Theory of Composite Shells

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Week - 05

Lecture - 02

Basics of MATLAB coding

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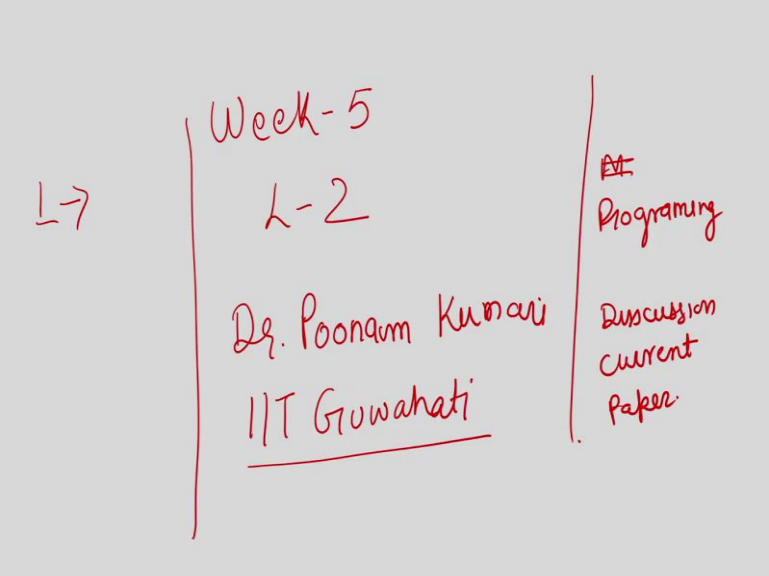

Dear learners welcome to week 05, lecture- 2. Till now, I covered the basic definition of differential geometry required for developing the shell equations. Then, I developed a partial differential set of governing equations based on first-order shear deformation theory.

And in the 4th week, I derived the governing equations for the special cases, like, cylindrical shell, spherical shell, circular plate, and so on.

In the 1st lecture of this week, I gave the basic guidelines to derive the governing equations for an infinite shell panel and the solution for a simply supported case. In today's lecture, first, I will explain to understand the recent research articles with this background.

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Engineering Structures

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A semi-analytical analysis of strength and critical buckling behavior of underwater ring-stiffened cylindrical shells

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ABSTRACT

In this paper, strength and buckling behavior analysis of a ring-stiffened cylindrical shell for the sightseeing submersible subjected to mechanical and thermal loads are studied. Based on the Donnell's assumption and classical shell theory, the governing equations of strength analysis for the ring-stiffened cylindrical shell are established, and the problems are solved by finite difference method (FDM) and Newmark-β method. Meanwhile, the governing equations of buckling behavior for the ring-stiffened cylindrical shell are set up in view of the energy method, which are resolved by the Ritz method. The research shows that the external load has the most significant influence on the strength of ring-stiffened cylindrical shell. Moreover, compared with other factors, the external load has a great influence on its strength, and the effect of shell thickness on the buckling load of

This is the recent paper having an article on a semi-analytical analysis of strength and critical buckling behavior of underwater ring stiffened cylindrical shells.

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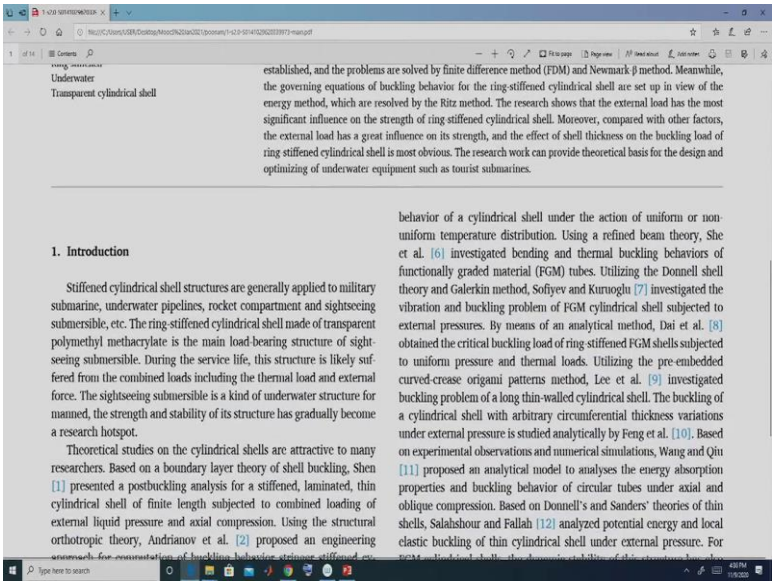

Underwater
Transparent cylindrical shell

established, and the problems are solved by finite difference method (FDM) and Newmark-β method. Meanwhile, the governing equations of buckling behavior for the ring-stiffened cylindrical shell are set up in view of the energy method, which are resolved by the Ritz method. The research shows that the external load has the most significant influence on the strength of ring-stiffened cylindrical shell. Moreover, compared with other factors, the external load has a great influence on its strength, and the effect of shell thickness on the buckling load of ring stiffened cylindrical shell is most obvious. The research work can provide theoretical basis for the design and optimizing of underwater equipment such as tourist submarines.

1. Introduction

Stiffened cylindrical shell structures are generally applied to military submarine, underwater pipelines, rocket compartment and sightseeing submersible, etc. The ring-stiffened cylindrical shell made of transparent polymethyl methacrylate is the main load-bearing structure of sight-seeing submersible. During the service life, this structure is likely suffered from the combined loads including the thermal load and external force. The sightseeing submersible is a kind of underwater structure for manned, the strength and stability of its structure has gradually become a research hotspot.

Theoretical studies on the cylindrical shells are attractive to many researchers. Based on a boundary layer theory of shell buckling, Shen [1] presented a postbuckling analysis for a stiffened, laminated, thin cylindrical shell of finite length subjected to combined loading of external liquid pressure and axial compression. Using the structural orthotropic theory, Andrianov et al. [2] proposed an engineering

behavior of a cylindrical shell under the action of uniform or non-uniform temperature distribution. Using a refined beam theory, She et al. [6] investigated bending and thermal buckling behaviors of functionally graded material (FGM) tubes. Utilizing the Donnell shell theory and Galerkin method, Sofiyev and Kuruoglu [7] investigated the vibration and buckling problem of FGM cylindrical shell subjected to external pressures. By means of an analytical method, Dai et al. [8] obtained the critical buckling load of ring-stiffened FGM shells subjected to uniform pressure and thermal loads. Utilizing the pre-embedded curved-crease origami patterns method, Lee et al. [9] investigated buckling problem of a long thin-walled cylindrical shell. The buckling of a cylindrical shell with arbitrary circumferential thickness variations under external pressure is studied analytically by Feng et al. [10]. Based on experimental observations and numerical simulations, Wang and Qiu [11] proposed an analytical model to analyses the energy absorption properties and buckling behavior of circular tubes under axial and oblique compression. Based on Donnell's and Sanders' theories of thin shells, Salahshour and Fallah [12] analyzed potential energy and local elastic buckling of thin cylindrical shell under external pressure. For

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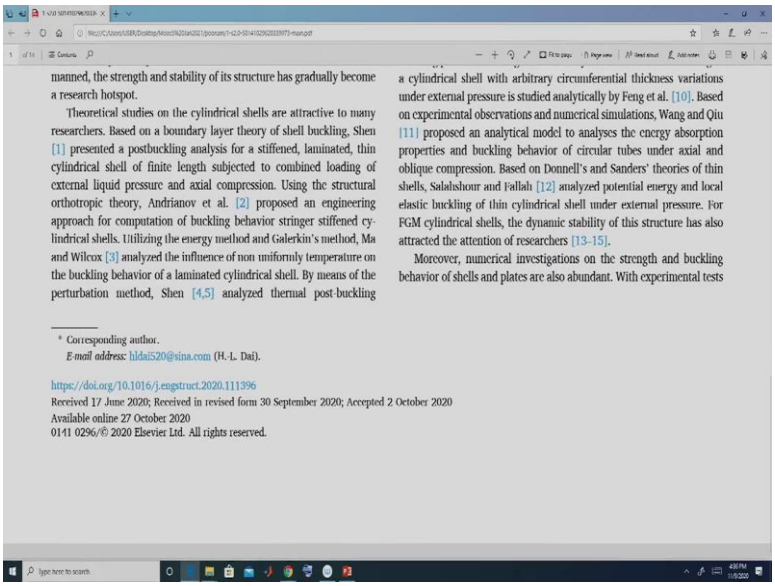

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Moreover, numerical investigations on the strength and buckling behavior of shells and plates are also abundant. With experimental tests

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I randomly chose this paper to make to understand whatever I explained to you.

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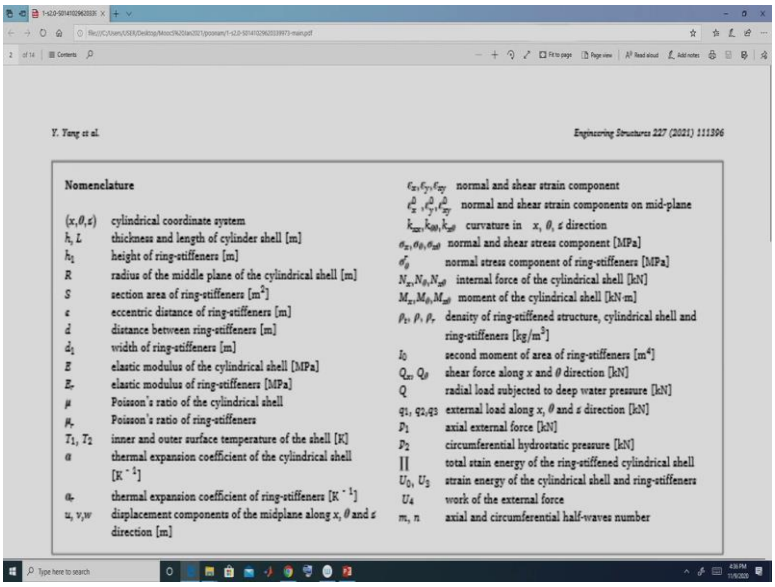

Nomenclature

(x,θ,z)	cylindrical coordinate system
h, L	thickness and length of cylinder shell [m]
h_2	height of ring-stiffeners [m]
R	radius of the middle plane of the cylindrical shell [m]
S	section area of ring-stiffeners [m^2]
e	eccentric distance of ring-stiffeners [m]
d	distance between ring-stiffeners [m]
d_2	width of ring-stiffeners [m]
E	elastic modulus of the cylindrical shell [MPa]
E_r	elastic modulus of ring-stiffeners [MPa]
μ	Poisson's ratio of the cylindrical shell
μ_r	Poisson's ratio of ring-stiffeners
T_1, T_2	inner and outer surface temperature of the shell [K]
α	thermal expansion coefficient of the cylindrical shell [K^{-1}]
α_r	thermal expansion coefficient of ring-stiffeners [K^{-1}]
u, v,w	displacement components of the midplane along x, θ and z direction [m]
$\varepsilon_x,\varepsilon_y,\varepsilon_{xy}$	normal and shear strain component
$\varepsilon_x^0,\varepsilon_y^0,\varepsilon_{xy}^0$	normal and shear strain components on mid-plane
$k_{xx},k_{\theta\theta},k_{x\theta}$	curvature in x, θ, z direction
$\sigma_x,\sigma_\theta,\sigma_{x\theta}$	normal and shear stress component [MPa]
σ_θ^r	normal stress component of ring-stiffeners [MPa]
$N_x,N_\theta,N_{x\theta}$	internal force of the cylindrical shell [kN]
$M_x,M_\theta,M_{x\theta}$	moment of the cylindrical shell [kN·m]
ρ_2, ρ, ρ_r	density of ring-stiffened structure, cylindrical shell and ring-stiffeners [kg/m^3]
I_0	second moment of area of ring-stiffeners [m^4]
Q_x, Q_θ	shear force along x and θ direction [kN]
Q	radial load subjected to deep water pressure [kN]
q_1, q_2,q_3	external load along x, θ and z direction [kN]
P_1	axial external force [kN]
P_2	circumferential hydrostatic pressure [kN]
Π	total stain energy of the ring-stiffened cylindrical shell
U_0, U_2	strain energy of the cylindrical shell and ring-stiffeners
U_4	work of the external force
m, n	axial and circumferential half-waves number

Like in this paper.

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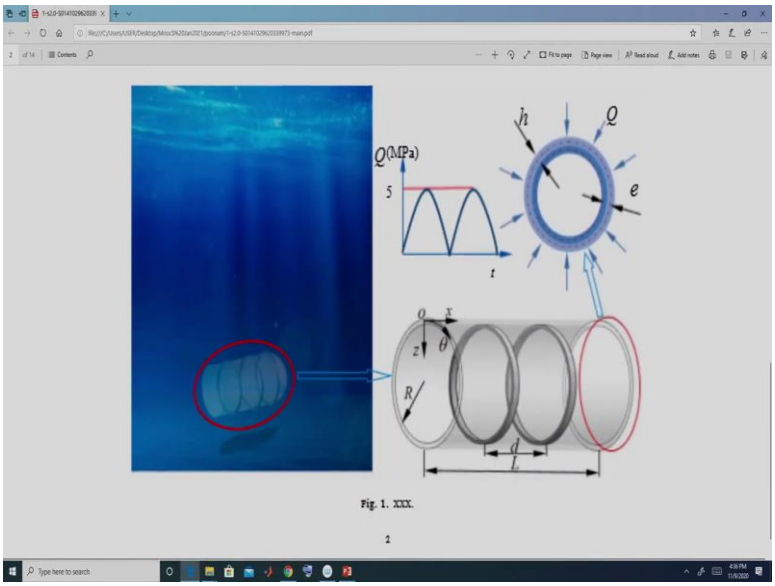

A cylindrical shell in which ring stiffeners are used as underwater pressure. We studied that outside of a tube, there is pressure over there. These are the recent articles.

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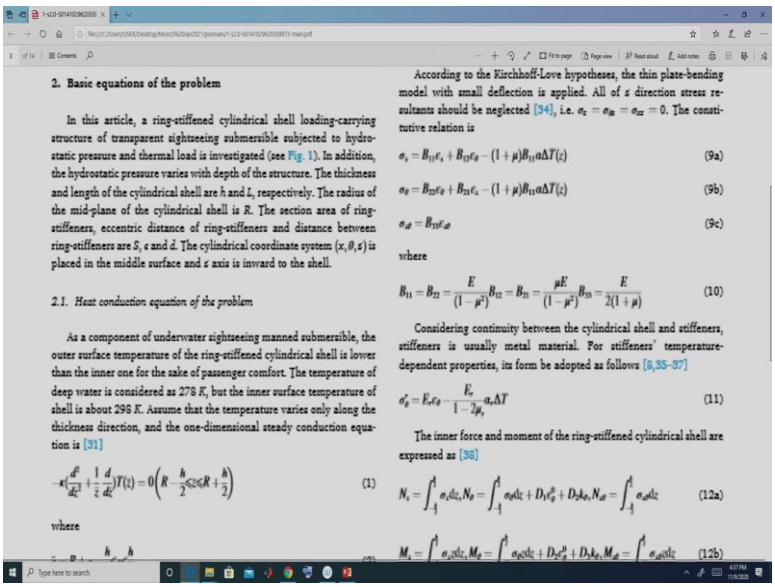

2. Basic equations of the problem

In this article, a ring-stiffened cylindrical shell loading-carrying structure of transparent sightseeing submersible subjected to hydrostatic pressure and thermal load is investigated (see Fig. 1). In addition, the hydrostatic pressure varies with depth of the structure. The thickness and length of the cylindrical shell are h and L, respectively. The radius of the mid-plane of the cylindrical shell is R. The section area of ring-stiffeners, eccentric distance of ring-stiffeners and distance between ring-stiffeners are S, e and d. The cylindrical coordinate system (x, θ, z) is placed in the middle surface and z axis is inward to the shell.

2.1. Heat conduction equation of the problem

As a component of underwater sightseeing manned submersible, the outer surface temperature of the ring-stiffened cylindrical shell is lower than the inner one for the sake of passenger comfort. The temperature of deep water is considered as 278 K, but the inner surface temperature of shell is about 296 K. Assume that the temperature varies only along the thickness direction, and the one-dimensional steady conduction equation is [31]

$$-\kappa\left(\frac{d^2}{dz^2}+\frac{1}{z}\frac{d}{dz}\right)T(z)=0\left(R-\frac{h}{2}\leqslant z\leqslant R+\frac{h}{2}\right) \quad (1)$$

where

According to the Kirchhoff-Love hypotheses, the thin plate-bending model with small deflection is applied. All of z direction stress resultants should be neglected [34], i.e. $\sigma_z = \sigma_{\theta z} = \sigma_{xz} = 0$. The constitutive relation is

$$\sigma_x = B_{11}\varepsilon_x + B_{12}\varepsilon_\theta - (1+\mu)B_{11}\alpha\Delta T(z) \quad (9a)$$

$$\sigma_\theta = B_{22}\varepsilon_\theta + B_{21}\varepsilon_x - (1+\mu)B_{11}\alpha\Delta T(z) \quad (9b)$$

$$\sigma_{x\theta} = B_{33}\varepsilon_{x\theta} \quad (9c)$$

where

$$B_{11} = B_{22} = \frac{E}{(1-\mu^2)}, B_{12} = B_{21} = \frac{\mu E}{(1-\mu^2)}, B_{33} = \frac{E}{2(1+\mu)} \quad (10)$$

Considering continuity between the cylindrical shell and stiffeners, stiffeners is usually metal material. For stiffeners' temperature-dependent properties, its form be adopted as follows [8,35–37]

$$\sigma'_\theta = E_r\varepsilon_\theta - \frac{E_r}{1-2\mu_r}\alpha_r\Delta T \quad (11)$$

The inner force and moment of the ring-stiffened cylindrical shell are expressed as [38]

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz, N_\theta = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\theta dz + D_1\varepsilon^0_\theta + D_3 k_\theta, N_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x\theta} dz \quad (12a)$$

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz, M_\theta = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\theta z dz + D_2\varepsilon^0_\theta + D_3 k_\theta, M_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x\theta} z dz \quad (12b)$$

We aim to have the basic governing equations. Initially, they gave the heat conduction equation which is in polar cylindrical coordinate.

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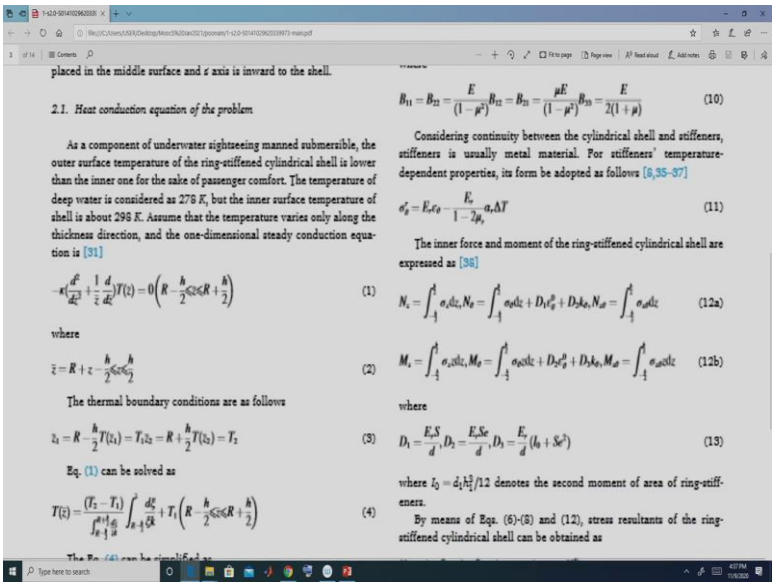

placed in the middle surface and z axis is inward to the shell.

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$$-\kappa\left(\frac{d^2}{d\bar{z}^2}+\frac{1}{\bar{z}}\frac{d}{d\bar{z}}\right)T(\bar{z})=0\left(R-\frac{h}{2}\leqslant\bar{z}\leqslant R+\frac{h}{2}\right) \tag{1}$$

where

$$\bar{z}=R+z-\frac{h}{2}\leqslant z\leqslant\frac{h}{2} \tag{2}$$

The thermal boundary conditions are as follows

$$\bar{z}_1=R-\frac{h}{2}T(\bar{z}_1)=T_1\bar{z}_2=R+\frac{h}{2}T(\bar{z}_2)=T_2 \tag{3}$$

Eq. (1) can be solved as

$$T(\bar{z})=\frac{(T_2-T_1)}{\int_{R-\frac{h}{2}}^{R+\frac{h}{2}}\frac{d\bar{z}}{\bar{z}}}\int_{R-\frac{h}{2}}^{\bar{z}}\frac{d\bar{z}}{\bar{z}}+T_1\left(R-\frac{h}{2}\leqslant\bar{z}\leqslant R+\frac{h}{2}\right) \tag{4}$$

$$B_{11}=B_{22}=\frac{E}{(1-\mu^2)}B_{12}=B_{21}=\frac{\mu E}{(1-\mu^2)}B_{33}=\frac{E}{2(1+\mu)} \tag{10}$$

Considering continuity between the cylindrical shell and stiffeners, stiffeners is usually metal material. For stiffeners' temperature-dependent properties, its form be adopted as follows [8,35–37]

$$\sigma_\theta'=E_s\varepsilon_\theta-\frac{E_s}{1-2\mu_s}\alpha_s\Delta T \tag{11}$$

The inner force and moment of the ring-stiffened cylindrical shell are expressed as [36]

$$N_s=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_s dz, N_\theta=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta dz+D_1\varepsilon_\theta^0+D_2k_\theta, N_{s\theta}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{s\theta}dz \tag{12a}$$

$$M_s=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_s zdz, M_\theta=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta zdz+D_2\varepsilon_\theta^0+D_3k_\theta, M_{s\theta}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{s\theta}zdz \tag{12b}$$

where

$$D_1=\frac{E_sS}{d}, D_2=\frac{E_sSe}{d}, D_3=\frac{E_s}{d}(I_0+Se^2) \tag{13}$$

where $I_0=\dot{a}_1h_1^3/12$ denotes the second moment of area of ring-stiffeners.

By means of Eqs. (6)-(8) and (12), stress resultants of the ring-stiffened cylindrical shell can be obtained as

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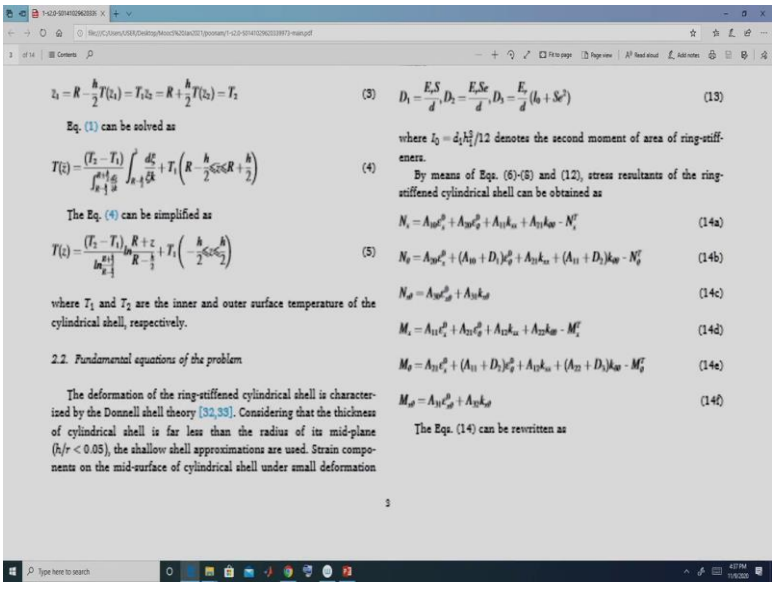

$$\bar{z}_1=R-\frac{h}{2}T(\bar{z}_1)=T_1\bar{z}_2=R+\frac{h}{2}T(\bar{z}_2)=T_2 \tag{3}$$

Eq. (1) can be solved as

$$T(\bar{z})=\frac{(T_2-T_1)}{\int_{R-\frac{h}{2}}^{R+\frac{h}{2}}\frac{d\bar{z}}{\bar{z}}}\int_{R-\frac{h}{2}}^{\bar{z}}\frac{d\bar{z}}{\bar{z}}+T_1\left(R-\frac{h}{2}\leqslant\bar{z}\leqslant R+\frac{h}{2}\right) \tag{4}$$

The Eq. (4) can be simplified as

$$T(\bar{z})=\frac{(T_2-T_1)}{\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}}}\ln\frac{R+z}{R-\frac{h}{2}}+T_1\left(-\frac{h}{2}\leqslant z\leqslant\frac{h}{2}\right) \tag{5}$$

where T_1 and T_2 are the inner and outer surface temperature of the cylindrical shell, respectively.

2.2. Fundamental equations of the problem

The deformation of the ring-stiffened cylindrical shell is characterized by the Donnell shell theory [32,33]. Considering that the thickness of cylindrical shell is far less than the radius of its mid-plane ($h/r<0.05$), the shallow shell approximations are used. Strain components on the mid-surface of cylindrical shell under small deformation

$$D_1=\frac{E_sS}{d}, D_2=\frac{E_sSe}{d}, D_3=\frac{E_s}{d}(I_0+Se^2) \tag{13}$$

where $I_0=\dot{a}_1h_1^3/12$ denotes the second moment of area of ring-stiffeners.

By means of Eqs. (6)-(8) and (12), stress resultants of the ring-stiffened cylindrical shell can be obtained as

$$N_s=A_{11}\varepsilon_s^0+A_{21}\varepsilon_\theta^0+A_{12}k_s+A_{22}k_\theta-N_s^T \tag{14a}$$

$$N_\theta=A_{21}\varepsilon_s^0+(A_{11}+D_1)\varepsilon_\theta^0+A_{12}k_s+(A_{12}+D_2)k_\theta-N_\theta^T \tag{14b}$$

$$N_{s\theta}=A_{66}\varepsilon_{s\theta}^0+A_{33}k_{s\theta} \tag{14c}$$

$$M_s=A_{11}\varepsilon_s^0+A_{21}\varepsilon_\theta^0+A_{12}k_s+A_{22}k_\theta-M_s^T \tag{14d}$$

$$M_\theta=A_{21}\varepsilon_s^0+(A_{11}+D_2)\varepsilon_\theta^0+A_{12}k_s+(A_{22}+D_3)k_\theta-M_\theta^T \tag{14e}$$

$$M_{s\theta}=A_{33}\varepsilon_{s\theta}^0+A_{33}k_{s\theta} \tag{14f}$$

The Eqs. (14) can be rewritten as

I explained to you section 2.2: fundamental equations of problem.

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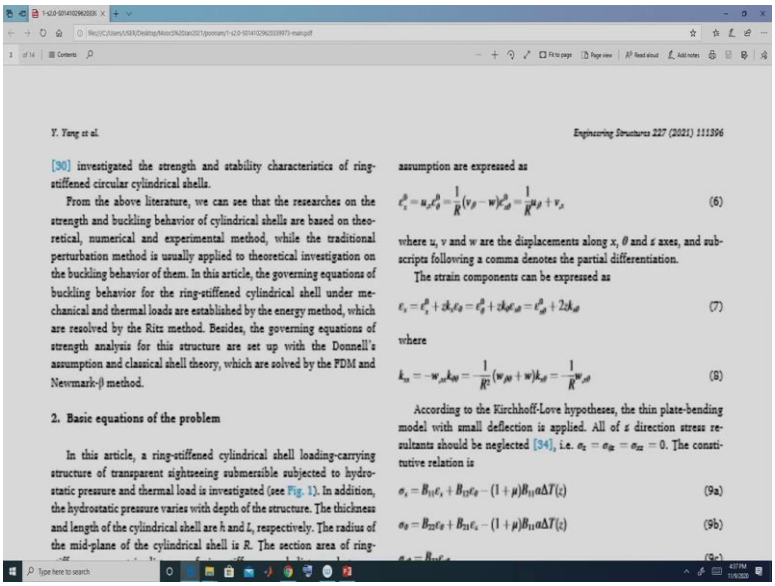

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[30] investigated the strength and stability characteristics of ring-stiffened circular cylindrical shells.

From the above literature, we can see that the researches on the strength and buckling behavior of cylindrical shells are based on theoretical, numerical and experimental method, while the traditional perturbation method is usually applied to theoretical investigation on the buckling behavior of them. In this article, the governing equations of buckling behavior for the ring-stiffened cylindrical shell under mechanical and thermal loads are established by the energy method, which are resolved by the Ritz method. Besides, the governing equations of strength analysis for this structure are set up with the Donnell's assumption and classical shell theory, which are solved by the FDM and Newmark-β method.

2. Basic equations of the problem

In this article, a ring-stiffened cylindrical shell loading-carrying structure of transparent sightseeing submersible subjected to hydrostatic pressure and thermal load is investigated (see Fig. 1). In addition, the hydrostatic pressure varies with depth of the structure. The thickness and length of the cylindrical shell are h and L, respectively. The radius of the mid-plane of the cylindrical shell is R. The section area of ring-

assumption are expressed as

$$\varepsilon_x^0 = u_{,x}\varepsilon_\theta^0 = \frac{1}{R}(v_{,\theta} - w)\varepsilon_{x\theta}^0 = \frac{1}{R}u_{,\theta} + v_{,x} \quad (6)$$

where u, v and w are the displacements along x, θ and z axes, and subscripts following a comma denotes the partial differentiation.

The strain components can be expressed as

$$\varepsilon_x = \varepsilon_x^0 + zk_x\varepsilon_\theta = \varepsilon_\theta^0 + zk_\theta\varepsilon_{x\theta} = \varepsilon_{x\theta}^0 + 2zk_{x\theta} \quad (7)$$

where

$$k_{xx} = -w_{,xx}k_{\theta\theta} = -\frac{1}{R^2}(w_{,\theta\theta} + w)k_{x\theta} = -\frac{1}{R}w_{,x\theta} \quad (8)$$

According to the Kirchhoff-Love hypotheses, the thin plate-bending model with small deflection is applied. All of z direction stress resultants should be neglected [34], i.e. $\sigma_z = \sigma_{\theta z} = \sigma_{xz} = 0$. The constitutive relation is

$$\sigma_x = \bar{B}_{11}\varepsilon_x + \bar{B}_{12}\varepsilon_\theta - (1+\mu)\bar{B}_{11}\alpha\Delta T(z) \quad (9a)$$

$$\sigma_\theta = \bar{B}_{22}\varepsilon_\theta + \bar{B}_{21}\varepsilon_x - (1+\mu)\bar{B}_{11}\alpha\Delta T(z) \quad (9b)$$

$$\sigma_{x\theta} = \bar{B}_{66}\varepsilon_{x\theta} \quad (9c)$$

Here, you can see that the strains ε_x, ε_θ, and $\varepsilon_{x\theta}$. In the previous lecture, I explained that ε_x was similar to ε_θ and $\varepsilon_{x\theta}$.

Because it is a complete shell, the derivative with respect to x is taken care. Now, using the displacement field. There we also use the Donnell shell theory which is a thick shell theory. Based on that displacement field, they obtained the strains and divide those strains into two parts. One is the membrane part and the other is the stretching part and then substituted back to σ_x, σ_θ, and $\sigma_{\theta\theta}$. We obtained an in-plain stress case and the definition of N_x, M_x all we have obtained in this paper.

You are now familiar with these approaches; therefore, you can understand at least in this paper the meaning of D_1, D_2, D_3 and so on. And then you see that N_x the constitutive relation for the stiffened cylindrical shell. A_{10} *and* A_{20} is corresponding to the membrane part then A_{11} *and* A_{21} is corresponding to the bending part.

Similarly, we have B_{11} *and* D_{11} . Depending upon the displacement field some more terms may come up. N_θ and $N_{x\theta}$, terms are coming.

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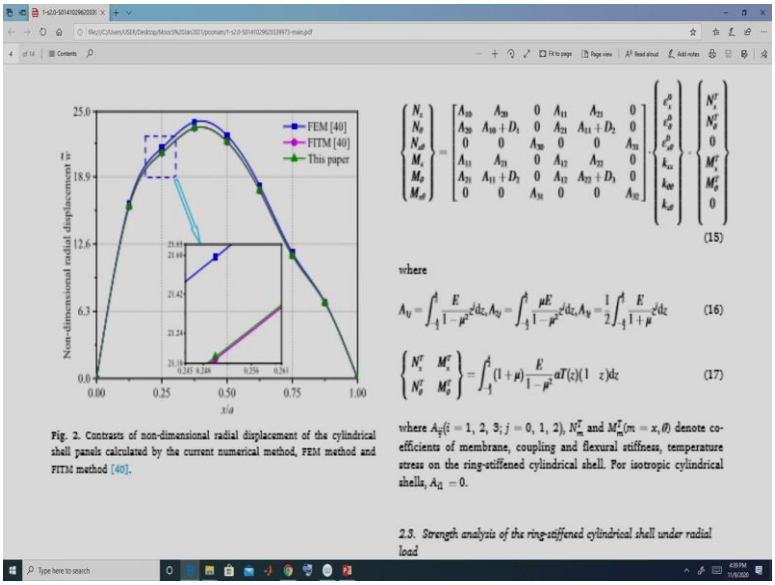

Fig. 2. Contrasts of non-dimensional radial displacement of the cylindrical shell panels calculated by the current numerical method, FEM method and FITM method [40].

where $A_{ij}(i = 1, 2, 3; j = 0, 1, 2)$, N_m^T and $M_m^T(m = x, \theta)$ denote coefficients of membrane, coupling and flexural stiffness, temperature stress on the ring-stiffened cylindrical shell. For isotropic cylindrical shells, $A_{11} = 0$.

And finally, the matrix of N_x and N_θ is represented like this. You can see the definition of A_{ij} . It is an isotropic shell, only E, young's modulus and μ , Poisson ratio is used to find.

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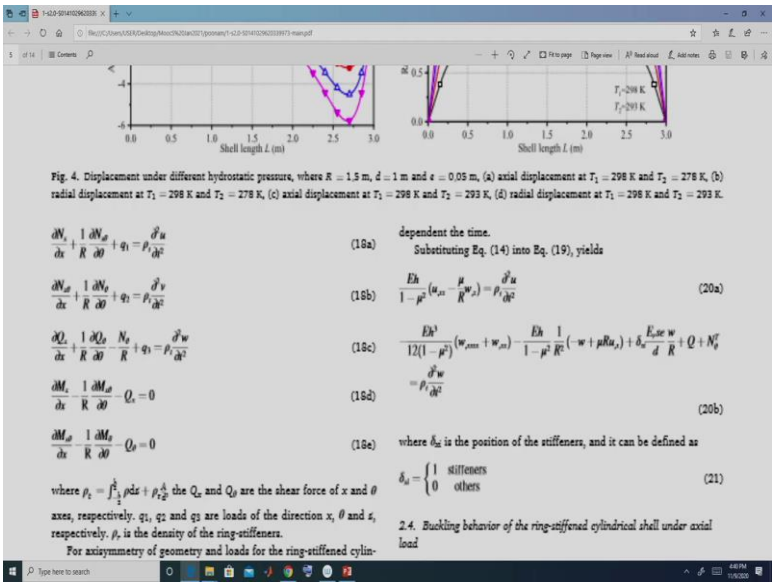

Fig. 4. Displacement under different hydrostatic pressure, where $R = 1.5$ m, $d = 1$ m and $\epsilon = 0.05$ m, (a) axial displacement at $T_1 = 298$ K and $T_2 = 278$ K, (b) radial displacement at $T_1 = 298$ K and $T_2 = 278$ K, (c) axial displacement at $T_1 = 298$ K and $T_2 = 293$ K, (d) radial displacement at $T_1 = 298$ K and $T_2 = 293$ K.

where $\rho_z = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz + \rho_z \frac{A_z}{ab}$, the Q_x and Q_θ are the shear force of x and θ axes, respectively. q_1, q_2 and q_3 are loads of the direction x, θ and z, respectively. ρ_z is the density of the ring-stiffeners.

For axisymmetry of geometry and loads for the ring-stiffened cylin-

where δ_{si} is the position of the stiffeners, and it can be defined as

You can see here, the 5 governing equations. It is a first-order shear deformation type

theory and you can see that the governing equations are similar. And these are the dynamic terms $M_x, M_{x\theta}, M_\theta, Q_1$ *and* Q_2. Whatever, I explained during the lectures. Now, I feel that anybody who is attending these lectures can understand the research articles.

The very important thing to work in any research area is first, to understand the existing literature, that what is given, and further moving with that. First, we have to understand the existing one and later on, we can apply our ideas.

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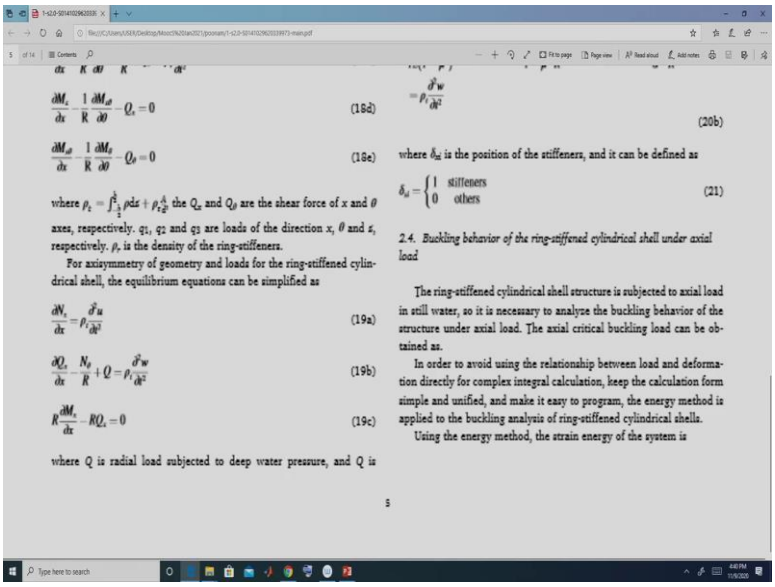

$$\frac{\partial M_x}{\partial x} - \frac{1}{R}\frac{\partial M_{x\theta}}{\partial \theta} - Q_x = 0 \tag{18d}$$

$$\frac{\partial M_{x\theta}}{\partial x} - \frac{1}{R}\frac{\partial M_\theta}{\partial \theta} - Q_\theta = 0 \tag{18e}$$

where $\rho_x = \int_{\frac{h}{2}}^{\frac{h}{2}} \rho dz + \rho_x \frac{A_x}{B_x}$ the Q_x and Q_θ are the shear force of x and θ axes, respectively. q_1, q_2 and q_3 are loads of the direction x, θ and z, respectively. ρ_r is the density of the ring-stiffeners.

For axisymmetry of geometry and loads for the ring-stiffened cylindrical shell, the equilibrium equations can be simplified as

$$\frac{\partial N_x}{\partial x} = \rho_r \frac{\partial^2 u}{\partial t^2} \tag{19a}$$

$$\frac{\partial Q_x}{\partial x} - \frac{N_\theta}{R} + Q = \rho_r \frac{\partial^2 w}{\partial t^2} \tag{19b}$$

$$R\frac{\partial M_x}{\partial x} - RQ_x = 0 \tag{19c}$$

where Q is radial load subjected to deep water pressure, and Q is

$$= \rho_r \frac{\partial^2 w}{\partial t^2} \tag{20b}$$

where δ_{ri} is the position of the stiffeners, and it can be defined as

$$\delta_{ri} = \begin{cases} 1 & \text{stiffeners} \\ 0 & \text{others} \end{cases} \tag{21}$$

2.4. Buckling behavior of the ring-stiffened cylindrical shell under axial load

The ring-stiffened cylindrical shell structure is subjected to axial load in still water, so it is necessary to analyze the buckling behavior of the structure under axial load. The axial critical buckling load can be obtained as.

In order to avoid using the relationship between load and deformation directly for complex integral calculation, keep the calculation form simple and unified, and make it easy to program, the energy method is applied to the buckling analysis of ring-stiffened cylindrical shells.

Using the energy method, the strain energy of the system is

5

We derived the five governing equations as shown in the article. They have now specialized axisymmetric geometry when the θ load is independent of θ. The five governing equations reduce into 3 equations and then they are solved for the buckling case.

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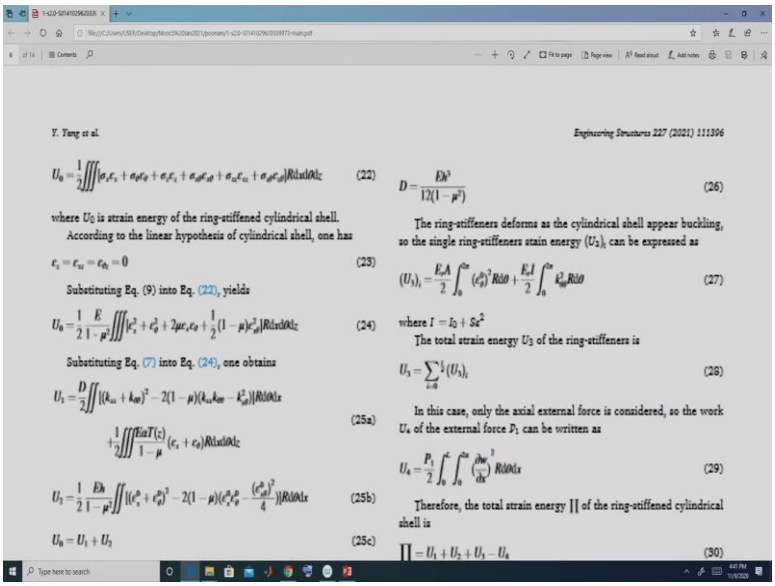
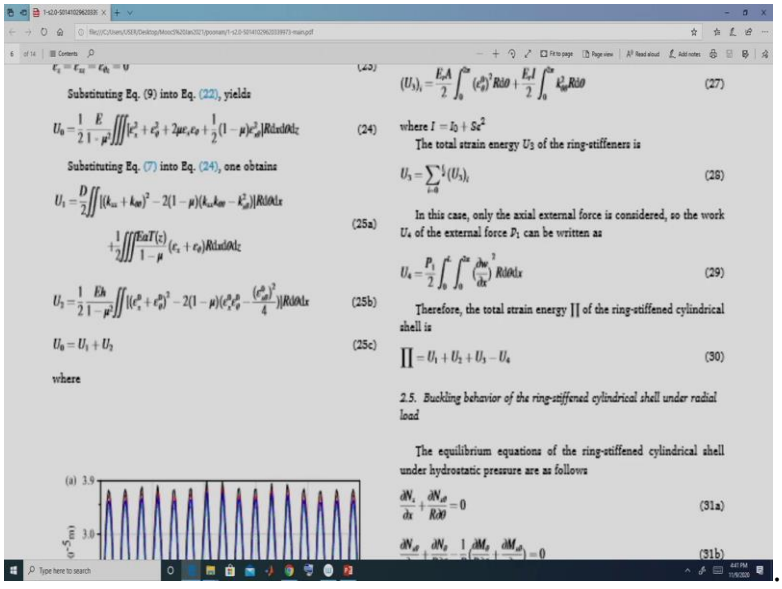


The purpose of explaining this is just to tell you that these are the recent research articles in the field of shell and one can understand that these are expressed like this. And you can see that the internal work done is expressed in the same way. We developed a very generalized form and it is the special case; cylindrical shell case.

(Refer Slide Time: 07:09)



Now, you can see that with the background of a doubly curved shell, you can understand the paper on the cylindrical shell, spherical shell, hyperbolic shell, conical shell, or any other shell. You can understand the state of art based on first-order shear deformation

theory and higher-order, the basic terms used, and to develop the governing equations.

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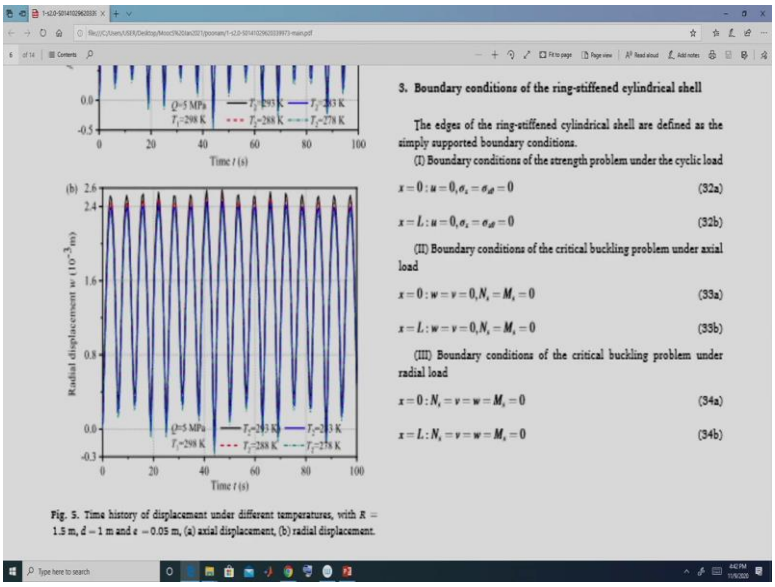

Boundary conditions are also shown here. You can see the boundary conditions in the cyclic and buckling case and. We have N_x, because only along the x-direction they can specify, along the θ direction the shell is closed. But when we talk about a finite shell panel then we have the boundary along the θ direction as well as x-direction.

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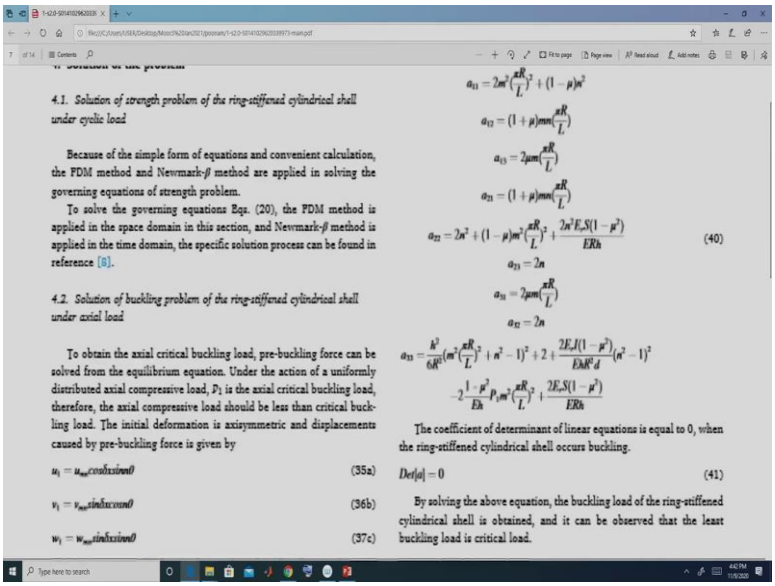

I would like to further go through this paper.

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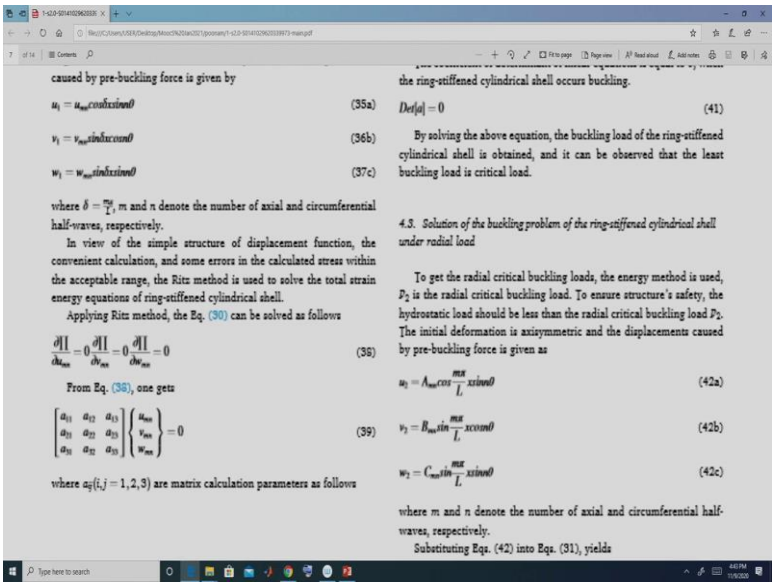

caused by pre-buckling force is given by

$$u_1 = u_{mn}\cos\delta x \sin n\theta \quad (35a)$$

$$v_1 = v_{mn}\sin\delta x \cos n\theta \quad (36b)$$

$$w_1 = w_{mn}\sin\delta x \sin n\theta \quad (37c)$$

where $\delta = \frac{m\pi}{L}$, m and n denote the number of axial and circumferential half-waves, respectively.

In view of the simple structure of displacement function, the convenient calculation, and some errors in the calculated stress within the acceptable range, the Ritz method is used to solve the total strain energy equations of ring-stiffened cylindrical shell.

Applying Ritz method, the Eq. (30) can be solved as follows

$$\frac{\partial \Pi}{\partial u_{mn}} = 0 \frac{\partial \Pi}{\partial v_{mn}} = 0 \frac{\partial \Pi}{\partial w_{mn}} = 0 \quad (38)$$

From Eq. (38), one gets

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \end{Bmatrix} = 0 \quad (39)$$

where $a_{ij}(i,j = 1,2,3)$ are matrix calculation parameters as follows

the ring-stiffened cylindrical shell occurs buckling.

$$Det|a| = 0 \quad (41)$$

By solving the above equation, the buckling load of the ring-stiffened cylindrical shell is obtained, and it can be observed that the least buckling load is critical load.

4.3. Solution of the buckling problem of the ring-stiffened cylindrical shell under radial load

To get the radial critical buckling loads, the energy method is used, P_2 is the radial critical buckling load. To ensure structure's safety, the hydrostatic load should be less than the radial critical buckling load P_2. The initial deformation is axisymmetric and the displacements caused by pre-buckling force is given as

$$u_2 = A_{mn}\cos\frac{mx}{L}x\sin n\theta \quad (42a)$$

$$v_2 = B_{mn}\sin\frac{mx}{L}x\cos n\theta \quad (42b)$$

$$w_2 = C_{mn}\sin\frac{mx}{L}x\sin n\theta \quad (42c)$$

where m and n denote the number of axial and circumferential half-waves, respectively.

Substituting Eqs. (42) into Eqs. (31), yields

The same way we represented the displacement field either in $\sin\theta$ or $\cos\theta$. So, Finally, substituting into the equation; A_{11} *and* A_{22} are coming as a function of M_{nn}.

The techniques may be different. Because it is a free vibration case there is no transverse loading. In that case, the right-hand side = 0. The eigenvalue problem is solved and frequencies are obtained for the first case and when you talk about buckling then the compressive load is applied.

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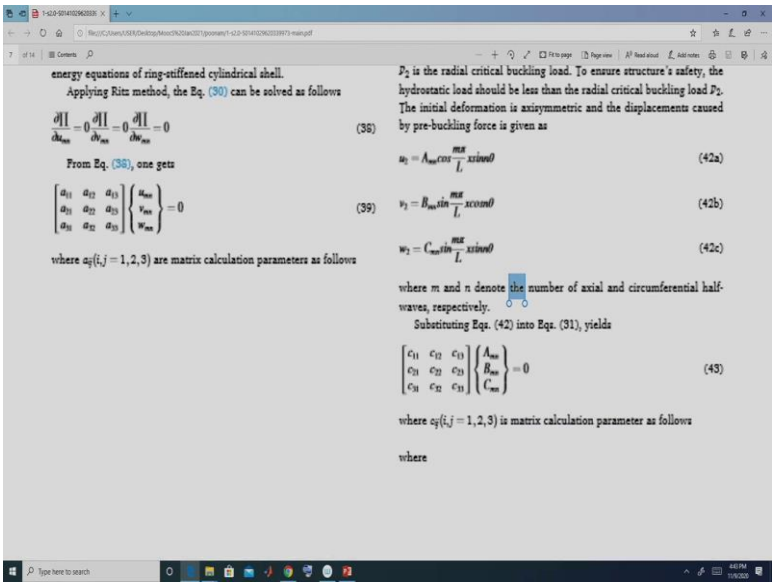

energy equations of ring-stiffened cylindrical shell.

Applying Ritz method, the Eq. (30) can be solved as follows

$$\frac{\partial \Pi}{\partial u_{mn}} = 0 \frac{\partial \Pi}{\partial v_{mn}} = 0 \frac{\partial \Pi}{\partial w_{mn}} = 0 \quad (38)$$

From Eq. (38), one gets

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \end{Bmatrix} = 0 \quad (39)$$

where $a_{ij}(i,j = 1,2,3)$ are matrix calculation parameters as follows

P_2 is the radial critical buckling load. To ensure structure's safety, the hydrostatic load should be less than the radial critical buckling load P_2. The initial deformation is axisymmetric and the displacements caused by pre-buckling force is given as

$$u_2 = A_{mn}\cos\frac{mx}{L}x\sin n\theta \quad (42a)$$

$$v_2 = B_{mn}\sin\frac{mx}{L}x\cos n\theta \quad (42b)$$

$$w_2 = C_{mn}\sin\frac{mx}{L}x\sin n\theta \quad (42c)$$

where m and n denote the number of axial and circumferential half-waves, respectively.

Substituting Eqs. (42) into Eqs. (31), yields

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{Bmatrix} = 0 \quad (43)$$

where $c_{ij}(i,j = 1,2,3)$ is matrix calculation parameter as follows

where

The techniques of solution are also given in the paper. Once, you understand the formulation you can understand the complete paper and you can proceed in your research.

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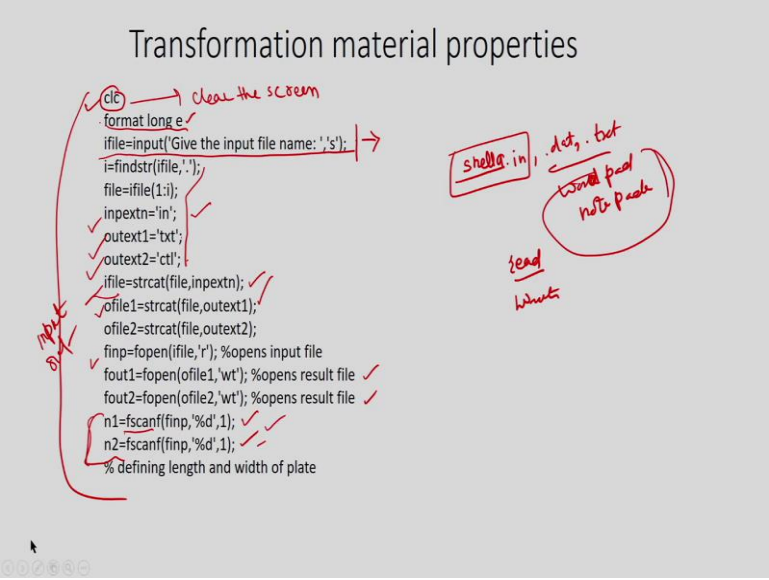

In this lecture, I briefly explained that how to understand a paper.

Now you may or may not be aware of developing a programme. What are the basic steps to write a code on a cylindrical shell? First, I will explain the basic steps then the transformation of material property which is very important.

Sometimes, we use these material properties modeling in CI software like abacus or any numerical software.

We need material properties in our hands or sometimes we do some experimentation or even we write our code. Inside that code, we include the subroutine that can we get the transformation of material property.

First, I will explain with the help of a MATLAB file. I have copy-pasted all MATLAB commands in these PPT slides. The very first command is clc which means clear the screen. Some of you may be master's students, early Ph.D. students, or research scholars.

So, you may have a background of coding you may not have the background of coding, but I would like to feel that for the sake of completeness when I explain the theory. So, I will briefly explain that how do we write a code and what are the basic steps.

You may or may not have a background in coding. But for the sake of completeness, I will briefly explain the basic steps to write a code. In structural engineering, specifically, when we do continuum mechanics, the command cls is used before writing any code.

Specifically in MATLAB, if we do not clear the screen, the other variables may be messed up.

The next command is format or format long e. Generally, for structural analysis, we use the format long e command. The purpose is that sometimes in the calculation we want the convergence checking, let us say, 10^{-9}, if it is not in long e format, then we cannot work with this type of convergence analysis. 10^{-2} and 10^{-3} is fine, but when you talk about more than 10^{-8}, then it is required. Depending upon the issues.

Generally, when you write a code for a plate or a shell, for any kind of analysis whether it is a static analysis or free vibration analysis, we should use the format long e command, because it is not just a simple calculation.

Sometimes, we have to express the exponential of something and it comes in very long numbers. Then, is input command: the ideal way; we can give input through many ways. When I give a programme to write to our master students at IIT Guwahati or to the research scholars, they generally write in a very hard form, they give the input just like e and mu and fix and then write a coding of this.

That is the one way that in the programme itself you give the material property and then process it. But that is not the soft form because every time you have to change in the code itself and if you do, there will be chance that some error may come.

We should write a programme in such a way that we should have minimum interference with outside information.

We should not work every time inside the programme, we should give the input through a file that is the ideal way of working in this field. Input data means the geometric parameters which mean the material properties in the layer-wise stacking sequence.

We should try to give through an input. This option is available in all kinds of languages whether you talk about MATLAB, Fortran, or C++. Whenever you do programming, please give input through a file. The idea behind that is as the information of your input data is given by you therefore, you can see any error.

We have to give the file extension, file name, and all these things. These are the standard formats that input file extension should be in ‘in’, whatever the file name. Let us say, we want to give a file name ‘shell a’, I will give a file name of a shella.in. Its extension will be ‘.in’. The other point is that if we give a ‘.in’, ‘.dat’ or ‘.txt’, all these files can be worked on in a word pad or on a notepad.

You can open it in ms word these will be in a zigzag format, which I have experienced. If you open in a notepad or in a WordPad, then you can work with those files easily. These formats are supported by both notepad and word pad ‘.in’, ‘.txt’, ‘.ctl’, you can open in these.

Input extension (inpextn) and output extension (outext) and even you can change these also. ifile=strcat(file,inpextn);

ofile1=strcat(file,outext1);

ofile2=strcat(file,outext2);.

Here, ifile means input file, ofile1 means output file number 1, and ofile2 means output file number 2.

Finp=fopen(ifile,’r’); %opens input file, this command is used to open an input file.

The program will ask to give the input file name, you have to give the input file name, and then it will open that file.

Fout1=fopen(ofile1,’wt’); %opens result file, Fout2=fopen(ofile2,’wt’); %opens result file, it opens the blank file of outputs. In the input file there may be some numbers that n1 or n2

n1=fsanf(finp,’%d’,1); and n2=fsanf(finp,’%d’,2);

In MATLAB the command fscanf is used to read, but if you work in a Fortran, it will read or maybe write if you want to print.

I would like to say that these commands are very much important to write any code.

You are giving data through the input file, you are getting data through the output file, and you are providing the extensions properly. So that you can open it in the basic notepad and word pad, this is supported for all kinds of operating systems. The next step is in that file, from here it has started, and up to reading the file, how much data is

required.

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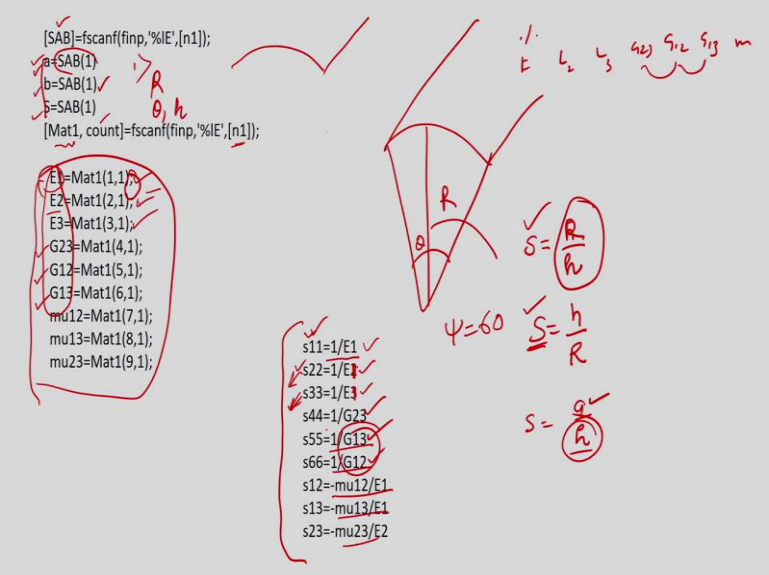

In the next file, it is reading a matrix called:

[SAB]=fscanf(finp,'%IE',[n1]);

fscanf means it is reading that matrix and giving the data,

a=SAB[1], b=SAB[1], and S=SAB[1].

a is written for a plate, but here for the arc length of like this what is the a and what is the b.

For the case of a shell, you can say that you are interested in terms of R, span angle, and thickness for a circular cylindrical shell panel. You may be interested in terms of a mean radius You can say that the very first element of this will be the mean radius.

Then, we are interested in the angle ψ = 60˚, 120˚, or any value, you can read through that. $S = \frac{R}{h}$.

In some literature, it is $\frac{R}{h}$ and in some, it is $\frac{h}{R}$, and in the case of the plate, it is $\frac{a}{h}$ (length upon the thickness of the shell). In the last lecture, I said that this is the radius to thickness parameter and it is required for non-dimensionalization, so we should do it.

Now, the material property; let us say, next is a matrix

[Mat1,count]=fscanf(finp,'%IE',[n1]);

n1 components are there and this is the counting. The first element of that matrix will be:

E1=Mat1(1,1);

The second element of that matrix will be:

E2=Mat1(2,1);

The third element will be:

E3=Mat1(3,1);

And others are:

G23=Mat1(4,1);

G12=Mat1(5,1);

G13=Mat1(6,1);

mu12=Mat1(7,1);

mu13=Mat1(8,1); and

mu23=Mat1(9,1), this is the assignment.

You must know that whenever you are giving data in an input file you should arrange it in such a manner that the first value should be E1 second value should be E2 third value should be E3 and so on. If you interchange these values, then the program will read something else. This is very much important to write some comments in the input file that what should be the first value and the other.

Once we read all the nine data for an orthotropic material then our purpose is to find the compliance matrix. The component of compliance matrix are:

s11=1/E1,

s22=1/E2,

s33=1/E3,

s44=1/G23,

s55=1/G13,

s66=1/G12,

s12=mu1/E1,

s13=mu13/E1, and

s23=mu23/E2.

When you put a semicolon at the end of the command, it will not pop up in the work window, but when you do not put a semicolon, it will come outside.

You can see the output window, the reason behind that is whatever you are writing you must check it. Sometimes, during writing a program instead of E2, you write again E1 here just by copying this may give you an error.

When you are checking for output of that, then you can say that ok this is wrong. Whenever, you write 2 lines, 3 lines, or 10 lines programme, you must check it with the help of a calculator or with the help of an excel file, do not assume that whatever you have written will be correct.

Sometimes, when there is an error in the program it looks ok, but once you are going to check it, you must check the input as well as output of the programme.

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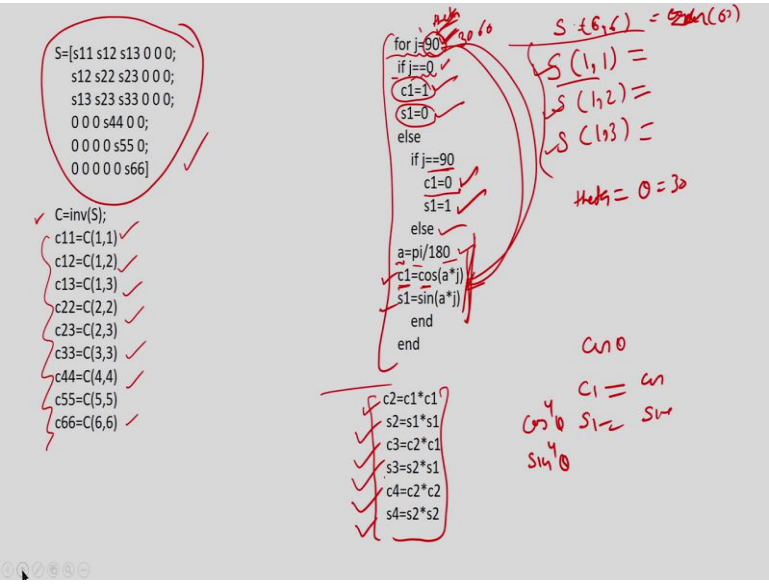

We have obtained the compliances of the matrix, then we can arrange them in an S

matrix.

These are the components.

$$S = \begin{bmatrix} s11 & s12 & s13 & 0 & 0 & 0; \\ s12 & s22 & s23 & 0 & 0 & 0; \\ s13 & s23 & s33 & 0 & 0 & 0; \\ 0 & 0 & 0 & s44 & 0 & 0; \\ 0 & 0 & 0 & 0 & s55 & 0; \\ 0 & 0 & 0 & 0 & 0 & s66; \end{bmatrix}$$

Because, in MATLAB you can arrange that in that 0, 0 and all these things say complete matrix form like this and you can write a subroutine also that you can arrange that s11. You first say that S is a matrix: S(6,6) then you can assign the elements over there like this S(1,1), S(1,2), S(1,3), and so on.

Writing a code has many ideas you can use in many ways because in MATLAB this is possible.

If sometimes these kinds of things are not possible, then we have to write in Fortran, in an anivental way like this. You assigned a 0s matrix of 6 by 6, there will be a 6 by 6 matrix which is having all 0s, and then you can assign, the nonzero elements of that.

The next is to find the stiffness C=inv(S) and assigning that

c11=C(1,1)

c12=C(1,2)

c13=C(1,3)

c22=C(2,2)

c23=C(2,3)

c33=C(3,3)

c44=C(4,4)

c55=C(5,5)

c66=C(6,6).

Now, we have to find the transformation required because when we go for fibre angle, it

is different, it is not 0, if it is 90, then we need to transform it.

At any angle, we have to transform this stiffness and compliances matrix, for that purpose this is the for loop.

In MATLAB for loop means:

for j = 90, if j = 0, then, c1=1 and s1=0.

Else if j = 90, then c1=0 and s1=1.

Here, I have done for j=90, this can be inputted through the input file, in a hard way. But if I do it for 30, 60, it will go to this loop:

a=pi/180, c1=cos(a*j), s1=sin(a*j).

j for the present case I have written 90, but you can give 30, 60. So, it is in hard form. This value j=90, we can assign as θ. So, θ we can read through the input file, the angle, or inside the fiber. We can put θ = 30˚. So, here theta = 30˚, c1 and s1 basically cos and sin is evaluated.

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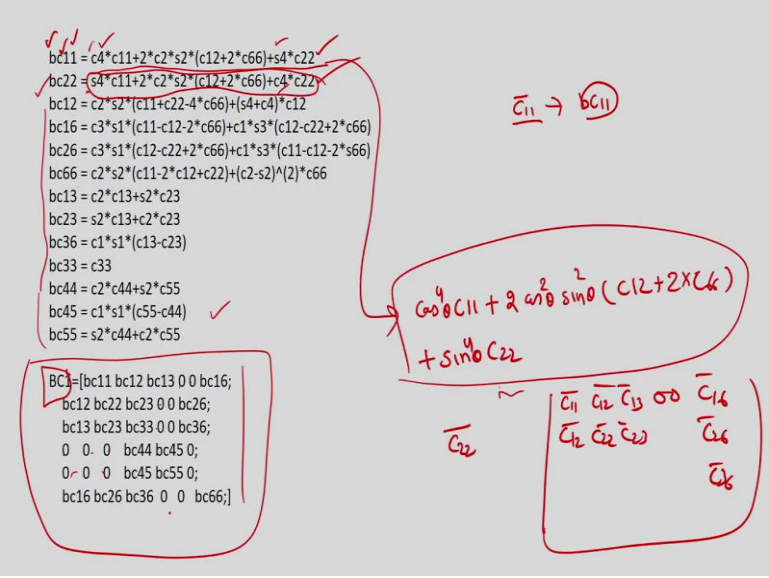


Now, for the transformation we need at least up to $\cos^4\theta$ and $\sin^4\theta$. So, $\cos^2\theta$, $\sin^2\theta$, $\cos^3\theta$, $\sin^3\theta$, $\cos^4\theta$, and $\sin^4\theta$.

This is the nomenclature:

c2=c1*c1,

s2=s1*s1,

c3=c2*c1,

s3=s2*s1,

c4=c2*c2,

s4=s2*s2.

Instead of writing cos and sin, we have written c1 and s1.

c1 = $\cos\theta$ and s1 = $\sin\theta$.

Then the transformation; here, bc11 = $\bar{C}_{11}$. So, we have written bc11.

bc11=c4*c11+2*c2*s2(c12+2*c66) +s4*c22,

The transformation of this formula, if I write in explicit form is

$\bar{C}_{11} = \cos^4\theta C_{11} + 2\cos^2\theta \sin^2\theta (C_{12} + 2xC_{66}) + \sin^4\theta C_{22}$.

These formulas are given in any theory of a plate book or mechanics of composites. There you can find the transformation formula and you have now written it in a code form.

Similarly, bc22 = $\bar{C}_{22}$.

Here, bc22=s4*c11+2*c2*s2(c12+2*c66)+c4*c22.

Instead of a cos, there will be sin and instead of a sin there will be cos, here the placement of sin and cos is changed now.

The other elements will be evaluated:

bc12=c2*s2(c11+c22-4*c66)+(s4+c4)*c12;

bc16=c3*s1(c11-c12-2*c66)+c1*s3(c12-c22+2*c66);

bc26=c3*s1(c12-c22+2*c66)+c1*s3*(c11-c12-2*s66);

bc66=c2*s2*(c11-2*c12+c22)+(c2-s2)^(2)*c66;

bc13=c2*c13+s2*c23;

bc23=s2*c13+c2*c23;

bc36=c1*s1(c13+c23);

bc33=c33;

bc44=c2*c44+s2*c55; bc45=c1*s1(c55-c44);

bc55=s2*c44+c2*c55.

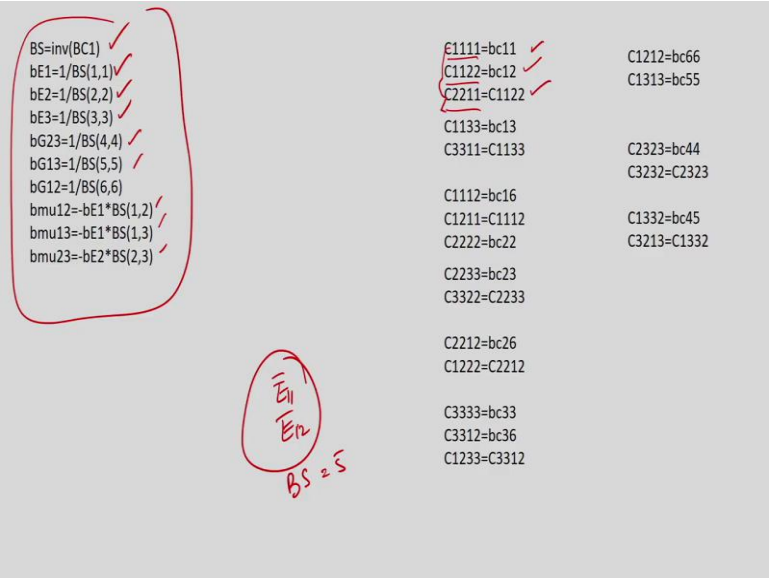
A generalized orthotropic material after the transformation is obtained.

In MATLAB, whatever elements are 0, I have put directly 0 here and developed a matrix BC1. $\bar{C}_1$ is now a matrix which is written like this, having 6 components and then compiled.

$$
\begin{aligned}
BC1 = [&bc11\ bc12\ bc13\ 0\ 0\ bc16;\\
&bc12\ bc22\ bc23\ 0\ 0\ bc26;\\
&0\ \ 0\ \ 0\ \ bc33\ \ 0\ 0\ bc36;\\
&0\ \ 0\ \ 0\ bc44\ \ bc45\ \ 0;\\
&0\ \ 0\ \ 0\ bc45\ \ bc55\ \ 0;\\
&bc16\ bc26\ bc36\ 0\ 0\ bc66;\ ]
\end{aligned}
$$

If it is not in MATLAB, you write in C or Fortran, then you cannot use this kind of function, you have to first define a matrix and its size and then its nonzero components, otherwise, you can say that all are zero.

(Refer Slide Time: 27:33)



And then BS = $\bar{S}$; BS=inv(BC1).

If somebody is interested in bE1. Instead of that E bar sometimes we are interested in the transformed engineering constant then directly you can use from there

bE1=1/BS(1,1),

bE2=1/BS(2,2),

bE3=1/BS(3,3),

bG23=1/BS(4,4),

bG13=1/BS(5,5),

bG12=1/BS(6,6),

bmu12=bE1*BS(1,2),

bmu13=bE1*BS(1,3),

bmu23=bE2*BS(1,3),

Ultimately, we can assign, c11, c1 2, and so on. The reason behind that is if you want to develop a model in abacus or Ansys. if you write an orthotropic material, they want data in a form like this.

C1111=bc11

C1122=bc12

C2211=C1122, and so on.

From this program, you can get all the material property and directly you can feed there 0˚ material property, 90˚ material property, or 30˚ material property. Definitely, in that software there are the options that first give a value means 0˚ material property.

And then they have an angle transformation. This is a very simple program you can use for directly using the material property.

(Refer Slide Time: 29:05)

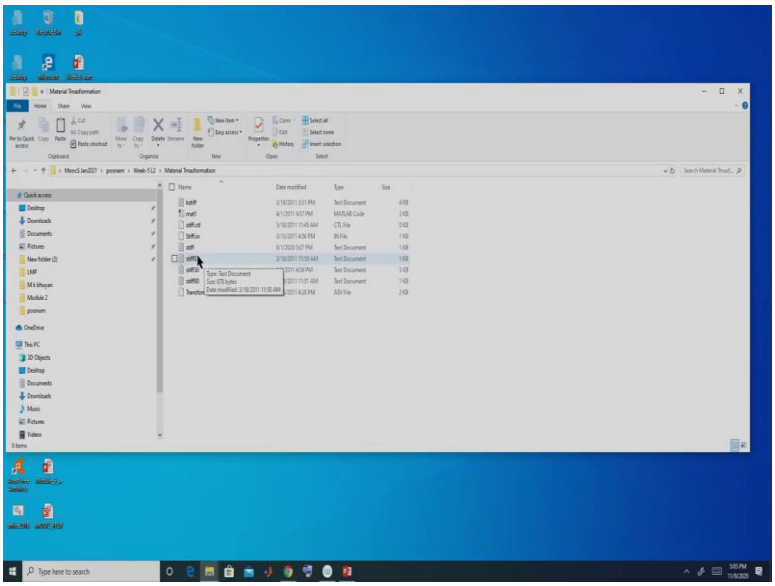
```
BC=[C1111 C1133 C1122  0    C1112  0  ;
  C3311 C3333 C3322  0    C3312  0  ;
  C2211 C2233 C2222  0    C2212  0  ;
  0   0   0    C1313 0    C1332;
  C1211 C1233 C1222  0    C1212  0  ;
  0   0   0    C3213  0   C3232;]


 end
fprintf(fout1,'\n   %12.8e  %12.8e  %12.8e  %12.8e  %12.8e
%12.8e',BC')


fclose(finp);
fclose(fout1);
```

Finally, $\bar{C}$ or BC is written here and this is the output having 6 values in a line.

$$BC = \begin{bmatrix} C1111 & C1133 & C1122 & 0 & C1112 & 0; \\ C3311 & C3333 & C3322 & 0 & C3312 & 0; \\ C2211 & C2233 & C2222 & 0 & C2212 & 0; \\ 0 & 0 & 0 & C1313 & 0 & C1332; \\ C1211 & C1233 & C1222 & 0 & C1212 & 0; \\ 0 & 0 & 0 & C3213 & 0 & C3232; \end{bmatrix}$$

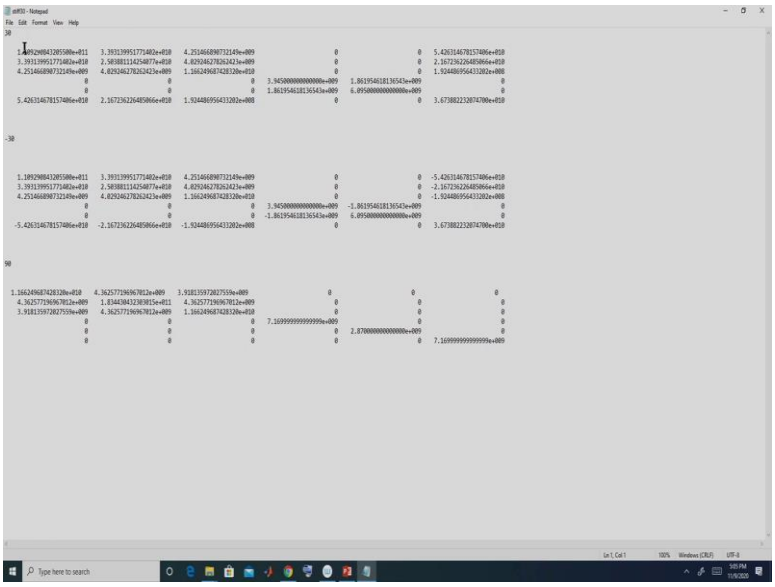
I would like to share one output file with you. So, you see the stiffness of 30.

(Refer Slide Time: 29:21)



I am sharing the file.

(Refer Slide Time: 29:25)

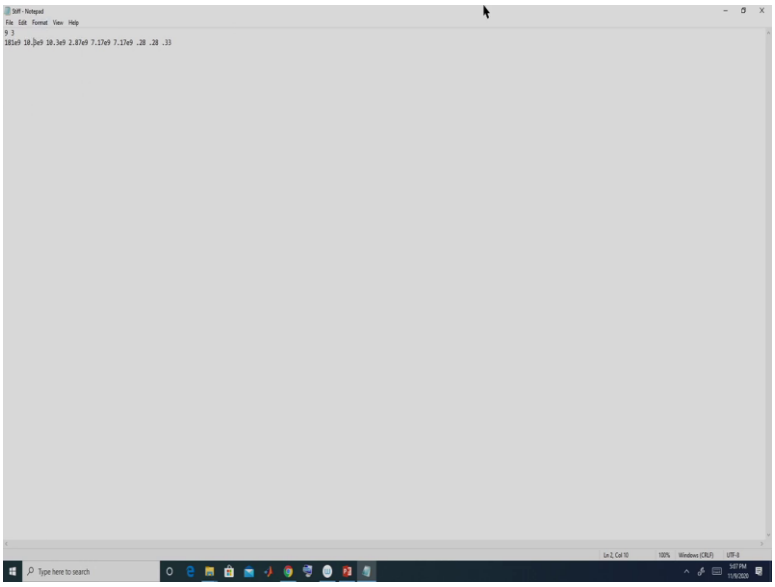


I am sharing the file. If you have provided the 30˚ angle then minus 30˚ and 90˚. You can get all the stiffness coefficients and it is written in long e format you can see that this is very much required.

You can directly use these material properties for your experimental purpose. For revaluing any property, these types of small codes are required, and even for developing a code, we require all these properties and the input file you can see that stiff.in. I am

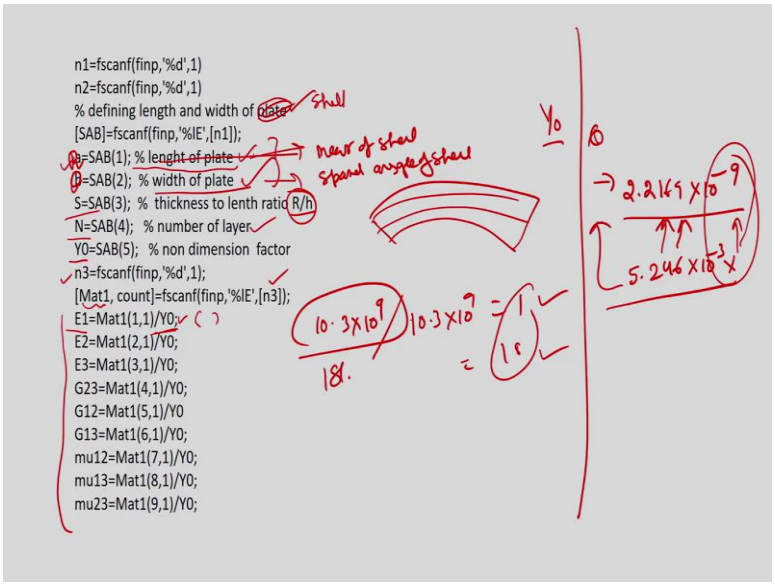
also sharing the input file.

(Refer Slide Time: 30:15)



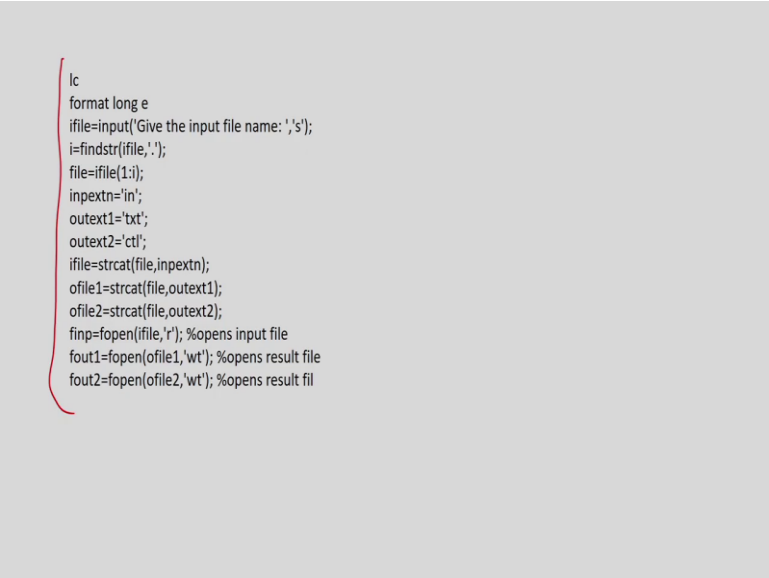
You can see that it is telling that we have to give first 9 constant and 181, each material properties are given that graphite-epoxy composes it 181e1, 181e2, 181e3 and so on. These properties will be used to transform 30˚, -30˚, 90˚, whatever you want to do.

(Refer Slide Time: 30:37)



Now, let us know, how to write a code for the shell case. First, I will explain up to the shell constitutive relations. I will explain the basic terms. Later on, you can make bigger code and more complex ones.

(Refer Slide Time: 30:59)



Following first 10 lines we will copy from the previous code as it is.

clc

format long e

ifile=input(‘give the input file name’,’s’);

i=findstr(ifile,’.’);

file=ifile(1:i);

inpextn=’in’;

outext1=’txt’;

outext2=’ctl’;

ifile=strcat(file,inpextn);

ofile1=strcat(file,outext1);

ofile2=strcat(file,outext2);

finp=fopen(ifile,’r’);%opens input file

fout1=fopen(ofile1,’wt’);%opens result file

fout2=fopen(ofile2,’wt’);%opens result file

Then, we can start writing. Now, I have written the comment over here that is very much important.

Whenever you write a code, if you do not write a comment even for yourself, after one or two years, you will not understand your code. You can have two copies in one you can write the comments so that you can understand the code later on. For confidentiality or some other point of view, you can delete those comments before giving to somebody.

We should write the comments unless it is very confidential if you do not want to give your data to someone. but if you are writing a general program then you should write a comment, it will be helpful for you and for others also. So that anybody can understand that code.

In this slide, the very first comment is % defining the length and width of the plate. Instead of a plate, we write %defining the length and width of a shell. a will be the length of the plate, b will be the width of the plate. Instead of 'a' let us say, R instead of 'b' let us say θ. Instead of

a=SAB(1);%length of a plate

b=SAB(2);%width of a plate

We can say,

R=SAB(1);%mean angle of a shell

θ=SAB(2);%span angle of a shell

You can write these comments. R/h is the thickness of the length ratio.

S=SAB(3); %thickness to length ratio

N=SAB(4);%number of layers, we want to give that how many layers are in that shell 1, 2, 4, 6, 10, then the most important command is Y0= SAB(5);%non dimension factor.

In the previous lecture, I have explained that non dimensionalisation constant.

We have to find non dimensionalisation factor, then n3=fscanf(finp,'%d'1); some other parameter for this reading a matrix

[Mat1,count]=fscanf(finp,'%IE',[n3]); it is having 9 components.

E1=Mat(1,1)/Y0;

E2=Mat(2,1)/Y0;

E3=Mat(3,1)/Y0;

G23=Mat(4,1)/Y0;

G12=Mat(5,1)/Y0;

G13=Mat(6,1)/Y0;

mu12=Mat(7,1)/Y0;

mu13=Mat(8,1)/Y0;

mu23=Mat(9,1)/Y0;.

Now, it is just becoming a number and its order is within or less than 100. It may be 20, 30 not more than that and then it may be 1 or 0.1. It will be easy to work with these things, instead of working with that $10^3 X 10^9$. If we divide it by, let us say $10^3 X 10^9$, it becomes 1 and if it is 180, it becomes 18. We can work with these numbers easily. This is the very important concept of non dimensionalisation, but you have to do it very carefully.

I would like to say that if you do not do other commands at least do the non dimensionalisation of material properties which is causing the main factor and once you get the results, what will be the factors coming up there? You have to multiply with that then only you can match with your results.

In the C software, abacus or Ansys, let us say, displacement is coming 2.2169 X 10^{-9}, but in your program displacement may be coming that 5.246 X 10^{-3}.

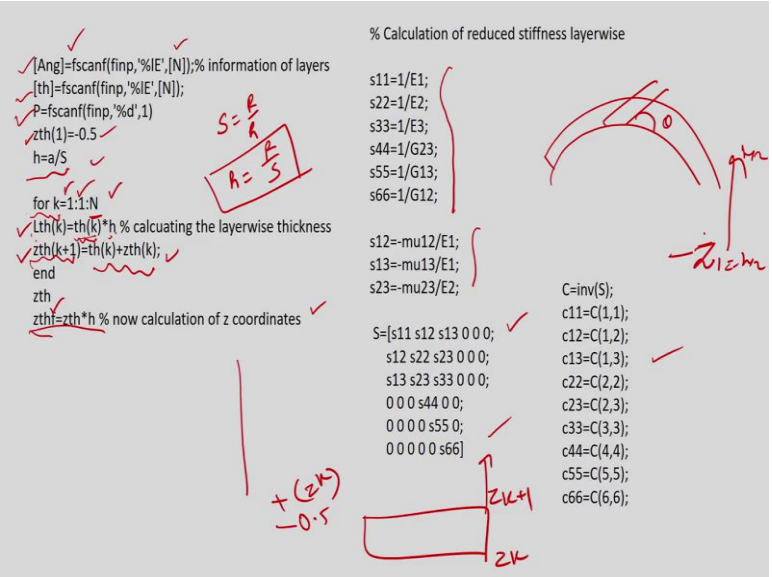
If you do the non dimensionalisation then you have to multiply with that and then only you can match with this. So, the factor with which you divide it you have to multiply it there to get the final solution.

Sometimes students say that their results are not matching. I find that they have non dimensionalised like this, but comparing the results with that. This abacus or Ansys has written by a very good programmer, these are very generalized ones. Whatever we write in our code we have to convert it back or you have to divide it by this.

So that we can match our results in that subroutine, to know what is going on inside the

code. But in our program, we know that we have divided this, so we have to take care. Through the governing equation, you can check out how it is moving ahead.

(Refer Slide Time: 36:01)



The next is the fiber angle: the angle fibers are making with respect to θ. For each layer there will be one angle. So, this information is read through here:
[Ang]=fscanf(finp,'%IE',[N]);%information of number of layers.

The θ is also read through here:

[th]=fscanf(finp,'%IE',[N]);

Then the loading information will be P=fscanf(finp,'%d',1);.

Zth(1)=0.5, then thickness can be find just like $S = \frac{R}{h}$.

Therefore, from here $h = \frac{R}{S}$.

We may provide the total thickness or inside that we may provide the thickness of each layer.

Zth(k+1)=th(k)+zth(k), here the thickness of the each layer is calculated.

Therefore, thickness of the each layer for k=1:1:N and

Lth(k)=th(k)*h%calculating the layer wise thickness.

You have given the thickness of the N X h. It tells you the layer wise thickness and then the coordinate of that. - 0.52 + the layer thickness, ZK , ZK + 1 and so on, it is arranged in a vector form

The thickness of z coordinates is calculated like here:

zthf=zth*h%now calculation of z coordinates. zth is a column row vector.

In this way, we can calculate the compliances like this:

s11=1/E1;

s22=1/E2;

s33=1/E3;

s44=1/G23;

s55=1/G13;

s66=1/G12;

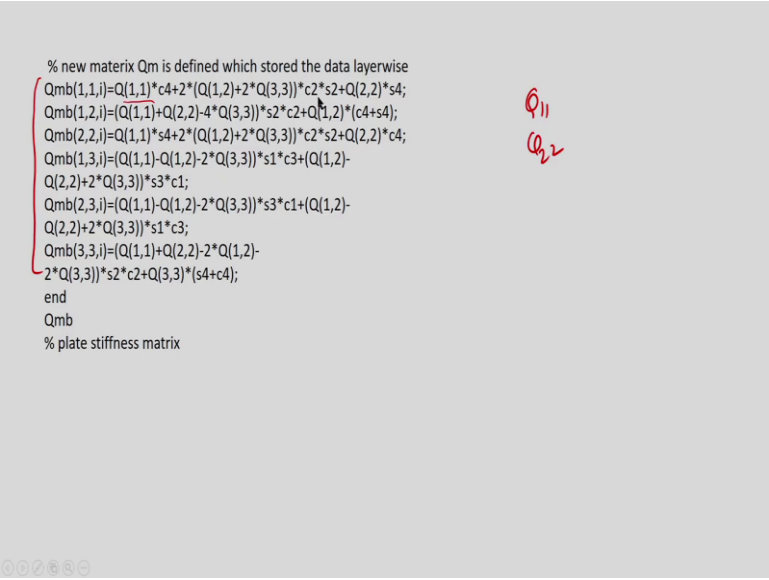
s12=mu12/E1;

s13=mu12/E1;

s23=mu12/E2;

S matrix and C matrix. Sometimes, we require formulation of C and sometimes formulation of S

(Refer Slide Time: 38:07)



Then, we have reduced stiffness,

% new matrix Qm is defined which store the data layer wise

Qmb(1,1, i)=Q(1,1)*c4+2*(Q(1,2)+2*Q(3,3))*c2*s2+Q(2,2)*s4;

Qmb(1,2,i)=(Q(1,1)+ Q(2,2)-4*Q(3,3))*s2*c2+Q(1,2)*(c4+s4);

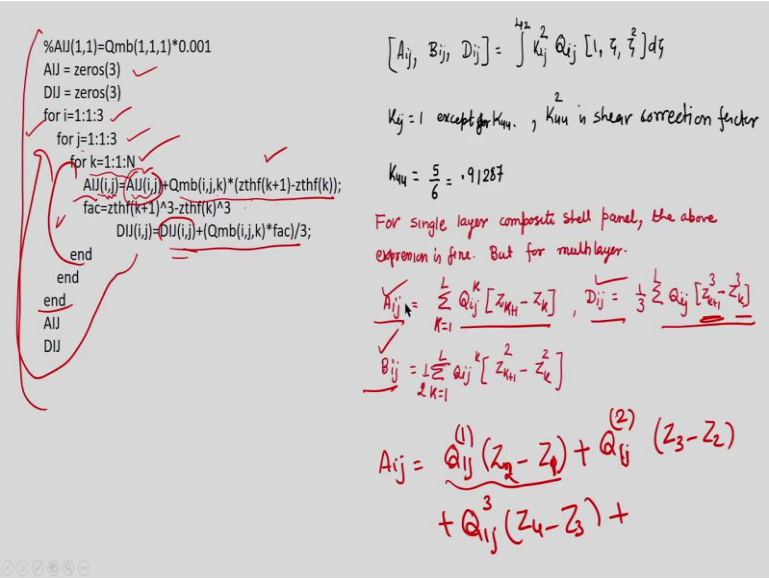
Qmb(2,2,i)=Q(1,1)*s4+2*(Q(1,2)+2*Q(3,3))*c2*s2+Q(2,2)*c4;

Qmb(1,3,i)=(Q(1,1)-Q(1,2)-2*Q(3,3))*s1*c3+(Q(1,2)- Q(2,2)+2* Q(3,3))*s3*c1;

Qmb(2,3,i)=(Q(1,1)-Q(1,2)-2*Q(3,3))*s3*c1+(Q(1,2)- Q(2,2)+2* Q(3,3))*s1*c3;

Qmb(3,3,i)=(Q(1,1)+ Q(2,2)-2* Q(1,2)-2* Q(3,3)*s2*c2+ Q(3,3)* (s4+c4);

(Refer Slide Time: 38:33)



First, we have to define what is Q1, Q2, and Q3 and then the transformation of those matrices.

Now, you have to find the matrix AIJ, BIJ, and DIJ. This is the subroutine:

%AIJ(1,1)=Qmb(1,1,1)*0.001,

AIJ=zeros(3),

DIJ=zeros(3)

For i=1:1:3,

for j=1:1:3,

for k= 1:1:N,

I have written this for a plate, A1, A2, A3, and so on.

Here, A_{ij} can be written in three layers.

$$A_{ij} = Q_{ij}(Z_2 - Z_1) + Q_{ij}(Z_3 - Z_2) + Q_{ij}(Z_4 - Z_3)$$

First layer is $Q_{ij}(Z_2 - Z_1)$,

$Q_{ij}(Z_3 - Z_2)$ is the second layer, and

$Q_{ij}(Z_4 - Z_3)$ is the third layer.

First, if you calculate then you have to add it. Next time when it comes in a loop, we add this factor:

Qmb(I,j,k)*(zthf(k+1)-zthf(k)),

Therefore, AIJ(i,j)=AIJ(I,j)+Qmb(I,j,k)*(zthf(k+1)-zthf(k));.

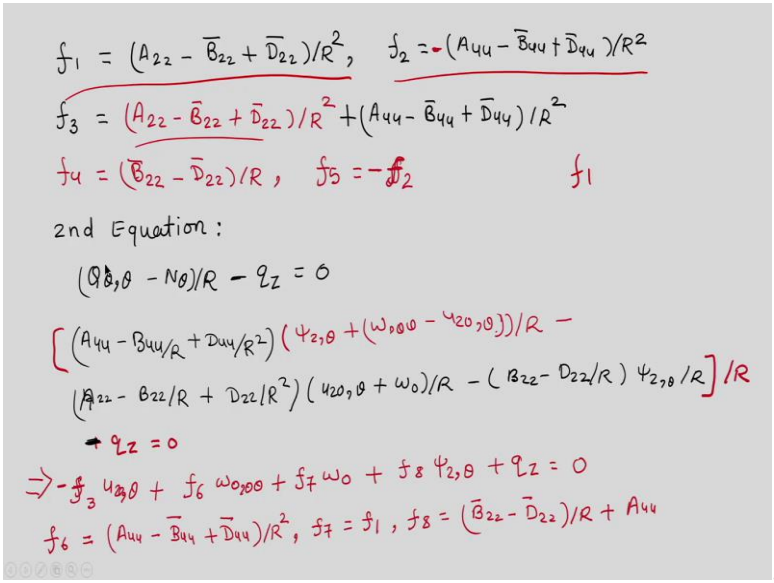
Then a factor for this will be:

fac=zthf(k+1)^3-zthf(k)^3.

Then, DIJ(I,j)=DIJ(I,j)+(Qmb(I,j,k)*fac)/3

In this way, we can write the A_{ij}, B_{ij}, and D_{ij}.

(Refer Slide Time: 40:25)



If we know A_{ij}, B_{ij}, and D_{ij}, then you can define the f matrix f_1, f_2, f_3, and so on. You can say $f_1 = \dfrac{\left(A_{22} - \bar{B}_{22} + \bar{D}_{22}\right)}{R^2}$,

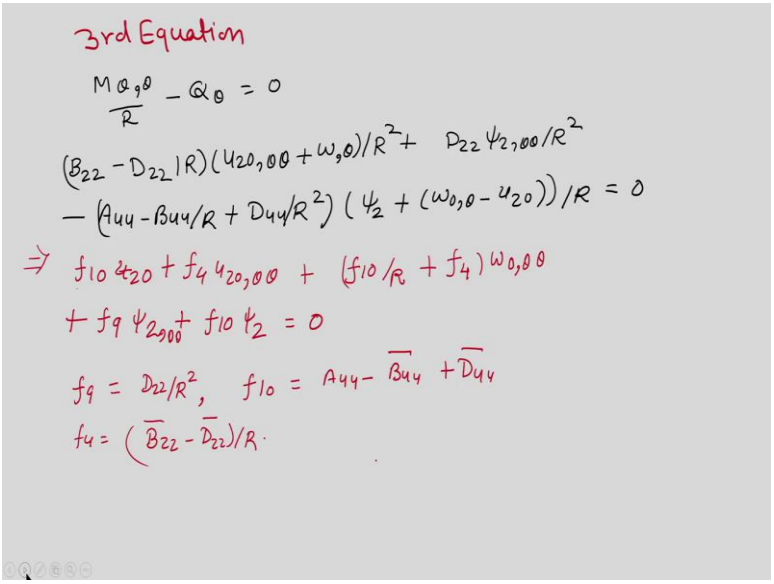
$f_2 = \dfrac{\left(A_{44} - \bar{B}_{44} + \bar{D}_{44}\right)}{R^2}$,

$$f_3 = \frac{\left(A_{22} - \bar{B}_{22} + \bar{D}_{22}\right)}{R^2} + \frac{\left(A_{44} - \bar{B}_{44} + \bar{D}_{44}\right)}{R^2},$$

$$f_4 = \frac{\left(\bar{B}_{22} + \bar{D}_{22}\right)}{R^2}, \quad f_5 = -\ f_2$$

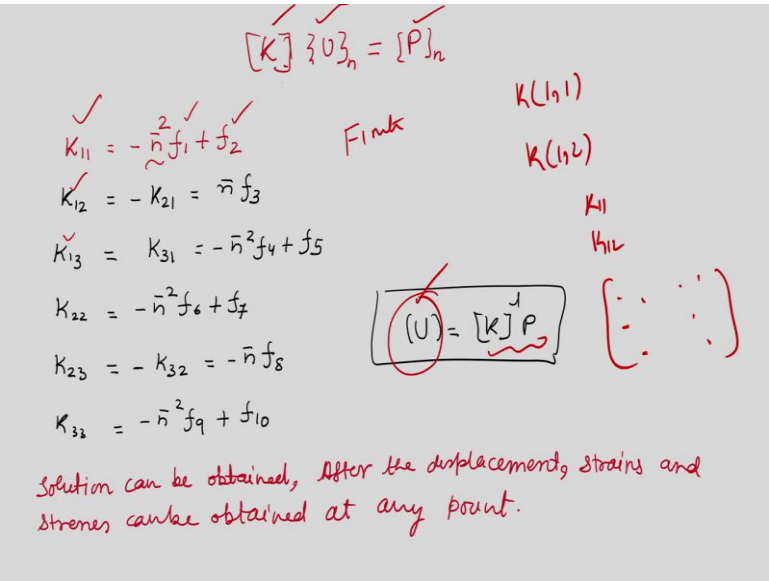
you can define these constants, once layer-wise is over. This will be a number, all constants you can define.

(Refer Slide Time: 40:55)



Similarly, $f_9 = \frac{D_{22}}{R^2}$ $f_{10} = A_{44} - \bar{B}_{44} + \bar{D}_{44}$.

(Refer Slide Time: 40:57)



Ultimately, after obtaining the value of f_1, f_2, f_3, we have to come to matrix K. Based on the simply supported boundary condition, you have to multiply with this and define now another matrix K 11:

$$K_{11} = -\bar{n}^2 f_1 + f_2 \ , \ K_{12} = -K_{12} = \bar{n} f_3 .$$

You can write that K matrix K (1,1), K (1,2) like that. You can say that K_{11}, K_{12} ultimately, make a big matrix wherever like this and finally, K inverse P will give you the displacement field $U = [K]^{-1} P$.

In this way, one can do the coding. The coding requires expertise, but I have just given a brief idea that how we can move ahead with the basic steps, and later on, you can write a big code. There may be after finding, there are many more steps to find the displacements at every z coordinate or at every theta value. One can find it.

With this, I end this lecture. In the next lecture, I will explain the finite shell formulation, developing a governing equation for a finite shell and the solution.

In the infinite shell, the solution, I gave is in sin and cos, but for the finite case, there will be three-four cases: the axisymmetric case, antisymmetric case, the journal bending case, etc. I will explain those things in the coming lectures.

Thank you very much.