Theory of Composite Shells

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Week – 01

Lecture – 02

Basic terminology in Shell

Welcome dear learners, to the course “Theory of Composite Shell”. Today is lecture 2 of week one.

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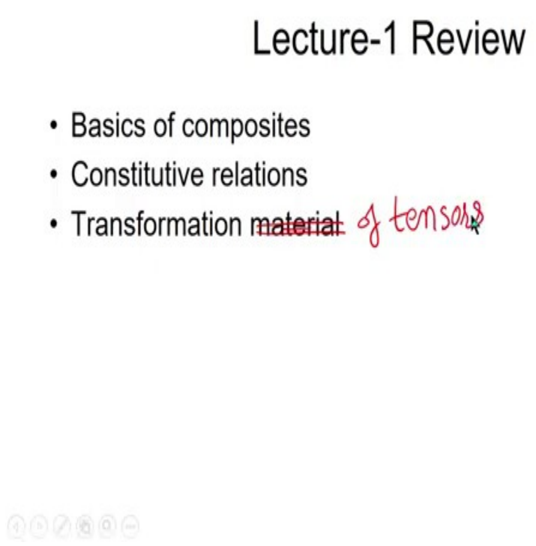

So, in the first lecture, I have explained, the basics of composites that the composites may be fibrous or the orientation of fibers maybe 0˚, 90˚, or angle apply, specifically for structural application. Then, I have discussed the constitutive relations and transformation materials. It is not a transformation material it is the transformation of tensors.

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So, there are extensive applications of shells in various fields i.e., civils, architecture, we have already discussed some of them. But, specifically large span roofs, liquid retaining structures like water tanks, containment shells of nuclear power plants, and concrete arch domes.

The reason to discuss this special or specific application is that because of its application in this course of the theory of shells. So, we can analyze these structures using the shell theories. In mechanical engineering, shell forms are used in piping systems. The piping system is a cylindrical shell, then the turbine disk, pressure vessels, like LPG storage tanks or oil tankers or any kind of pressure vessels.

The aircraft, missiles, rockets, ships, all are excellent examples of shell elements in aeronautical and marine engineering. If, you see that this is the dome-shaped roof of a famous building Infosys Pune, and then the famous Hawa Mahal in Jaipur. You can look into their windows and doors.

It is an arch type or dome type. In any ancient building, you will find that there is half arch form. So, it can take more load and it also looks beautiful, that is the reason it can take more load. If you have a flat system then we require more thickness, but if it is in a curved one with less material, it can take more stress.

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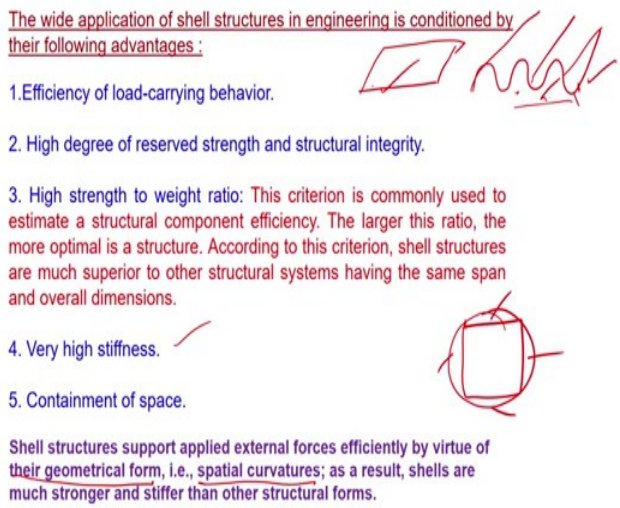

Why shell structures are used? It has excellent efficiency of load-carrying behavior. A very famous example, if this is a thin sheet say, a corrugated sheet when you have some structure of aluminum or steel for roofing purpose. This curvature makes them stiffer. Even the thickness remains the same, but for a roof of a structure, these corrugated sheets are preferred mostly as compared to flat sheets because flat sheets cannot take more load.

So, just because of the curvature, there have a more load-carrying capacity. Then, the high degree of reserved strength and structural integrity, the most important part is high strength to weight ratio, because it can take more load. So, we can go with thinner sheets, we do not need thick sheets. These have very high stiffness and the containment of space, basically a sphere you know that it can have maximum volume compared to other bounded volumes.

A square tank will hold a larger volume than just a cuboid structure. Shell structure supports applied external forces efficiently, by virtue of their geometrical form. The main reason is spatial curvature. If this curvature is not present, it cannot support. So, because of this curvature, these are stronger and stiffer than other structural forms.

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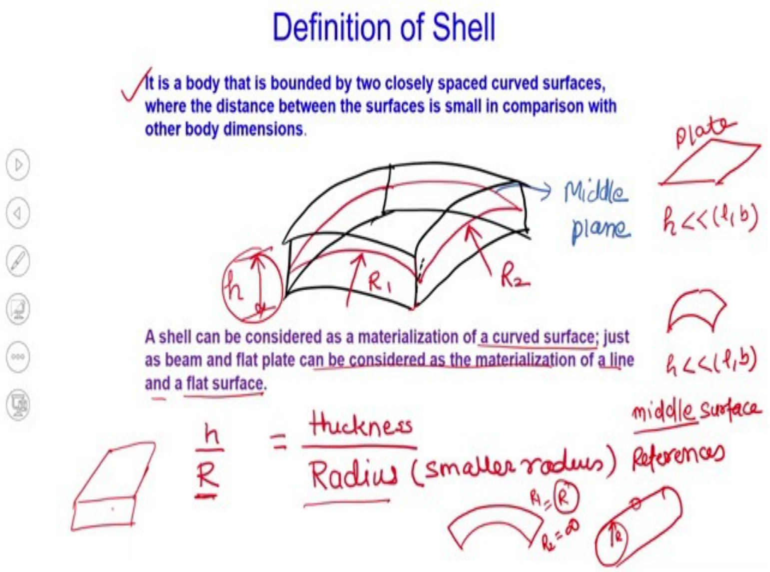

So, let us come to our topic, how do you define a shell? Sometimes students get confused between the plate and shell. We can say that both are two-dimensional or three-dimensional bodies, let us talk about thin shells or thin plates when the thickness is extremely less as compared to length and width. The same is true with the shell, that thickness is extremely less as compared to length and width.

But the important part is that in the case of the plate, the surface is flat, there is no curvature. But, in the case of a shell, there is curvature. Curvature may be in one direction or two directions. The shell theories or the governing equations are derived in terms of the middle surface or reference surface of the shell. That is the most important part in the case of shell theories. The middle surface is sometimes called a reference surface.

So, now the shell is defined in terms of a reference surface that, if a body that is bounded by two closely spaced curved surfaces, where the distance between the surfaces is small in comparison with the other body dimension. In this figure, you can see that red enclosement, it is a reference surface or middle surface.

If, you make another surface, let us say + h by 2, and bottom - h by 2, and close it, it will become a shell element. A shell can be considered as a materialization of a curved surface, a very essential component. It should be curved, its surface should be curved, it may be in one direction or maybe in two directions, but it must have a curve. Like beam and plate can be considered as a materialization of a line and flat surface. For example, if you say that in designing extrusion or revolution.

In the case of plate, there is a flat surface and then you can extrude it in the third direction. But for the case of a shell, it should have some curvature and then you extrude it, it will give you a finite element or a shell panel kind of thing. Then, how do you define if a particular shell is thin or thick? what are the bases?

So, this thickness is divided by the radius of the shell. Generally, if it is a double-curved surface, so, we will have two radii R_1 and R_2. Here, we are going to put the smaller radius. For example, if we have a cylinder, then in one direction the radius is R; the other surface is flat in that direction radius R_2 is an infinity flat line. So, that thinness can be find out through

$\frac{h}{R_0}$ or $\frac{h}{\infty}$.

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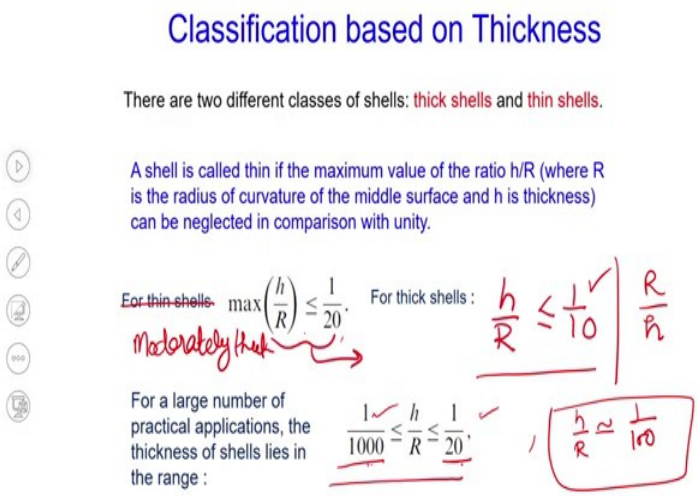

Let us see, how do you define a thin shell or a thick shell

If, $\frac{h}{R} \le \frac{1}{10}$, or vice versa

If you want to write $\frac{R}{h}$ 10, but generally we used to write in terms of $\frac{h}{R}$. If it is less than or equal to $\frac{1}{10}$, then it is considered as a thick shell.

For a thin shell, $\frac{1}{10} \leq \frac{h}{R} \leq \frac{1}{20}$ and like that. So, that is known as moderately thick.

For very thin, $\frac{1}{1000} \leq \frac{h}{R} \leq \frac{1}{20}$ So, the h by R ratio will be between 1000 and 20.

It has been found, that for a large number of practical applications, thickness of the shell lies between this. So, specifically, in most of the cases $\frac{h}{R} \simeq \frac{1}{100}$, that is the reason the thin shell theories are preferred mostly to analyze the behavior of shells.

But there may be some applications like machine components or turbine disks, where the thickness is not small, compared to its radius. Then, we can go for thick shells, but in nature and different applications, in 99 percent of cases, shells are thin. So, in the literature, many shell theories have been developed for the thin shell cases.

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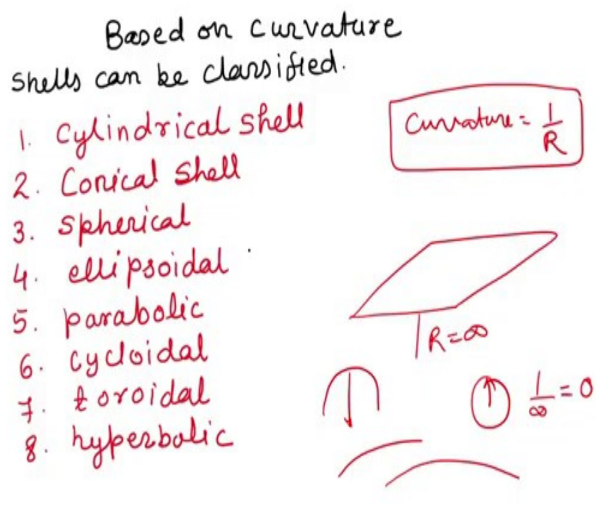

As I have discussed that based on the curvature or based on the radius of curvature. Curvature is nothing, but the inverse of the radius 1 upon by R. What does it tell? It tells you that how deep this is.

For example, in a flat surface, the radius is infinity. So, 1 by infinity gives you 0. So, it is a flat surface, if it is very far. it will be slightly shallow. If, it is near it will be deep like that circle, a very small circle. So, it tells you, the shell is a measure of deepness, whether the shell is deep, or shallow, or flat.

So, based on the curvature we can define, cylindrical shells, conical shells, spherical shells, ellipsoidal, parabolic, cycloidal, toroidal, and hyperbolic shells. So, these are different types of shells. So, in this course, we will develop generalized governing equations. So that we can get the special equations for all kinds of shells.

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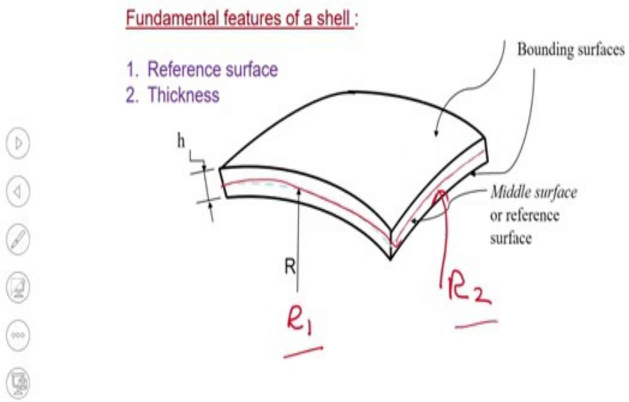

The fundamental features of a shell, the basic entities or the variables, very first is the reference surface. So, the blue line is the reference surface. So, shell geometry is defined by its reference surface. Whatever the form of the reference surface, just above and below we will add the thickness, and then the shell will be formed.

So, the shape or the curvature is given to the middle surface defines the shape of a shell, then the thickness of the shell. So, in one direction radius is R_1 and in the second direction radius is R_2. These top and bottom surfaces are known as bounding surfaces.

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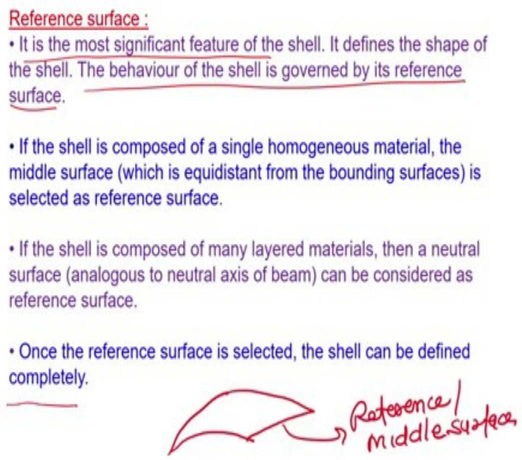

So, I would like to explain the reference surface, it is the most significant feature of the shell. It defines the shape of the shell. The behavior of the shell is governed by its reference surface. This is the important part if we are making some error for defining a reference surface, then the theory will not be accurate. So, you have to define the reference surface accurately.

If the shell is composed of a single homogeneous material, then the middle surface is selected as a reference surface. Shell is made of isotropic material and there is variation. So, we can say that the middle surface equidistance from the bounding surface is known as the reference surface. If a shell is composed of many-layered materials like composite materials, which is our study part here, then a neutral surface can be considered as a reference surface.

Once the reference surface is selected, then the shell can be defined completely. The very first step is to select a proper reference surface. For a cylindrical, conical, or spherical shell first, you have to define a proper reference surface, then you can proceed further.

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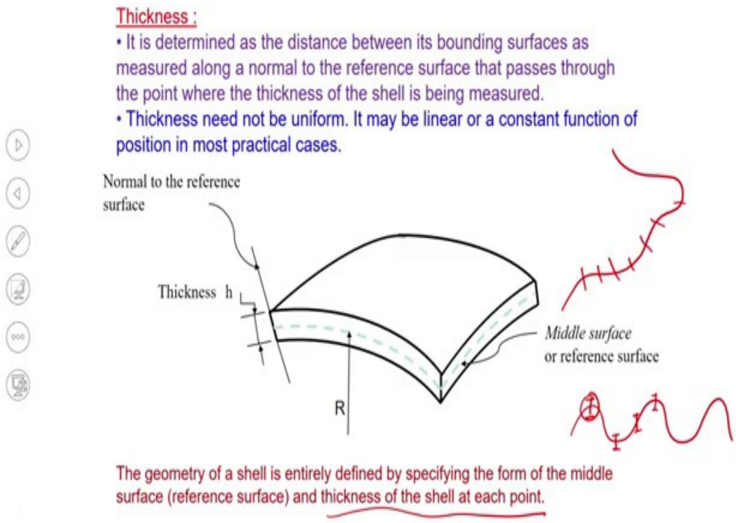

I have told you that what is the thickness? But I would like to emphasize that, it is determined as the distance between two bounding surfaces. It is not always that this thickness will remain the same, it has no boundation, it may change if we go from here to here, but from the middle surface upper bound and lower will be from equidistance.

Let us say, going from here to here the surface is going like this. So, the same way the bounding surfaces will be up and down. So, measured along a normal to the reference surface, that passes through a point, where the thickness of the shell is being measured. Thickness need not to be uniform. It may be linear or a constant function of position in most of the practical applications.

So, it is not that thickness is measured from other references from every point normal to that from two bounding surfaces this will be the thickness and here again and so on. The geometry of a shell is entirely defined by the middle surface and thickness of the shell at each point. We have to clearly define the changes in thickness. Either this is uniform or it is changing or it is a constant function it may be the same or it may not be.

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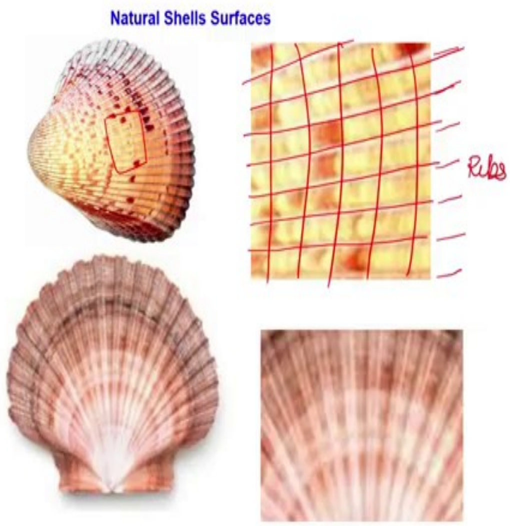

You see the figures of natural shell surfaces. These we call shells and you can see the different shell structures I have taken one patch and magnified it here. I want to show you that these are the lines curved lines, that later on, I will explain. If we want to study a shell, we have to prepare a reference element and these grids along with this, whatever the curvature will come. And, then we will define some parameters and finally, able to understand or develop governing equations.

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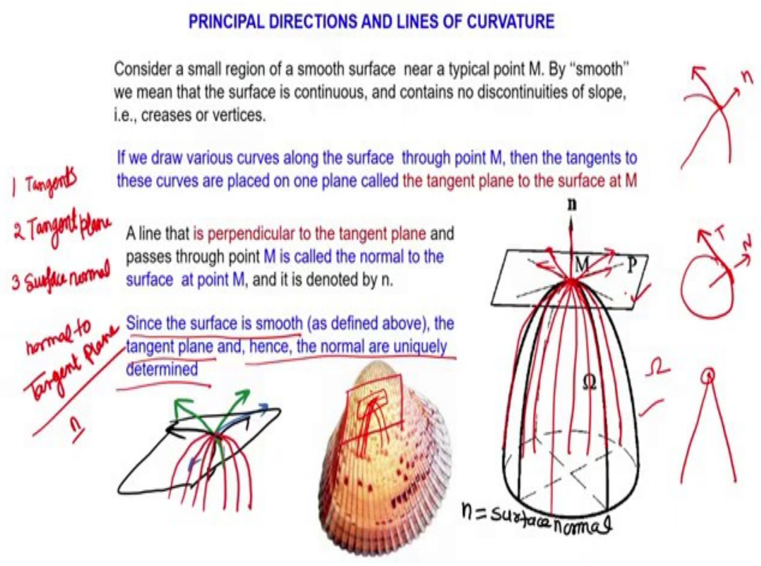

So, the very first concept is the principal directions or lines of curvature. Like here these are the lines of curvatures, which you see on the natural shells. Though they are acting as a stiffener, they are making the shell stiffer, because their thickness is slightly more as compared to base thickness. So, these are also known as lines of curvature or curvature lines.

Consider a small region of a smooth surface, let us say we take this surface. So, in this figure, a shell is described here and its domain is omega. So, there are lines like you see on natural surfaces. So many lines will pass through.

Let us say we have taken the top or you can take any point a point M. And, at this point the surface is "smooth". Then, what do you mean by smooth? It means, the surface is continuous, and there is no discontinuity of slopes, no vertices. We have to select an area, where it is smooth with no vertices, no discontinuity. If we talk about the cone, we cannot take it, because here the slope is changing. The surface is not continuous and this is making vertices.

So, we can draw various curves along the surface through this point. So, from this point, we can draw curves like this 1, 2, 3, and so on. Then, the tangent to this curve, let us say in a circle this is my line.

So, this is a curve and from the center, we are drawing a line. So, there will be two things, tangent to this, tangent lines, and one will be the normal. the Same thing is here over the surface for each line there will be one tangent, let us say for this line this tangent, for this line this tangent, and so on.

Then tangent to these curves is placed on one plane. And, that plane is known as the tangent plane to the surface at M. For example, if you take any point M here and through that you pass all the curvatures curves and find out the tangent planes. So, for a particular curved line, there will be one tangent and all the tangents for all the curved lines will lie in one plane. And, that plane is known as the tangent plane to this surface.

Now, a line perpendicular to this tangent plane is a very important concept in a shell, the concept of the surface normal, not just normal. In a curve, in a one-dimensional curve, or a two-dimensional case, it is the tangent and this is the normal direction, outward normal, or inward normal. But, in a case of a shell or a surface, it will be a tangent plane. And, a line perpendicular to this tangent plane will be denoted by n and it is known as the surface normal.

So, it is not normal because each curve will have one tangent and normal, but over this plane, there will be one normal. So, this is known as the surface normal. Since the surface is smooth and tangent plane; hence the normals are uniquely determined. There will be a single normal, not so many normals.

So, now I have discussed two things, one tangent plane, tangents, and surface normal. I am going to write it here, tangents, then tangent planes and third is surface normal or normal to tangent plane, it is denoted by n.

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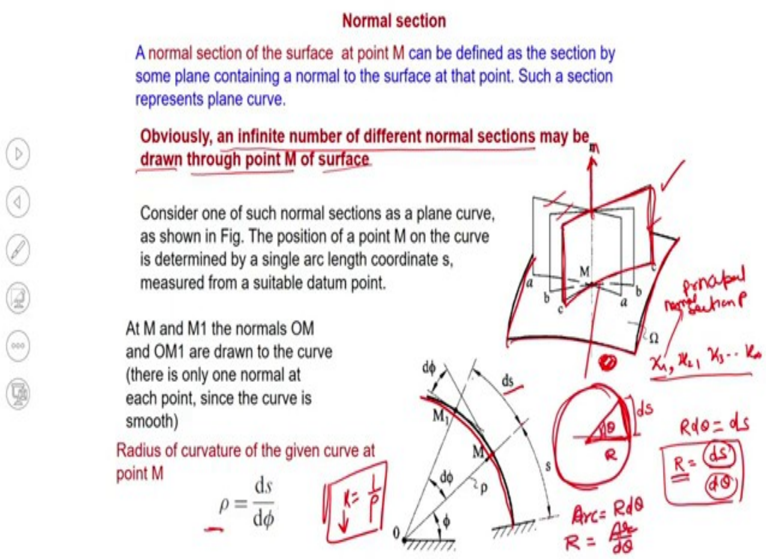

Now, we can say that this is one normal we can draw the sections, which means cutting sections, which passes through this normal. This is our surface, which I have drawn already. And, this is our normal n. Can we draw a section plane, which contains this normal? Yes, through the curved lines through which it is going, so, we can say that this is one of the section planes, which cuts that surface.

We can see that we can draw infinite normal sections, which will have the normal and cutting the surface. So, it is written that an infinite number of different normal sections may be drawn through the point of a figure. Here the concept of the radius of curvature is explained, then after that I will come to principal sections, principal normal section.

So, I would like to explain it here that for each normal section, there will be one radius of curvature. So, the radius of curvature is denoted by kappa (K). Let us say this for K_1, K_2, K_3, so on and K_∞. For each normal section, there will be a corresponding radius of curvature.

Now, the concept of the principal normal section. So, there will be at least two normal sections, for which one will be maximum and one will be minimum. So, the corresponding maximum radius of curvature will be known as principal sections.

So, before coming to this concept, the radius of curvature comes into the picture. How do you define a radius of curvature? So, first of all, we will see through a circle, let us say in a circle, it is our center and at any point, it goes from here to here this is our arc length and radius let us say it is dθ. So, Rdθ is your arc length which is ds.

So, $$R = \frac{ds}{d\theta}$$

It will give you the radius if somebody tells you the arc length and the subtended angle, that change in angle, then you can find out the radius.

So, in one dimensional, a curve is there and it is moving from point M to M' and making an angle dϕ. And, this change in angle from here to here will be dϕ and from here to here is s, and change in arc length is ds.

Then, rho $\rho = \frac{ds}{d\phi}$ ϕ, if we reverse it, it will be kappa (K) and $K = \frac{1}{\rho}$

So, kappa (K) is the radius of curvature inverse of the radius is known as the radius of curvature, which is written here as curvature.

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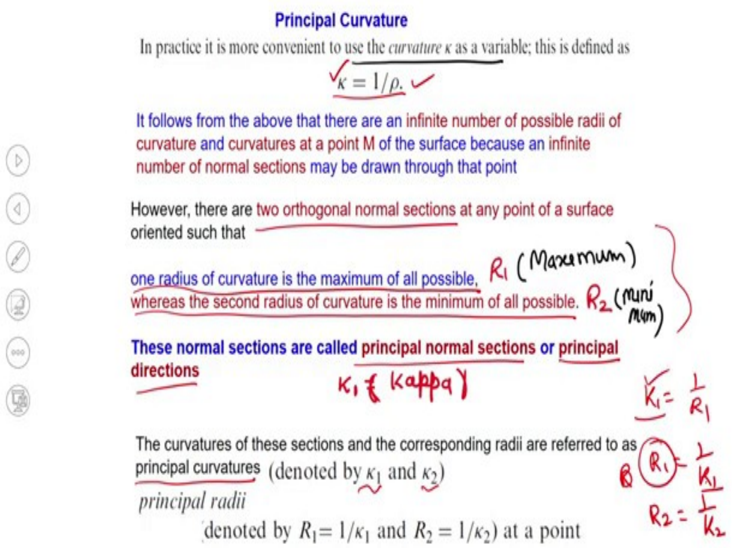

So, we do not call it a radius of curvature we call it curvature. And, previously we call it a radius of curvature means, this is the radius basically, it is the curvature, the radius of that curvature is ρ or in a circle it is R, it is $\frac{ds}{d\phi}$,

and the curvature measure of deepness which is $\frac{1}{\rho}$.

I have discussed that an infinite number of possible radii of the curvature and curvatures at a point M of the surfaces, because an infinite number of normal sections may be drawn. So, there will be two orthogonal normal sections means, perpendicular at any point of a surface such that one radius of curvature is the maximum of all possible whereas, the second radius of curvature is the minimum of all possible.

So, these normal sections are called principal normal sections or principal directions. Generally, the shell surfaces are defined in terms of principal normal sections, and principal directions, and curvature or radius of curvature. The curvature of these sections corresponding radii are referred to as principal curvatures R_1 and R_2 are the radius of curvatures and K_1 and K_2 are curvatures or sometimes we call them principal curvatures.

And, they are vice versa you can say that, if $K_1 = \frac{1}{R_1}$ then you can say that $R_1 = \frac{1}{K_1}$. If you know the curvature, you can find out the radius. If you know the radius then you can find out the curvature.

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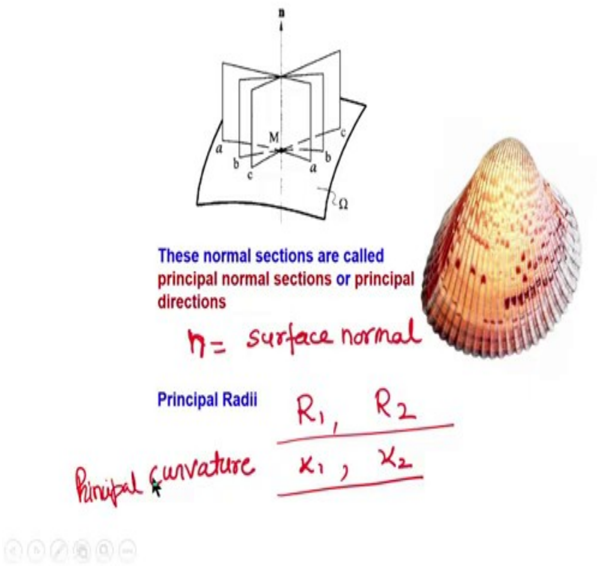

So, these normal sections are principal normal sections, principal radii, and principal curvatures.

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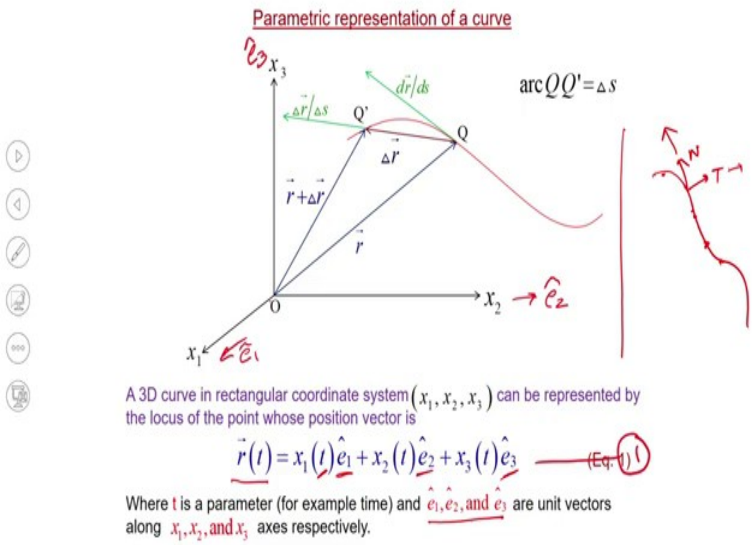

Now, we are trying to explain the parametric representation of a curve in three-dimensional coordinate systems; rectangular coordinate system. Let us say, the curve red line curve is moving and the original system is defined that x_1, x_2 and x_3, and it is moving from Q-to-Q'.

So, from the origin the position vector is $r + \Delta r$.

Then the position vector can be defined as $r(t) = x_1(t)\hat{e}_1 + x_2(t)\hat{e}_2 + x_3(t)\hat{e}_3$

In the case of curves basically, I think if you are at undergraduate levels, that **n** and **t** normal and tangent path system that, it changes with the path the curves positions are defined with the help of normals and tangents. So, at different locations, it varies.

So, the unit vector is associated with changes along the path. That is why it is not constant these are vectors these changes along the length.

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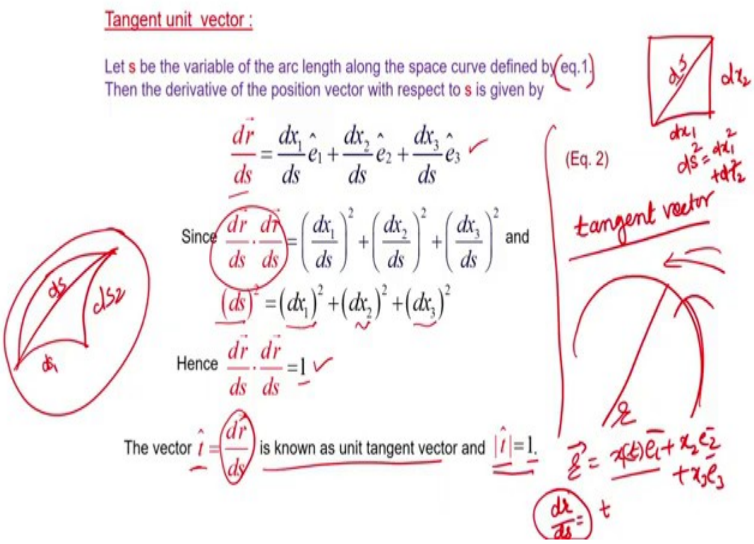

Their magnitude remains the same, but the direction changes. Then, the concept of the tangent unit vector.

So, $\frac{dr}{ds} = \frac{dx_1}{ds}\hat{e}_1 + \frac{dx_2}{ds}\hat{e}_2 + \frac{dx_3}{ds}\hat{e}_3$.

We are interested to find out the definition of the tangent vector.

Because, we have seen a curve whose position vector is defined in such a way that

$$r(t) = x_1(t)\hat{e}_1 + x_2(t)\hat{e}_2 + x_3(t)\hat{e}_3$$

you put position vectors and the $\frac{dr}{ds}$ you find it out, with respect to s it is our curve parameters. So, we are going to find out $\frac{dr}{ds}$, it is nothing but a tangent.

We are going to find it out by

$$\frac{dr}{ds}\cdot\frac{dr}{ds} = \left(\frac{dx_1}{ds}\right)^2 + \left(\frac{dx_2}{ds}\right)^2 + \left(\frac{dx_3}{ds}\right)^2$$

So, $(ds)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2$

Then, $\frac{dr}{ds} \cdot \frac{dr}{ds} = 1$.

We can say that this is known as the unit tangent vector, and the magnitude is 1.

It is nothing, but in two dimensional you see just for the case of a plate. If this length is dx_1, this length is dx_2, and this let us say ds. So, it will be $(ds)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2$. But, for the case of a curved one, it is not so simple.

So, you have to find out this. It will be again ds_1 and ds_2 and this is ds. So, dx_1, dx_2 in three-dimensional space, it is in two-dimensional case, same we can explain in terms of the 3-dimensional case. We are interested to find out this for a curved one, what will be the ds. So, before that, we must know what is that unit vector? It is defined by $\frac{dr}{ds}$.

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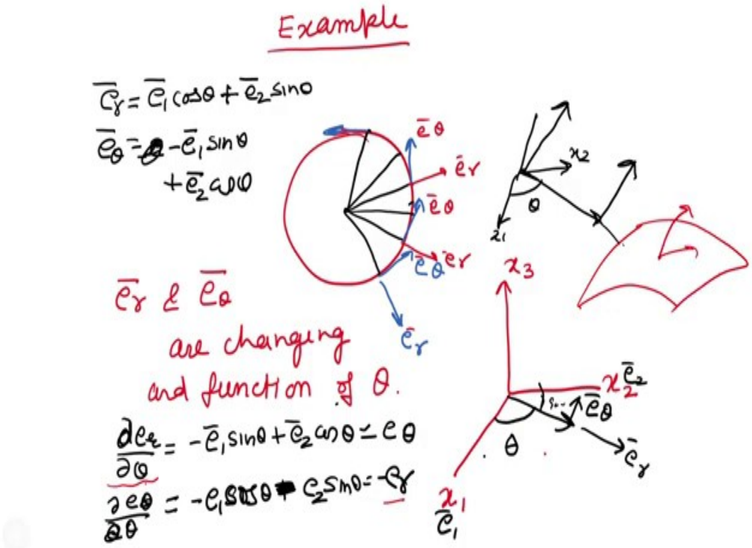

So, I have tried to explain that in the case of a circle that e_θ and e_r unit vector along the radial direction, unit vector along θ direction. So, these are the function of theta (θ) and it can change. So, $\frac{\partial e_r}{\partial \theta} = e_\theta$ and $\frac{\partial e_\theta}{\partial \theta} = -e_r$.

So, at the undergraduate level, it has been explained nicely for the case of a cylindrical coordinate system. I would like to say that similarly here for the case of shell surfaces, these have curvatures. So, we can define the tangent vectors 1 and 2

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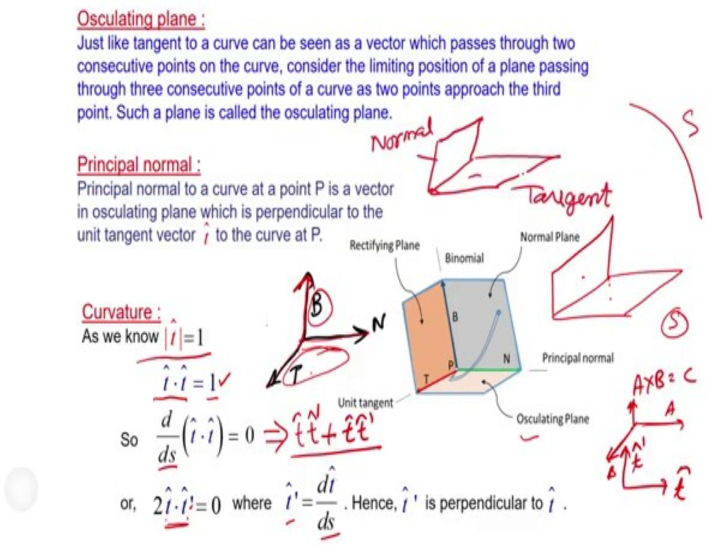

Now, the concept of osculating planes. For example, that you know we have defined a tangent plane, then we defined a normal section or a normal plane. So, perpendicular to this there will be another plane that is known as a bi-normal plane. If there is the direction in a right-hand thumb rule or like 2 vectors, a and b when you are saying cross product of that, it gives you direction c. In the perpendicular direction, it moves in that way.

So, the same way here instead of just two vectors here these are planes, one is a tangent plane and a normal plane, perpendicular to that there will be a bi-normal plane. And, this bi- normal plane, just opposite (mirror image the plane) will exist, that plane is known as osculating planes. This plane will touch all these three planes or the bottom mirror plane known as the kissing plane also.

Principal normal; already we have discussed that A normal corresponding to the maximum radius of curvature is known as principal normal. So, we know that $\left|\hat{t}\right| = 1$ and $\hat{t}.\hat{t} = 1$. So, if we differentiate this with respect to s.

$$\frac{d}{ds}\left(\hat{t}.\hat{t}\right) = 0$$

First of all, if I explicitly write that first function and the differentiation of second function and differentiation is denoted by just a dash plus first, the second function as it is and, the differentiation of the first function. So, basically $2\hat{t}.\hat{t}' = 0$, what does it mean?

It means $\hat{t}' = \frac{d\hat{t}}{ds}$, ($\hat{t}$) is perpendicular to ($\hat{t}'$).

So, $\hat{t}'$ is nothing, but it is d$\hat{t}'$ by ds. So, tangent, tangent vector, perpendicular vector exists that is $\hat{t}'$ We are going to use this type of basics later on, initially we have to derive the basic relations. The meaning of $\hat{t}$ with respect to the curve. So, if s is a curve length basically along x direction and s is moving.

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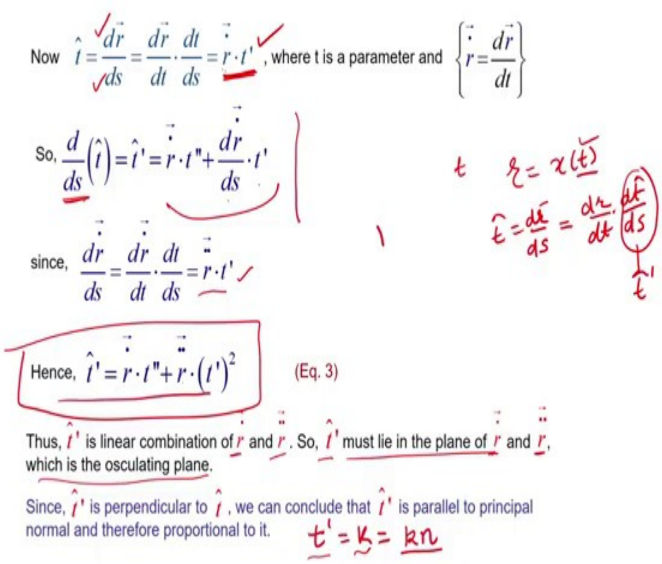

Then, what is $\hat{t}$? $\hat{t} = \frac{dr}{ds}$, which was initially defined, then after that, we have defined, $\hat{t} = \frac{dr}{ds}$. If you know $r = x(t)$.

So, $\hat{t} = \frac{dr}{ds}$, but here r is function of t and using the chain rule we can say that

$$\hat{t} = \frac{dr}{ds} = \frac{dr}{dt}.\frac{dt}{ds} = \dot{r}.t'$$

We can say that dr by dt will be time derivative or parameter derivative of r dot t dash. So, then we are differentiating this with respect to **s**. So, we will say that,

$$\frac{d}{ds}\left(\hat{t}\right) = \hat{t}' = \dot{\vec{r}}.t' + \frac{d\dot{r}}{ds}.t'$$.

So, from there dr by ds again find out $\ddot{r}.\left(t'\right)$, finally, we got this equation,

$$\hat{t}' = \dot{r}.t'' + \ddot{r}.\left(t'\right)^2$$.

The reason for deriving this equation is that it will help us to find out the normal curvature or the definition of curvature. How do you define a curvature in terms of a normal and some variable?

The differentiation of tangent vector **t** with respect to **s** can be represented at second differentiation and first differentiation $\dot{r}$ and $\dot{r}$. So, it is showing that the meaning of this equation (3) is $\hat{t}'$, a linear combination of $\dot{r}$ and $\ddot{r}$. $\hat{t}'$ must lie in the plane of $\dot{r}$ and $\ddot{r}$ which is an osculating plane. Here, we can say that $\hat{t}'$ is related to a principal normal and curvature, it can be written as K (kappa) function and normal unit normal.

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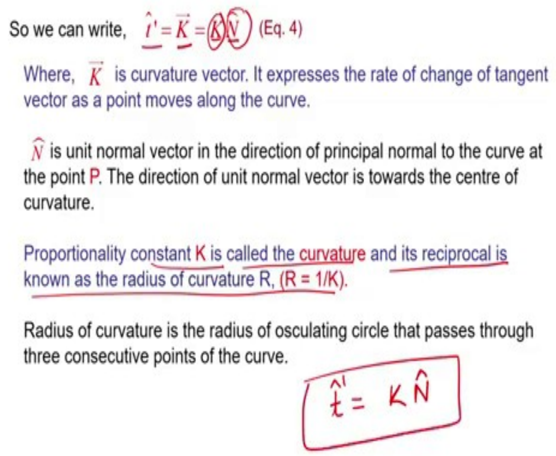

So, in the next slide, it is clearly explained, $\hat{t}'$ can be represented as

$$\hat{t}' = K = K\hat{N}$$

K is a curvature vector and $K\hat{N}$, here $\hat{N}$ is a unit normal vector. So, the direction of principal normal to this curve at a point is defined.

K is called curvature and its reciprocal is known as the radius of curvature. So, it is called curvature. So, $\hat{t}'$ can be written as curvature into the unit normal vector. It is a very important relation; we will use it later on.

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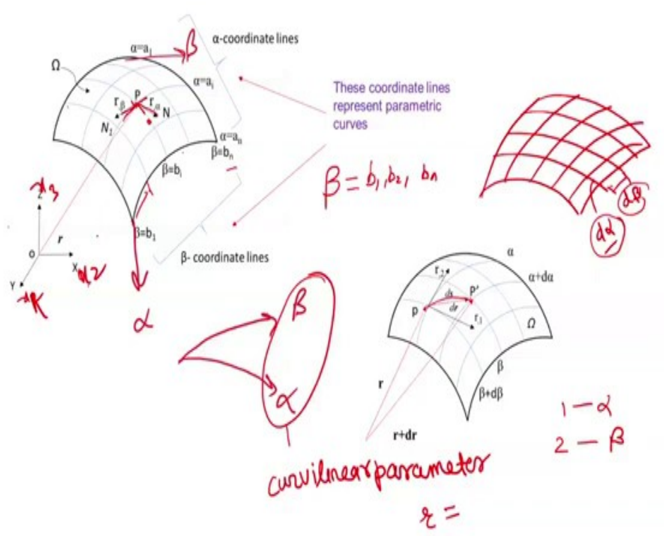

Now, define a shell element, which is having curvatures in both directions. And, alpha (α) and beta (β) are curvilinear parameters; these are the sub curvilinear parameters. So, from any reference, you can choose a point P and the most important part make a grid along the curves.

So, let us say from here to here this distance is b to b_1, b_2, b_3 and b_n. So, β curvilinear coordinate takes value from b_1, b_2, up to b_n. Similarly, the curvature lines α we are drawing these lines so that a grid can be made over there, over the surface. So, these are the α lines that take value from α_1, α_2, α_3, and so on to α_n.

So, you draw the line parallel to this or like that. So, let us say this will be dβ and this will be dα, change in small α, change in small β. So, at a point, p corresponding to that there will be two meeting curves. So, corresponding to this curve, there will be one tangent, and corresponding to this there will be one tangent.

So, it is clear. from a position r that we can define like this, but if we take a small increment, let us say this point P to it moves to P', then we are interested to find out this ds. So, the radius of curvature r,1; basically 1 means α and 2 means β. So, here it is written as r,α and r,β, sometimes it can be written as r,1 and r ,2. So, from position vector is r and position vector r plus tr. So, can we define that r or dr.

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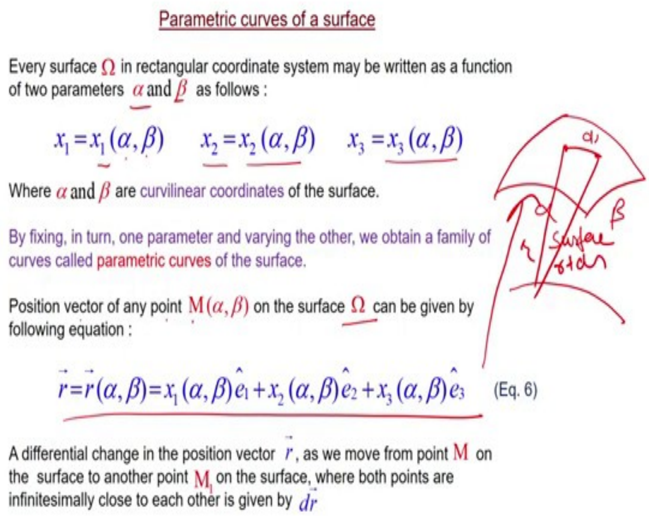

So, every surface in the rectangular coordinate system may be written in two parameters α and β.

Here it can be written as $x_1 = x_1(\alpha,\beta)$ $x_2 = x_2(\alpha,\beta)$ $x_3 = x_3(\alpha,\beta)$

So, we can say that x is equal to x_1 which is a function of α and β. If you remember, previously we said that for a single curve x is a function of t. Now, I am saying for a surface this x is a function of α and β, x_2 is also a function of α and β, x_3 is also a function of α and β So, the position vector of any point at M on this surface can be given by this equation.

$$r = r(\alpha,\beta) = x_1(\alpha,\beta)\hat{e}_1 + x_2(\alpha,\beta)\hat{e}_2 + x_3(\alpha,\beta)\hat{e}_3$$

So, first, we have defined a position vector for a single curve, now we have defined a position vector for a surface. Now, a differential change in position r hat, so, let us see it move from 0.1 to 0.2 change in that dr position vector r plus dr. We are interested to find out what is dr first?

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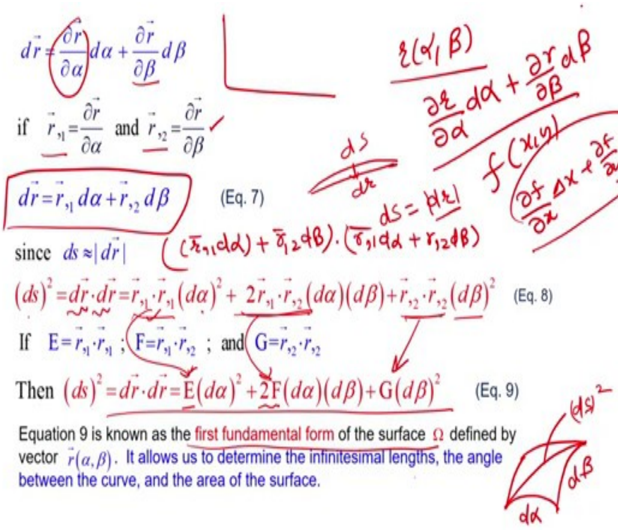

So, dr can be find out through

$$dr = \frac{\partial r}{\partial \alpha} d\alpha + \frac{\partial r}{\partial \beta} d\beta$$

It is very simple. So, dr using the chain rule of differential, $r(\alpha, \beta)$ can be found out by :

$$\frac{\partial r}{\partial \alpha} \partial \alpha + \frac{\partial r}{\partial \beta} \partial \beta \quad d\beta$$.

Same way $f(x, y)$ can be found out by: $\frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y$

So, dr can be written like this. Where, $\frac{\partial r}{\partial \alpha} = \vec{r}_{,1}$ and $\frac{\partial r}{\partial \beta} = \vec{r}_{,2}$

So, it is the new equation

$$dr = r_{,1}\, d\alpha + r_{,2}\, d\beta$$

This is our ds and this is our dr. So, we say that for the smallest one, it is going to take the same.

So, $ds \approx |dr|$ which means,

$$(ds)^2 = dr.dr = r_{,1}.r_{,1}(d\alpha)^2 + 2r_{,1}.r_{,2}(d\alpha)(d\beta) + r_{,2}.r_{,2}(d\beta)^2$$

This quantity is replaced by E ($E = r_{,1}.r_{,1}$). And, this quantity is replaced by F ($F = r_{,1}.r_{,2}$),

and this term is replaced by G ($G = r_{,2}.r_{,2}$). So, this equation (9) is known as the first form of fundamental surfaces, it allows us to determine the small change in the length and angle between the curves and area of the surfaces, which is the very first equation of the surfaces.

Basically, what does it tells let us say these are the double-curved surface and this is $d\alpha$, we have moved very small, and what is this change in arc ds square?

So, $(ds)^2 = dr.dr = E(d\alpha)^2 + 2F(d\alpha)(d\beta) + G(d\beta)^2$.

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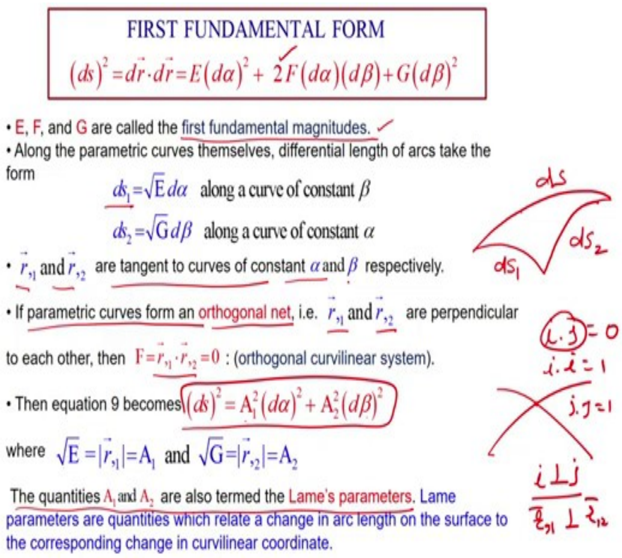

Again, if we say that E F G is called the first fundamental magnitudes. So, ds_1, that arc length along one direction $ds_1 = \sqrt{E}d\alpha$. So, over this the arc length ds_1 and ds_2.Arc length in one direction, arc length in the second direction, can be represented by $\sqrt{E}d\alpha$. Arc length in the second direction can be represented by $ds_2 = \sqrt{G}d\beta$. $r_{,1}$ and $r_{,2}$ are the tangent to the curves of the constant α and β so basically for these curves if you make a grid, over that α and β.

If the parametric curves form an orthogonal net then $r_{,1}$ is perpendicular to $r_{,2}$, then this 2 F will be 0, because $, F = r_{,1} . r_{,2} = 0$. If you talk about rectangular coordinate system, i.j is 0, i.i is 1, similarly j.j is 1. So, i.j is 0, it is 0 because when i is perpendicular to j. Same way, if tangent vector 1 is perpendicular to tangent vector 2, then we can say F is 0 and it reduces to a very simple form 2 dimensional.

For the orthogonal system: $(ds)^2 = A_1^2(d\alpha)^2 + A_2^2(d\beta)^2$.

Where $\sqrt{E} = |r_{,1}| = A_1$ and $\sqrt{G} = |r_{,2}| = A_2$. Because, it is defined in such a way that A_1 square, A_2 square. These A_1 and A_2 are lame's parameters. Sometimes, we call it distortion parameters, why we call it distortion parameters you see that if it is a flat case.

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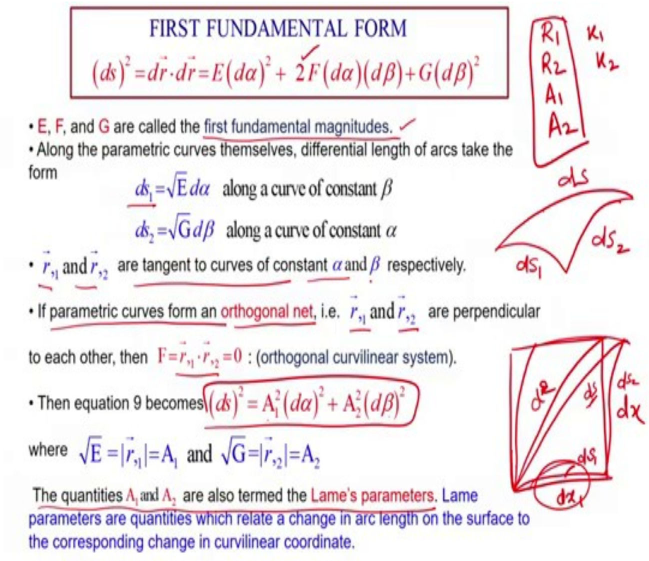

Let us say dx_1 and dx_2. Now, you are saying dx_1 is slightly modified like that, changing something. So, we are interested to that this change is changing basically arc length along s_1, arc length along s_2. So, this is your ds and this is your dr.

So, this tells you this distortion in this case is A_1. A_1 is a distortion parameter or measure of distortion, A_2 is also a measure of distortion arc length along the second direction. So, we can find out the arc length of a curved surface. And, A_1 and A_2 are very important parameters to define the shell surfaces.

So, the governing equations or the shell theories are developed in terms of r_1, r_2, A_1, and A_2 these are the four parameters or sometimes you may say that K_1 and K_2. So, we need a radius of curvature, then we need lame parameters. So, lame parameters are found out through the first fundamental form of surfaces, and if our curvilinear parameters are orthogonal. So, it may be orthogonal it may not be orthogonal. Generally, for the structural application, we use that orthogonal curvilinear system, where this F can be 0.

So, with this, I end our second lecture. In the next lecture, I will explain the second fundamental of surfaces which is related to the curvature.

Thank you.