**Theory of Composite Shells**

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**Week - 08**

**Lecture - 01**

**Development of three-dimensional solution**

Dear learners welcome to the course Theory of Composite Shell, week-08, lecture-01. In this lecture, I shall explain the Development of a three-dimensional solution for a cylindrical shell.

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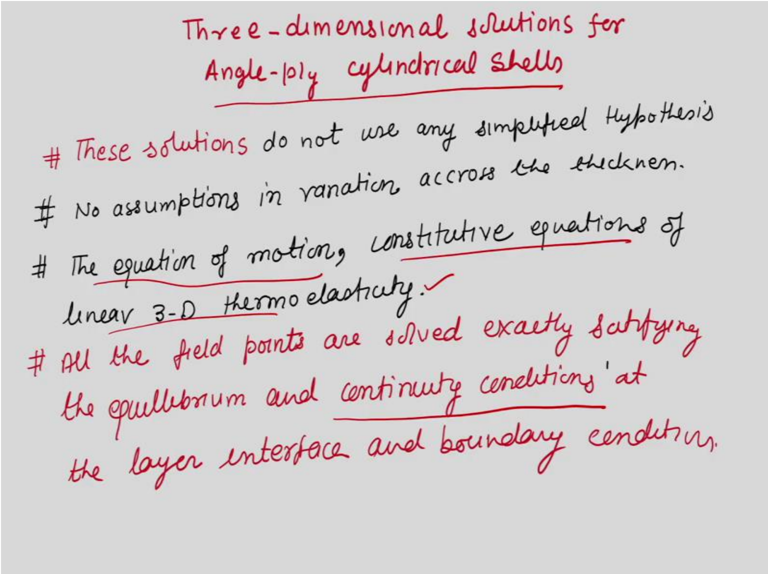


In the previous lectures, I explained the solution of doubly curved shells and singly curved shells, using the first-order shear deformation theories.

We may have higher-order shear deformation theories or these days we call refined shell theory. Every year researchers develop a new shell theory that predicts the special shell behaviours or some special applications.

In two-dimensional theories, we assume the displacement field; if we talk about a first-order shear deformation case, we assume a displacement field is linear varying across the thickness. We are assuming that u, v, w follows a linear variation; w is constant along the thickness, and u and v follow linear variation along the thickness. If we talk about a third-order shell theory or higher-order shell theory, we assume that the displacement follows a cubic or trigonometric variation across the thickness.

Two-dimensional shell theories have many advantages and are very easy to implement their corresponding finite element solution, finite different solution, or the DQM solution. Apart from the two-dimensional shell theories, there are three-dimensional shell theories. The purpose of developing the three-dimensional shell theory is that it gives a very accurate estimation of all the stresses and displacement.

The important part is we do not take any assumptions along the thickness or how the displacement will vary. From the three-dimensional solutions, we get how displacement varies along the thickness, the shear stresses at the interfaces, the slope or the exact magnitude of the shear stresses.

Through three-dimensional solutions, we obtain these stresses. And these three-dimensional solutions act as benchmark solutions which means we can assess the accuracy of other two-dimensional solutions by comparing their results with the three-dimensional solutions. In this way, three-dimensional solutions are used and they act as a benchmark.

Even some of you may use the commercially available software based on the finite element, in that software also if you are modelling the first time you have to validate because just by tinkering with one thing you will get some result. But who will tell the results you are getting through that software are right or wrong?

We have to first validate our results or model with our existing literature, whether it is two-dimensional or three-dimensional solutions and then we can proceed with a complex case. In this lecture, I shall explain an angle ply cylindrical shell. Here, we will use the equation of motion, constitutive equations of the linear 3D thermoelasticity case.

I will explain under thermoelasticity loading that the shell is subjected to mechanical as well as thermal loading. Just to give the feel, we can also develop shell solutions for the thermal loading in the previous solutions are also valid for shell solutions, where we can take temperature as a loading variable.

All the field points are solved exactly satisfying the equilibrium and continuity conditions. One major point here is in the case of two-dimensional solutions we assume that layers are perfectly bonded. But we do not satisfy any interface continuity conditions in two-dimension because we just integrate over the thickness and we find the effective value of $Q_{ij}$ stiffness or shell constitutive relations constants $A_{11}A_{12}A_{22}$. But in the present case, each layer has variables.

If we have a 10 layer, at the interface by applying a concept of perfect bonding we satisfy the interface continuity conditions and we solve the exact variation.

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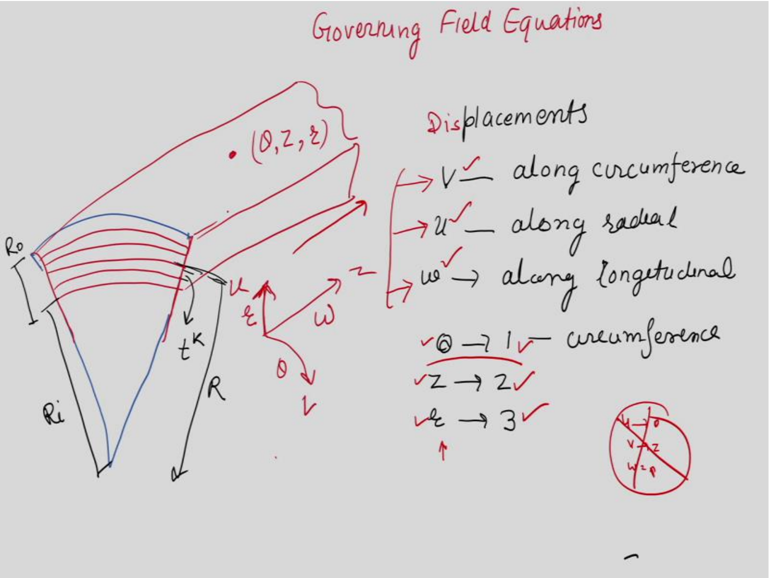


This is the geometry along the z-direction it is very long. $\theta$ along the circumferential direction, r along the thickness direction, and z along the longitudinal direction.

Our coordinate system is $\theta$, z, and r;

where, $\theta$ = 1, z = 2, and r = 3

Because $\theta$ is the circumferential direction here the displacement is denoted as v.

A radial direction is thickness direction, here the displacement is denoted as u. Along the longitudinal direction, displacement is denoted as w.

Do not get confused with the displacement field denotations. I have tried to follow one of my Ph.D. thesis papers so that in conformance with that v, u, and w, but in the present course we have taken that u along $\theta$, v along z, and w along radial directions, but for the present three-dimensional solutions we are taking v along the circumference, u along the radial, and w along the longitudinal direction.

This is also important whenever you are going to read any journal article or a paper, you first see that the displacement field exists because anybody can take any notation, I will take u along the x-axis or v along the x-axis. It depends upon the displacement field.

Sometimes most researchers, scholars, and students make mistakes, they do not go through the paper seriously and try to validate their results and found that results are not matching or match with a different variable. We have to be very much careful about the displacement variables along the coordinate axis.

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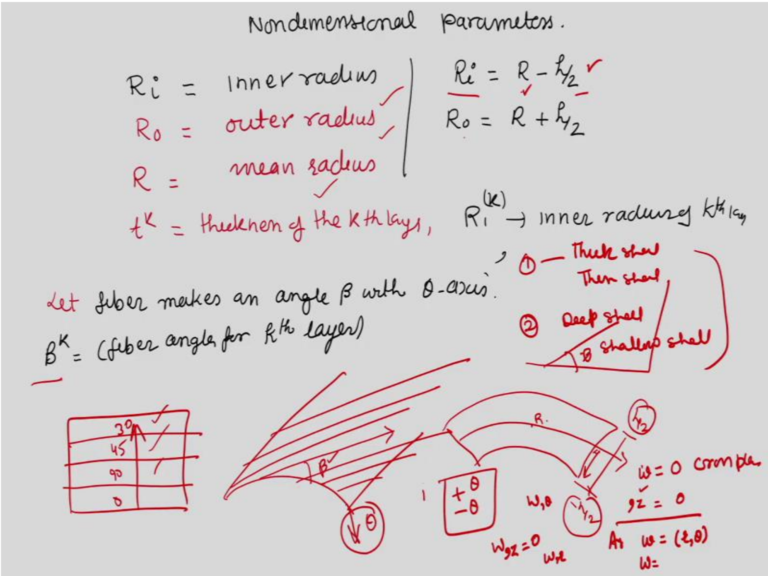


Then $R_i$ is denoted as an inner radius, $R_0$ is denoted as an outer radius, R is a mean radius. If the shell panel thickness is $-\frac{h}{2}$ to $+\frac{h}{2}$ coordinate system is such that then,

$$R_i = \mathrm{R} - \frac{h}{2} \text{ and } R_0 = \mathrm{R} + \frac{h}{2}.$$

Because we are talking about it is an angle ply cylindrical shell, therefore, fiber angle that it makes an angle $\theta$ or $\beta$ with respect to $\theta$ coordinate. These are the fibers making an angle $\beta^{\circ}$ with respect $\theta$ axis. In the case of a plate, it makes with the x-axis. Our angle is $\beta$ and it is denoted as $B^k$, each layer may have a different orientation.

For example, we may say that our layup may be 0˚, 90˚, 45˚, 30˚, and so on. If a ply layup contains angles only 0 and 90˚, then it is known as a cross ply shell panel. And if it has a combination of $+\theta$ and $-\theta^{\circ}$, then it is known as a symmetric angle ply layoff.

And if it has any angle of 30˚, 45˚, 90˚, 0˚, then it is known as an asymmetric angle ply layer. The present formulation is valid for any kind of layups, it may be symmetric, it may be anti-symmetric or it may be an asymmetric layup. In some of the research papers that even in the title itself it is written that it is symmetric angle ply laminates or anti-symmetric angle ply laminates.

Initially, they take formulation in such a way that they do not consider the coupling. But the present formulation is valid for thick shells as well as thin shells, then deep shells as well as shallow shells.

If you talk about a two-dimensional shell theory; some formulations are developed only for deep shells, some are developed for shallow shells, some gives very accurate solution for thin shells, and some gives solutions for thick shells.

But the present three-dimensional solution is valid for a thick, thin, deep as well as shallow shells angle ply or cross-ply, we can get all the results. We assumed that along z-direction panel is very long, it is a case of generalized plane strain case.

And it is an angle ply, if it will be a cross-ply, w = 0 for a cross-ply case and the derivative along the z-direction is going to vanish. But for the case of an angle ply, we say w is a function of r and $\theta$ direction and it is a constant in the z-direction.

Therefore, $w,_z$ = 0, but $w,_r$ and $w,_\theta$ will exist. This is the major difference when we are going to develop a solution for angle ply shell panels. That w is not 0 here means the deflection along the longitudinal direction.

If you remember in the previous case, when we studied using the two-dimensional shell theory, we assumed that the deflection along the z-direction = 0, but for the case of an angle ply, it cannot be 0. We take w is a function of r and $\theta$, it is not a function of z-direction.

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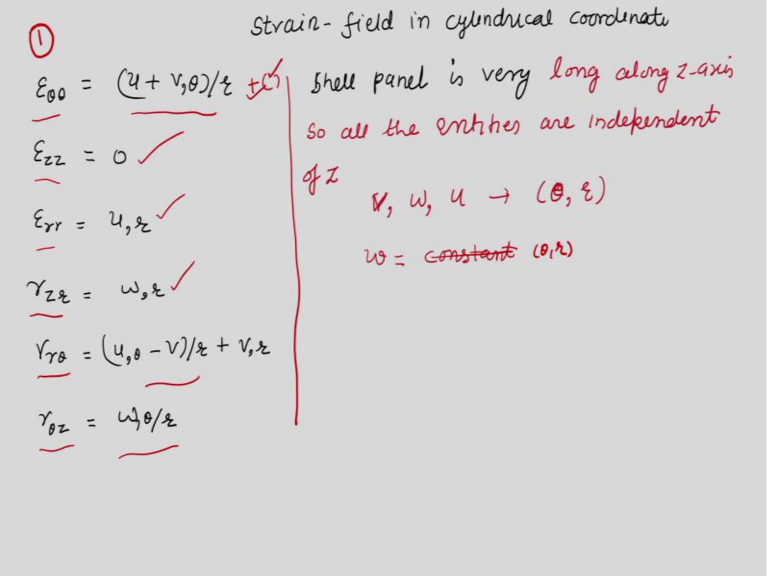


The very first step is to find the suitable strain field relations in a cylindrical coordinate system, and identify the suitable strain field by considering the assumption that all the entities are independent of z and w is a function of $\theta$ and r.

When w (r, $\theta$), then

$$\varepsilon_{\theta\theta} = \frac{(u + v,_{\theta})}{r} + ()$$

$$\varepsilon_{zz} = 0$$

$$\varepsilon_{rr} = u,_{r}$$

$$\gamma_{zr} = w,_{r}$$

$$\gamma_{r\theta} = \frac{(u,_{\theta} - v)}{r} + v,_{r}$$

$$\gamma_{\theta z} = \frac{w,_{\theta}}{r}.$$

Readers or learners if you want to develop a spherical shell panel; obviously, you have to find out a suitable strain displacement field. If you developed for different like a conical shell panel then you have to identify a suitable strain field. In this lecture, I am going to explain to you the state of the art. Depending upon your requirement you can use these equations.

The very first step is the strain field equations. Now, we are taking the linear strain field relations. If we are interested to solve a problem of buckling or a problem in the non-linear domain then we can consider the non-linear part of the strain also.

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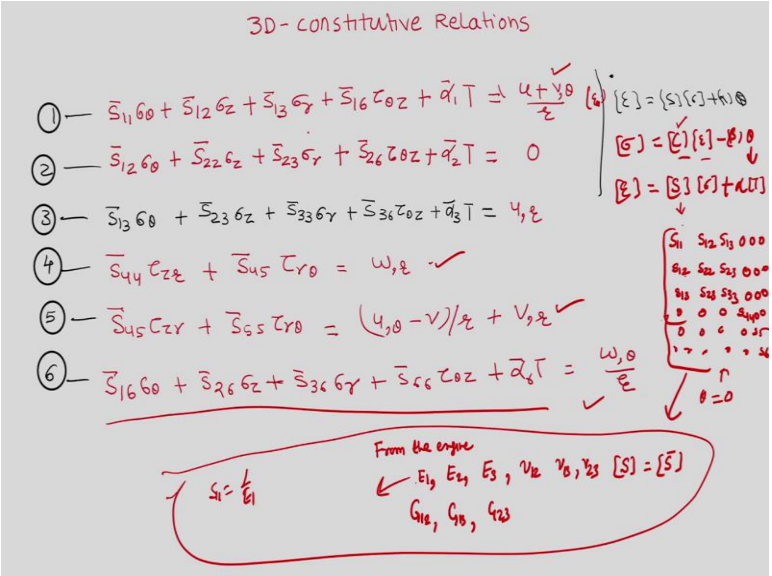


Next is the three-dimensional constitutive relations. The three-dimensional constitutive relations can be written as $[\sigma] = [C][\varepsilon] - [\beta]\theta$ is temperature.

If you are interested to find in terms of a strain then $[\varepsilon] = [S][\sigma] + \alpha$ [T].

Again, when we are going to say that our composites are orthotropic materials, not isotropic materials, that case S matrix will look like this:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}$$

This is the compliance matrix for an orthotropic material when the angle = $\theta^\circ$ material axis. But if the fiber makes an angle at any $\theta^\circ$ then we have to transform this matrix. It becomes $\bar{S}$ .

I have written $\bar{S}_{11}\sigma_\theta + \bar{S}_{12}\sigma_z + \bar{S}_{13}\sigma_r + \bar{S}_{16}\sigma_{\theta z} + \bar{\alpha}_1 T = \frac{u + v,_\theta}{r}(\varepsilon_{\theta\theta})$

And $\varepsilon_{\theta\theta}$ is expressed in terms of displacements.

Then, we have $\varepsilon_{zz}$, we have written it explicitly:

$$\bar{S}_{12}\sigma_\theta + \bar{S}_{22}\sigma_z + \bar{S}_{23}\sigma_r + \bar{S}_{26}\sigma_{\theta z} + \bar{\alpha}_2 T = 0.$$

$$\varepsilon_{rr} = \bar{S}_{13}\sigma_\theta + \bar{S}_{23}\sigma_z + \bar{S}_{33}\sigma_r + \bar{S}_{36}\sigma_{\theta z} + \bar{\alpha}_3 T = u,_r$$

$$\gamma_{zr} = \bar{S}_{44}\tau_{zr} + \bar{S}_{45}\tau_{r\theta} = w,_r$$

$$\gamma_{r\theta} = \bar{S}_{45}\tau_{zr} + \bar{S}_{55}\tau_{r\theta} = \frac{u+v}{r}v,_r$$

$$\gamma_{\theta z} = \bar{S}_{16}\sigma_\theta + \bar{S}_{26}\sigma_z + \bar{S}_{36}\sigma_r + \bar{S}_{66}\sigma_{\theta z} + \bar{\alpha}_6 T = \frac{w,_\theta}{r}$$

From there we can find these relations. From a coding or programming point of view, we have to obtain the value of $\bar{S}_{11}$, $\bar{S}_{12}$, $\bar{S}_{13}$ or $\bar{\alpha}_1$, $\bar{\alpha}_2$, $\bar{\alpha}_3$.

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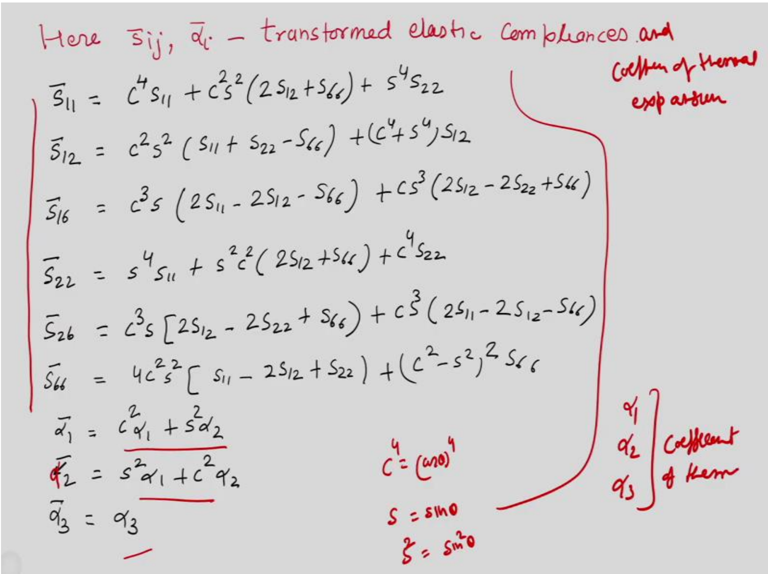


These are the transformed elastic compliances and coefficient of thermal expansion. $\bar{S}_{11}$ can be found out. These relations are given in any book of mechanics of composite, but for the sake of completeness I have presented here $\bar{S}_{11} = C^4$.

$C^4 = (\cos\theta)^4$ and S = $\sin\theta$ .

Whenever you say $S^2$, it means $\sin^2\theta$, $\cos^2\theta$ or $(\cos\theta)^4$ and so on. $\bar{S}_{11}$, $\bar{S}_{12}$, $\bar{S}_{16}$ their transformation is given here. One can write a small MATLAB program to obtain the value of $\bar{S}_{11}$, $\bar{S}_{12}$, or one can inside a program make a subroutine in which they can find the stiff compliance matrices.

Then, $\bar{\alpha}_1 = C^2\alpha_1 + S^2\alpha_2$

$\bar{\alpha}_2 = S^2\alpha_1$

$\bar{\alpha}_3 = \alpha_3$

Where $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}_3$ are the coefficients of thermal expansion.

One major difference is that in the present case, we try to formulate the governing equation using the compliance matrix $S_{11}, S_{12}$ ..., there are some papers in the journal where we try to develop governing equations using the stiffness $C_{11}, C_{12}, C_{12}....$ It does not make much difference, but for the present case it is very easy to formulate in the form of $S_{11}, S_{12}$ ...

The reason behind that is from the engineering constant like Young's modulus $E_1, E_2, E_3, \mu_{12}, \mu_{13}, G_{12}, G_{13}, \textit{ and } G_{23}$, we can easily obtain the value of $S_{11} = \frac{1}{E_1}$. From the engineering constants, we can find the compliance matrix very easily and by inverting the compliance matrix we can find the stiffness matrix.

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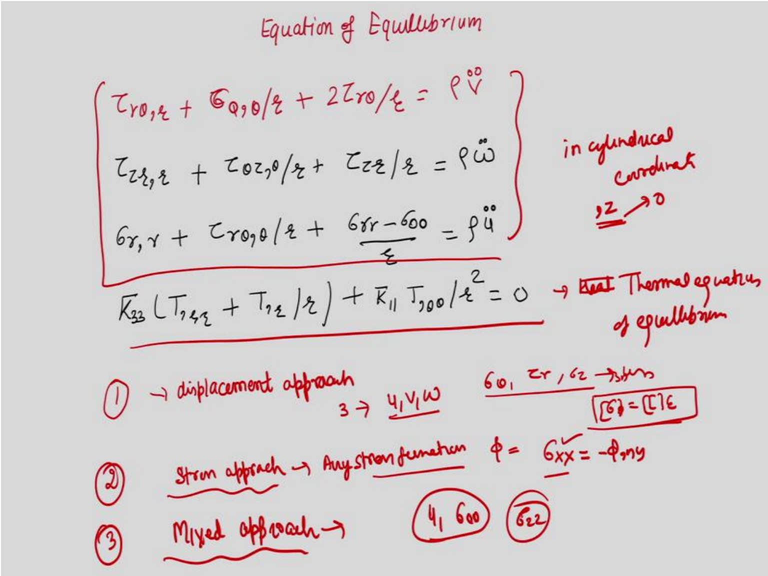


The third step is rewriting the equation of equilibrium. The following three are the equations of equilibrium in the cylindrical coordinate system considering the z derivative is neglected here.

$$\tau_{r\theta,r} + \frac{\sigma_{\theta,\theta}}{r} + \frac{2\tau_{r\theta}}{r} = \rho\ddot{v}$$

$$\tau_{zr,r} + \frac{\tau_{\theta z,\theta}}{r} + \frac{\tau_{zr}}{r} = \rho\ddot{\omega}$$

$$\sigma_{r,r} + \frac{\tau_{r\theta,\theta}}{r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\ddot{u}$$

This equation, $\bar{K}_{33}\left(T,_{rr} + \frac{T,_{r}}{r}\right) + \bar{K}_{11}\frac{T,_{\theta\theta}}{r^2} = 0$ , is in your thermal equations or sometimes we call it a heat conduction equation thermal equation of equilibrium.

Here you see that the temperature equation is the 4th equation and there is no temperature in these three equations. I will explain later also that whenever we are going to solve a problem of a thermoelastic case in three-dimensional their thermal problem and mechanical problems are decoupled.

This means we can solve the temperature equation independently, and then once we know the temperature variation that variation, we can input it into the mechanical equation as a load variable is a known temperature and we can solve the mechanical equations. Once we know the temperature variation then we can solve the thermoelastic problem.

First, we solve a temperature thermal problem the variation of temperature across the thickness and how the temperature varies through different layers from bottom to top. And then this temperature is taken as a loading variable because we know the temperature variation then for a known temperature, we can find the stresses. If a body is subjected to this amount of temperature what will be the stresses will be the displacement in the body are the equations of equilibrium.

Even for the three-dimensional solutions, the 1st technique is the displacement base type approach. In this approach, these 3 equations of equilibriums are used. Here, $\tau_{r\theta}$, $\sigma_{\theta}$ are replaced and their derivatives are replaced and ultimately these 3 equations are expressed in terms of u, v, and w.

$\sigma_{\theta\theta}$, $\tau_{\theta}$, $\sigma_{r}$ using the constitutive relation 3D constitutive relations, if we use that type of approach from that case the $[\sigma] = [C]\varepsilon$, this type of constitutive relations help. When there is a displacement approach then in the terms of stiffness the governing equations are expressed, and then sometimes it is second-order or a fourth-order.

The partial differential equations are solved exactly. This is one of the approaches. The second approach is known as the stressed approach, which is not using this one.

In that approach, the Michael Beltrami equation or array stress function, where let us say, $\phi$ is an array function that satisfies the sum that if we talk in terms of rectangular coordinate stress $\sigma_{xx} = -\varphi,_{yy}$ similarly in the cylindrical coordinate system.

We can say that $\sigma_{xx}$ or $\sigma_{yy}$ and substituting it here, one can get the governing equation in terms of $\phi$.

And the third approach is the mixed approach. In most cases, a mixed approach is referred. The reason behind considering the mixed approach is that displacement, as well as stress, are considered a variable. Like in the Levy type of solutions.

Because the boundary conditions are the mixed type in which displacement, as well as stresses, are specified. If we talk about a simply supported case; in that case, u as well as $\sigma_{\theta\theta}$ or $\sigma_{zz}$ are specified, means we need to satisfy the boundary condition in terms of stresses as well as displacement.

And further the mixed base approach leads to an ordinary differential equation, a first-order differential equation. That way it is easy to solve a first-order differential equation as compared to a fourth-order, fifth-order, or third-order differential equation.

If we go in a displacement-based approach, in that case, it may be a second-order or a third-order differential equation, but if we go for a mixed base approach then it will be a first-order differential equation.

Therefore, I will follow a mixed approach which is easy to work with and gives more accurate solutions about stresses as well as displacement. In a cylindrical coordinate system, the equation of equilibrium and the thermal equation is written.

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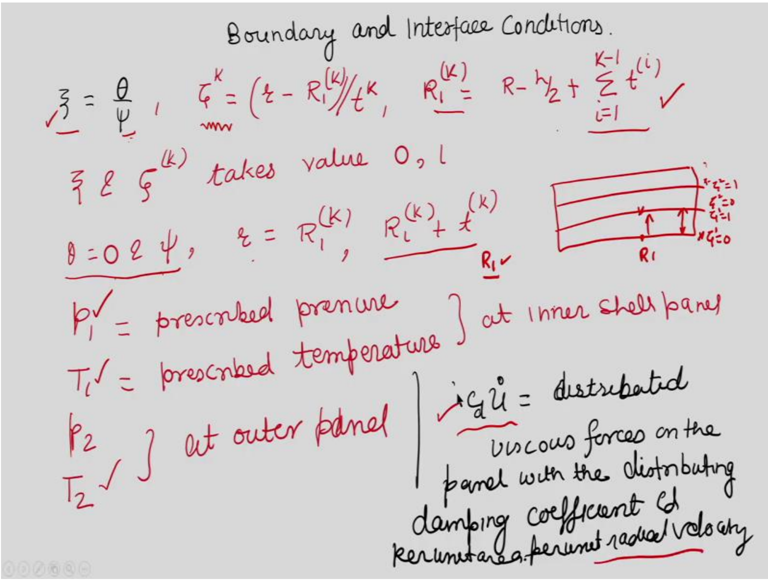


Now, we are going to define the non-dimensionalized coordinates. Along the circumferential direction $\xi = \dfrac{\theta}{\psi}$, along the circumferential it takes values 0 and 1. And a non-dimensionalized coordinate along the thickness direction here is $\varsigma^k$. $\varsigma^k$ is a variable that takes values 0 to 1 inside a layer.

Let us say if you have three layers. In this first layer at the bottom $\varsigma = 0$, at here $\varsigma = 1$. Then, if we talk about the second layer, then again $\varsigma = 0$, it will take for the second layer 1. Same where the third layer, it = 0 and 1. In each layer, it varies from 0 to 1. And its coordinates are defined like this $\frac{\left(r-R_1^{(k)}\right)}{t^k}$.

$$R_1^{(k)} = R - \frac{h}{2} + \sum_{i=1}^{k-1} t^{(i)}.$$

$R_1^{(k)}$ is the inner radius of the kth layer first and if you want to find the radius and if you add the thickness of that layer that will give you the inner radius of the layer. It will vary from 0 to $\psi_\theta$ and r, r varies from $R_1^{(k)}$ the inner radius of that kth layer plus the outer radius of that kth layer $t^k$.

The prescribed pressure let us say $p_1$ and $T_1$ are mechanical pressure; $T_1$ is the temperature at the inner shell panel, then $p_2$ and $T_2$ are prescribed pressure at the top of the shell panel and $T_2$ is the temperature at the outer shell panels. We can also study the force vibration or the vibration under damping.

If you are going to consider in that case $C_d\dot{U}$, where it can be treated as a distributed viscous force on the panel with a distribution damping coefficient of $C_d$ and $\dot{U}$ is the radial velocity.

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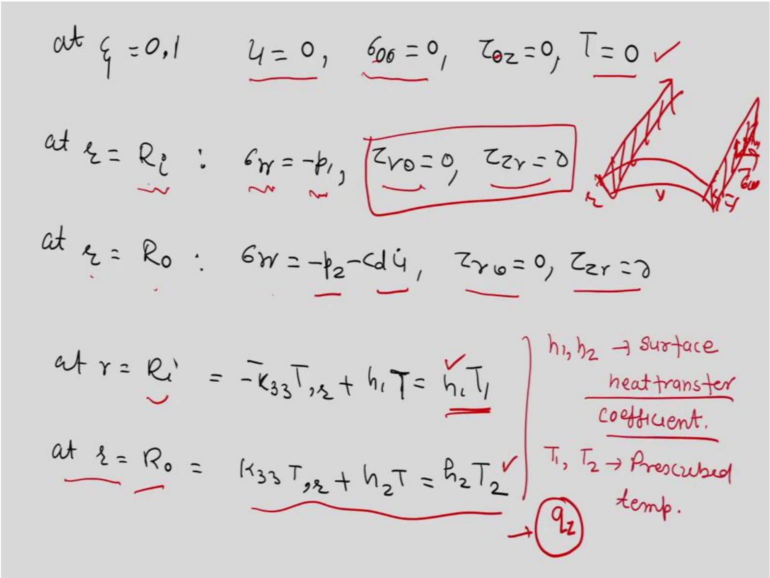

The boundary conditions at $\xi$ = 0 and 1, i.e., where $\xi$ = 0 and this $\xi$ = 1. We consider that it is simply supported, therefore, radial displacement u = 0 and $\sigma_{\theta\theta}$ = 0, $\tau_{\theta z}$ = 0, and temperature on this face is going to be 0. $\sigma_{\theta\theta}$, $\tau_{\theta z}$, u, and temperature, over this face following variables are to be specified.

Now, we are talking about at the bottom of the shell panel bottom surface r = $R_i$ :

$\sigma_{rr} = -p_1$ and $\tau_{r\theta} = \tau_{zr}$ = 0.

Similarly, at r = $R_0$ :

$\sigma_{rr} = -p_2 + -C_d\dot{U}$ , if you are considering if you do not want to consider it will going to be 0, $\tau_{rw}$ and $\tau_{zr}$ = 0.

Now, the thermal loading: the temperature at the inner panel $R_i = -\bar{K}_{33}T,_r + h_1T = h_1T_1$ or just a temperature can be applied and then at R = $R_0 = K_{33}T,_r + h_2T = h_2T_2$ .

Thermal loading can be of many types, either you prescribe only temperature or you prescribe only heat conduction or that qz.

The heat conduction equation can be prescribed in many senses. Here, we are saying that either it may be just a temperature you can prescribe $T_1$ and $T_2$ absolute temperature or you can prescribe in terms of $h_1T_1$ and $h_2T_2$, where $h_1$ and $h_2$ are surface heat transfer coefficient, and $T_1$ and $T_2$ are the prescribed temperatures.

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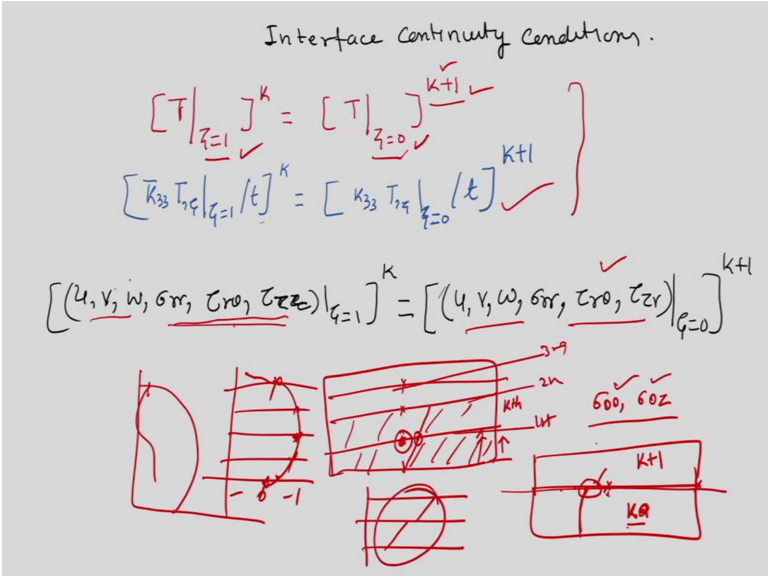


Now, we talk about the interface continuity conditions. If we are talking about a layered panel, in that case at the interface, this is known as 1st interface, 2nd interface, and 3rd interface. If we solve an equation of this layer and an equation of this layer, at the interface they should match. The temperature obtained from the bottom layer; let us say, the temperature is varying like this, here in the next we start from here.

There will be no kinkiness and no slope change. The temperature at the interface must match.

AT $\varsigma = 0 \& 1$. If this is the Kth interface this will be K - 1 or if you say this is Kth layer and K + 1.

K +1th layer bottom face will be $\varsigma$ = 0 and the top face of the Kth layer will be $\varsigma$ = 1.

At the interface $\left[T/_{\varsigma=1}\right]^{k} = \left[T/_{\varsigma=0}\right]^{k+1}$.

Similarly, if qz is written in terms of $\left[\bar{K}_{33}T,_{\varsigma}/_{\varsigma=1}/t\right]^{k} = \left[K_{33}T,_{\varsigma}/_{\varsigma=0}/t\right]^{k+1}$ t.

This is about thermal continuity.

Now, if we talk about the displacement and transfer stresses,

$$\left[\left(u,v,w,\sigma_{rr},\tau_{r\theta},\tau_{zr}\right)/_{\varsigma=1}\right]^{k} = \left[\left(u,v,w,\sigma_{rr},\tau_{r\theta},\tau_{zr}\right)/_{\varsigma=0}\right]^{k+1}$$

The displacement should be continuous. They cannot jump. Using the concept of perfect interface bonding, u, v, w and transfer stresses need to be continuous from the bottom to the top layer.

If you want to see the graph, if $\tau_{r\theta}$ is coming like this or 0 here at the top and bottom, they are going to be 0 -1. Top and bottom are going to be satisfied and in between layers also there will be no jump they will be continuous over the system.

But the rest of the variables, what are these? They may be $\sigma_{\theta\theta}$, they may be $\sigma_{\theta z}$, they may jump when they go from one layer to another layer but the transfer stresses will be continuous.

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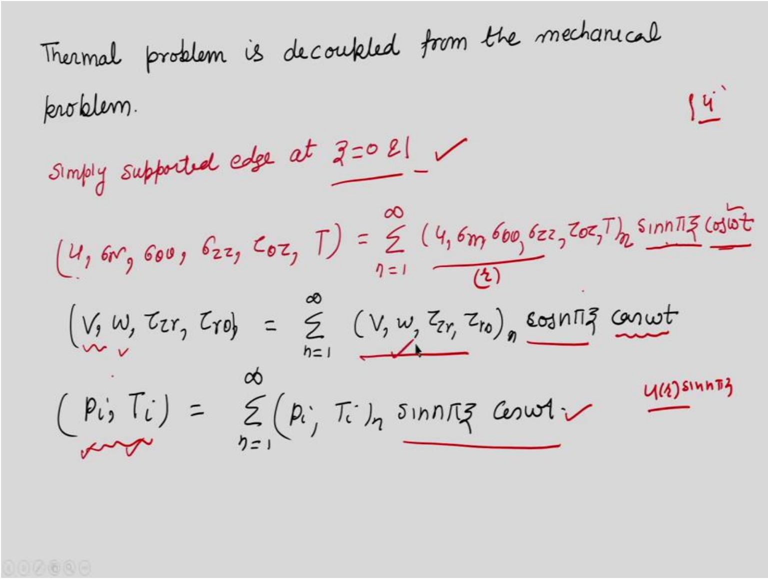


As I have discussed the thermal problem is decoupled from the mechanical problem, and we can solve a thermal equation independently. Now, that is parallelly subjected to simply supported case $\varsigma$ = 0 & 1:

$$\left(u,\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\tau_{\theta z},T\right)=\sum_{n=1}^{\infty}\left(u,\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\tau_{\theta z},T\right)_n \sin n\pi\xi \cos\omega t\ .$$

If you remember in the case of governing equations, we have used $\rho\ddot{u}$ time derivative. We are going to solve the free vibration problem, force vibration problem as well as static problems altogether. And the rest of the variables:

$$\left(v,w,\tau_{zr},\tau_{r\theta}\right)=\sum_{n=1}^{\infty}\left(v,w,\tau_{zr},\tau_{r\theta}\right)_n \cos n\pi\xi \cos\omega t\ .$$

And the same way the mechanical loading and thermal loading

$$\left(p_i, T_i\right) = \sum_{n=1}^{\infty} \left(p_i, T_i\right)_n \sin n\pi\xi \cos \omega t .$$

If we assume along the $\theta$ direction in sine and cosine form., these variables will be a function of r only. I will say $u(r)\sin n\pi$ .

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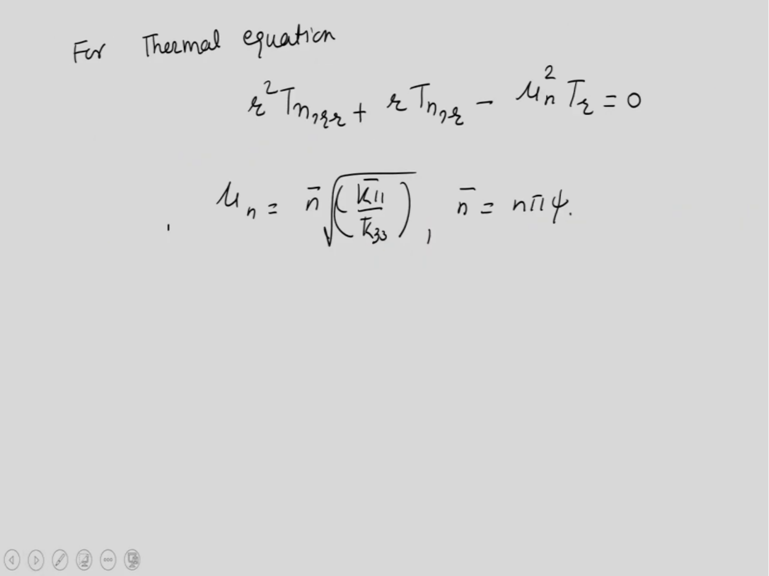


If we substitute all these things into the constitutive relations and other equations of equilibrium and do some mathematical simplification, we say that $\sigma_{\theta\theta}$ , $\sigma_{zz}$ , and $\tau_{\theta zn}$ , they can be expressed in terms of displacement and $\sigma_{rr}$ and temperature.

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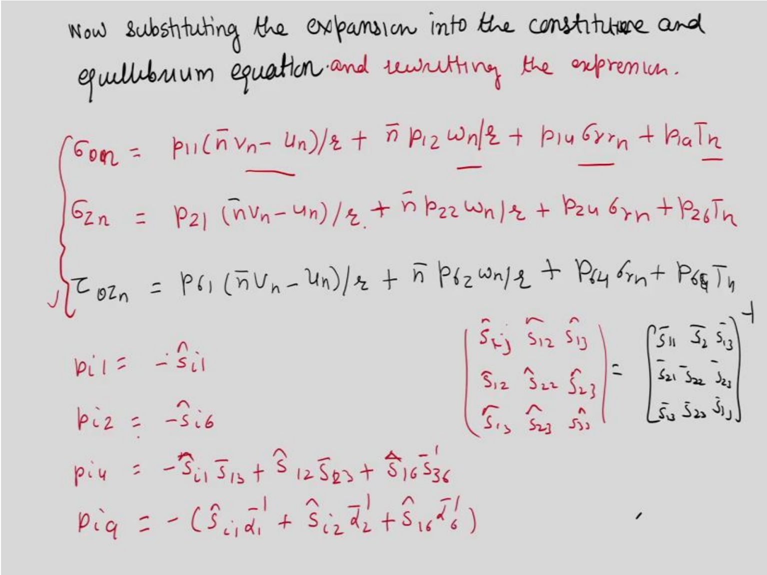


Displacements and $\sigma_{rr}$ maybe our primary variables can be independent variables and this $\tau_{\theta zn}$ is the dependent variable. $\sigma_{\theta n}$, $\sigma_{zn}$ and $\sigma_{\theta zn}$ can be expressed in terms of displacement and stresses.

$$\sigma_{\theta n} = p_{11}\frac{(\bar{n}v_n - u_n)}{r} + \bar{n}p_{12}\frac{w_n}{r} + p_{14}\sigma_{rn} + p_{16}T_n$$

$$\sigma_{zn} = p_{21}\frac{(\bar{n}v_n - u_n)}{r} + \bar{n}p_{22}\frac{w_n}{r} + p_{24}\sigma_{rn} + p_{26}T_n$$

$$\sigma_{\theta zn} = p_{61}\frac{(\bar{n}v_n - u_n)}{r} + \bar{n}p_{62}\frac{w_n}{r} + p_{64}\sigma_{rn} + p_{66}T_n$$

These can be dependent variables.

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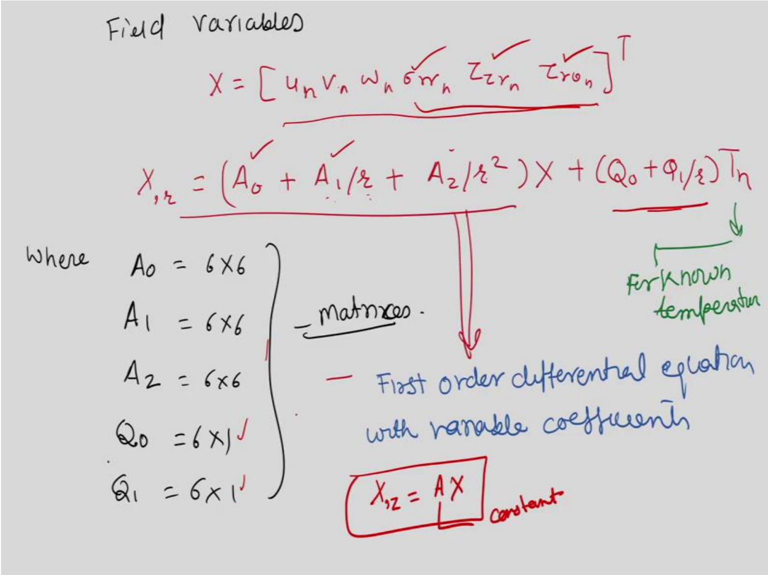


Now, we can say that our total field variables will be all three displacement and the stress components which appears on the boundary and the top and the bottom of the shell panels where these are $\sigma_{rr_n}$, $\tau_{zr_n}$ and $\tau_{r\theta_n}$. We can express others in terms of these 6 variables:

$$X = \left[u_n \, v_n \, w_n \;\; \sigma_{rr_n} \;\; \tau_{zr_n} \;\; \tau_{r\theta_n}\right]^T$$

Ultimately, rewriting all three equations of equilibrium from there the derivative of r is taken on the left-hand side, and other variables are put on the right-hand side. Similarly, these 3 left constitutive relations that u, v, w in which we have a derivative with respect to $u,_r$, $v,_r$, $w,_r$ keeping them on the left-hand side and the rest of the variable on the right-hand side.

If we have $\sigma_\theta$ or $\tau_{\theta z}$, we convert it into this form. Ultimately, doing all mathematical simplification leads to a first-order differential equation with a variable coefficient. Here $A_0$, $A_1$, and $A_2$ are the 6 by 6 matrix, $Q_0$ and $Q_1$ are the 6 by 1 column matrix. Here you see $\frac{A_1}{r}$, $\frac{A_2}{r^2}$, r is taken commonly. $A_1$ is containing a matrix,

$X,_r = \left(A_0 + \frac{A_1}{r} + \frac{A_2}{r^2}\right)X + \left(Q_0 + \frac{Q_1}{r}\right)T_n$ is the first-order differential equation with variable coefficient. And this equation is most difficult to solve. And these are the temperature coefficients. There are several ways researchers have tried to solve this equation. For the case of a plate when we developed a three-dimensional solution in that case of the plate it becomes $X_{,z} = AX$ .

For the first-order differential equation with a constant coefficient, the exact solution can be obtained. But first-order differential equation with a variable coefficient exact solution is not possible even till the date algorithm is not maintained. Therefore, we can solve either by a power series or by some other techniques, but each technique has some advantages as well as disadvantages.

I shall discuss the solution techniques in lecture 02 of week 8.

Thank you very much.