Theory of Composite Shells

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Week – 08
Lecture - 01
Introduction to composite materials

Hi everyone. This is our course "Theory of Composite Shells". Hope you have already done that course of "Theory of plates" in the previous year. This is an 8-week course. I am Dr. Poonam Kumari, presently working as an Associate Professor in the Department of Mechanical Engineering.

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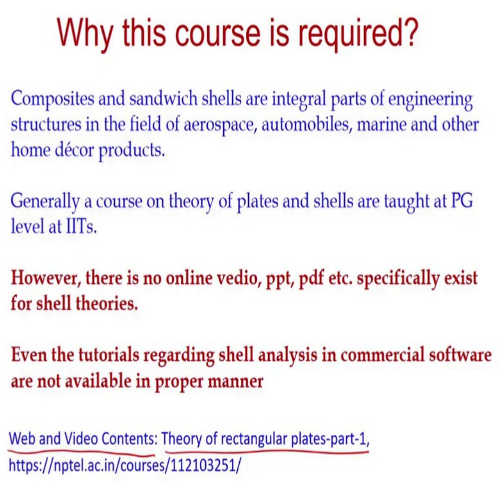

Before going into the content of this course, is anybody wants to know why this course is required? Why you should do this course? Specifically, this course is for postgraduate students, Ph.D. students, and working faculty members in different colleges with their universities.

As you are aware that composite and sandwich materials developed in the 1970s. After that their applications were in many fields like aerospace, automobile, marine, and other home decor products.

Specifically, if you talk about shell generally in India, most of the students are aware of the plate theory that how to develop a three-dimensional model or a two-dimensional model for a plate, but the shell is also a fundamental element and extensively used in almost all engineering structures whether you talk about the aerospace automobile and so on.

As I have already told you that this course comes under the theory of plate and shell wherein first part plate equations are expressed and another part shell equations are expressed at PG levels in various IITs. I got many requests that we should have a course on the shell. There is no online video or courses available specifically for the shell theories.

Even the tutorial regarding the shell analysis in commercial software if you talk about that, FEM Software like Utakabi ABAQUS forensics or any other software where we used to do finite element analysis of the structure. So, detailed tutorials that how to model a shell or its theory are not available in a proper manner.

So, my last course was the theory of rectangular plate part 1 in which I have developed a classical theory for a rectangular plate and its solution. Similarly, in this course I will develop state of art means how to develop shell equations, so, which is slightly different from the plate.

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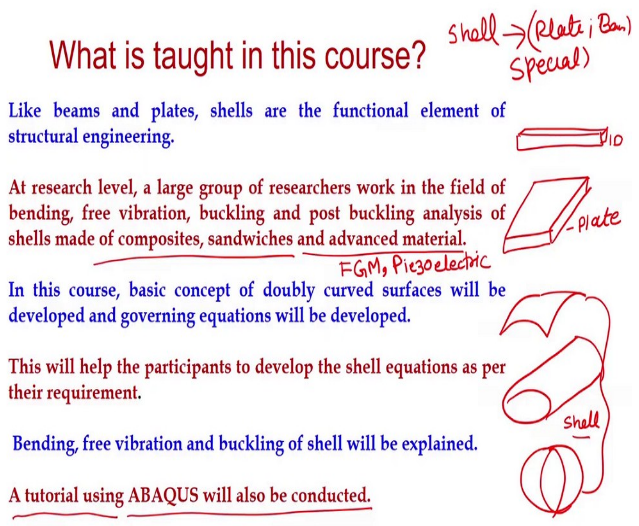

What the basic things I am going to teach in this course? Like beams plate, shells are also a functional or fundamental element of structural engineers as we say that it is a beam. I am just going to slightly review that one-dimensional structure is a beam.

If one dimension is very long compared to the other 2 dimensions then we can treat it as a beam. When 2 dimensions are comparable and the third dimension is very small and the most important part is that surface is flat then we treat it as a plate. But if we have a curvature that the surface is not flat it may be singly curved like one curve or it may be like a sphere double-curved system then all these comes under the shell.

So, for example, if somebody can develop a governing equation for a shell plate and beam will be a special case. So, if you can get the governing equation for a shell then you can easily deduce the plate and beam equations.

So, the shell is the higher-order element in functional elements. Even its formulation is slightly difficult at the research level. A large group of researchers works in the field of bending, free vibration, buckling, post-buckling analysis of shells made of composites and sandwiches or advanced materials like these days we call like functionally graded materials FGM or some more advanced materials piezoelectric materials that shell is made of for some smart materials. So, we are interested to study the behavior.

In this course, the basic concept of doubly curved surfaces will be developed and governing equations will be developed. The governing equations will be developed in such a way that they will be useful for all doubly curved shells specifically regular shells. If you talk about known regular surfaces then the equations become furthermore complex, but we will try to develop as general as possible.

Further, this course will help the participants to develop the shell equations as per their requirements. If you talk about the shell theories, a large number of articles have been reported in this field and I think more than 50 books are devoted to the shell. So, it is a very highly researched area.

Now, whether you talk about the thin shell, thick shell, composite shells, or sandwich shells, several books, several research papers are available, and interestingly many theories are also very high as compared to the plate theories. The reason behind that, in the shell theory if you include one term it gives you an entirely different solution if you reduce one.

Like sometimes you want to do only membrane analysis, you want to do an axis-symmetric bending of the shell, sometimes journal bending of the shell or different kind of shells. So, in this way, some terms one may consider in the strain displacement or one takes two terms or three terms only. So, it gives you a variety of shell theories.

Starting the very first theory was Love and Kirchhoff shell theory which is a thin shell theory, later on, it has been developed for considering the shear-like Mindlin and basically, Ressinor or Flugge number of researchers have worked in this field. In the last, I will share one tutorial in ABAQUS that how to develop a shell model in ABAQUS and conduct the analysis.

It is slightly different than the plate because here the coordinate system is different. So, we have to sometimes convert our coordinate system to get our local elements or initially defining the shell structures, and so on.

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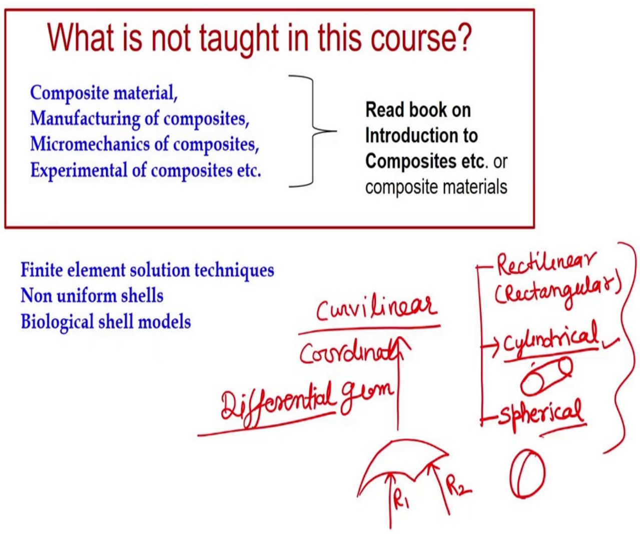

I would like to clear in the very first lecture that what is not taught in this course or I am not going to explain. If you talk about the discussion about the composite materials or that various manufacturing issues about the composites or the micro mechanics behavior of composites or the experimental analysis of composites or the finite element solutions, non-uniform shells, and the biomechanics shell models that is not the scope of this course.

This is the first course that just to presents a state of art from scratch that how to drive a shell equation using a doubly or curvilinear coordinate system. So, there are curvilinear coordinates, curvilinear. So, before going just you may be aware or may not be like, you have gone through first rectilinear coordinate system **x, y, z** or sometimes you called it a rectangular coordinate system which is generally used to study the plates and beam.

Then we have a cylindrical coordinate system and a spherical coordinate system.So, these are the special coordinate system. In a cylindrical coordinate system, only the curvature is in one direction to analyze the cylinder. The cylinder is also a special shell.

So, if you want to study a solution for a composite cylinder then, you can use a cylindrical coordinate system. If you want to analyze a sphere then, you can use a spherical coordinate system. But, if you have any curve in both two directions that the radius of curvature let us say, R_1 in one direction

And in the second direction radius of curvature is R_2. Then, we cannot drive the governing equations using this set of the system then we have to go to the curvilinear coordinate system. Sometimes this is differential geometry. So, in mathematics, you will find a course devoted to differential geometry.

So, in this course, I will briefly describe the governing equation that we used or the constitutive relations used for the composite that we will discuss in brief. But the details of the composite materials will not be taught in this course.

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Week	Module name and contents to be covered	No. of lectures planned
1	Curvilinear coordinate system and various fundamental of surfaces Assignment-1	3
2	Classification of shell theories Assignment-2	3
3	Development of governing equations	3
4	Use of shell constitutive relations and special cases Assignment -3	3
5	Navier solutions for Cylindrical shells under bending load Assignment -4	3
6	Navier solution to axisymmetric, unsymmetrical cases Assignment -5	2
7	Free vibration and buckling of cylindrical shells and basic development for Levy solutions Assignment -6	3
8	Introduction to three dimensional solutions of cylindrical shell Assignment -7	2

These are the modules. This course is divided into 8 modules. As we are aware this is an 8-week course. So in the 1st week, the curvilinear coordinate system and various forms of fundamental surfaces will be covered then, after this there will be one assignment and one tutorial.

In the 2nd week, I will discuss the various classification of shell theories and the basic strain displacement relations, and their reduction to the orthogonal system. Then the development of governing equations will be taken up in the 3rd week. In week 4, use of shell constitutive relations and derive the special cases like from a journal partial differential equation of a doubly curved shell to it may be a cylinder or it may be a sphere or it may be a conical shell or it may be some other kind of a shell.

Then, we will derive the Navier solution for special shells and Navier solution to axisymmetric unsymmetrical cases specifically the bending solution. In the 7th week, the free vibration solution, buckling solution of a cylindrical shell, and basic development for Levy solution will also be discussed.

In the 8th week for getting the accurate behavior, two-dimensional theories are ok. These are easy to develop less computational efforts are required and most preferred, but three-dimensional solutions are also required sometimes for a thick shell or we want to verify that how many two-dimensional solutions are accurate. Whatever assumptions we are taking, definitely in the two-dimensional solutions we make some assumptions. We want to verify the accuracy of our solution.

In this course, I will briefly explain the three-dimensional solution for a cylindrical shell and how to develop these three-dimensional solutions and the accuracy of the two-dimensional theories with respect to the three-dimensional solutions.

In this way, the lectures are planned. I would like to tell you that it is a very systematic flow from 1st lecture to 2nd week or 3rd week. If we are not able to follow in 1st week then we cannot understand 2nd week or 3rd-week lectures. So, we have to do systematically all the lectures then we will be able to understand.

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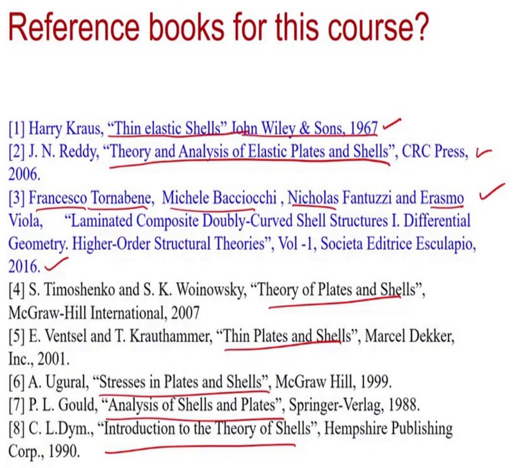

Reference books for this course are "The Harry Kraus book; Thin elastic Shells", this is a very old book but the basic governing equations are presented very nicely in this book. One can try this book. Then, the next book "J. N. Reddy; Theory and Analysis of Elastic Plates and Shells", the11th chapter of this book is devoted to the solution of shells.

As I have told you that there is a large number of books on the shell and recently a book by Tornabene Michele Nicholas and Erasmo "The Laminated Composite Doubly Curved Shell Structure, Differential Geometry, Higher-Order Structural Theory". This is the recent book in which derivation of governing equations or various geometrical forms are discussed.

Then the Timoshenkos book "Theory of Plate and Shell" and there is another book "Thin Plate and Shell". Some of the chapters are devoted to shells. Similarly, A. Ugural, Gould, Dym. Some old books are also available, one can go through these books, but I am going to follow that Harry Kraus and Reference number 2 and 3 mostly.

If required some topics are given in reference number books 5, 6, 7. Definitely, in between the course or during the lecture, I will discuss which topic is taken from which book.

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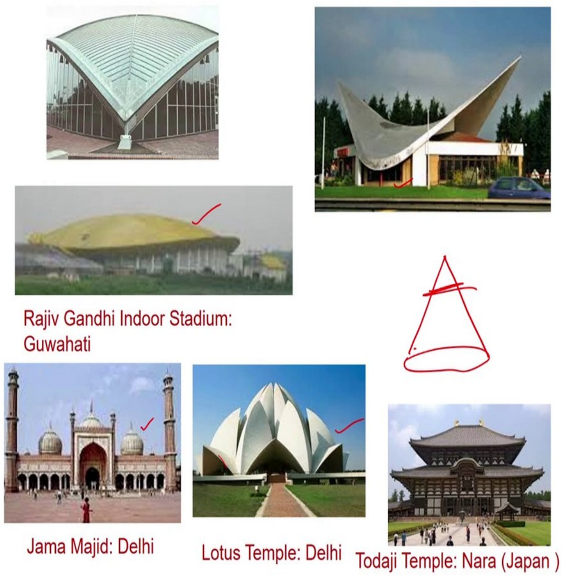

These are the structural application of the shell. You see that roof of all the buildings is beautiful shells. Here in Guwahati, Rajiv Gandhi Indoor Stadium like a turtle. So, turtle shaped roof is there, and then tom-shaped roofs and Lotus Temples curves. Specifically, if you talk about China and Japan their roof structures are slightly different as compared to ours. Generally, we have conical shape structures or frustum-type structures.

So, most of the roofs are in a shell form. It is a very general experiment that if you take a simple A4 paper and put some weight over this it will not be able to hold it, but if you make it of cylindrical shape then it can take that weight. So, due to the curvature, these can take more weight and these have more resistance toward the stresses.

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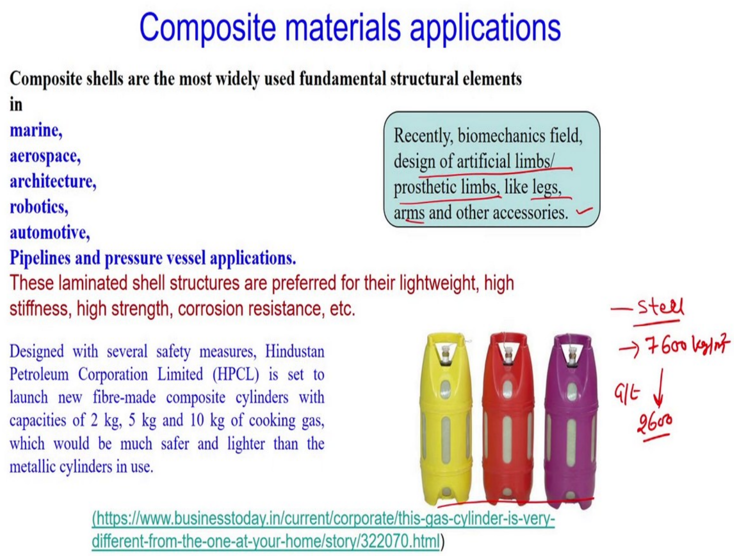

Composite materials application: we have already discussed that it is used in marine, aerospace, robotic, automobiles. Specifically, nowadays you see that composite pipes or pressure vessels are also developed. Recently in the field of biomechanics, the design of artificial limbs and prosthetic limbs like legs, arms, and other accessories are developed.

I have taken from the internet that you can see that these are standard LPG storage cylinders made of composites, and these are used for storing the cooking gas 2 kg, 5 kg, and 10 kg capacity. So, the reason for choosing the composite material over traditional material i.e., traditional materials like steel

It is our traditional material, but we want to go for composite material. It is very light; means, density is very small. If we talk about in a tentative sense that steel density is 7600 kg per meter cube in general and if you talk about a glass epoxy then it is just 2600.

It is one-third of that. So, a lot of savings. That is why it is the first preference in the aerospace industry because we are cutting a small weight led to a huge economic benefit. but Definitely, there are some limitations also that it fails suddenly.

Now people are trying to enhance it because it acts as a brittle material. So, trying to sort it out can we increase life? or can we find out some symptoms before the failure? this is the major disadvantage of this composite material that it fails suddenly, it does not give any plastic flow or very less plastic flow of the material.

So, this material cannot be even be used for some impact kind of loading where the loading is sudden or the blast loading till date. Research is going on in this direction how can we develop this composite material for such applications through using some different material combinations or some different directions in that way we are doing the research.

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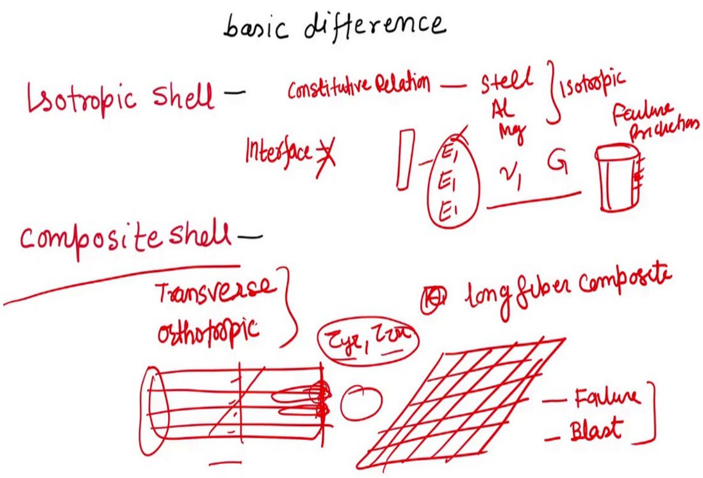

What we do first? What is the difference between isotropic and composite shells? why are we interested to study a composite shell, the analysis of a composite shell, and what is the basic difference? The constitutive relations that an isotropic material has if you talk about steel, aluminum, magnesium are isotropic materials. They have the same property microscopy. If you take a bar and subject to that, in all directions, they give the same Young's modulus.

So, because of this property, their formulation becomes very easy. So, only one Young's modulus or mu and then G; so, that number of equations are reduced and mathematically complexity also reduced. The next is no concept of interface. Like, if you say that a cylinder is made of steel, over the thickness there is no concept of interfacing.

So, from bottom to top, the smooth variation of stresses or smooth variation of displacement; definitely, can give you a smooth variation of stresses, a smooth variation of displacements and failure can be predicted very easily; failure prediction is easy. But, for the case of composite material, it is no longer isotropic, if you say that because, for structural application fibrous, long fiber composites are preferred.

For example, if I take this lamina, like fiber may be a cross ply, and so on. They are no longer an isotropic material. They are minimum at least if you talk about transverse isotropy or orthotropic material. So, because of these materials if the material is becoming orthotropic or anisotropic the governing equation becomes complex because we cannot take out common things.

Now, we have to solve. It is mathematically more complex and, in a laminate, the concept of interface comes into the picture. In each layer material property is different. Because of this, a huge stress jump is over the interfaces that specifically transverse stresses **y, z**, and **z, x** increases.

Though we can say that we apply the concept of perfect interface they will remain continuous, but their intensity becomes high at the interface or specifically at the boundaries where it starts first. This is the typical thing that we want to know how this stress behavior or displacement behavior in this composite. Because it is no longer isotropic.

Definitely, it will have different behavior and if we can develop accurate behavior or analysis of stresses then we would like to predict its failure. Even that in composite materials failure theories are not settled till date. We cannot say that 100 percent that this failure theory will work for this configuration. We have to try this for composite.

But, in isotropic materials that 5 or 6 theories the maximum stress, maximum shear, then maximum principle, strain energy, octahedral. They tell you the failure behavior, but in the composite the yield strength in compression and tension is different they are not the same. So, in this way the application of even the failure concept is difficult.

So, nowadays that two major areas; one is that failure analysis and then impact or blast analysis. The main reason that somebody may be good that how to develop a solution for an isotropic shell that not be directly extended to the composite shell.

We have to consider the concept of orthotropy and the concept of interfaces and yes definitely the shear stresses that we have to consider those simple classical theories will not give you that accurate behavior of stresses for the case of composites. So, there is a need to understand that how to develop the shell theories when shell is made of composite materials.

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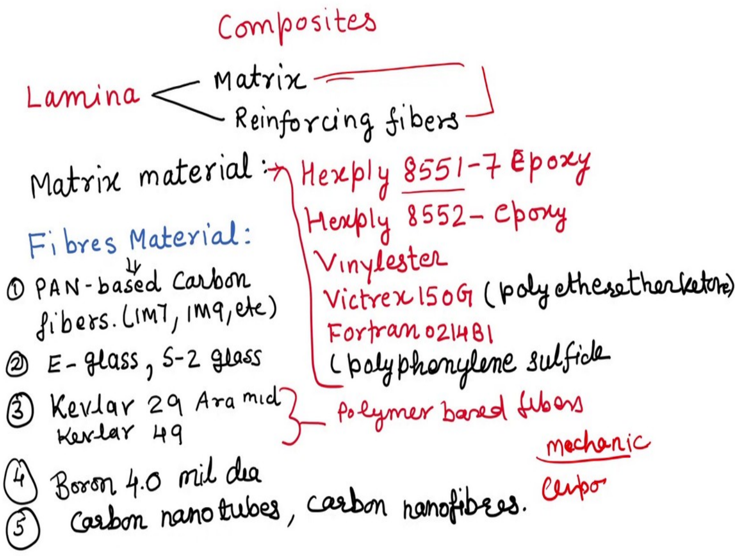

Before going into the actual shell governing starting equations, I would like to first discuss briefly the composite materials. Like in composite materials two basic quantities are required. One is the matrix, second is reinforcing fibers. So, these are the two components that we required for making a composite.

During the interview or any viva, we use to ask what is the difference between a composite and an alloy. Alloy is a chemical composition, in which the solid solution of two metals is mixed. So, you can see they are different metals through a microscope, but in composites, they are not mixed chemically, heterogeneously. So, through a naked eye, you can see the fibers, you can see the matrices through some temperature. So, they work before recrystallization temperature before that.

So, the very basic concept is that in composite those two materials are placed together, but they are physically intact through some hot press and so on, but in alloy, there is a solid solution and you cannot see through a naked eye which material is this, you cannot differentiate.

The next question comes up that what are the matrix materials? Some students do the course of mechanics of composite materials or some other. So, if somebody asks that do you know the commercial name of the matrix material. So, epoxy, generally it is a general commercial name, but one has some actual number is that Hexply 855178552.

So, epoxy is a general matrix material used for developing composites then vinyl ester, then polyether, then polyphenylene sulfide. So, these are the sum matrix materials, the details of which you can find out in any mechanics of composite books. What is fiber?

Fibers are made of glass, boron. You know that these days' carbon fiber materials contain carbon fiber. Carbon fibers are the hardest fiber. Then E-glass or glass epoxy and for army applications that Kevlar aramid fibers. So, Kevlar epoxy materials are a composite material then boron.

Boron is a hard material, and recently the concept of carbon nanotube and carbon nanofiber comes. So, using the concept of carbon nanotube and carbon nanofibers we develop the composites for different applications.

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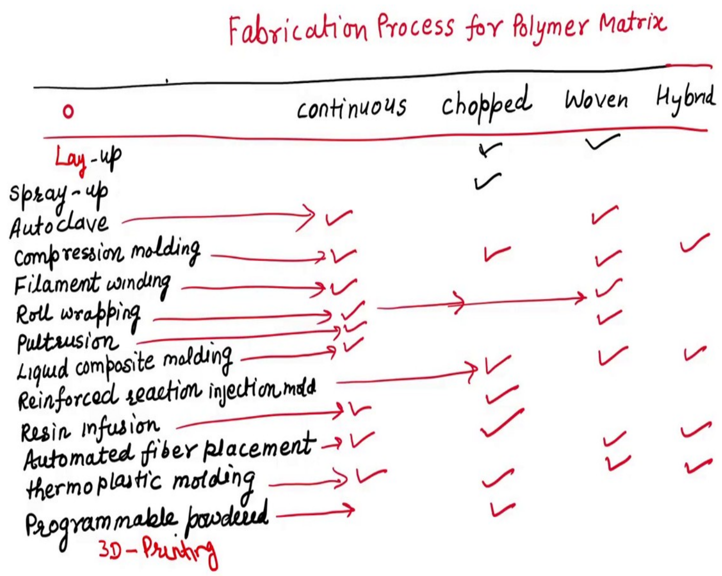

Then, what are the different fabrication processes? There is a polymer matrix where the matrix is a polymer and other maybe some glass, boron, the carbon they act as a fiber. So, a very simple method is that spraying up and layup basically, simple layup method; spray up method, autoclave techniques, compression moldings, filament winding. So, through the filament windings, we can develop a cylindrical shell made of composites, roll wrapping pultrusion, liquid composite molding, reinforced reactions, resin infusion.

So, recently the concept of 3D printing of composites comes into the picture. The techniques are developing that can make a composite using the 3D printers. So, we can further classify the composite based on the fibers. If the fibers are continuous or fibers are chopped or fibers are woven like a textile material or hybrid concept like they may be some chopped some continuous and so on.

So, we have that different method from that we can develop the composite through a layup we can use the chopped fiber, but if we want to use a continuous for that we need autoclave, compression molding, or filament winding. So, in that we can use the continuous fibers. The tick sign is there that which technique gives on with which type of fiber so that we can develop the polymer matrices.

So, details of these manufacturing techniques can be found in the composite material, fabrication techniques where details are given that how to develop. Why I have given this slide? The reason behind that once you develop a theoretical solution ok, but some of you may be interested in working on an experimental part of this composite. You want to make a composite shell and want to study at different loading or for different purposes.

So, you must be aware that these are the techniques available and basically the theoretical models that we can compare with the experimental ones. Let us see some basic ideas. These are the techniques and details that can be found out there and you can.

Later on, if somebody is interested to develop some experimental setup using or developing the composites like you have developed a theory you find that it is giving good behave means stresses or that very means different loading if it is showing safe in that particular theoretical model then you want to assess through experimental whether your theoretical model is giving accurate prediction or not then you can try for experimental one.

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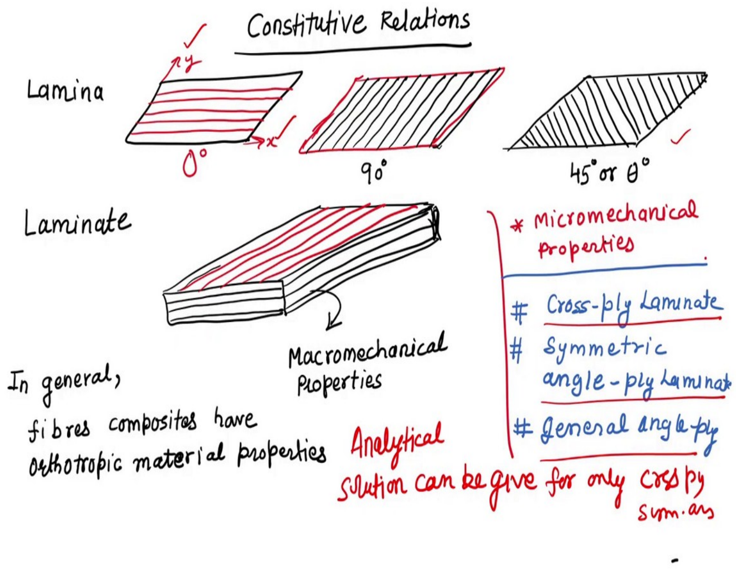

In composites, a very thin sheet is called the lamina in which continuous fibers are laid. In the very first that fibers are aligned along with their geometry. Let us say it is x-direction and it is y-direction and fibers are aligned along the x-direction then we called it is 0 degree to the x-axis. If fibers are perpendicular to this direction then we called it a 90-degree layer. If fiber is at any angle or 45 then we call it angle ply lamina.

So, these are the single laminas, and after that, these laminas are stacked together and pressed hard to make a laminate. So, it may be a combination of 0, 90, or any angle. Based on that we can categorize these laminates as cross-ply laminate, symmetric angle ply laminates, and general angle ply laminates.

In cross-ply laminates, we will have a combination of 0 and 90. They may be symmetric, they may be antisymmetric, they may be 0 /0 /0/ 0 /90/ 90/ 0/ 90 like that and in general angle ply. So, most of the theories we are going to develop here by analytical solutions.

So, analytical solutions can be presented for only cross-ply and symmetric angle ply. For a general angle ply, the analytical solutions cannot be developed. So, for that, if you want to study a laminate in which a general angle ply scheme is there. You cannot get the analytical solution then finite element will be used to get the solution.

Then for that purpose, you have to develop any numerical technique like finite element very famous or boundary element or these days differential quadrature method DQM. So, using those techniques you can get the solution for a general angle ply, but in this course, I will develop an analytical solution that will be useful only for cross-ply composites and symmetric angle ply composites.

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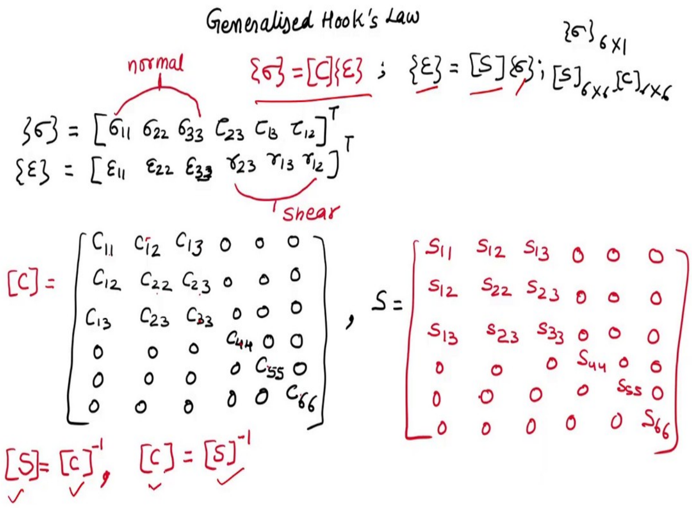

So, at the very start of a formulation first of all we must know what is the constitutive relation for this material. If we want to solve or develop a solution for a composite shell or a piezoelectric shell or a functionally graded shell or an isotropic shell for every material first, we want to know what is the constitutive relation very first information then we have to start some solution.

So, constitutive relations for a three-dimensional case. In the three-dimension we called it generalized Hook's law

$$\{\sigma\} = [C]\{\varepsilon\}$$

Where, σ is a stress component, ε is the strain component and C is the stiffness of the matrix. If you want to know that strain tensor in terms of stress then it will be S and it is known as compliance.

So, our composite material is orthotropic. So, I have taken this C for an orthotropic material that gives you these 9 constants; like 1, 2, 3, 4, 5, 6, 7, 8, 9. So, these 9 constants are required to define an orthotropic material. Sometimes there is a transverse isotropic material that in the transverse direction the material properties are the same. So, for that, we need five engineering constants.

So, this information we need first. We are going to develop, let us say our material is orthotropic and this is our constitutive relations. This is our stiffness matrix and compliance matrix. So, once you know any material either in terms of C or in terms of S you can get another. If you know in terms of C take the inverse of C it will give you the S component. If you know the S component then you can find out the C component.

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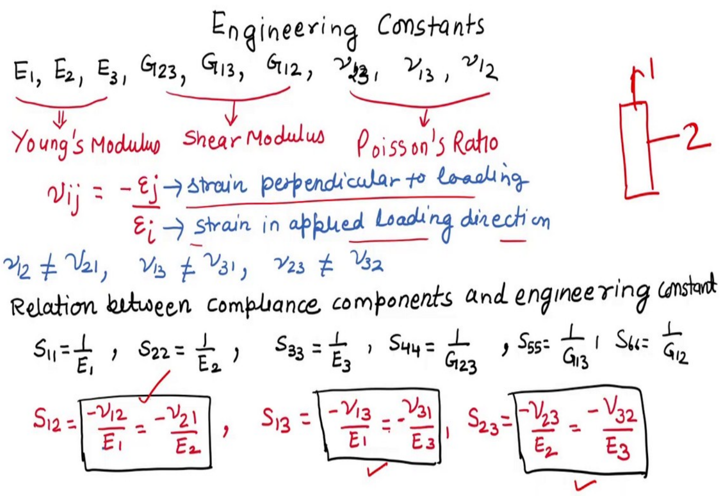

In real practice or when we do experimentation that we want to get the material properties these are evaluated in terms of engineering constant Young's modulus E_1, E_2, E_3 or in terms of shear modulus or Poisson's ratio. We do not get the first information through experiments. We do not get S_{11} or C_{11}. We get E_1, E_2, E_3 G_{23} then we convert into compliance or stiffness.

Here for a composite material since it is an orthotropic material mu$_{12}$ the Poisson ratio not equal to mu$_{21}$. I would like to explain what do you mean by mu$_{12}$.

$$\mu_{ij} = \frac{-\varepsilon_j}{\varepsilon_i}$$

Where, epsilon j (ε_j) is the strain perpendicular to the loading and epsilon i (ε_i) is the strain in the applied loading direction.

For example, if you take this test strip and this is one direction loading. You load it in the one direction and this is your second direction. So, you first get the strain in the second direction divided by the strain in one direction will give you mu_{12}. If you load into the second direction, then you first get the strain in the first direction and divided by $epsilon_2$ then you will get mu_{21}. So, in this way, we will get Poisson's ratio.

So, these engineering constants are related to the compliances. Definitely, we can also relate these things to the stiffness also, but their relation with the compliance is very simple and we can even remember it.

Like, $S_{11} = \frac{1}{E_1} \qquad S_{22} = \frac{1}{E_2} \qquad S_{33} = \frac{1}{E_3}$

These are the normal, and then S_{44}, S_{55}, S_{66} are the shear modulus. Then we can obtain the S_{12}, S_{13}, S_{23}.

$$S_{12} = \frac{-\mu_{12}}{E_1} = \frac{-\mu_{21}}{E_2}$$

For example, sometimes we want to know what is the value of mu_{21}. Then using this relation if we know mu_{12}, E_1, E_2, mu_{21} is calculated. Similarly, using these relations the Poisson ratio either 13 or 31 can be calculated. So, through an experiment we calculate only 1 mu_{12} let us say and mu_{21} can find it out through the relations. Similarly, if we calculate mu_{13}, mu_{31} can find out using this relation and so on.

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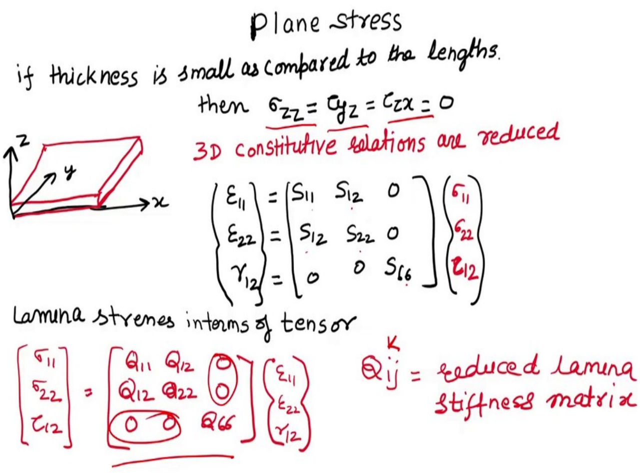

A lamina is very thin, its two dimensions are comparable, and if we talk about a laminate that is also thin not very thick, and is two-dimensional comparable. Then the concept of plane stress works very nicely. If the thickness is small as compared to the length and it is loaded in transverse directions, then we can say that since the thickness is small material cannot take stress in the third direction.

So, by using that assumption σ_{zz} τ_{yz} and τ_{zx} can be 0. So, if we use this assumption three-dimensional constitutive relations are reduced to in this form ε_{11}, ε_{22}, γ_{12} and these are our non-zero compliances.

Again, if we are interested to find out the stress in terms of strain, they are represented by Q. Here most of the theories that Q_{ij} is used, it is a standard notation Q_{ij} is a reduced lamina stiffness matrix. So, when we analyze a composite, we take the property of one lamina, not whole laminate one lamina property and second lamina; so, basically Q_{ijK} we integrate or sometimes in layer-wise theories we may not integrate.

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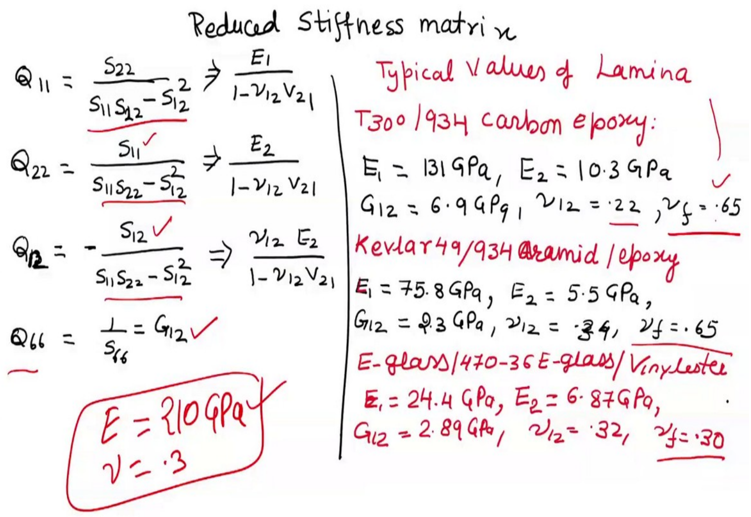

So, how do we get this Q_{11}, Q_{22}, Q_{12}, Q_{66}? So, using this formula. It is very simple to remember.

$$Q_{11} = \frac{S_{22}}{S_{11}S_{12} - S_{12}^{2}}$$

So, that is known as the denominator. It remains the same.

$$Q_{22} = \frac{S_{11}}{S_{11}S_{12} - S_{12}^{2}}.$$

Here, the denominator remains the same or you can find out in terms of engineering constants. Q_{66} is nothing but just G_{12}.

So, this much information is required for developing a theory in the composite shells or composite plate. So, this material information is required first we must have the information about the Q_1, Q_{12}, Q_{66}, we must have information about these things. Once we know then we can move to the development of our shell theories.

So, typical values of laminas. We must have some idea like if somebody tells you about a steel material. By default, you will take 210 GPa Young's modulus then mu 0.3 like that or 0.25, 29. So, this is the range, you can choose it that if somebody says ok this is steel material, so, tentative this.

I would like to give you the typical values of a lamina. If you talk about a carbon epoxy lamina for that case Young's modulus in one direction will be 131 GPa in the second direction 10.3 and shear modulus will be 6.9 and mu_{12} will be 0.2. This information; what is V_f?

V_f is the volume of fiber. So, if we make a composite in which the carbon fiber volume is 0.65 means, 65 percent is fiber and 35 percent is matrix material epoxy then you will get this type of engineering constant. Then if we talk about a Kevlar epoxy it will be 75 E_1, E_2 5.5, G_{12}. So, it is less I would like to say that hard material than epoxy glass, E glass or vinyl ester then it will be 24, 6.8, 2.8 and 0.32; here volume of fiber is 0.3, the volume of fiber is 0.6.

So, once you like to procure some composite material from a vendor, it will clearly say that what is your volume fraction and the material properties. Let us say it may be carbon epoxy, but if your volume fraction is less than they have chosen 0.3 or 0.4 volume fraction of fiber then this matter property will change it is not that always it will be the same. So, it is highly dependent upon the volume fraction of fiber.

So, depending upon the application it is not that we have to choose always 65 percent, 70 percent, no. It depends upon your application you can choose your fiber volume fraction or your material that is why these composite materials are called tailor-made material; means, you can design material according to your requirement there may be some applications you have done the stress analysis found that your stress is not going much. So, you can use a less strong material you need not to use big means like in steel we cannot tailor. This Young's modulus is 210 GPa.

If you are loading and such that it is just coming 1 means a factor of safety coming 10, you cannot do tailor-made it will be heavy, but you can tailor-made using the composites. Less fiber if you need that material should absorb more energy. So, the matrix should be a high amount of matrix so that epoxy can absorb more material. So, in this way, we can tailor-made the material.

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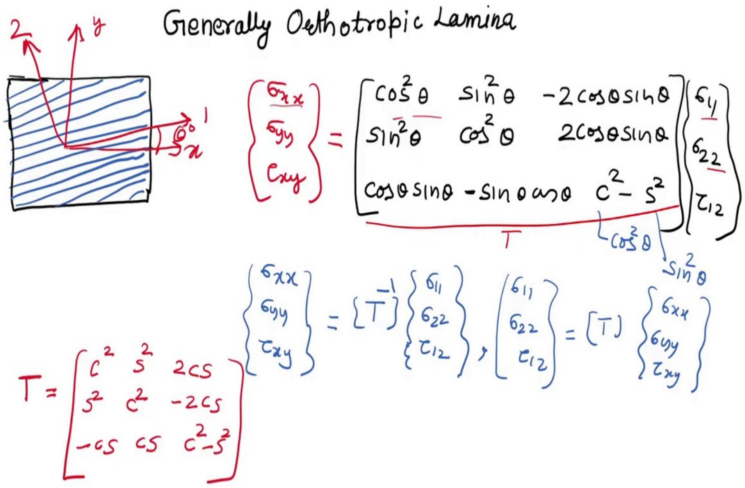

Now, there is a concept. In the previous case this Q_1, Q_2 we have seen this matrix here is 0. This is for a cross-ply laminate, but if you say no, my laminate is angle ply then you have to transform it, like in angle ply laminate.

First of all, get the information that how to transfer the stresses. Let us say 1 and 2 directions we know. If you remember in the undergraduate courses' solid mechanic courses, it is shown that how to express that σ_{xx}, find out at any angle σ_{11}, σ_{22} to find out that principle stresses concept at any angle.

So, using the Mohr circle theory or the direct formula that how to transform from one coordinate system to another coordinate system if it is rotated and origin remains same; so, for that case σ alone σ geometrical axis if fiber is at one direction. So, using these relations the stresses can find it out.

So, this matrix is known as the transfer matrix. Once we know because this is second-order sigma stresses are the second-order tensor. So, it has square terms like that.

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$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12}+2Q_{66})s^2c^2$$

$$\bar{Q}_{12} = (Q_{11}+Q_{22}-4Q_{66})s^2c^2 + Q_{12}(c^4+s^4)$$

$$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12}+2Q_{66})s^2c^2$$

$$\bar{Q}_{16} = (Q_{11}-Q_{12}-2Q_{66})c^3s - (Q_{22}-Q_{12}-2Q_{66})cs^3$$

$$\bar{Q}_{26} = (Q_{11}-Q_{12}-2Q_{66})cs^3 - (Q_{22}-Q_{12}-2Q_{66})c^3s$$

$$\bar{Q}_{66} = (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})c^2s^2 + Q_{66}(s^4+c^4)$$

But compliances stiffness is 4th order tensor. So, for that, we use this transfer matrix twice and C_4 S_4. C_4 means, cos 4 times θ, sinθ 4 times, S square means sin square θ, C_2 means cos square θ. So, we can find out Q_{16}. Transformed if you say that from 0˚ to 90˚, 45˚.

You use these relations. These are also required. If you want to analyze the angle ply laminate then for angle ply lamina if our fiber is making an angle with the geometrical axis, let us say theta then what are the stiffness property in the geometrical axis? For example, let us say this is my lamina and it is having some angle, but we are loading in one direction and two directions. So, how do you get E_1, E_2? So, they will find it out using this transformation kind of thing.

So, with this our first lecture is over. In the next lecture, I will start what are the different shells that how do you call it thin and thick and various differential geometry, I would like to say the basic concept of differential geometry used in the shell theories; the concept of surface normal, the concept of principle radius of curvature and the concept of principle normal so that we can develop the governing equation in more general form.

Thank you very much.