Theory of Composite Shells

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Week - 06

Lecture - 03

Development of Levy type finite shell under static and free vibration case

Dear learners welcome to Week 6, Lecture 3. In this lecture, I shall explain the solution technique and mix type formulation of static bending of finite shell and free vibration of the finite shell under Levy type support conditions.

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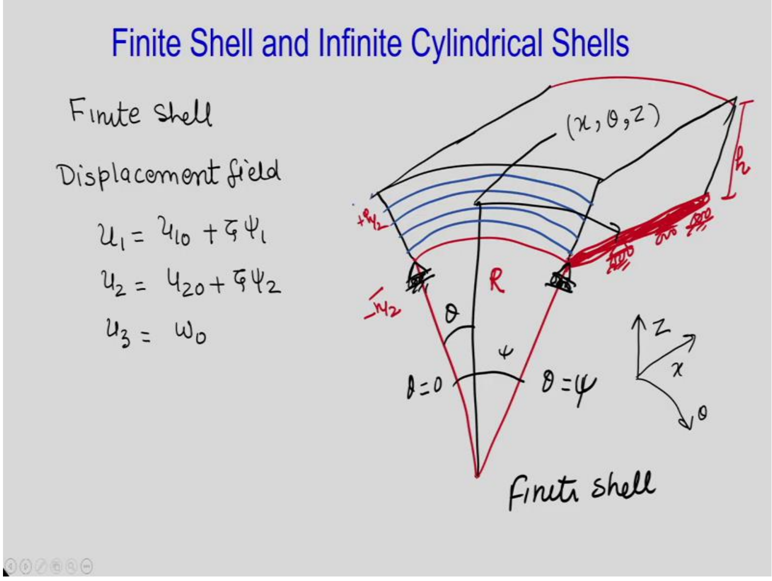

Previously, we considered this finite shell and followed the displacement;

$u_1 = u_{10} + \varsigma\psi_1$, $u_2 = u_{20} + \varsigma\psi_2$, *and* $u_3 = w_0$.

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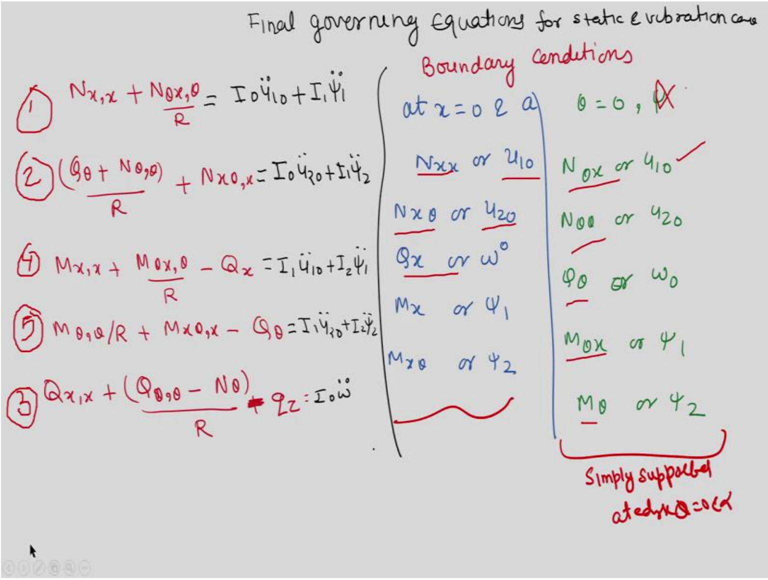

The partial differential equations and boundary conditions are as follows:

$$Partial \quad differential \quad equations:$$

$$N_{x,x} + \frac{N_{\theta x,\theta}}{R} = I_0 \ddot{u}_{10} + I_1 \ddot{\psi}_1 \quad equation(1)$$

$$\frac{\left(Q_\theta + N_{\theta,\theta}\right)}{R} + N_{x\theta,x} = I_0 \ddot{u}_{20} + I_1 \ddot{\psi}_2 \quad equation(2)$$

$$M_{x,x} + \frac{M_{\theta x,\theta}}{R} - Q_x = I_1 \ddot{u}_{10} + I_2 \ddot{\psi}_1 \quad equation(4)$$

$$\frac{M_{\theta,\theta}}{R} + M_{x\theta,x} - Q_\theta = I_1 \ddot{u}_{20} + I_2 \ddot{\psi}_2 \quad equation(5)$$

$$Q_{x,x} + \frac{Q_{\theta,\theta} - N_\theta}{R} - q_z = I_1 \ddot{w}_0 \quad equation(3)$$

$$Boundary\, conditions:$$

$$At\; x = 0 \,\&\, a:$$

$$N_{xx} \;\; or \;\; u_{10}; \;\; N_{x\theta} \;\; or \;\; u_{20}; \;\; Q_x \;\; or \;\; w_0; \;\; M_{xx} \;\; or \;\; \psi_1; \;\; M_{x\theta} \;\; or \;\; \psi_2$$

$$At\; \theta = 0 \,\&\, \alpha:$$

$$N_{x\theta} \;\; or \, u_{10}; \;\; N_{\theta\theta} \;\; or \;\; u_{20}; \;\; Q_\theta \;\; or \;\; w_0; \;\; M_{x\theta} \;\; or \;\; \psi_1; \; M_{\theta\theta} \;\; or \;\; \psi_2$$

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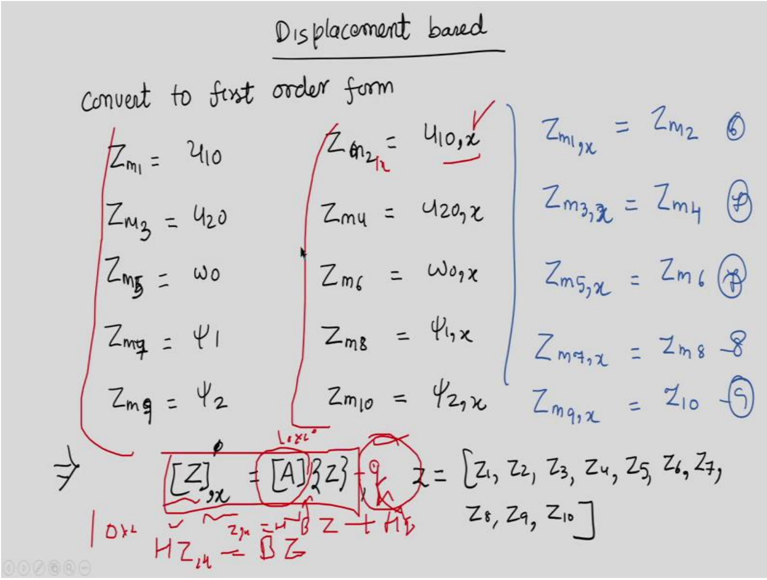

Using the concept of displacement-based approach: I explained the static bending of Levy-type boundary conditions for the finite cylindrical shell. In the previous lecture, I explained that we can define new variables so that we can convert those into first-order differential equations:

$$Z_{m1} = u_{10}; \quad Z_{m2} = u_{10,x}; \quad Z_{m3} = u_{20}; \quad Z_{m4} = u_{20,x}; \quad Z_{m5} = w_0; \quad Z_{m6} = w_{0,x}$$
$$Z_{m7} = \psi_1; \quad Z_{m8} = \psi_{1,x}; \quad Z_{m9} = \psi_2; \quad Z_{m10} = \psi_{2,x}$$

By doing so, we get 10 first-order differential equations:

$$Z_{m1,x} = Z_{m2} \quad equation(6)$$
$$Z_{m3,x} = Z_{m4} \quad equation(7)$$
$$Z_{m5,x} = Z_{m6} \quad equation(8)$$
$$Z_{m7,x} = Z_{m8} \quad equation(9)$$
$$Z_{m9,x} = Z_{m10} \quad equation(10)$$

And $[Z]_{,x} = [A]\{Z\} + \bar{q}_n$ is the non-homogeneous equation.

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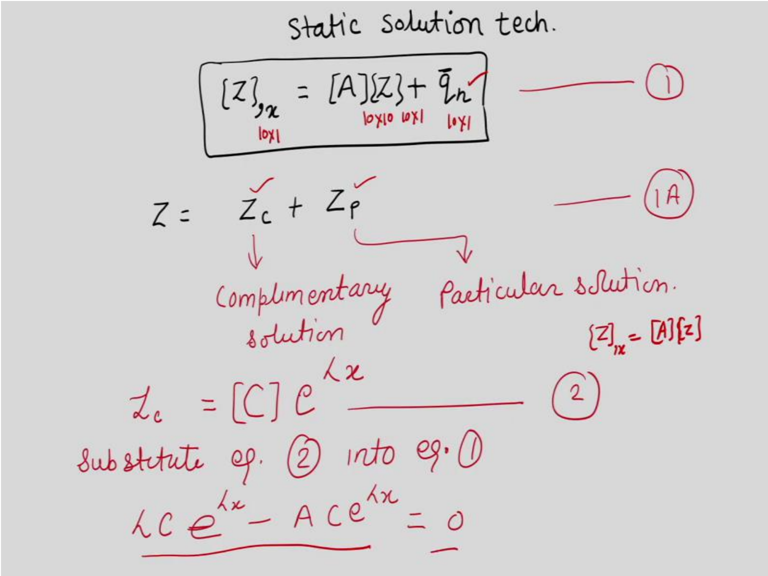

Where A is a 10 by 10 matrix and Z is 10 by 1 matrix and $\bar{q}_n$ is also 10 by 1 matrix, $\bar{q}_n$ is a modified loading. In this way, we got a final governing equation which is having a derivative along the x-direction.

These are the set of 10 ordinary differential equations and the solution can be represented like this $Z = Z_c + Z_p$.

First, we will consider, $[Z]_{,x} = [A]\{Z\}$.

We can assume the complementary solution $Z_c = [C]e^{\lambda x}$. When we substitute this equation into $[Z]_{,x} = [A]\{Z\} + \bar{q}_n$ this equation, then it leads to an eigenvalue equation:

$\lambda Ce^{\lambda x} - ACe^{\lambda x} = 0$

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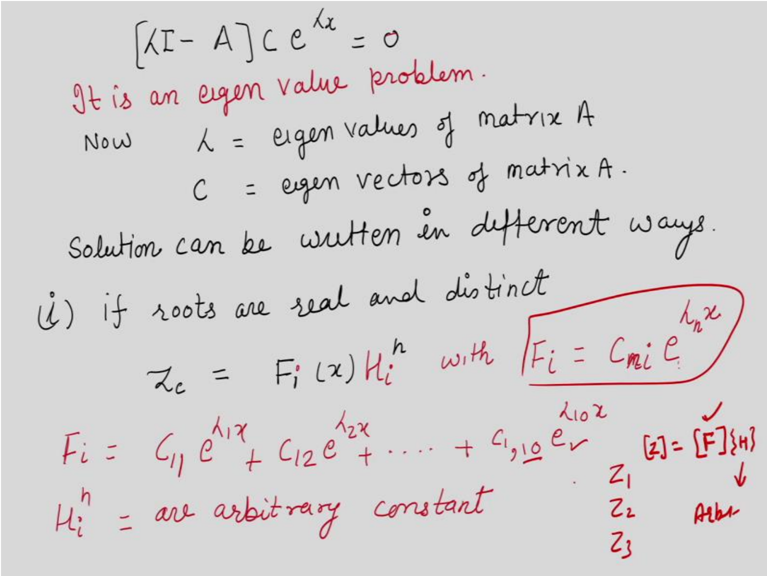

It is an eigenvalue, where λ is the eigenvalue of a matrix A and C are the eigenvectors of a matrix A. If we know the matrix A, we can find the eigenvalues and eigenvectors and we can write the complementary solutions.

The solution can be written in different ways. The very standard solution is Pagano type solution, where these solution techniques are given in any higher engineering mathematics book, specifically the book by Kraljic.

In that book, the solution for a simultaneous first-order differential equation is written in three different ways.

First case: if the roots are real and distinct; then we can assume a complementary solution $Z_c = F_i(x) H_i^n$, where, H_i^n are arbitrary constants and F_i is a function of eigenvectors and eigenvalues. $F_i = C_{11}e^{\lambda_1 x} + C_{12}e^{\lambda_2 x} + C_{13}e^{\lambda_3 x} + \dots\dots + C_{10}e^{\lambda_{10} x}$ 87695/7

In this way, all complementary solutions can be written.

Ultimately, the solution $[Z] = [F]\{H\}$, where, H is the arbitrary constant and F is known to us because we know the eigenvalues and eigenvectors and at any x location we can find.

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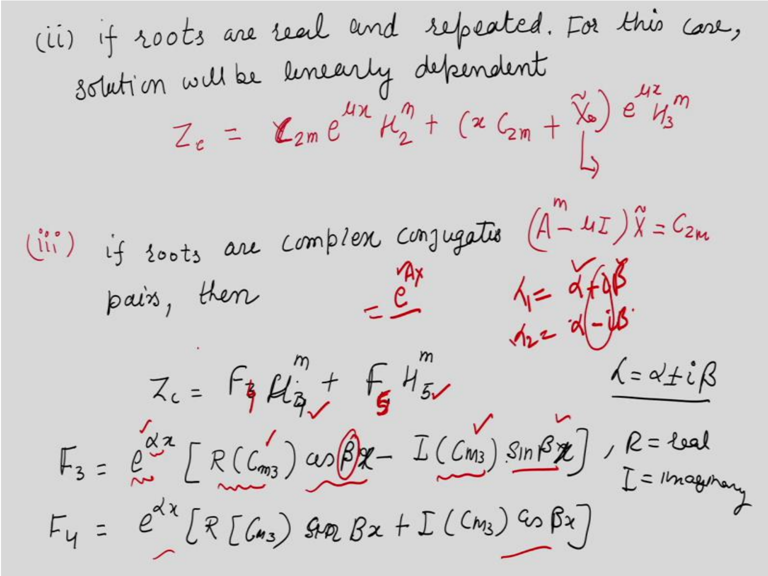

Second case: If roots are real and repeated; in that case, the solution will be linearly dependent and if you say that there are 2 roots and they are the same, therefore, 2nd and 3rd constant will be the same and xC_{2m} will be linearly dependent. In this way, the solution can be written.

Third case: if the roots are complex conjugates; it is generally for a structural problem, if we think about a composite shell the most of the time roots are complex conjugate or real and distinct. In that case, we can write a solution in this form because let us say, one root $\lambda_1 = \alpha + i\beta$ and another $\lambda_2 = \alpha - i\beta$. Here, α and β remain same only with + and $-$ sign the root is differentiated. If the root is such that then we can write a solution $Z_c = F_4 H_4^m + F_5 H_5^m$. Up to three cases we take.

$F_3 = e^{\lambda x}\left[R(C_{m3})\cos\beta x - I(C_{m3})\sin\beta x\right]$, where, α is the real part of the root and real part of that eigenvector and $\cos\beta x$. β is the imaginary part of the root and $-$ of the imaginary part of eigenvector and $\sin\beta x$. and F_4 can be written like this: $e^{\lambda x}\left[R(C_{m3})\sin\beta x + I(C_{m3})\cos\beta x\right]$.

There are many ways to write the solutions in Professor J. N. Reddy's book, it is written in terms of cos hyperbolic and sin hyperbolic.

Here, we have written directly in terms of exponential, and in some other way, one can use the state-space technique. If you can get the exponential of a matrix Ax, then the first-order differential equation where A is a matrix will be the solution. There are some techniques to find the solutions. I have explained the very standard technique used to write the solutions.

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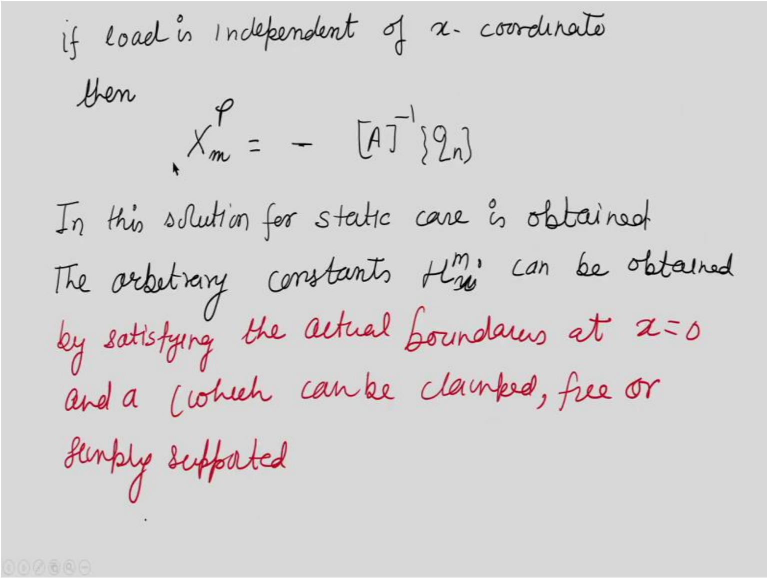

If we say that the load is independent of the x coordinate in that case the particular solutions can be written like this: $X_m^p = -[A]^{-1}\{q_n\}$.

In this way, we can get the solution for a static case and arbitrary constants H_n^m can be obtained. By satisfying the actual boundary condition; x = 0 and x = a, which may be clamped, free, or simply supported. In this way, the displacement base Levy type solution is presented.

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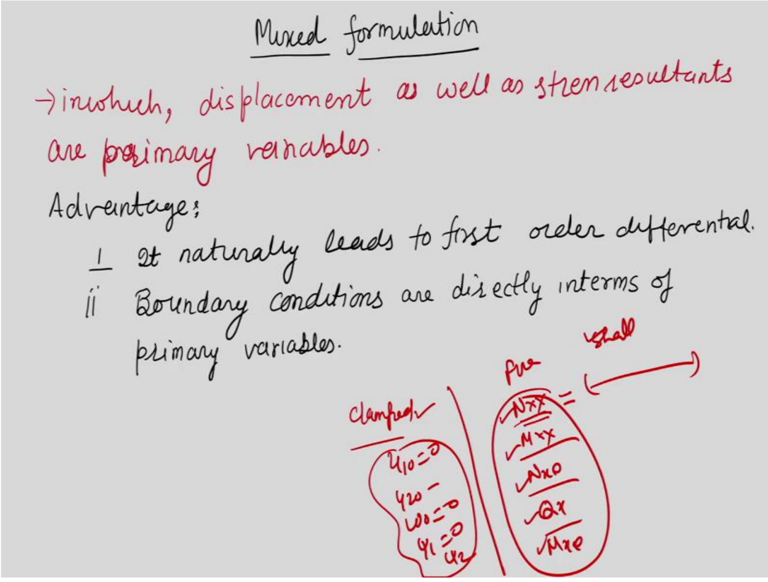

Now, there is another technique “mixed formulation”, which I have already discussed in the previous lecture. Again, I am just going to review that. The mixed formulation technique is the technique in which we take displacement as well as stresses as our primary variables. Stress resultants are the primary variables. The first advantage of choosing a mixed formulation over a pure displacement-based formulation is it naturally leads to a first-order differential equation.

In the previous case, for the case of displacement, we have to identify or define a new variable so that we can convert the second-order differential equation into a first-order differential equation. But if we choose a mixed formulation approach then it naturally converts first-order differential equation.

The next advantage is that the boundary conditions are directly satisfied in terms of primary variables. For example, if the boundary condition is clamped only then your displacement-based formulation leads to first-order form because the primary variables $u_{10} = u_{20} = w_0 = \psi_1 = \psi_2 = 0$.

But if the edge is free, in that case, we have to define N_{xx}, M_{xx}, $N_{x\theta}$, $M_{x\theta}$, and Q_x, these are not our primary variables.

We have to satisfy these stress resultant conditions through the shell constitutive relations which are again slightly typical, but in a mixed formulation these stress resultants are primary variables, we can directly satisfy the boundary conditions.

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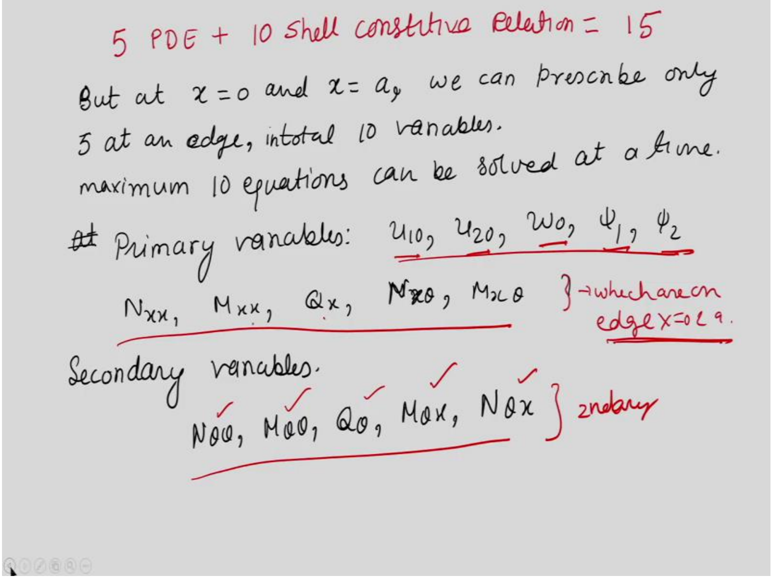

We have 5 partial differential equations and 10 shell constitutive relations. Overall, we have 15 equations. At an x = 0 and x = a, we can prescribe only 5 equations, maximum we can solve 10 variables at a time. Therefore, we have to choose 10 primary variables in such a way that we can satisfy the boundary conditions.

The variables which are prescribed over the edge x = 0 and x = a will be considered as the primary variables. The other variables which are not in this are considered secondary variables. For our case: u_{10}, u_{20}, w_0, ψ_1, and ψ_2 are the 5 displacement variables and N_{xx}, M_{xx}, $N_{x\theta}$, $M_{x\theta}$, and Q_x, are 5 stress resultants.

These are considered as primary variables because N_{xx}, M_{xx}, $N_{x\theta}$, $M_{x\theta}$, and Q_x, will be specified at an edge x = 0 and x = a.

$N_{\theta\theta}, M_{\theta\theta}, M_{\theta x}, N_{\theta x}, and\ Q_\theta$ will be considered as secondary variables. The very important part of this development is that you need not to convert your partial differential equations into displacement form.

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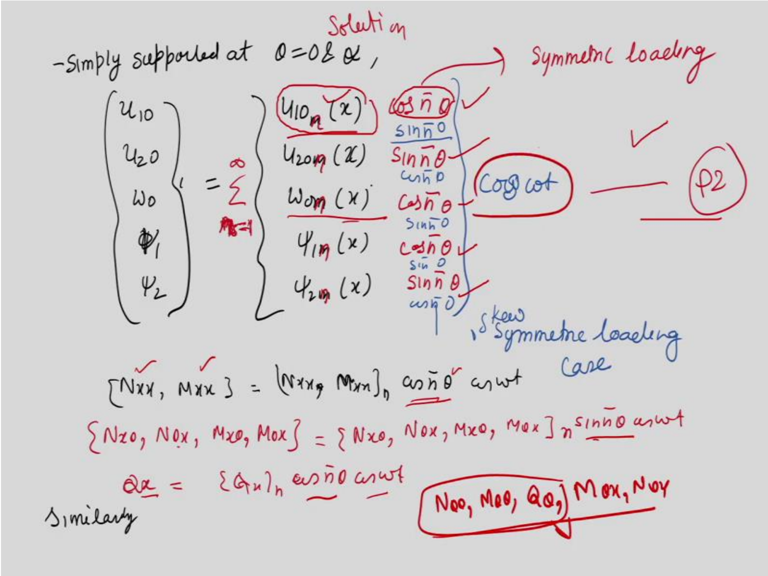

Let us first consider θ = 0 and α is simply supported like previously. We have assumed our displacement field like this:

$$\begin{bmatrix} u_{10} \\ u_{20} \\ w_0 \\ \psi_1 \\ \psi_2 \end{bmatrix} = \sum_{ms=1}^{\infty} \begin{bmatrix} u_{10n}(x) & \cos\bar{n}\theta \\ & \sin\bar{n}\theta \\ u_{20n}(x) & \sin\bar{n}\theta \\ & \cos\bar{n}\theta \\ w_{0n}(x) & \cos\bar{n}\theta \\ & \sin\bar{n}\theta \\ \psi_{1n}(x) & \cos\bar{n}\theta \\ & \sin\bar{n}\theta \\ \psi_{2n}(x) & \sin\bar{n}\theta \\ & \cos\bar{n}\theta \end{bmatrix} \cos\omega t \quad equation-p2$$

Like our displacement variables we have other stress resultants N_{xx}, M_{xx}, $N_{x\theta}$, $M_{x\theta}$, and Q_x, these are the primary stress resultant variables. Therefore, we need to express these also in terms of sine and cosine θ. Using the concept of shell constitutive relations, we can directly say that $\left[N_{xx}, M_{xx}\right] = \left[N_{xx}, M_{xx}\right]_n \cos\bar{n}\theta\cos\omega t$ because w_0 is expressed. Then $\left[N_{x\theta}, N_{\theta x}, M_{x\theta}, M_{\theta x}\right] = \left[N_{x\theta}, N_{\theta x}, M_{x\theta}, M_{\theta x}\right]_n \sin\bar{n}\theta\cos\omega t$.

$Q_x = \left[Q_x\right]_n \cos\bar{n}\theta\cos\omega t$.

Similarly, the other variables $N_{\theta\theta}, M_{\theta\theta}, M_{\theta x}, N_{\theta x}, and\ Q_\theta$ can be represented. These were also represented as $\cos\bar{n}\theta$ and this $\cos\omega t$ is for time variation.

We know that we have assumed the variation along θ direction like this and the rest is the variable of x. Now we have only that function of x. Wherever derivative θ comes, there we can use the Fourier expansion. Now, we have 5 governing partial differential equations, and the boundary conditions that we have satisfied by taking simply supported case at an edge θ = 0 and α .

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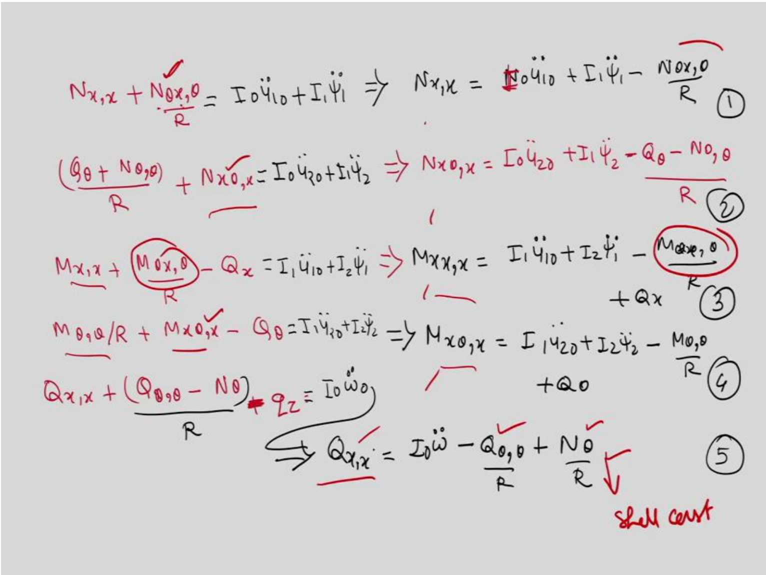

We can see that the equation (1) can be expressed as:

$$N_{x,x} = I_0\ddot{u}_{10} + I_1\ddot{\psi}_1 - \frac{N_{\theta x,\theta}}{R}$$

Equation (2) will be:

$$N_{x\theta,x} = I_0\ddot{u}_{20} + I_1\ddot{\psi}_2 - \frac{Q_\theta - N_{\theta,\theta}}{R} \;;$$

Equation (3):

$$M_{xx,x} = I_1\ddot{u}_{10} + I_2\ddot{\psi}_1 - \frac{M_{\theta x,\theta}}{R} + Q_x$$

Equation (4):

$$M_{x\theta,x} = I_1\ddot{u}_{20} + I_2\ddot{\psi}_2 - \frac{M_{\theta,\theta}}{R} + Q_\theta$$

Equation (5):

$$Q_{x,x} = I_1\ddot{w}_0 - \frac{Q_{\theta,\theta}}{R} + \frac{N_\theta}{R}$$

Here, $Q_{\theta,\theta}$ and N_θ will be expressed using the shell constitutive relation. Ultimately, all x derivatives will be on the left-hand side and θ derivatives on the right-hand side.

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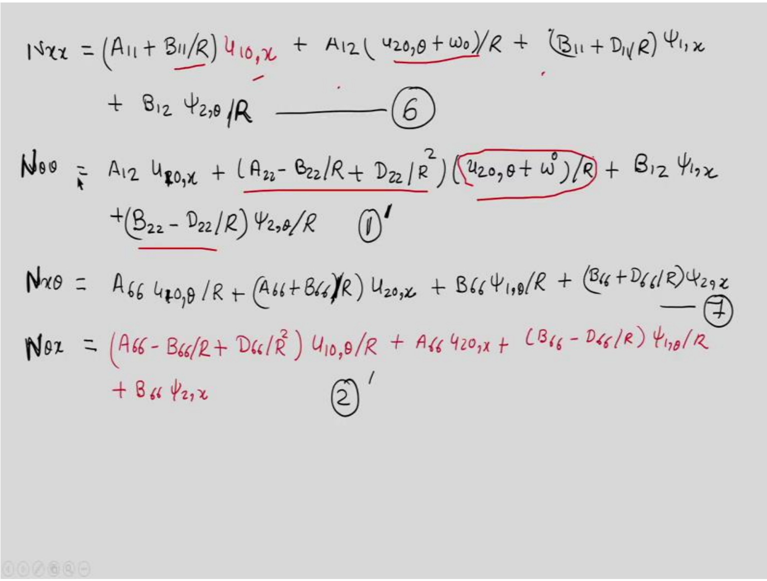

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Similarly, from the first equation:

$N_{xx} = \left(A_{11} + \frac{B_{11}}{R}\right)u_{10,x} + A_{12}\frac{(u_{20,\theta} + w_0)}{R} + \left(B_{11} + \frac{D_{11}}{R}\right)\psi_{1,x} + B_{12}\frac{\psi_{2,\theta}}{R}$, we can find out these things $N_{\theta\theta}$, $N_{x\theta}$, $N_{\theta x}$.

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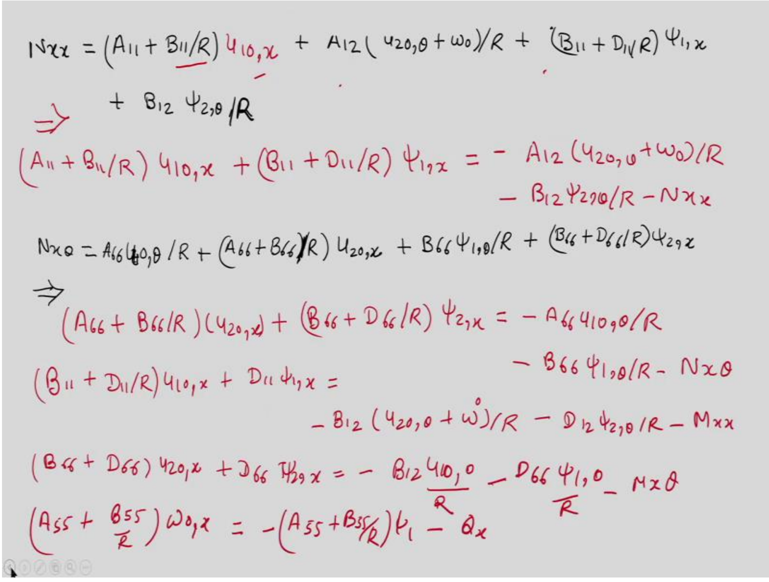

From here, we can say:

$$\left(A_{11}+\frac{B_{11}}{R}\right)u_{10,x}+\left(B_{11}+\frac{D_{11}}{R}\right)\psi_{1,x} = -A_{12}\frac{\left(u_{20,\theta}+w_0\right)}{R}-B_{12}\frac{\psi_{2,\theta}}{R}-N_{xx}\,.$$

From this equation:

$$N_{x\theta}=A_{66}\frac{u_{10,\theta}}{R}+\left(A_{66}+\frac{B_{66}}{R}\right)u_{20,x}+\frac{B_{66}\psi_{1,\theta}}{R}+\left(B_{66}+\frac{D_{66}}{R}\right)\psi_{2,x}\,,$$

we can say, $\left(A_{66}+\frac{B_{66}}{R}\right)u_{20,x}+\left(B_{66}+\frac{D_{66}}{R}\right)\psi_{2,x} = -A_{66}\frac{u_{10,\theta}}{R}-\frac{B_{66}\psi_{1,\theta}}{R}-N_{x\theta}$

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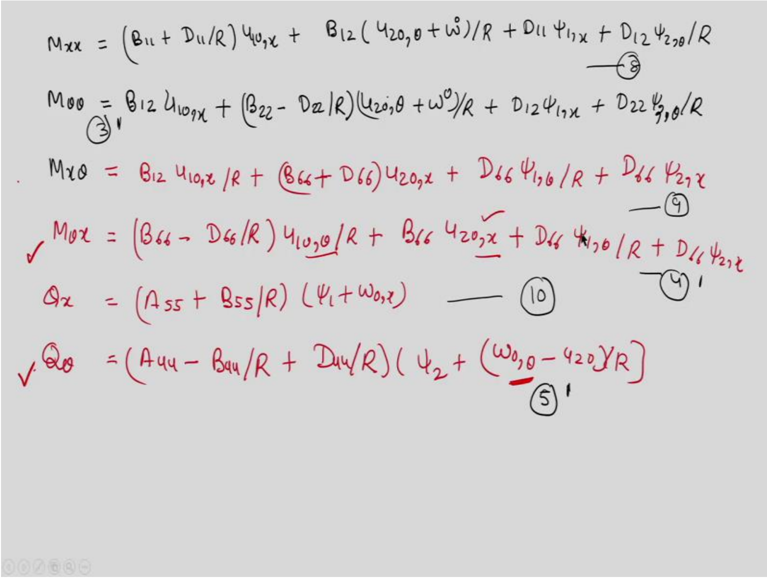

From equation: $M_{xx} = \left(B_{11} + \frac{D_{11}}{R}\right)u_{10,x} + B_{12}\frac{\left(u_{20,\theta} + w_0\right)}{R} + D_{11}\psi_{1,x} + D_{12}\frac{\psi_{2,\theta}}{R}$, we can say,

$$\left(B_{11} + \frac{D_{11}}{R}\right)u_{10,x} + D_{11}\psi_{1,x} = -B_{12}\frac{\left(u_{20,\theta} + w_0\right)}{R} - D_{12}\frac{\psi_{2,\theta}}{R} - M_{xx}.$$

From equation: $M_{x\theta} = B_{12}\frac{u_{10,x}}{R} + \left(B_{66} + D_{66}\right)u_{20,x} + \frac{D_{66}\psi_{1,\theta}}{R} + D_{66}\psi_{2,x}$, we can say,

$$\left(B_{66} + D_{66}\right)u_{20,x} + D_{66}\psi_{2,x} = -B_{12}\frac{u_{10,x}}{R} - \frac{D_{66}\psi_{1,\theta}}{R} - M_{x\theta}.$$

Similarly, from the equation: $Q_x = \left(A_{55} + \frac{B_{55}}{R}\right)\left(\psi_1 + w_{0,x}\right)$, we can say, $\left(A_{55} + \frac{B_{55}}{R}\right)w_{0,x} =$

$$-\left(A_{55} + \frac{B_{55}}{R}\right)\psi_1 - Q_x.$$

We will substitute these variables; you can see that there is θ derivative. Along the x-axis there will be one derivative.

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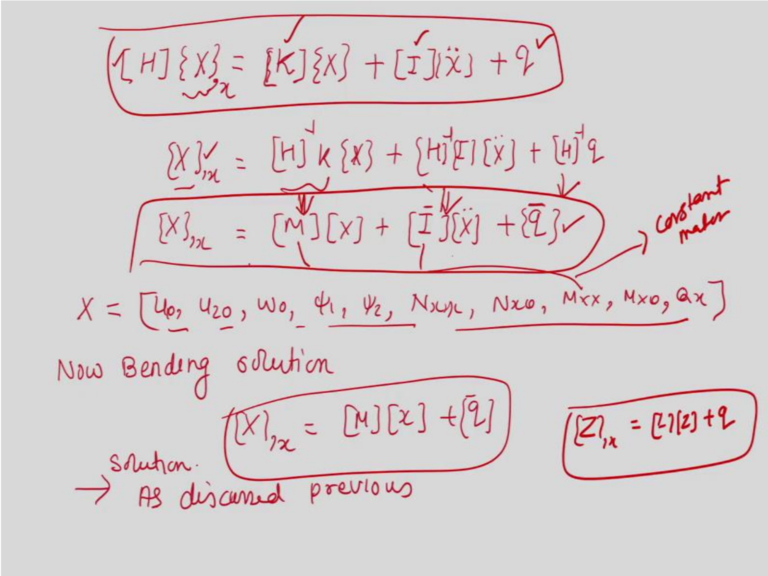

By doing all mathematical simplifications, we get a set of governing equations that looks like this: $[H]\{X\}_{,x} = [K]\{X\} + [I]\{\ddot{X}\} + q$. Let us say H is a matrix, the coefficient of $\{X\}_{,x}$ primary variables. In this case, here the coefficient of $[K]$ = 1, the coefficient of $[I]$ = 1, and the coefficient of $q = 1$.

Here, you can say that the coefficient is $A_{11} + \dfrac{B_{11}}{R}$ and here the coefficient is $B_{11} + \dfrac{D_{11}}{R}$. In this way, the matrix H is defined as a matrix K and inertia matrix I and a load vector q. Now, we can find $[X]_{,x}$.

$$[X]_{,x} = [H]^{-1}[I]\{\ddot{X}\} + [H]^{-1} q$$

This is the modified matrix:

$$[X]_{,x} = [M]\{X\} + [\bar{I}]\{\ddot{X}\} + \{\bar{q}\}.$$

Now, this is the first-order differential equation with non-homogeneous constant or non-homogeneous first-order differential equations, where M and I are constant matrices which is very important.

The solution is readily available. You know that if we are going to solve a bending case,

$X = u_{10}, u_{20}, w_0, \psi_1, \psi_2, N_{xx}, N_{x\theta}, M_{xx}, M_{x\theta}, \textit{and } Q_x$. For the case of a static solution, there will be no dynamic term. This equation $[X]_{,x} = [M][x] + [q]$ is similar to $[Z]_{,x} = [L][Z] + q$.

We can get this solution the same way, the only difference is that in this X we have a displacement as well as stress resultant. In the previous case, in the case of Z we have only displacement and Z_1, Z_2, Z_3, and so on, these are the second-order derivatives.

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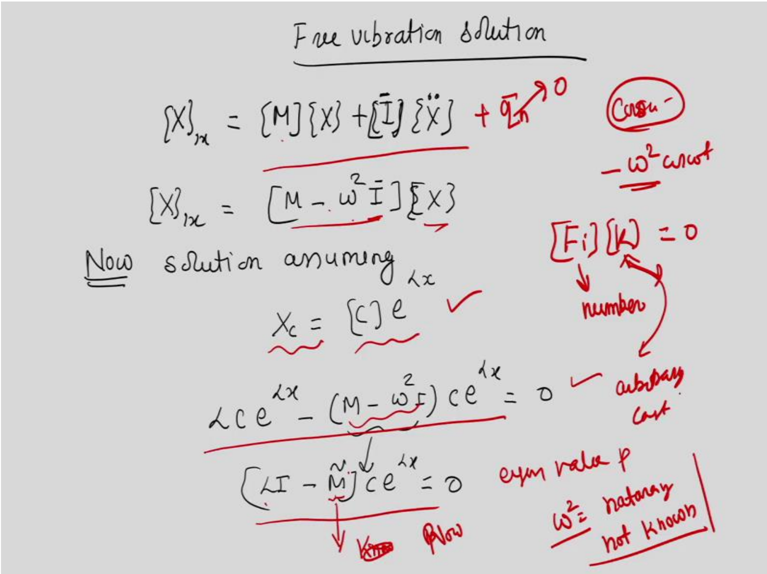

For the case of free vibration problem, $[X]_{,x} = [M]\{X\} + [\bar{I}]\{\ddot{X}\} + \bar{q}_n$, where $\bar{q}_n = 0$.

Only the time derivative we have assumed that cos ω t. If we take the double derivative, it becomes ω^2 and cos ω t.

Ultimately, $[X]_{,x} = \left[M - \omega^2 \bar{I}\right]\{X\}$.

Here, we can assume a solution in the same way as in the case of complementary solution. In a complementary form $X_c = [C]e^{\lambda x}$ and then substitute it here, it becomes an equation like this:

$$\lambda C e^{\lambda x} - \left(M - \omega^2 I\right) C e^{\lambda x} = 0 .$$

Again, we are going to modify that in this form: $\left[\lambda I - \tilde{M}\right] Ce^{\lambda x} = 0$, which is an eigenvalue problem. Here, the major difference from the static to this case is that in the previous case, $\tilde{M}$ was known to you.

Now, for the dynamic case, ω is the natural frequency or a fundamental frequency of the shell which is not known to us. We do not know what is the natural frequency of the system.

I will explain the basic idea to solve such kind of system. Let us assume that $\left[\lambda I - \tilde{M}\right] Ce^{\lambda x}$ = 0, this is an eigenvalue problem.

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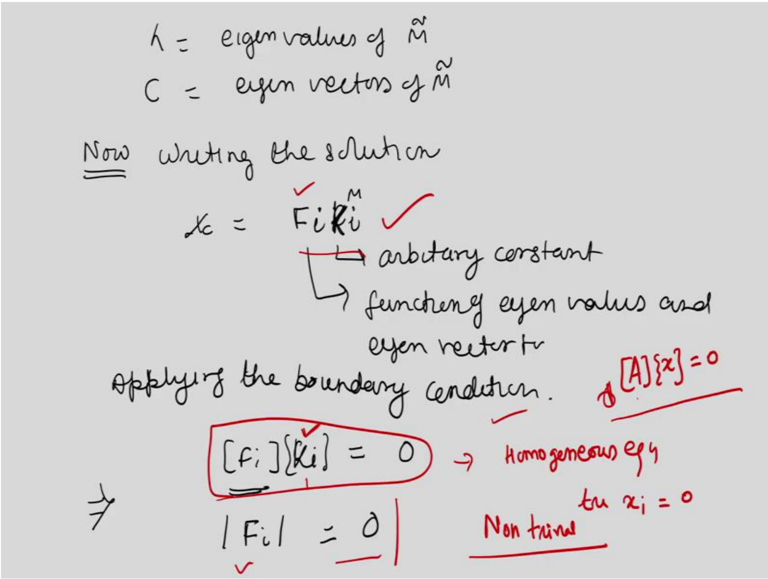

For that case, λ is the eigenvalue of $\tilde{M}$ and C is the eigenvector of $\tilde{M}$, then we can write the complementary solution $X_c = F_i K_i^m$, where K_i^m is our arbitrary constant and F_i is the function of eigenvalues and eigenvectors.

Then applying the boundary conditions, if we write a solution ultimately, either you apply a displacement-based boundary condition or a stress-based boundary condition. you say that right hand side that. If it is clamped then all variables are going to be 0, if it is free then also all variables are going to be 0.

It leads to this equation: $[F_i][K_i] = 0$.

This is a homogeneous equation. Let us say, $[A]\{x\}$ = 0, in that case, the trivial solution i.e., x_i = 0 and the non-trivial solution will be 0. If a system is having a non-trivial solution which means a unique solution and if it is a matrix then the determinant of that matrix $|F_i|$ = 0.

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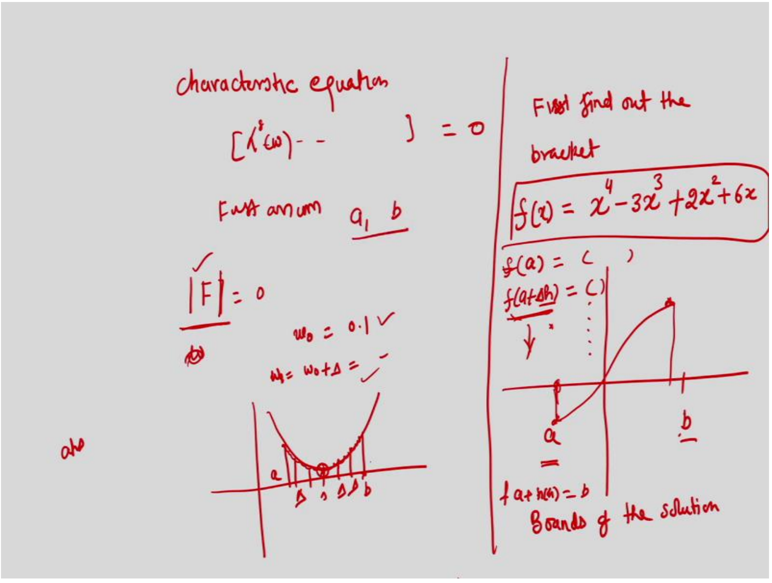

If we pose this condition, ultimately, it reduces to a characteristic equation. Though it will be in equation of λ^8 and something, ultimately it will be a function of ω that is going to be 0. How do we solve this kind of problem? This kind of problem can be solved. First, assume that ω is the solution.

Let us say, ω is the solution, then $|F_i|$ = 0, that is the check. We will first assume a solution; I will explain with the help of some mathematics.

Let us say, $f(x) = x^4 - 3x^3 + 2x^2 + 6x$.

We are interested to find the solution to this equation. First, find a and b that are known as bounds of the solutions. If we know the bounds of the solution then we can say that a function is having some negative value here and or in this zone or maybe here and positive value here.

It may go like this; it may have some 0 values. We will start from any value; we can take an initial guess. For the present case, we will take an initial guess from the simply supported case or we can start from 0.01.

Let us say, you have chosen some value $f(a+\Delta h)$ then evaluate it. Till you evaluate it will change the sign. When it changes the sign then you will assign that $fa+n(h)=b$, the function is changing the sign. For the present case, this matrix F = 0.

We will assume ω_0 = 0.01 and then ω_1 = ω_0 + Δ some functions and we are going to evaluate it.

We can notice the change in the sign. Whenever there will be a change in sign $-$ sign will come up or sometimes, we can find the absolute value. When there will be absolute value then it will form like this that this will be a and this will be b that from this function values are decreasing and then start increasing. When it starts increasing that value is taken as Δ.

If you take the absolute value, it will become like this that a minimum value we are interested to find will be the solution of our system. First, we will find the bounds of solutions a and b.

If you know the bounds of the system then the 90% problem is solved, because your Δ increment is very small. It depends upon you sometimes it will be 0.1, 0.02, or 0.05.

If the bound function is continuously varying there is no discontinuity then we can say that solution will exist between a and b. Now, we have different techniques. If it is a single variable; for the case of ω, we can apply the bisection technique.

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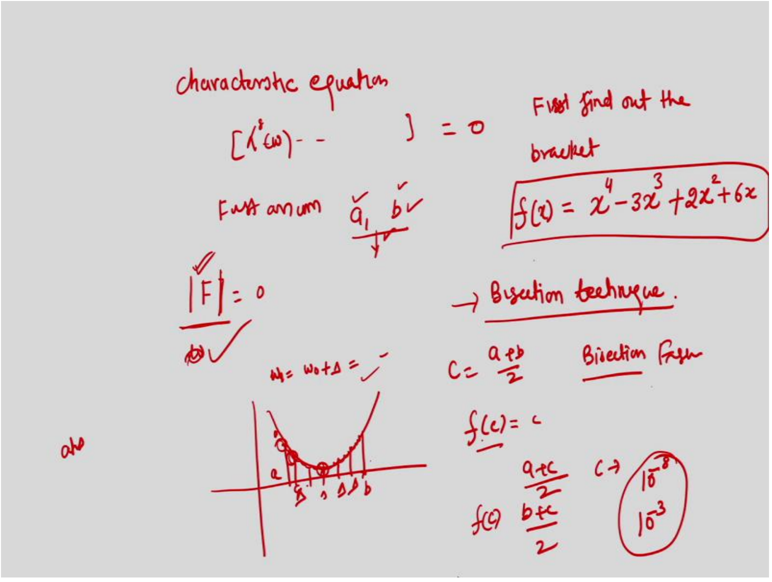

There you can say that another guess is $\frac{a+b}{2}$, average of that and you can evaluate the function value which means you can evaluate the determinant of that matrix and you can check the sign. If this c is coming here and this is going down again means this will be the guess. If it is going up, you have to take a as your starting point. Bisection technique is a very standard technique taught in undergraduate engineering mathematics.

Again, if you check the sign, it is negative or positive, then $\frac{a+c}{2}$ or $\frac{b+c}{2}$ depending upon the sign you can decide and calculate.

Ultimately, you can find a particular value c for which it will be a solution, but it is a numerical solution, it will not be directly 0 will be having some 10^{-8} or 10^{-3}, whatever you define the accuracy at 0 levels. If that meets your criteria, then you can say that frequency is found.

Once this frequency is found then you can start finding the second frequency. How to know whether this frequency corresponds to which mode, bending and other. In that case, once you know the frequency, we aim to find this K_i. By doing this, we obtained $|F_i| = 0$.

We obtained ω. Now ω is known to us. Now, we have $[F_i]$ matrix in which everything is a number. Previously, it was having ω and K = 0. From these K arbitrary constants, we can find using the method of least square or using the method of super inverse or pseudo inverse.

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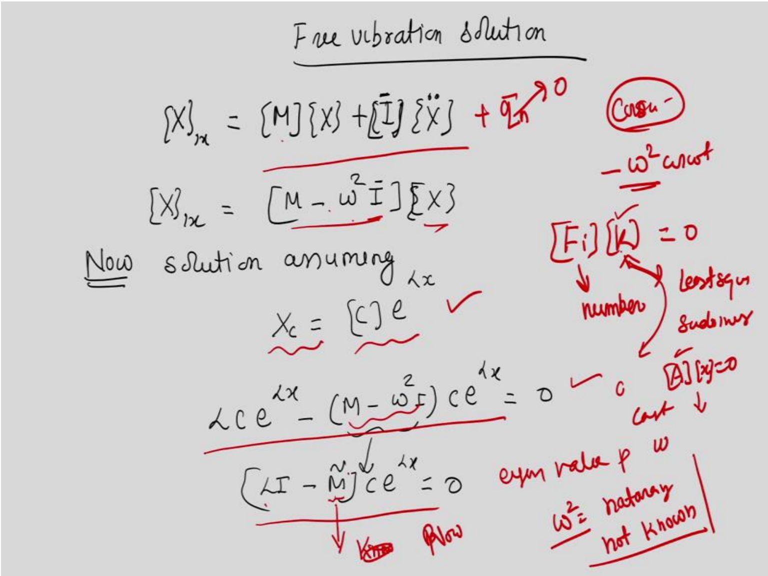

These methods are generally used to find the non-trivial solution of a homogeneous system of linear algebraic equations. We can find out this K, if you substitute in that value of K, you can find the mode shapes.

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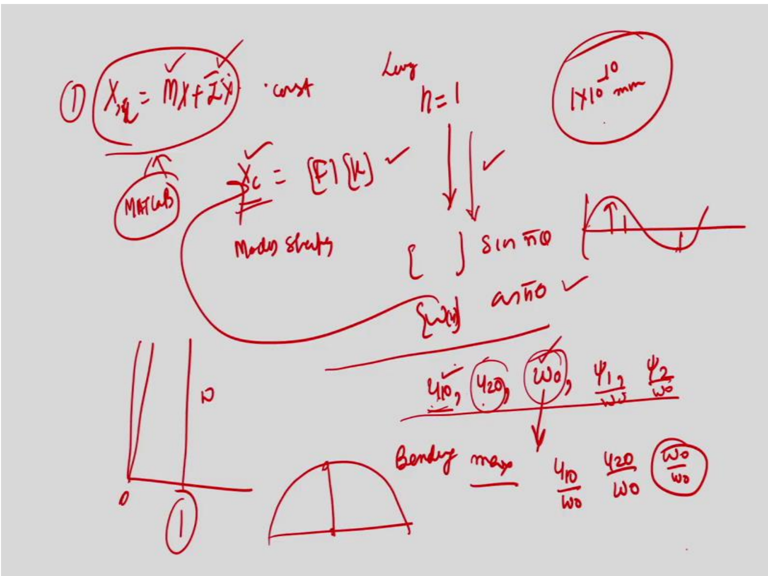

Once you find the value of all constants, then you can say $X_c = [F][K]$ and that will give you the mode shapes. Ultimately, you multiply with something $\sin\bar{n}\theta$ or $\cos\bar{n}\theta$ if it is $[w_{cx}]$.

Now depending upon the actual value of u_{10}, u_{20}, w_0, ψ_1, and ψ_2, we will check the maximum value of this. Out of these variables, only one will be maximum sometimes two will be maximum, depending upon the types of modes.

For the case of bending mode only w_0 is going to be maximum and other variables are going to be less. If you plot those mode shapes, w_0 will be maximum and others will be 0. The most important part of plotting mode shape is that you will divide other variables with w_0 value. For example, this deflection is coming 1 into 10^{-10} mm in terms of like that.

We divide with all these things. Ultimately $\frac{\omega_0}{\omega_0}$ becomes 1. You will find that in most of the cases the magnitude is showing 1. When you see that a mode shape of a shell such that then the magnitude is coming 1, because of that we divide non-dimensionalization again. If u_{20} is maximum, then you have to divide all the variables with u_{20}.

If u_{10} is maximum then you have to divide all the variables with the u_{10}. In this way, we can find the mode shapes and the frequencies. The most important part is that for the case of Levy for a particular value of n. Let us say n is equal to 1 you can find an infinite set of frequencies. Initially, you will get all bending frequencies because the bending frequency is the lowest frequency.

After that you will get stretching frequencies, then you will get the shear and coupling. All kinds of frequencies you can find. After doing the free vibration problem of a Levy support cylindrical shell, one can apply similar techniques to solve the problem of a spherical shell, conical shell or a doubly curved shell.

I have explained with the help of first-order differential equation. If you go for higher-order shell theories there you will have more numbers of equations and large numbers of simultaneous equations that can be solved. One more important part is that we got this equation $[X]_{,x} = MX + \bar{I}\ddot{X}$ and so on.

You need not go by the technique I explained, in MATLAB you can solve this ordinary differential equation. You can solve this ordinary differential equation in COSMOS or chromosome, you have to just provide a matrix $[M]$, matrix $[\bar{\psi}]$ and matrix $[\bar{M}]$.

From a theoretical point of view, you have to prepare a $[M]$ and $[I]$ very accurately with a proper sign with every consideration. Once you prepare the matrix $[M]$ and $[I]$, you can solve it by any technique, and these days we have some more techniques. Recently, I have gone through some research articles that shell problems are solved using DQM; the differential quadrature method.

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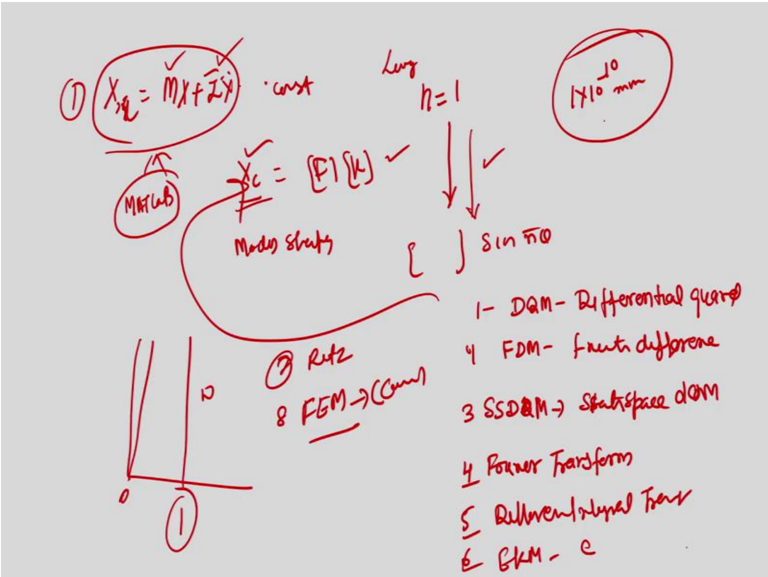

Then we have FDM which is an old technique finite differential method. Then SSDQM; state-space DQM, then we have a Fourier transform, then there is a differential integral transform and we have that EKM; extended Kantorovich method and then we have a technique Ritz and the very standard is finite element technique which is a general-purpose finite element method.

Generally, in all commercial software's these finite element techniques are applied. These are the techniques used to solve shell equations. The only issue with that is we have to prepare accurately the matrix $[M]$ or the matrix $[\bar{I}]$ or the matrix $q\bar{n}$, the rest of the part can be taken care. In this way, we can solve a Levy-type cylindrical shell or arbitrary supported shell using the EKM technique or the Navier type. The static and free vibration problem is solved.

In the next lectures, I shall explain the buckling of a cylindrical shell and the three-dimensional solutions of the cylindrical shells. I have developed the two-dimensional solutions, similarly, we have a three-dimensional solution where we do not assume anything. In the present case, we assume that our displacements u_{10}, u_{20}, or w_0 is varying linearly across the thickness.

But, in the three-dimensional case, we do not assume this, we directly solve the variation. Three-dimensional solutions act as benchmark solutions and these two-dimensional solutions are assessed or compared with the three-dimensional solutions for accuracy and because two-dimensional solutions are simple and these can be generalized for many cases.

We used to mostly develop the two-dimensional solutions for this shell. In the very starting, I explained that for the case of shell, for the general application, the radius to thickness is very less like 100 or 200. In that case, our first order or the classical shell theory is applicable, but when we have thick or composite shells there are some machine components where the shell is thick compared to its length and widths, we can use the higher-order shell theories.

The most important part is that in the first-order shell theory and classical shell theory, the concept of shear stress is not taken. But in the case of composites, though it is thin, still the concept of shear stress comes into the picture because of different material properties in each layer. For that case, we have to go for higher-order theories at least FSDT or TOT or some higher-order theories.

With this, I would like to say thank you very much.