Theory of Composite Shells

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Week – 04
Lecture – 03
Workout problems

Dear learners welcome to lecture-03 of week-04. In this lecture, I shall do some more formulations for the special cases of the shell of revolutions. I have solved the problem of a cylindrical shell, special cases, spherical shells, or conical shells. But in the initial lectures, where I said the surface of revolution, several shell geometries can be formed with the general formulation.

If we can develop the governing equations for the shell of revolution, then those set of equations or the solutions are valid for all kind of shells which can be generated through the shell of revolutions. In week-03, we have developed the governing equations in a general form doubly curved shell. Then in week-04, lecture-01 and lecture-2, I have derived the special cases like membrane theory of shells.

If you see in the literature, you will find the shell theories, after the initial formulation can be divided into two categories, one is membrane theory and another is the moment theory. Some problems can be solved using the concept of membrane theory and some problems can be solved using the concept of moment shell theory.

In the membrane shell theory, we assume that there is no bending and no shear, only three equations come, shear force and moments will be 0, and only in-plane stress resultants like N_{11}, N_{22}, and N_{12} plays in the membrane shell theory.

In the moment shell theory, the effect of bending is taken care of, and the moments are considered, and it is slightly typical. Already, in the week- 04 lectures, I have explained, when to use the membrane shell theory and the moment shell theory.

In this lecture, I shall explain in a slightly more elaborative way, first the membrane shell theory for a shell of revolution, and one problem will be solved, and later the formulation for a moment shell theory will be done.

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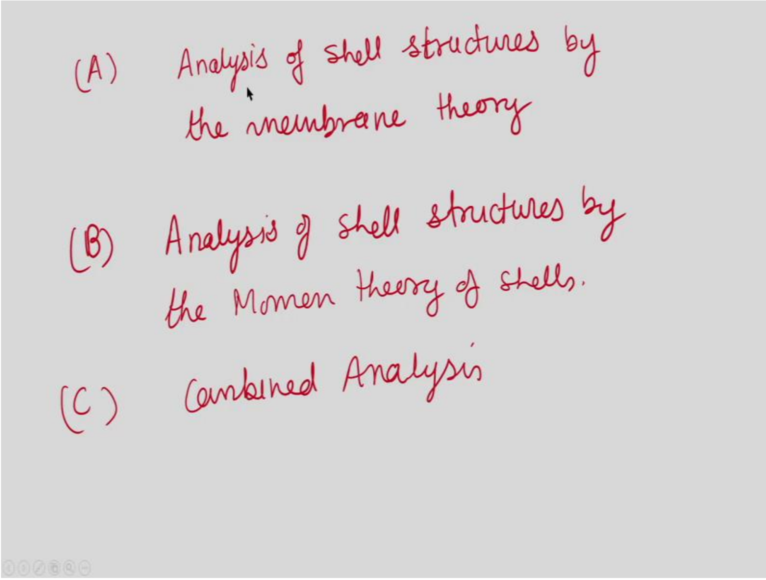

We will do an analysis of the shell structures by membrane shell theory, moment theory, and combined shell theory also because in a structure, you will find that there are some places where membrane shell theory gives accurate results, and some places where we have joints and a change in curvature or thickness, at those special locations moment shell theory works.

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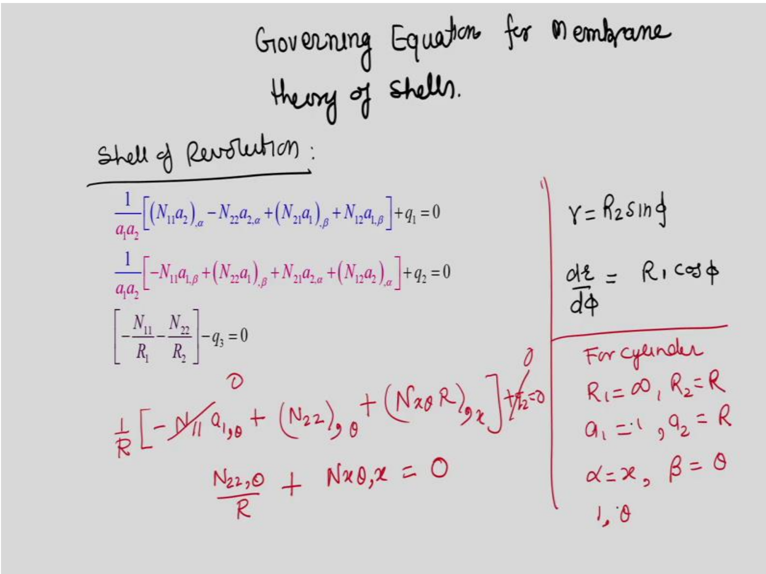

Governing equations for membrane theory of shells:

$$\frac{1}{a_1 a_2}\left[\left(N_{11}a_2\right)_{,\alpha} - N_{22}a_{2,\alpha} + \left(N_{21}a_1\right)_{,\beta} + N_{12}a_{1,\beta}\right] + q_1 = 0 \quad equation(1)$$

$$\frac{1}{a_1 a_2}\left[-N_{11}a_{1,\beta} + \left(N_{22}a_1\right)_{,\beta} + N_{21}a_{2,\alpha} + \left(N_{12}a_2\right)_{,\alpha}\right] + q_2 = 0 \quad equation(2)$$

$$\left(-\frac{N_{11}}{R_1} - \frac{N_{22}}{R_2}\right) - q_3 = 0 \quad equation(3)$$

These are the general equations for a doubly curved shell. These are for the case of membrane theory of shells, but if we are interested to convert these equations for the shell of revolution, for that case, r is taken as $R^2 \sin\theta$. And the change of r with respect to ϕ, $\frac{dr}{d\phi} = R_1 \cos\phi$. This already we have discussed in previous lectures, $a_1 = R_1$ and $a_2 =$ r.

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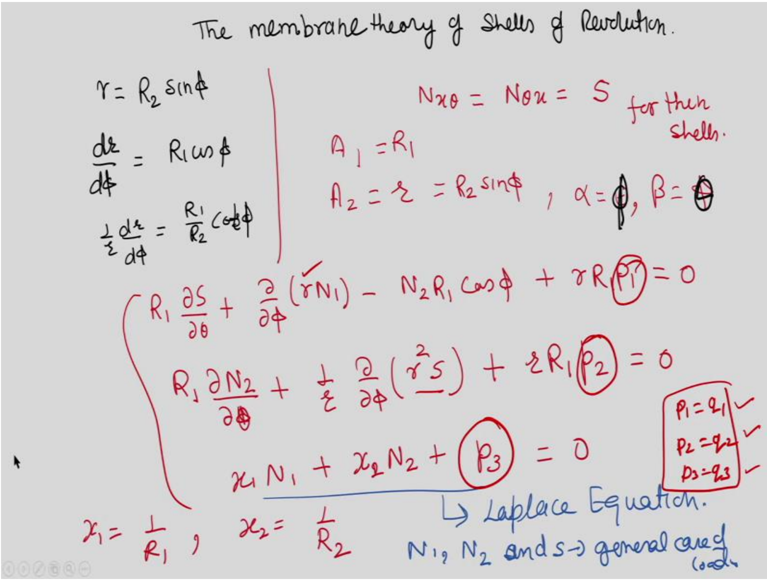

And if we substitute all the parameters here, then finally, it becomes an equation like this $N_{\theta x} = N_{x\theta} = S$.

In the membrane theory of shells, we assume that the shell is very thin, $N_{x\theta}$ and $N_{\theta x}$ are considered the same. In most of the book it is represented by S, or we can say shear in-plane stress resultants or N_{12} or N_{21}.

Here, $\alpha = \phi$ and $\beta = \theta$.

If we take into consideration these parameters and substitute it in those governing equations for the membrane theory of shells, that will lead to these three equations.

$$R_1 \frac{\partial S}{\partial \theta} + \frac{\partial}{\partial \phi}(rN_1) - N_2 R_1 \cos\phi + rRp_1 = 0 \quad equation\,(1)$$

$$R_1 \frac{\partial N_2}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial \phi}(r^2 S) + rR_1 p_2 = 0 \quad equation\,(2)$$

$$\kappa_1 N_1 + \kappa_2 N_2 + p_3 = 0 \quad equation\,(3)$$

In the last equation, $\kappa_1 N_1 = \dfrac{N_1}{R_1}$,

$\dfrac{1}{R_1} = \kappa_1$, and $\dfrac{1}{R_2} = \kappa_2$,

Therefore, (kappa 1) κ_1 and (kappa 2) κ_2 are written here.

One more change here is that initially we were using $q_1,\ q_2,\ and\ q_3$, whenever I go for a book of a thin elastic plate and shell they have used, $p_1,\ p_2,\ and\ p_3$ instead of $q_1,\ q_2,\ and\ q_3$. To keep consistent with the literature or books I use $p_1,\ p_2,\ and\ p_3$, but do not get confused as these are the same. Here, we obtained three equations, and equation (3) is called the Laplace equation. We want to know the solution to this equation.

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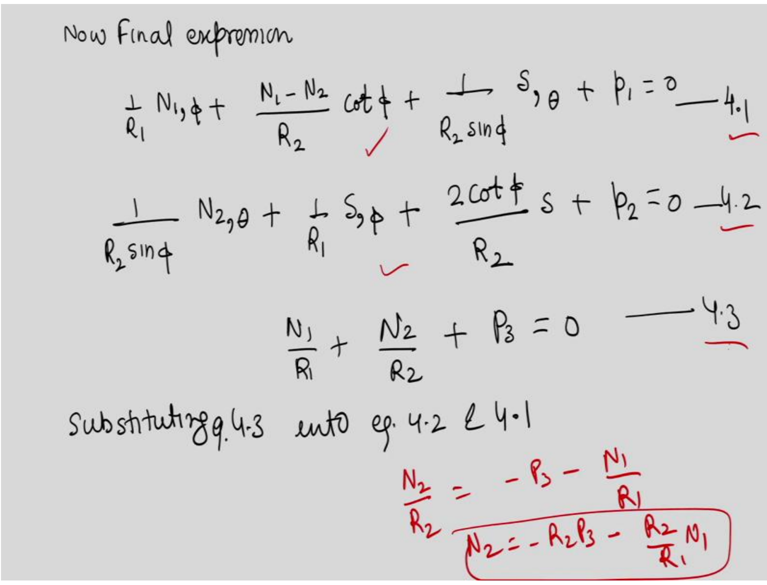

The final expression is written in terms of r, you know that r is equal to $R_2 \sin\phi$, if you further substitute those equations, then the final expression will look like this.

$$\frac{1}{R_1} N_{1,\phi} + \frac{N_1 - N_2}{R_2} \cot\phi + \frac{1}{R_2 \sin\phi} S,_{\theta} + p_1 = 0 \qquad \textit{equation } (4.1)$$

$$\frac{1}{R_2 \sin\phi} N_{2,\theta} + \frac{1}{R_1} S,_{\phi} + \frac{2\cot\phi}{R_2} S + p_2 = 0 \qquad \textit{equation } (4.2)$$

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + p_3 = 0 \qquad \textit{equation } (4.3)$$

If we substitute equation (4.3) in equations (4.1) and (4.2), that $\frac{N_2}{R_2}$ can be written as:

$$\frac{N_2}{R_2} = -p_3 - \frac{N_1}{R_1} \quad \text{and} \quad N_2 = -R_2 p_3 - \frac{R_2}{R_1} N_1$$

We can substitute this ‘ N_2 here in equation (4.1) and (4.2) in terms of N_1. These are the equations.

$$\frac{1}{R_1} N_{1,\phi} + N_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \cot\phi + \frac{1}{R_2 \sin\phi} S,_{\theta} = -p_1 - p_3 \cot\phi \qquad \textit{equation } (4.4)$$

$$-\frac{1}{R_2 \sin\phi} N_{1,\theta} + \frac{1}{R_1} S,_{\phi} + \frac{2\cot\phi}{R_2} S = \frac{1}{\sin\phi} + p_{3,\theta} - p_2 \qquad \textit{equation } (4.5)$$

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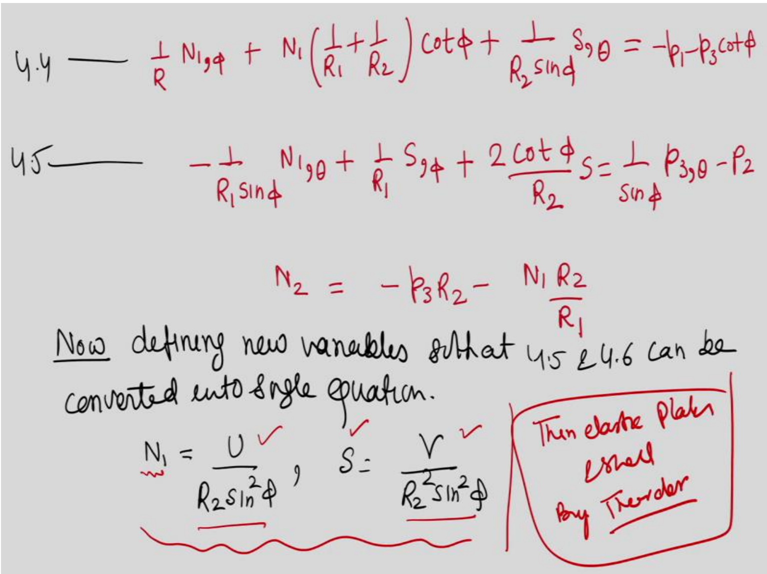

Now, we can define new variables. why are we doing these things? We want to convert all these three equations either into two variables or a single variable so that we can get the solution easily. We can solve these equations no problem, but we need to find some technique that is well-explained in the book “Thin Elastic Plate and Shells” by Theodor.

Already, I have given all these things in the references. We can assume:

$$N_1 = \frac{U}{R_2 \sin^2 \phi} \qquad S = \frac{V}{R_2{}^2 \sin^2 \phi}$$

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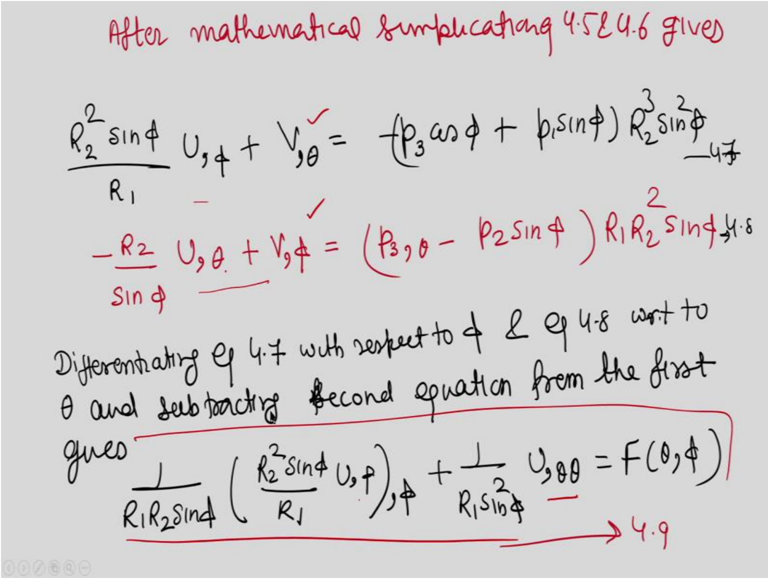

If we use this expression and substitute it into this, ultimately, these three equations can be converted into two equations like this.

$$\frac{R_2 \sin\phi}{R_1} U_{,\phi} + V_{,\theta} = -\left(p_3 \cos\phi + p_1 \sin\phi\right) R_2^3 \sin^2 \phi \qquad \textit{equation } (4.7)$$

$$-\frac{R_2}{\sin\phi} U_{,\theta} + V_{,\phi} = \left(p_{3,\theta} + p_2 \sin\phi\right) R_1 R_2^2 \sin\phi \qquad \textit{equation } (4.8)$$

Finally, we get these equations (4.7) and (4.8) having variables U and V, then we take differentiation with respect to θ in equation (4.7) and in (4.8) equation with respect to ϕ. And subtracting the second equation from the first equation will give you this single equation.

$$\frac{1}{R_1 R_2 \sin\phi}\left(\frac{R_2^2 \sin\phi}{R_1} U_{,\phi}\right)_{,\phi} + \frac{1}{R_1 \sin^2 \phi} U_{,\theta\theta} = F(\theta,\phi) \qquad \textit{equation } (4.9)$$

We can further deduce it in terms of a single variable, $U_{,\phi}$ and $U_{,\theta\theta}$, variable V can be eliminated from those equations. Then, this equation can be solved using the standard techniques. And once we know variable U, we can find the variable V by substituting it in equation (4.7) or (4.8). And then we can find the normal stresses and shear stresses. In this way, we can solve the problems.

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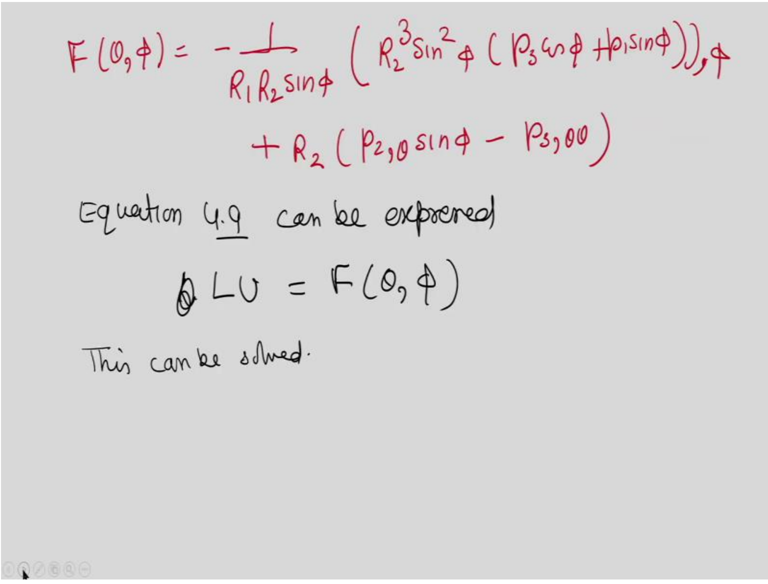

Where $F(\theta,\phi)$ is a loading function and that can be represented as:

$$F(\theta,\phi)=\frac{1}{R_1R_2\sin\phi}\left(R_2^3\sin^2\phi\left(p_3\cos\phi+p_1\sin\phi\right)\right)_{,\phi}+R_2\left(p_{2,\theta}\sin\phi-p_{3,\theta\theta}\right).$$

This is the way the shell of revolution problem is solved. If you substitute the real values of R_1, R_2, and ϕ, then you can get the solution of all kinds of shells which are developed using the shell of revolution.

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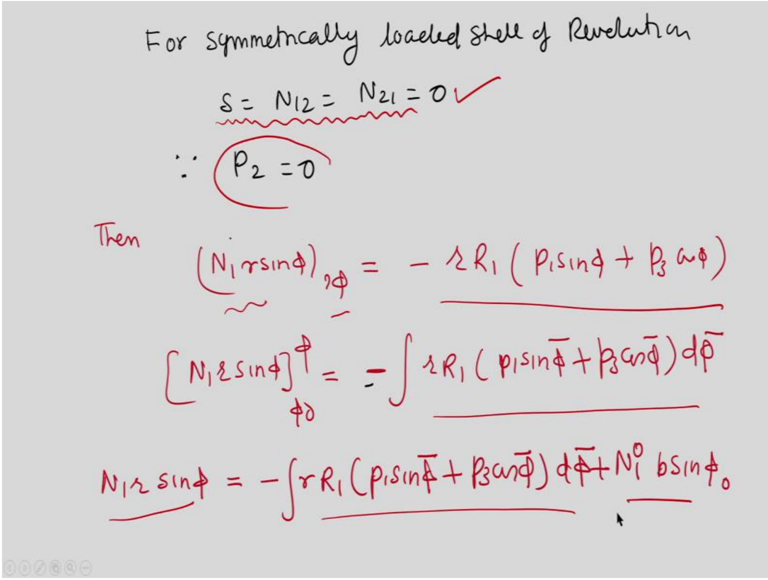

Now, there is another case, for the symmetrically loaded shell of revolution which means the shells are generated through the revolution. They are symmetric around the θ, and if we assume that loading is also symmetric, for that case it is not dependent on $\theta, \because p_2 = 0$. Further, shear components $N_{12} = N_{21} = 0$.

If this is the situation, then from that equation, $(N_1 r \sin\phi)_{,\phi}$ can be represented as:

$-rR_1 (p_1 \sin\phi + p_3 \cos\phi)$.

If you integrate with respect to ϕ, then,

$$(N_1 r \sin\phi)_{\phi_0}^{\phi} = -\int rR_1 \left(p_1 \sin\bar{\phi} + p_3 \cos\bar{\phi} \right) d\bar{\phi}$$

And ultimately, $N_1 r \sin\phi$ will be:

$-\int rR_1 \left(p_1 \sin\bar{\phi} + p_3 \cos\bar{\phi} \right) d\bar{\phi} \; + \; N_1^0 b \sin\phi_0$ some integrating constant.

From here, we can find out N_1 for an axially symmetrically loaded shell, where N_1^0 is the applied in-plane stress at the boundary.

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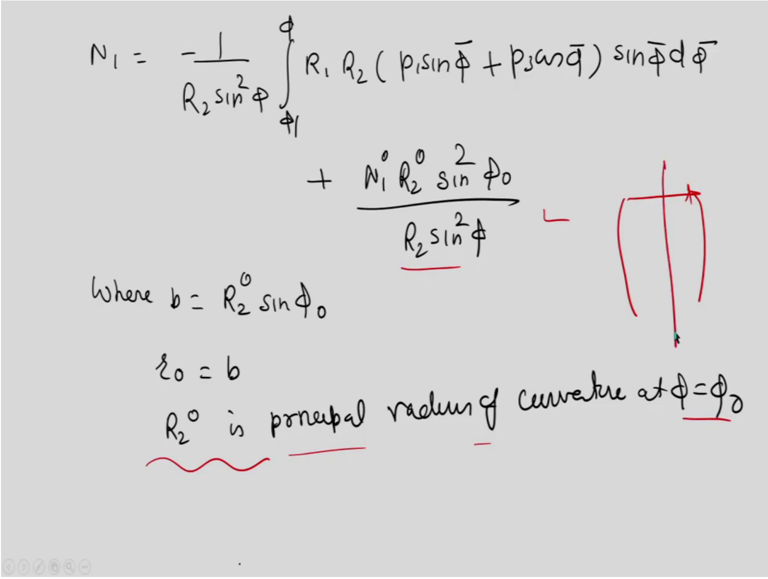

Ultimately, N_1 can be represented like this.

$$-\frac{1}{R_2\sin^2\phi}\int_{\phi_1}^{\phi} R_1R_2\left(p_1\sin\bar{\phi}+p_3\cos\bar{\phi}\right)\sin\bar{\phi}\,d\bar{\phi}+\frac{N_1^0R_2^0\sin^2\phi_0}{R_2\sin^2\phi}.$$

For an open kind of shell, when r_0 = b and b = $R_2^0\sin\phi_0$, where then R_2^0 is the principal radius of curvature at $\phi=\phi_0$. For the closed case, some of the terms are going to be 0.

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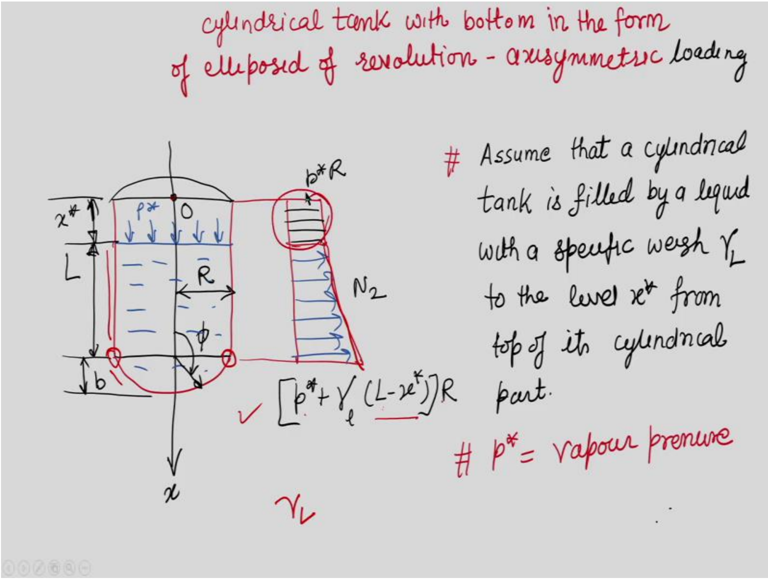

Let us solve a problem, a cylindrical tank with a bottom in the form of an ellipsoid of revolution- axisymmetric loading. You can see, here, the bottom part is ellipsoidal in shape and the upper part is a cylindrical tank. It consists of two shells - one is cylindrical, another is an ellipsoidal shell. The membrane solution will be valid up to these lines because at the junctions there will be a drastic change in curvature and the solution will not be valid.

We assume that it is filled with water or some liquid and the height of the ellipsoidal cylinder is b, the length of the cylindrical cylinder is L, and the vapor pressure is p^0, and that distance is denoted as x^0. And the origin is here. The cylindrical tank is filled, the specific weight of a liquid is γ_l.

We can say that the total load or a total pressure is acting at any point can be represented as:

p $\left[p^* + \gamma_l \left(L - x^*\right)\right] R$.

First, we will discuss, the cylindrical shell and then the ellipsoidal shell. When L = $2x^*$, then p^* = 0, only vapor pressure is there, but after $\left(L - x^*\right)$, pressure will follow this rule.

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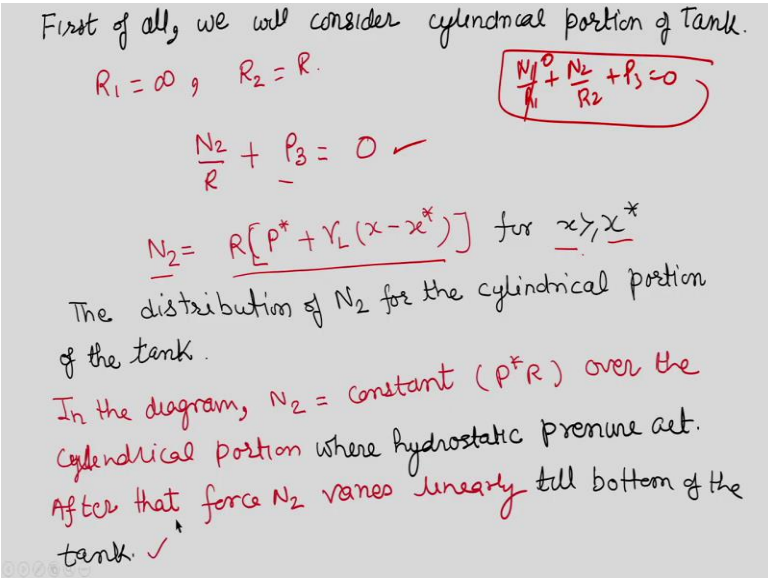

In the cylindrical portion of the tank $R_1 = \infty$ and $R_2 = R$. The very first equation is:

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + p_3 = 0$$

Here, $R_1 = \infty$, this $\frac{N_1}{R_1}$ will not participate.

Therefore, $\frac{N_2}{R} + p_3 = 0$, it is valid for when $x > x^*$.

The distribution of N_2 for the cylindrical portion can be represented like this.

$$N_2 = R\left[p^* + \gamma_l\left(x - x^*\right)\right]$$

In the previous figure, the distribution of N_2 is varying linearly as L is increasing up to here, but it is constant over the cylindrical portion where hydrostatic pressure acts after the force. And N_2 varies linearly till the bottom of the tank.

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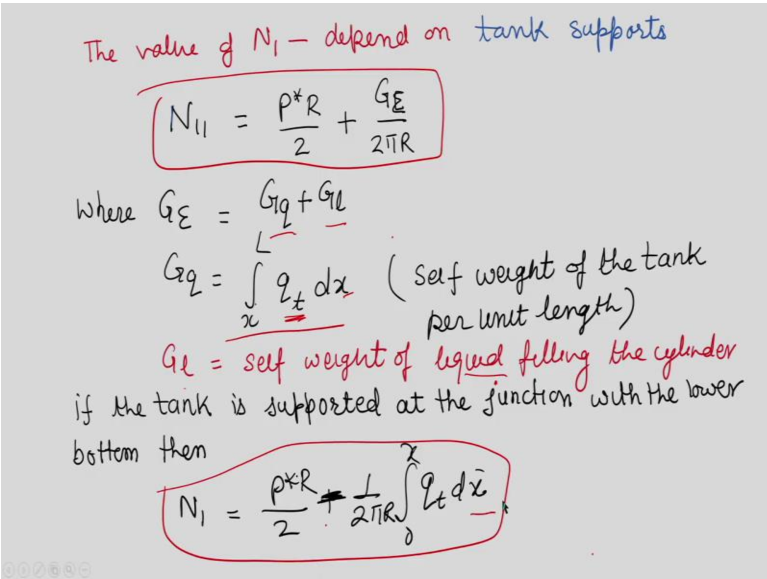

Now, we want to find the value of N_1, in the second equation, N_{12} = 0, we are substituting the value of $N_{2,\theta}$, I will go back to slide at 6:10, equation (4.1), S = 0.

For axisymmetric, this portion $\frac{1}{R_1} S,_{\phi} + \frac{2\cot\phi}{R_2} S$ = 0.

$N_{2,\theta}$ + p_2 = 0, it will identically satisfy, so we will directly go to the first equation. N_1

can be found by solving this equation (4.1).

I am directly writing that N_{11} can be represented as:

$$N_{11} = \frac{p^* R}{2} + \frac{G_\Sigma}{2\pi r}$$

Where, G_Σ contains two parts G_q and G_l

G_q is the self-weight of the tank per unit length. In the cases, where self-weight is neglected, there we will not consider G_q.

If the self-weight is considered, then $G_q = \int_x^L q_t dx$

Here, q_t is the self-weight per unit length. We can find these things. Then, G_l is the self-weight of the liquid filling the cylinder. If the tank is supported at the junction with the lower bottom, then N 1 can be represented as:

$$\frac{p^* R}{2} - \frac{1}{2\pi r} \int_0^x q_t d\overline{x} .$$

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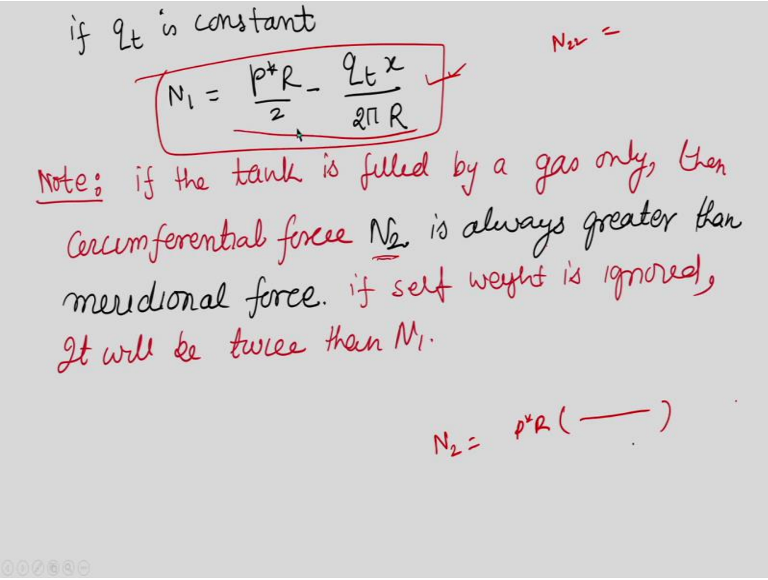

If q_t is constant, then finally integration can be solved. I would like to say that circumferential stress N_{11} will be:

$$\frac{p^{*}R}{2}-\frac{q_t x}{2\pi r}.$$

If the tank is filled by gas only, then the circumferential force N_2 is always greater than the meridional force. If we neglect the self-weight, then,

$$\frac{q_t x}{2\pi r} = 0 \text{ and } N_2 = 2N_1.$$

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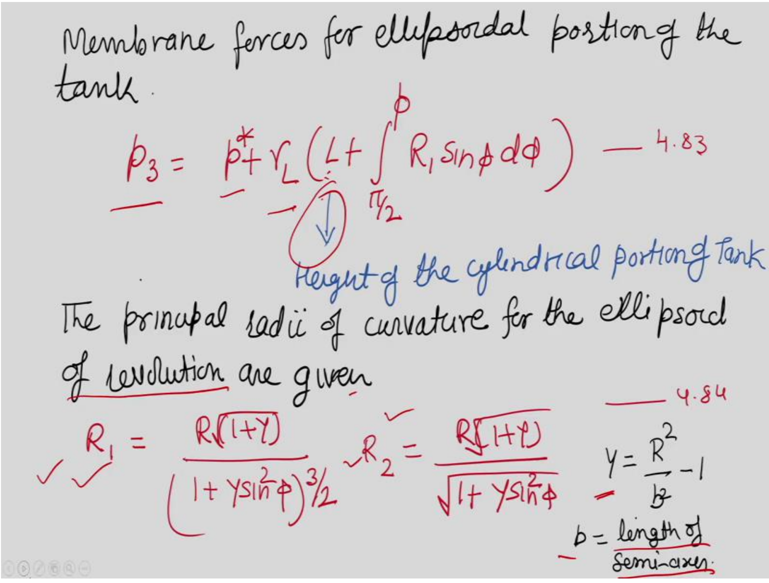

Now we are talking about the stresses in the ellipsoidal portion, for that, first, we have to find the total load acting in the third direction. p_3 is:

$p^{*}+\gamma_l\left(L+\int_{\pi/2}^{\phi} R_1 \sin\phi\, d\phi\right)$ equation (4.83), length multiplied by γ_l will give you the total pressure acting on that tank.

The principal radius of curvature for a shell of revolution is given in terms:

$$R_1=\frac{R\sqrt{1+\gamma}}{\left(1+\gamma\sin^2\phi\right)^{3/2}} \text{ and } R_2=\frac{R\sqrt{1+\gamma}}{\sqrt{1+\gamma\sin^2\phi}}$$

Where $\gamma=\frac{R^2}{b^2}-1$,

b is the length of a semi-axis, and R is the radius of the cylinder. In this way, the radius of curvature is known to us.

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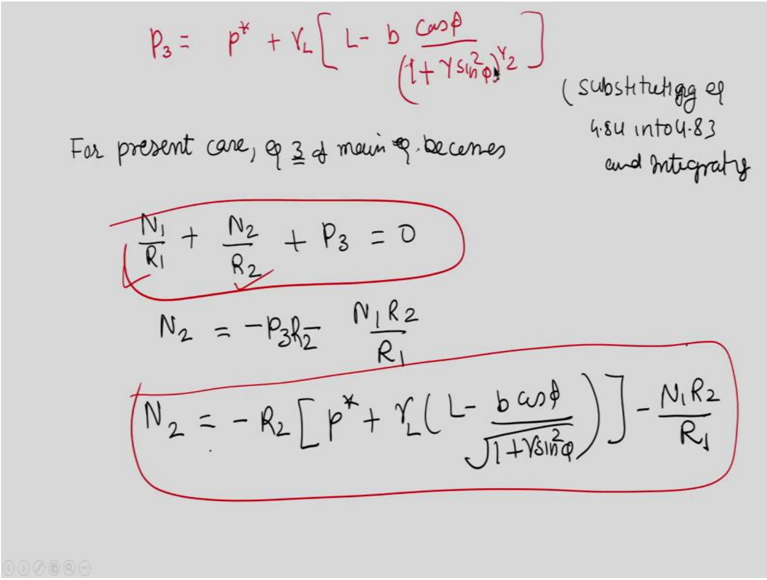

And then we can find the value of N_1 by substituting in the first equation.

$\frac{N_1}{R_1}+\frac{N_2}{R_2}+p_3=0$. R_1 and R_2 are known to us, we can find N_2:

$$N_2=-R_2\left[p^*+\gamma_L\left(L-\frac{b\cos\phi}{\sqrt{1+\gamma\sin^2\phi}}\right)\right]-\frac{N_1R_2}{R_1},$$

And ultimately, p_3 can be represented as:

$$p^*+\gamma_L\left(L-\frac{b\cos\phi}{\sqrt{1+\gamma\sin^2\phi}}\right)$$

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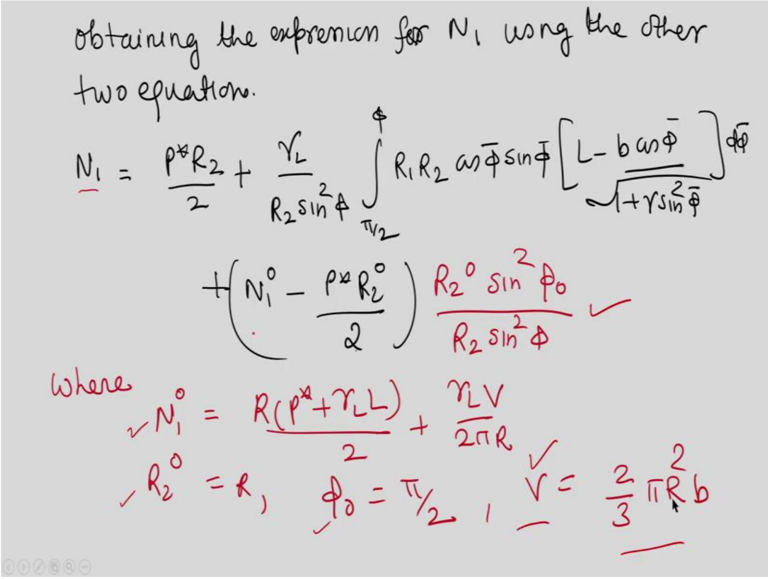

If we substitute this expression using those governing equations, we have already found these things. In the present case, some more terms come up as per the loading conditions and the final expression of N_1 will be:

$$\frac{p^* R_2}{2} + \frac{\gamma_L}{R_2 \sin^2 \phi} \int_{\pi/2}^{\phi} R_1 R_2 \cos\bar{\phi} \sin\bar{\phi} \left(L - \frac{b \cos\bar{\phi}}{\sqrt{1+\gamma \sin^2 \phi}} \right) d\bar{\phi} + \left(N_1^0 - \frac{p^* R_2^0}{2} \right) \frac{R_2^0 \sin^2 \phi_0}{R_2 \sin^2 \phi}.$$

N_1^0 is $\frac{R\left(p^* + \gamma_L L\right)}{2} + \frac{\eta V}{2\pi R}$;

R_2^0 = R; $\phi_0 = \frac{\pi}{2}$; and V (volume of the ellipsoid) = $\frac{2}{3}\pi R^2 b$.

N_2 and N_1 are also obtained for the ellipsoidal case. Substituting the actual value of R_2 γ_L, we can find the stresses in the cylindrical tank.

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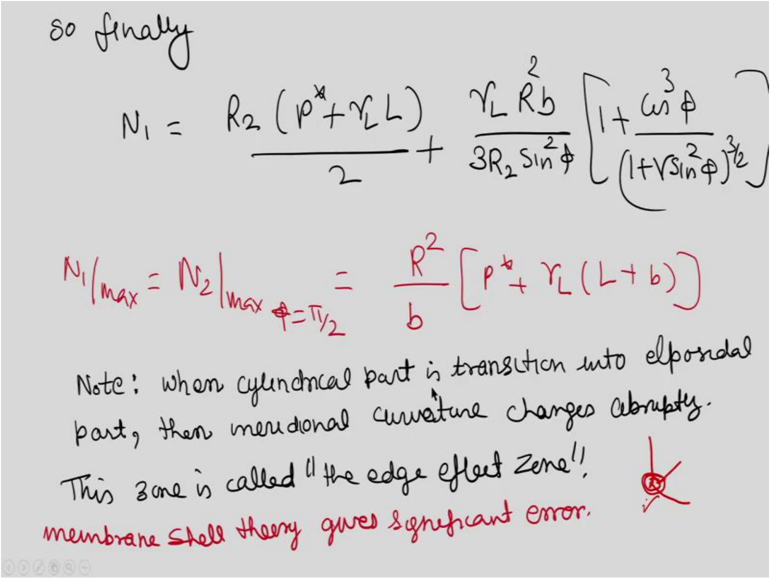

The final expression will be:

$$N_1 = \frac{R_2\left(p^* + \gamma_L L\right)}{2} + \frac{\gamma_L R^2 b}{3R_2 \sin^2 \phi} \frac{1+\cos^3 \phi}{\left(1+V\sin^2 \phi\right)^{3/2}}$$

When the cylinder part is transitioned, into the ellipsoidal part, then, the meridional curvature changes abruptly. This zone is called the edge effect zone. When the cylindrical portion is connected to an elliptical, this zone is called the end zone. In this case, membrane shell theory does not give accurate results which means the membrane shell theory is valid slightly away from that zone.

If you see, in this zone, we are not able to find the solution. And some high stresses or bending moments present in this zone. And we should know for designing of this kind of special case, it may crack from these joints. We have to find the moments and couples at this joint, the moment theory of shells works in this field.

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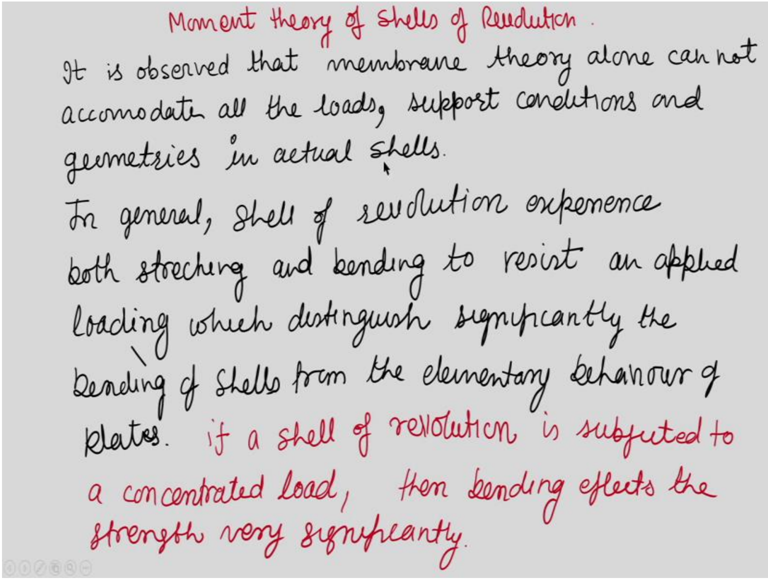

It is observed that membrane theory alone cannot accommodate all the loads, support conditions, and geometries in the actual shells. In general, the shell of revolutions experiences both stretching and bending to resist an applied loading which distinguishes significantly the bending of shells from the elemental behaviour of the plate.

If the shell of the revolution is subjected to a concentrated load or the boundary conditions, its strength affects very significantly. Load carrying capacity affects when it is subjected to bending effects.

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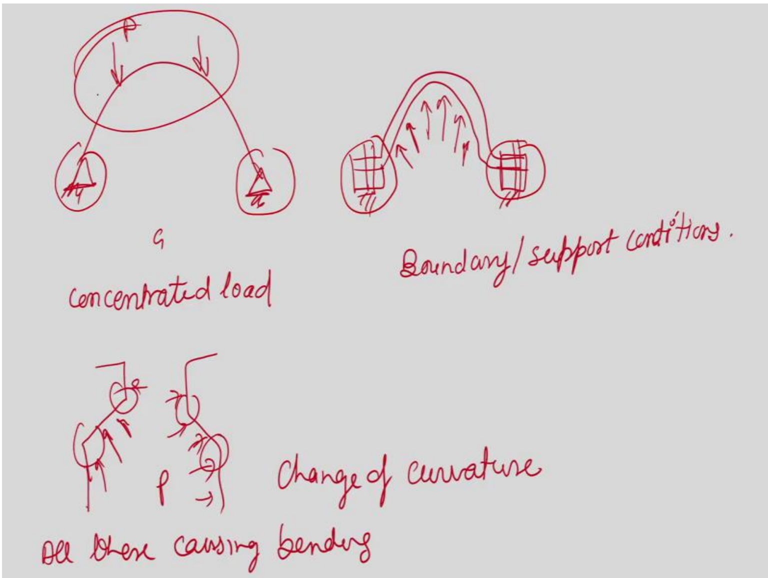

These are some examples of the shell though the boundaries are perfect, but a concentrated load is applied, which will cause a bending effect and a moment effect. And you see at the axis these are clamped, flanges are there.

This is also called an edge effect or boundary effect. Then if a structure is made of a composition of two or more shells of revolutions like previously, we have done that cylindrical shell and ellipsoidal shell. Now, in this kind of container, where curvature changes or thickness changes, this kind of bending stress may exist.

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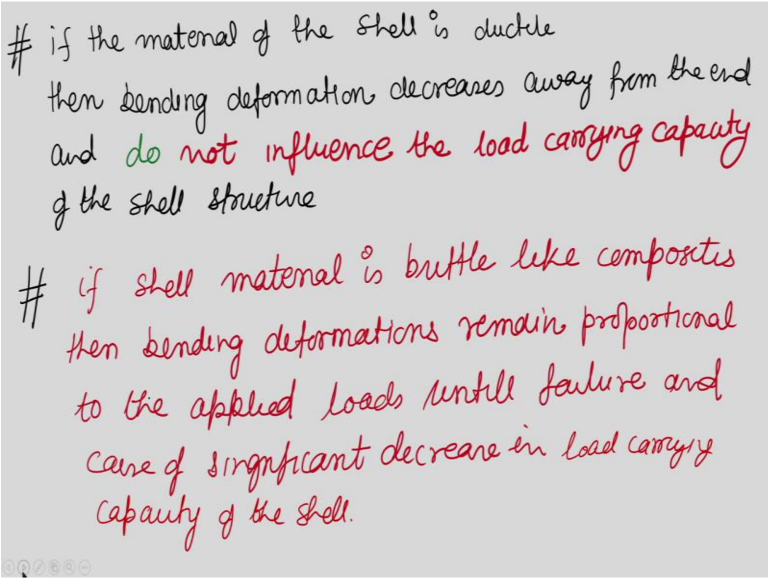

If the material of the shell is ductile, then bending deformation decreases away from the end and do not influence the load-carrying capacity of the shell structure. But if shell material is brittle like a composite, then bending deformation remains proportional to the applied load until failure, so it will cause a significant decrease in the load-carrying capacity of the shell.

That is why, when we talk about a composite shell, most of the time we used to get the solution means we do not go for a membrane theory of shells, or a moment we want to solve a complete shell equation.

The reason behind that is in the composite shells, the very basic ingredient is the material properties change abruptly from layer to layer. This change in material property and poisson's ratio is different, cause deformation. Due to that, bending may take place. If it is made of an isotropic material, then this issue is not there that means delamination may take place.

So, for the case of a composite shell, preferably, we try to solve all the complete equations, we do not simplify only using the moment shell theory or the membrane shell theory. At the interfaces and the axis, these problems further enhance or amplify for the case of composite shells.

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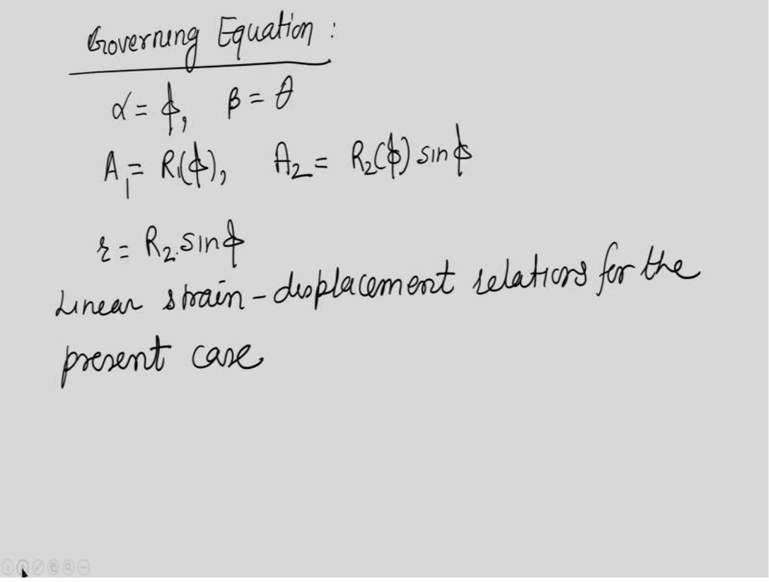

So, in this case, $\alpha = \phi$ and $\beta = \theta$. I would like to derive the first sets of governing equations for the moment theory of shells, and then its solution. For the case of the shell of revolutions, these parameters $A_1 = R_1(\phi)$, $A_2 = R_2(\phi)\sin\phi$, $r = R_2 \sin\phi$ are known to you. First, we will find the linear strain-displacement relations for the present case.

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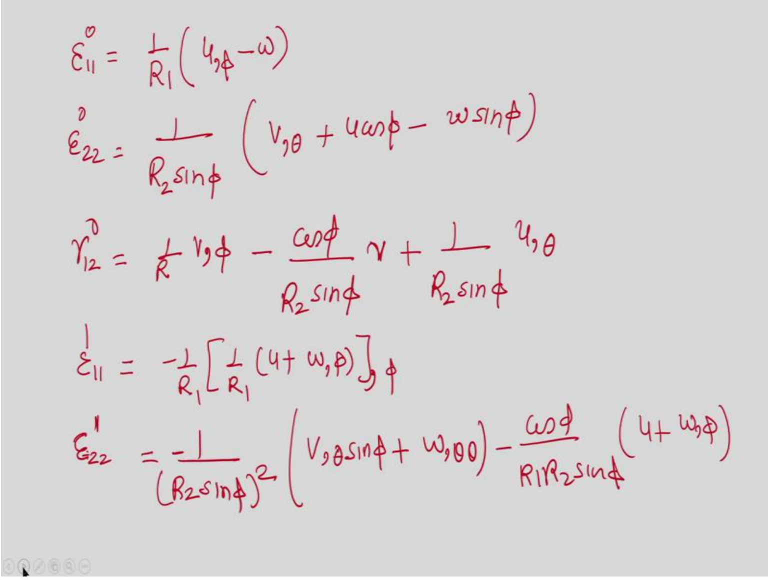

$$\varepsilon_{11}^{0} = \frac{1}{R_1}\left(u,_{\phi} - w\right) \;\; ;$$

$$\varepsilon_{22}^{0} = \frac{1}{R_2 \sin\phi}\left(v,_{\theta} + u\cos\phi - w\sin\phi\right);$$

$$\gamma_{12}^{0} = \frac{1}{R} v,_{\phi} - \frac{\cos\phi}{R_2 \sin\phi} v + \frac{1}{R_2 \sin\phi} u,_{\theta}$$

$$\varepsilon_{11}^{1} = \frac{1}{R_1}\left[\frac{1}{R_1}\left(u + w,_{\phi}\right)\right]_{,\phi} \;\; ;$$

$$\varepsilon_{22}^{1} = \frac{1}{\left(R_2 \sin\phi\right)^2}\left(v,_{\theta}\sin\phi + w,_{\theta\theta}\right) - \frac{\cos\phi}{R_1 R_2 \sin\phi}\left(u + w,_{\phi}\right); \text{ and}$$

$$\varepsilon_{12}^{1} = \frac{1}{R_2 \sin\phi}\frac{\cos\phi}{R_2 \sin\phi}\frac{\partial w}{\partial\theta} - \frac{1}{R_1} w,_{\theta\phi} - \frac{1}{R_1} u,_{\theta} - \frac{\sin\phi}{R_1} v,_{\phi} + \frac{\cos\phi}{R_2} v$$

These are the strain displacement relations for the shell of revolutions.

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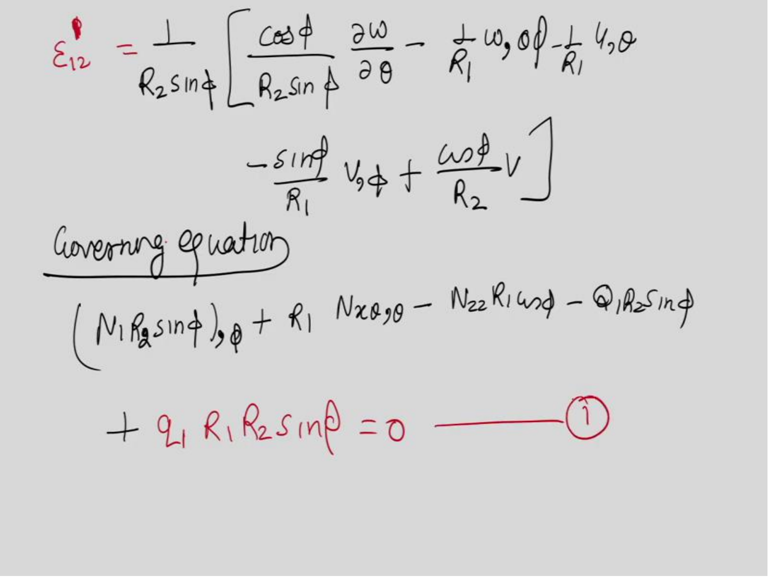

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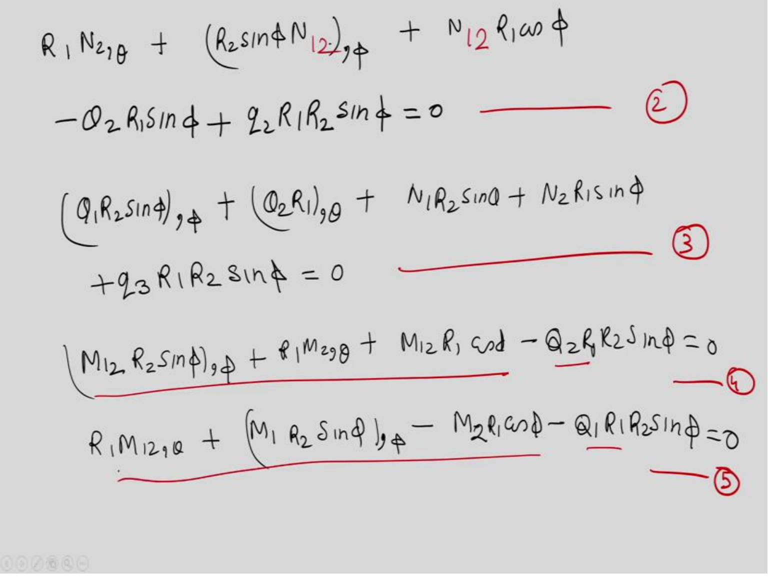

$$\left(N_1R_2\sin\phi\right)_{,\phi} - N_{22}R_1\cos\phi + R_1N_{x\theta,\theta} + Q_1R_2\sin\phi + q_1R_1R_2\sin\phi = 0 \ \ equation(1)$$

$$N_{2,\theta}.R_1 + \left(R_2\sin\phi N_{12}\right)_{,\phi} + N_{12}R_1\cos\phi - Q_2R_1\sin\phi + q_2R_1R_2\sin\phi = 0 \ \ equation(2)$$

$$\left(Q_1R_2\sin\phi\right)_{,\phi} + \left(Q_2R_1\right)_{,\phi} + N_1R_2\sin\phi + N_2R_1\sin\phi + q_3R_1R_2\sin\phi = 0 \ \ equation(3)$$

$$\left(M_{12}R_2\sin\phi\right)_{,\phi} + M_{12}R_1\cos\phi + R_1M_{2,\theta} - Q_1R_1R_2\sin\phi = 0 \ \ equation(4)$$

$$M_{12,\theta}.R_1 + \left(R_2\sin\phi M_1\right)_{,\phi} - M_2R_1\cos\phi - Q_2R_1R_2\sin\phi = 0 \ \ equation(5)$$

These are the five governing equations, which contain all the variables all moments, and in-plane stress resultants for the shell of revolutions.

In the previous case, when we are talking about the membrane theory of shells, we have neglected the moments and shears. But now we have taken all the variables together, and substitute the value of R_1, R_2, a_1, and a_2

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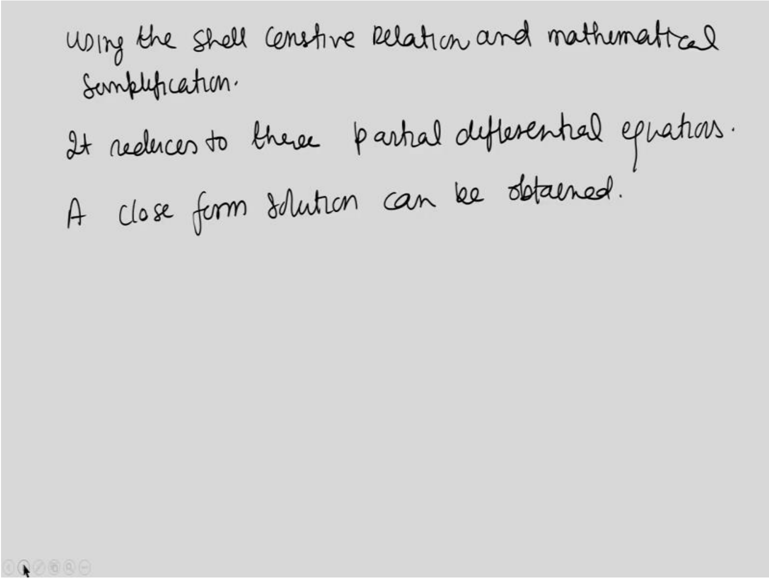

Now, using the shell constitutive relations and mathematical simplification. Q_2 can be expressed from equation (5) and Q_1 is expressed from equation (4). Substituting into the first three equations, it reduces to three partial differential equations.

$$\left(N_1 R_2 \sin\phi\right)_{,\phi} - N_2 R_1 \cos\phi - Q_1 R_2 \sin\phi + q_1 R_1 R_2 \sin\phi = 0$$
$$\left(Q_1 R_2 \sin\phi\right)_{,\phi} + N_1 R_2 \sin\phi + N_2 R_1 \sin\phi + q_3 R_1 R_2 \sin\phi = 0$$
$$\left(M_1 R_2 \sin\phi\right)_{,\phi} + M_2 R_2 \cos\phi - Q_1 R_1 R_2 \sin\phi = 0$$

A closed-form solution can be obtained for such cases, it is very difficult and more complex.

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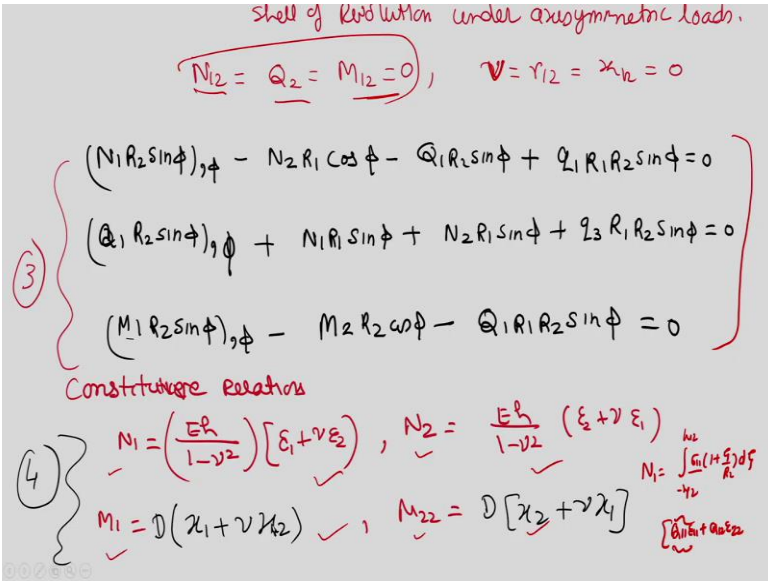

In the most general cases, the shell of revolutions under axis-symmetric load case, the displacement along second interaction is 0, and $N_{12} = Q_2 = M_{12} = 0$. If we assume such cases, these three equations reduce like this. And the constitutive relations for the isotropic case can be written like this.

$$N_1 = \left(\frac{Eh}{1-\mu^2}\right)(\varepsilon_1 + \mu\varepsilon_2) \quad N_2 = \left(\frac{Eh}{1-\mu^2}\right)(\varepsilon_2 + \mu\varepsilon_1)$$

$$M_1 = D(k_1 + \mu k_2) \quad M_2 = D(k_2 + \mu k_1)$$

But you are aware that for the case of a composite, N_1 can be written as:

$$N_1 = \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{11}\left(1 + \frac{\varsigma}{R_2}\right) d\varsigma$$

$\sigma_{11} = Q_{11}\varepsilon_{11} + Q_{12}\varepsilon_{12}$. For the elastic case, N_1, N_2, M_1, and M_2 are written like this. Where k_1 and k_2 are the curvature part of the strain displacement relations.

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Strain-displacement Relation

$$5\left\{\begin{aligned} &\varepsilon_1 = \frac{1}{R_1}\left(u_{,\phi} - w\right), \quad \varepsilon_2 = \frac{1}{R_2}\left(u\cot\phi - w\right) \\ &\chi_1 = -\frac{1}{R_1}\left[\frac{1}{R_1}\left(u + w_{,\phi}\right)\right]_{,\phi} \quad \chi_2 = -\cot\phi\,\frac{1}{R_1 R_2}\left(u + w_{,\phi}\right) \\ &V_\phi = \frac{u}{R_1} + \frac{1}{R_1} w_{,\phi} \end{aligned}\right.$$

↳ angle of rotation.

12 eqn.

Here, for the axis-symmetric case:

$$\varepsilon_1 = \frac{1}{R_1}\left(u_{1,\phi} - w\right);\ \varepsilon_2 = \frac{1}{R_2}\left(u \cot\phi - w\right);$$

$$k_1 = -\frac{1}{R_1}\left[\frac{1}{R_1}\left(u + w,_{\phi}\right)\right] \text{ and } k_2 = -\cot\phi \frac{1}{R_1 R_2}\left(u + w,_{\phi}\right).$$

And a new variable V = $\dfrac{u_1}{R_1} + \dfrac{1}{R_1} w,_{\phi}$.

Now, we have now 12 equations.

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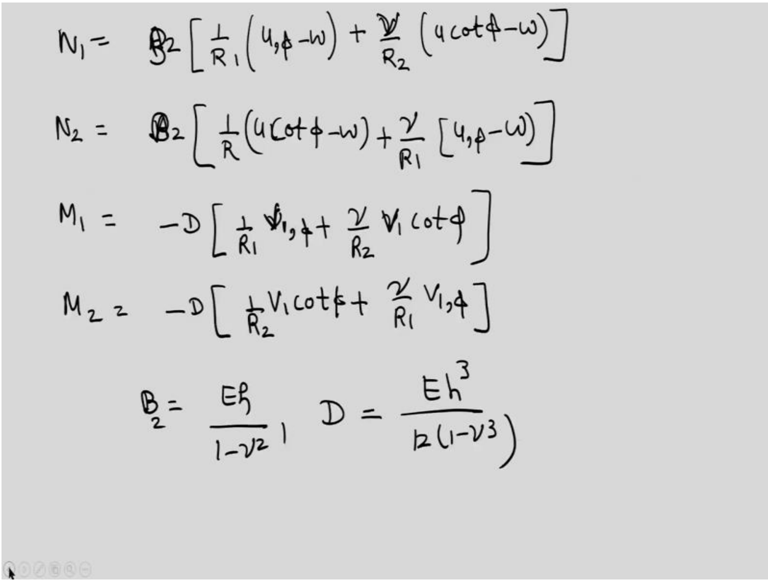

This further can be written in terms of actual components N_1, N_2, M_1 and M_2 by substituting all these things.

$$N_1 = B_2\left[\frac{1}{R_1}\left(u,_{\phi} - w\right) + \frac{\mu}{R_2}\left(u\cot\phi - w\right)\right],$$

$$N_2 = B_2\left[\frac{1}{R_2}\left(u\cot\phi - w\right) + \frac{\mu}{R_1}\left(u,_{\phi} - w\right)\right],$$

$$M_1 = -D\left[\frac{1}{R_1}V_{1,\phi} + \frac{\mu}{R_2}V_1\cot\phi\right] \text{, and}$$

$$M_2 = -D\left[\frac{1}{R_2}V_1 \cot\phi + \frac{\mu}{R_1}V_{1,\phi}\right].$$

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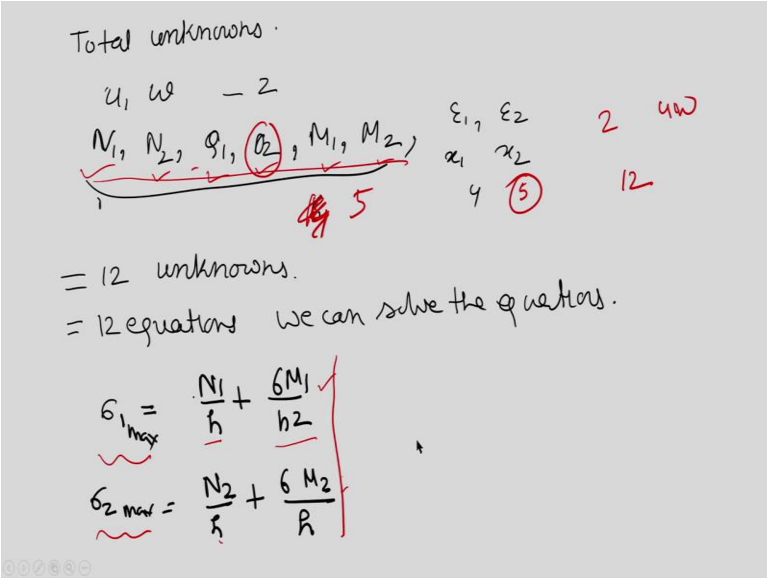

Now, we have two displacements u and w, five stress resultants N_1, N_2, M_1, M_2, Q_1, and Q_2. We have taken Q_2 = 0, therefore, we have 5 strains and 12 equations. We can solve a set of equations. In the 5th week, I will explain that if you substitute these components N_1, N_2, M_1 and M_2 into the basic governing equations.

These three governing equations can be expressed in terms of primary displacement variables u, w, and V. These will be expressed like that. Three equations can be solved as partial differential equations. one solution can be obtained if it is based on the boundary conditions. The solutions can be expressed in terms of a trigonometry series or power series or some other kind of a series and the solution is obtained.

From those solutions:

$$\sigma_{1\max} = \frac{N_1}{h} + \frac{6M_1}{h_2}$$

$$\sigma_{2\max} = \frac{N_2}{h} + \frac{6M_2}{h} \quad \sigma_{1\max}$$

In this way, the stresses are obtained.

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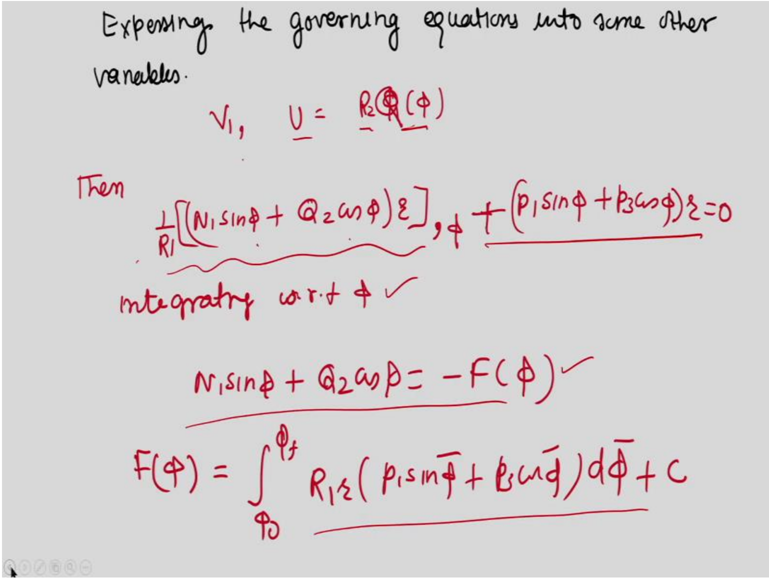

We are assuming some other variables let us say V_1 and U = $R_2 Q(\phi)$.

Here Q_1 = 0, substituting this variable into the equations, ultimately, these three equations will be converted into one equation:

$$\frac{1}{R_1}\left[\left(N_1 \sin\phi + Q_2 \cos\phi\right) r\right]_{,\phi} + \left(p_1 \sin\phi + p_3 \cos\phi\right) r = 0 .$$

$$N_1 \sin\phi + Q_2 \cos\phi = -F(\phi)$$

If we integrate with respect to ϕ, that leads to this equation

$$F(\phi) = \int_{Q_0}^{Q_f} R_1 r \left(p_1 \sin\bar{\phi} + p_3 \cos\bar{\phi}\right) d\bar{\phi} + C ,$$

Where F is the loading parameter.

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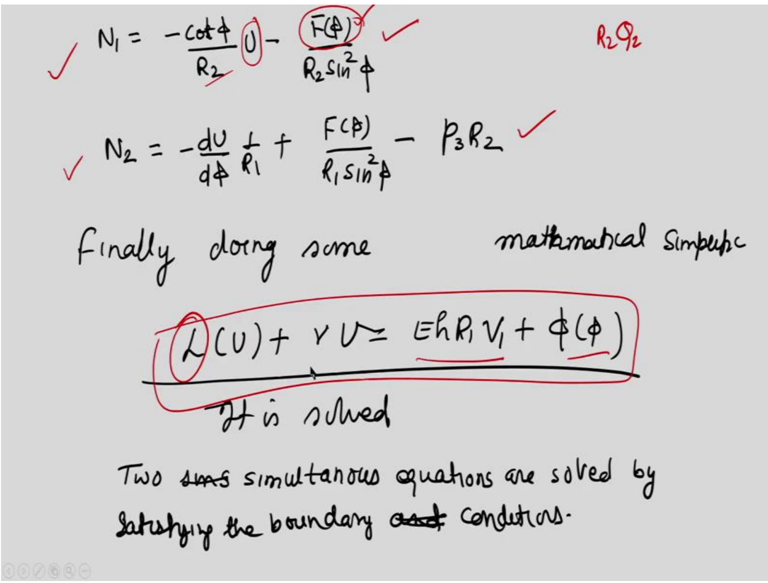

And from here, N_1 and N_2 can be written like this:

$$N_1 = -\frac{\cot\phi}{R_2}U - \frac{F(\phi)}{R_2 \sin^2\phi} \text{ and } N_2 = \frac{du}{d\phi}\frac{1}{R_1} + \frac{F(\phi)}{R_1 \sin^2\phi} - p_3 R_2 .$$

Finally, doing some mathematical simplifications, N_1 and N_2 will lead to two parameters V and U. U is a 2 by 2 matrix. Simultaneously these two equations will look like this:

$$L(U) + vU = EhR_1V_1 + \phi(\phi).$$

This is the value and this L is the differential operator, we can solve such kind of system.

The purpose of describing these things is if we get the N_1 and N_2, by substituting the value of the radius of curvature R_2, the loading parameter $F(\phi)$, and the value of U:

$$U = R_2 Q_2 .$$

We can get the expression of N_1 stress resultant in the first direction and the expression of stress resultant in the second direction N_2.

Similarly, once you can solve these stress resultants U and V, then by using all those things, we can get all the moments and strains.

With this, the basic methodology is explained here. But when you are going to solve a problem, depending upon the boundary conditions $F(\phi)$ will be a different loading parameter, the rest of the equations will remain the same. In week-05, lecture-01, I shall explain the complete formulation from the scratch for a cylindrical infinite or finite shell, their governing equations, and the solution techniques.

In these lectures, the basic equations in terms of stress resultants, there are some problems in the shell theories that can be directly solved using the same stress resultant equations. But in general, if we can convert into a primary displacement form this that is the more general form, we can solve all kind of a problem.

The general approach is to convert into a primary displacement form, and then get the solution for that. After that, we can get the stress resultants. With this, I would like to end this lecture here.

Thank you very much.