 means from 1 to 200 whatever the cell number comes it will come it here, give a comma whether it is greater or equal to 8.

So, you will count if it is greater or equal to 8 otherwise you will not count and whatever that count is will come here.

So, let us do it here. So, this will be equal to COUNTIF, so, already COUNTIF is here, so, COUNTIF from this range till it goes to 200.

Then give a comma, double quote, greater than equal to 8, double quote close, bracket close and enter.

So, COUNTIF from enter.
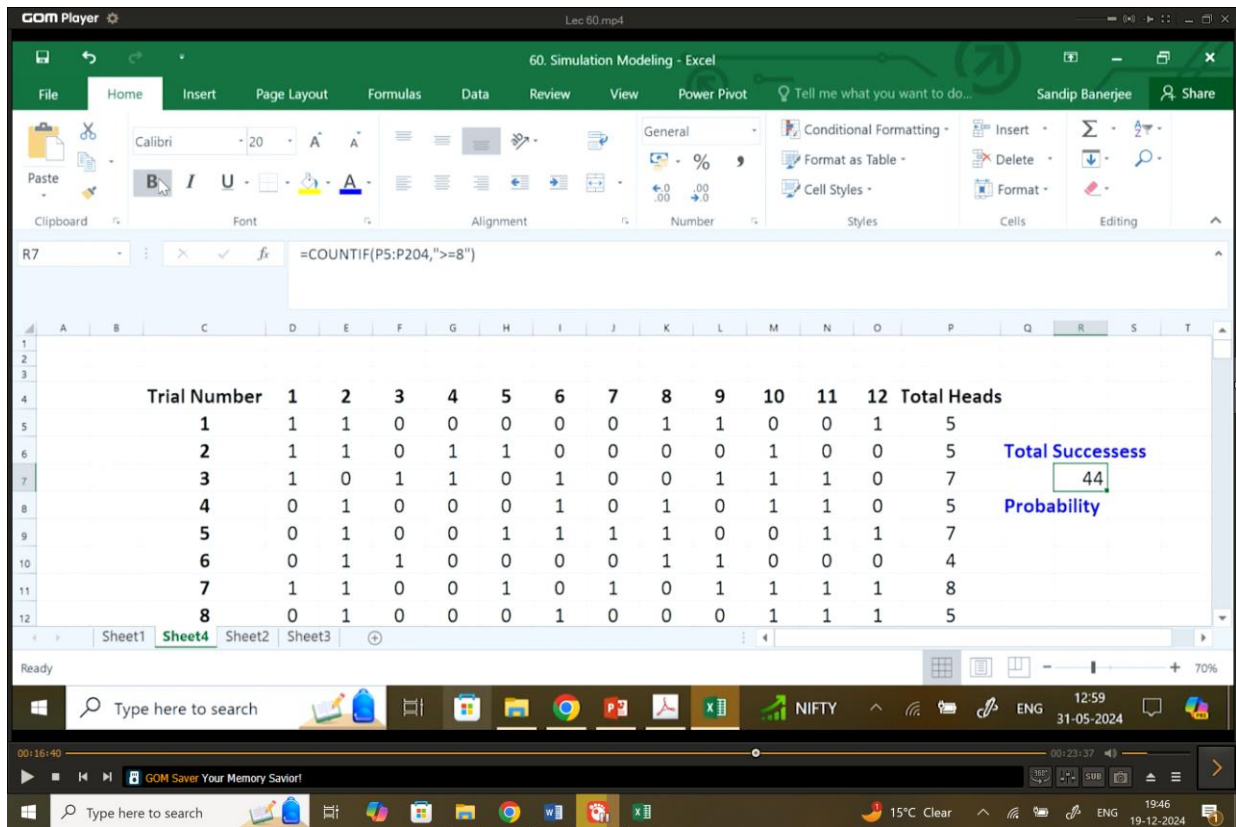
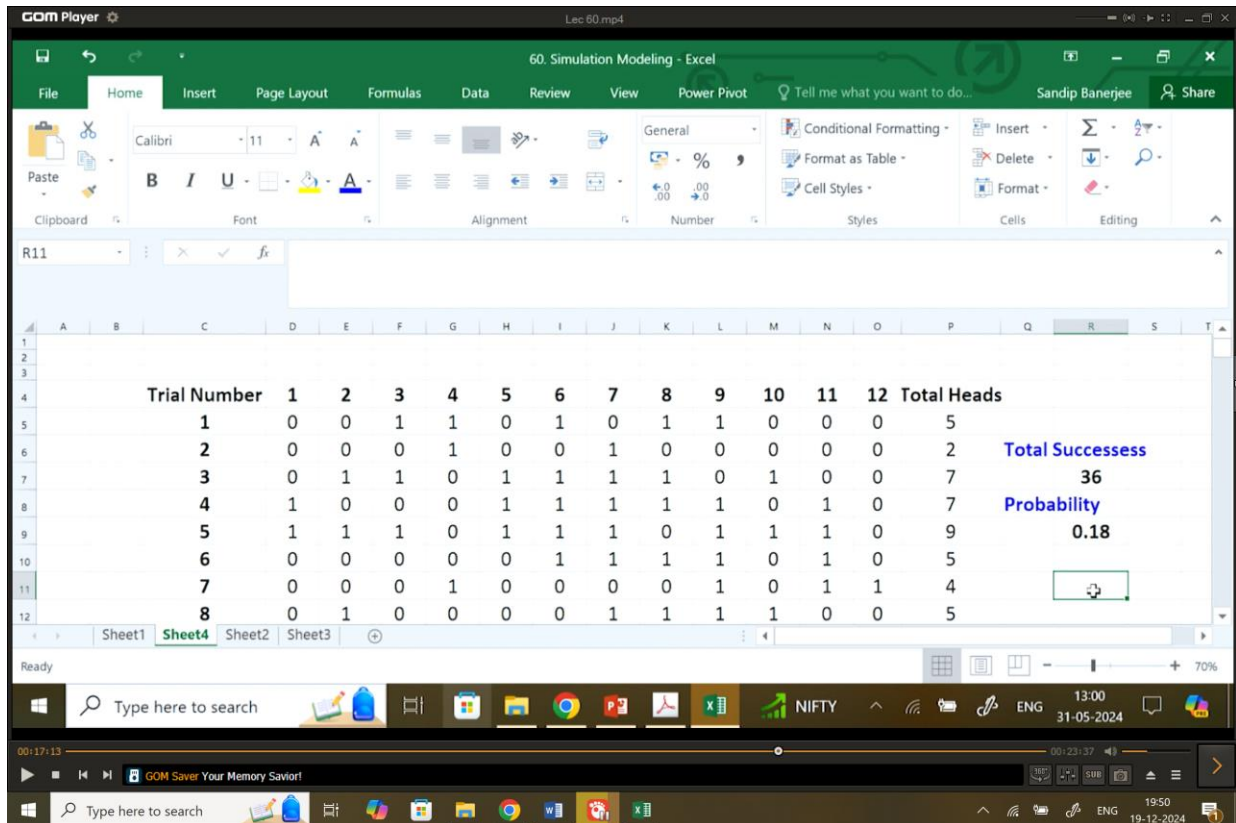So, COUNTIF from P5 to P04, if it is greater or equal to 8 and COUNTIF comma greater or equal to 8 yes.

And then the probability that is equal to the total success divided by the number of trials that is 200 and enter.

Now, if you want to see various results you just click it here and press F9.

So, you can see your probability of success and as I told you we have calculated that probability value to be 0.19.

So, these values will come you know close to 0.19.

So this is 0.16,0.17,0.19,0.18 again sometimes a bit odd 0.285 and that is how you do your mathematical simulation.

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	Total Heads	
1	0	0	1	1	0	1	0	1	1	0	0	0	5	
2	0	0	0	1	0	0	1	0	0	0	0	0	2	Total Successess
3	0	1	1	0	1	1	1	1	0	1	0	0	7	36
4	1	0	0	0	1	1	1	1	1	0	1	0	7	Probability
5	1	1	1	0	1	1	1	0	1	1	1	0	9	0.18
6	0	0	0	0	0	1	1	1	1	0	1	0	5	
7	0	0	0	1	0	0	0	0	1	0	1	1	4	
8	0	1	0	0	0	0	1	1	1	1	0	0	5	

So let me quickly explain this one more time.

So what you are doing is you are taking a coin and you are flipping it 12 times.

So these 12 numbers have come here

The next thing is this is trial number this is the first trial.

So, if it is a head you consider the head to be the success you attribute 1 to it if it is a tail it is a failure attribute 0 to it and the command to this is rand between 0, 1.

So, it is a random number between 0 and 1.

So, this will keep changing.

So, you write this and drag the whole cursor up to 12, and then you calculate what is the total number of heads.

So, this is 6 which is the sum of from this point to this point.

Once you have done with one trial you highlight this whole thing and drag up to 200 of them it can be 500 also the more the trials, the better result you get, and then comes the total success.

So, success means that this is the count of head for one trial.

So, in 200 trials how many heads which is greater or equal to 8 is here.

Because the problem says that I have to find the probability of at least 8 means 8, 9, 10, 11 and 12. So, there will be 8, there will be 9, 10, 11 and 12.

So, you count those and put the value here.

So, you can see here what is happening the command is COUNTIF. It will start from this particular cell and it will go up to those 200 trials.

So, that is why counting from here to here and what you are going to count which is greater or equal to the number 8.

So, it will consist of 8, 9, 10, 11 and 12 and that number has come to 46 in this case, and the probability is the total success divided by the total number of trials which is 0.23 in this case.

If you want to keep it changing you just keep the cursor here and press F9 like this and when you press this F9 make sure that what kind of command your keyboard allows like for my case I have to press the function key and then press F9 in your laptop it can be some different or in your computer it can be some different.

So as I click on this F9 I can see that various values of the probability it is generating.

So with that we come to an end of this last lecture which is this simulation modeling.

So this is just a start which I gave it to you which is different from the deterministic modeling.

So that if you have interest, you can then cultivate that and go and move forward.

This is the final video of this course.

I hope you have enjoyed learning excelling with mathematical modeling and feel you have new and exciting knowledge to bring back to your educational setting.

In conclusion, let us take a look at the key features of the entire course.

You have started your journey with the introduction, importance and limitations of mathematical modeling.

Next, given a realistic problem, you have learned how to build a mathematical model from scratch, followed by hands-on training with Microsoft Excel, mostly solving algebraic equations, systems of linear and nonlinear equations, first order differential equations, and difference equations.

I next move on to the stability analysis and phase plane analysis of the models.

I have divided our course into two parts namely continuous models and discrete models.

We have studied 21 continuous models and 16 discrete models.

While discussing all these models, we learned how to create the model, how to explain each term of the model and their relevance, find the equilibrium points and hence check for its stability, trying to find the solution analytically and then numerically obtain graphs using Microsoft Excel and match the solution with the analytical one.

Finally, we have explained the graphical representation of the solution in the context of the problem.

At the end, we also talked about empirical modeling, estimation of system parameters, and simulation modeling to give you a new flavor.

During your learning process, you may have faced difficulties.

Please do not hesitate to ask questions in the open discussion forum, no matter how trivial it may sound.

This will clear your droughts and strengthen your basics.

Please be sure to submit all the assignments according to the course deadline.

Then we will assess through your submission of the assignments whether you have demonstrated a mastery of the knowledge of this course or not.

Thank you for your participation and I wish you good luck for your examination.