Theory of Composite Shells

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Lecture - 28
Free vibration of a composite cylindrical shell

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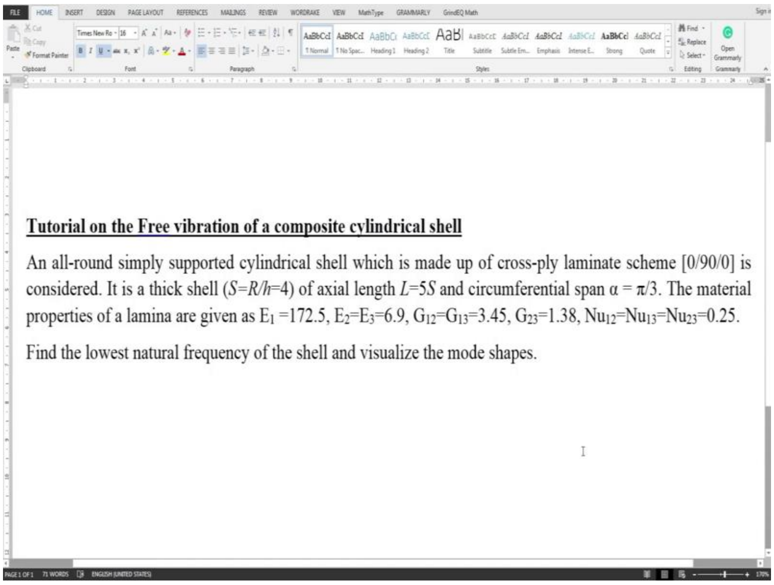

Welcome to the tutorial on the Free Vibration analysis of a Composite Cylindrical Shell. An all-round simply supported cylindrical shell which is made up of cross-ply laminate scheme (0/90/0) is considered.

It is a thick shell of $S = \frac{R}{h} = 4$.

And is of axial length L, and circumferential span α . The material properties of a lamina are that of an orthotropic material. Find the lowest natural frequency of the shell and visualize the mode shapes.

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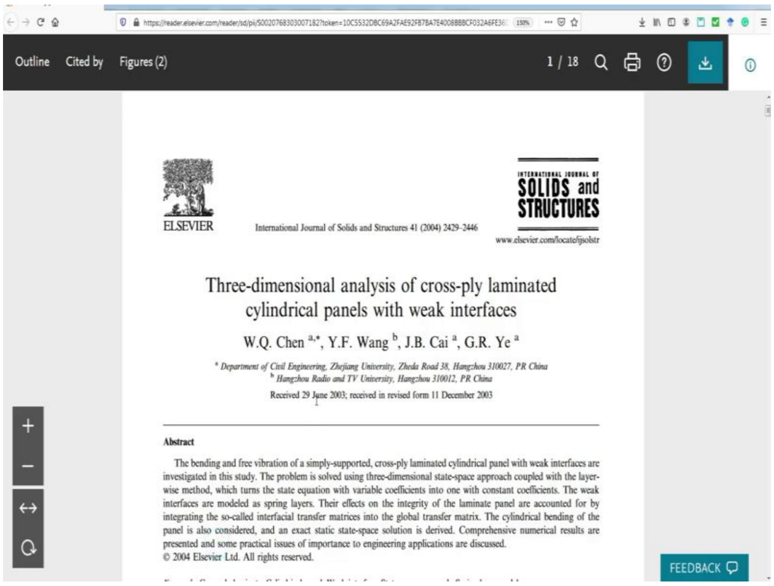

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Three-dimensional analysis of cross-ply laminated cylindrical panels with weak interfaces

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Abstract

The bending and free vibration of a simply-supported, cross-ply laminated cylindrical panel with weak interfaces are investigated in this study. The problem is solved using three-dimensional state-space approach coupled with the layer-wise method, which turns the state equation with variable coefficients into one with constant coefficients. The weak interfaces are modeled as spring layers. Their effects on the integrity of the laminate panel are accounted for by integrating the so-called interfacial transfer matrices into the global transfer matrix. The cylindrical bending of the panel is also considered, and an exact static state-space solution is derived. Comprehensive numerical results are presented and some practical issues of importance to engineering applications are discussed.

The results are compared with those presented by Chen et al in this paper.

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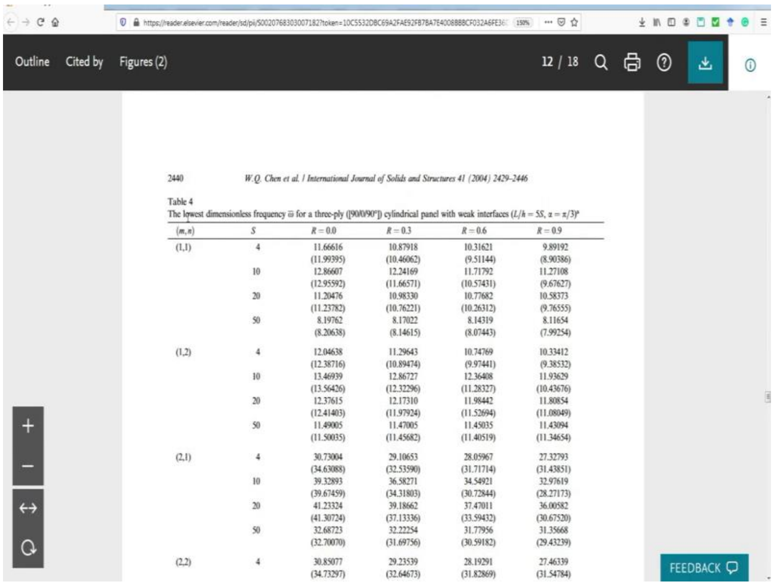

Table 4
The lowest dimensionless frequency $\bar{\omega}$ for a three-ply ([90/0/90°]) cylindrical panel with weak interfaces ($L/h = 5S$, $\alpha = \pi/3$)[a]

(m,n)	S	$R = 0.0$	$R = 0.3$	$R = 0.6$	$R = 0.9$
(1,1)	4	11.66616 (11.99395)	10.87918 (10.46062)	10.31621 (9.51144)	9.89192 (8.90386)
	10	12.86607 (12.95592)	12.24169 (11.66571)	11.71792 (10.57431)	11.27108 (9.67627)
	20	11.20476 (11.23782)	10.98330 (10.76221)	10.77682 (10.26312)	10.58373 (9.76555)
	50	8.19762 (8.20638)	8.17022 (8.14615)	8.14319 (8.07443)	8.11654 (7.99254)
(1,2)	4	12.04638 (12.38716)	11.29643 (10.89474)	10.74769 (9.97441)	10.33412 (9.38532)
	10	13.46939 (13.56426)	12.86727 (12.32296)	12.36408 (11.28327)	11.93629 (10.43676)
	20	12.37615 (12.41403)	12.17310 (11.97924)	11.98442 (11.52694)	11.80854 (11.08049)
	50	11.49005 (11.50035)	11.47005 (11.45682)	11.45035 (11.40519)	11.43094 (11.34654)
(2,1)	4	30.73004 (34.63088)	29.10653 (32.53590)	28.05967 (31.71714)	27.32793 (31.43851)
	10	39.32893 (39.67459)	36.58271 (34.31803)	34.54921 (30.72844)	32.97619 (28.27173)
	20	41.23324 (41.30724)	39.18662 (37.13336)	37.47011 (33.59432)	36.00582 (30.67520)
	50	32.68723 (32.70070)	32.22254 (31.69756)	31.77956 (30.59182)	31.35668 (29.43239)
(2,2)	4	30.85077 (34.73297)	29.23539 (32.64673)	28.19291 (31.82869)	27.46339 (31.54784)

We can see the lowest dimensionless frequencies have been listed in this table.

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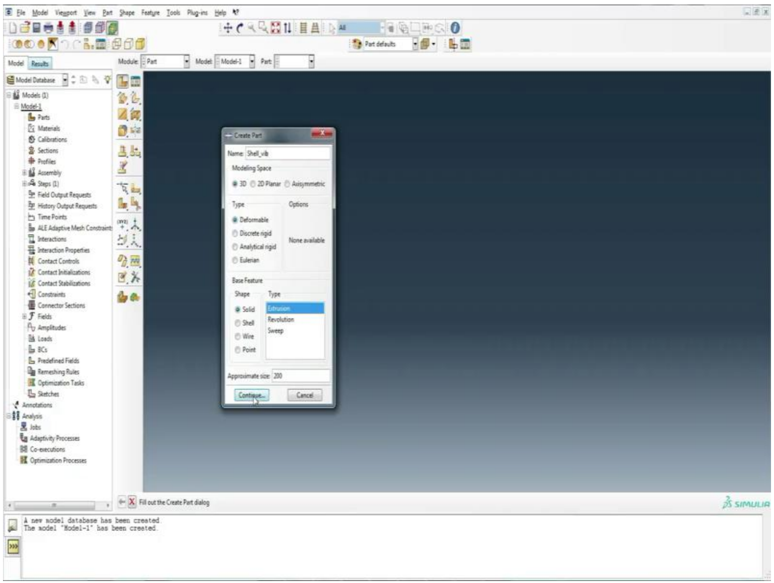

We model the shell, create a new model database and set the working directory. Create the new part which is named as shell vibration.

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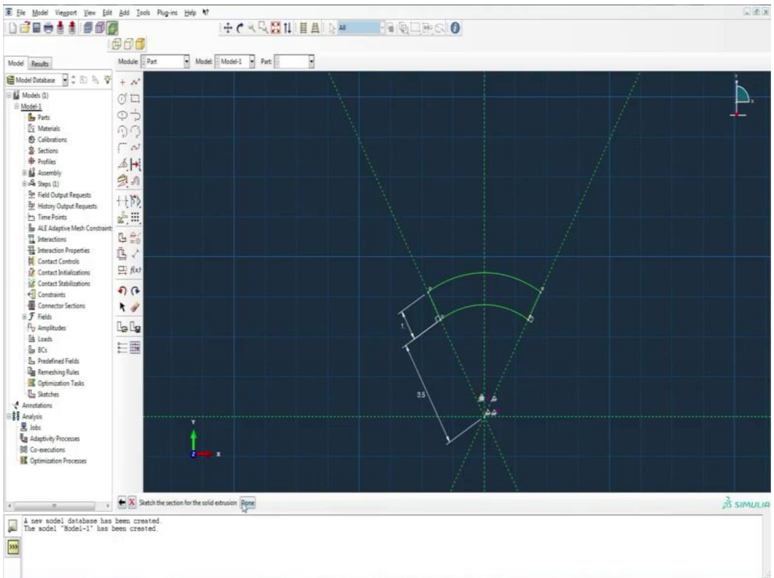

And these are the construction lines for initializing the model as the circumferential span $\alpha = \frac{\pi}{3}$. The construction lines are at an angle of $\frac{\pi}{3}$ and an arc is created at a distance of 3.5 unit which is corresponds to the inner surface of the shell. And the thickness is taken as 1 unit. Close the sketch for extrusion and it is extruded as 5S units i.e., 20 units.

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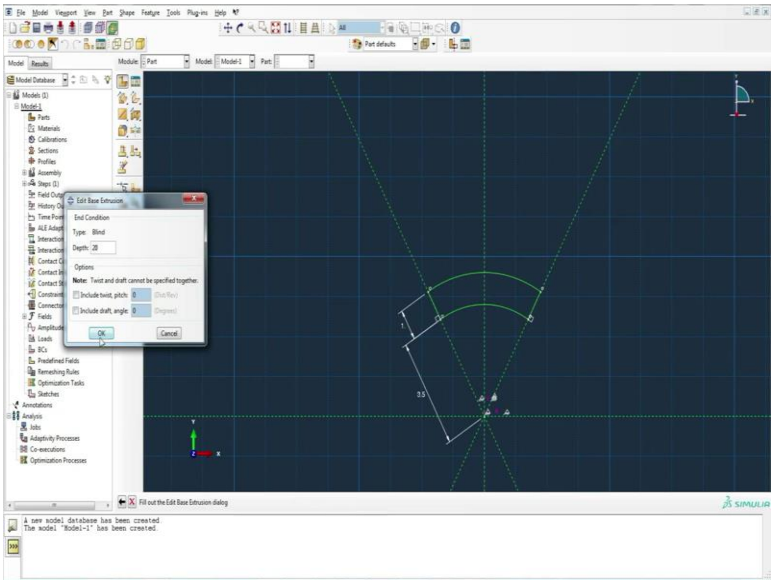

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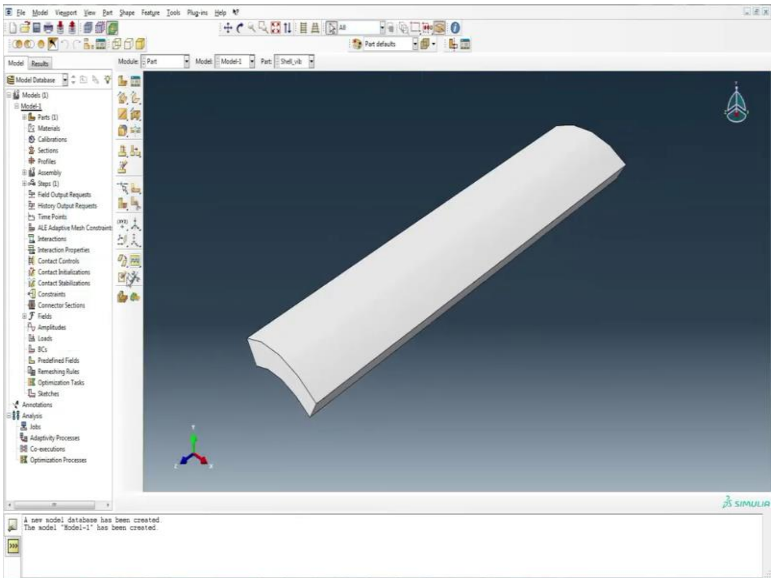

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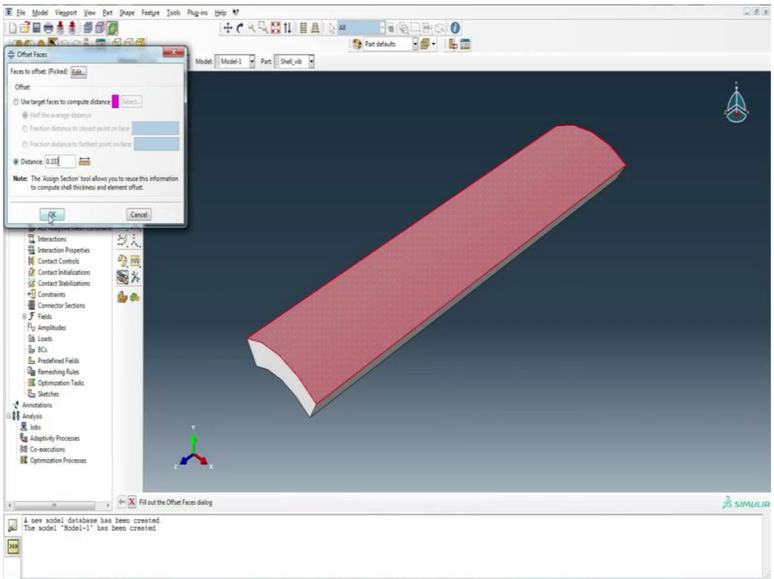

Then, the plies are created through an offset tool as previously explained.

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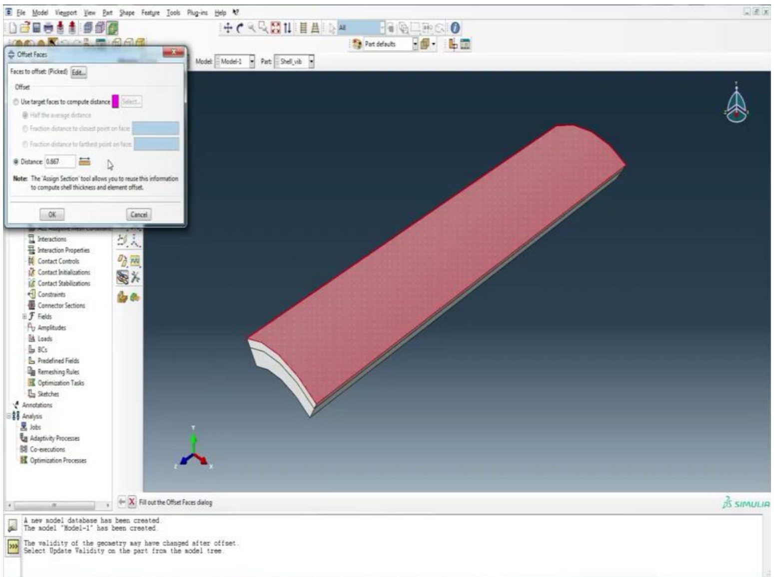

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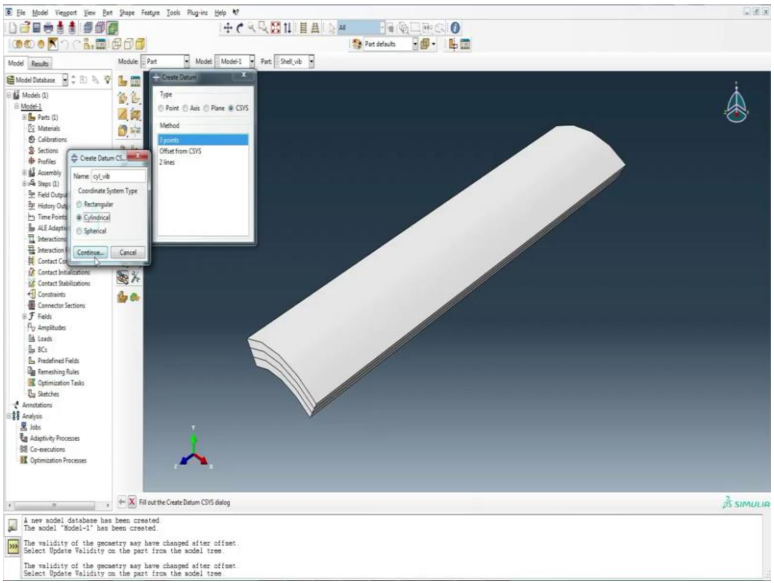

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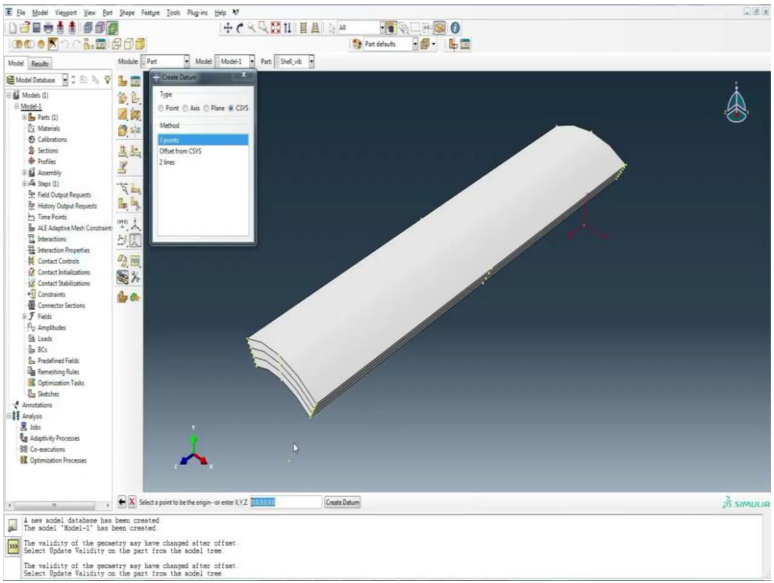

And a cylindrical coordinate system is also provided.

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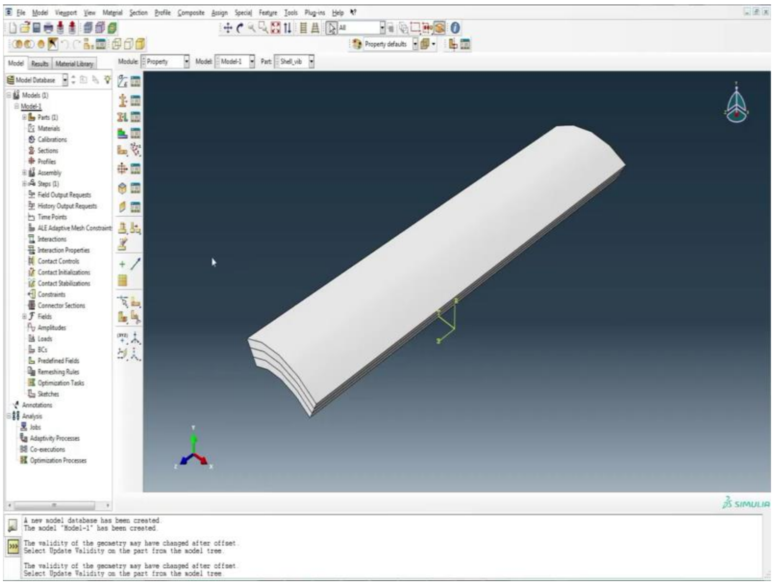

Then, in the property module the material properties are assigned to the plies.

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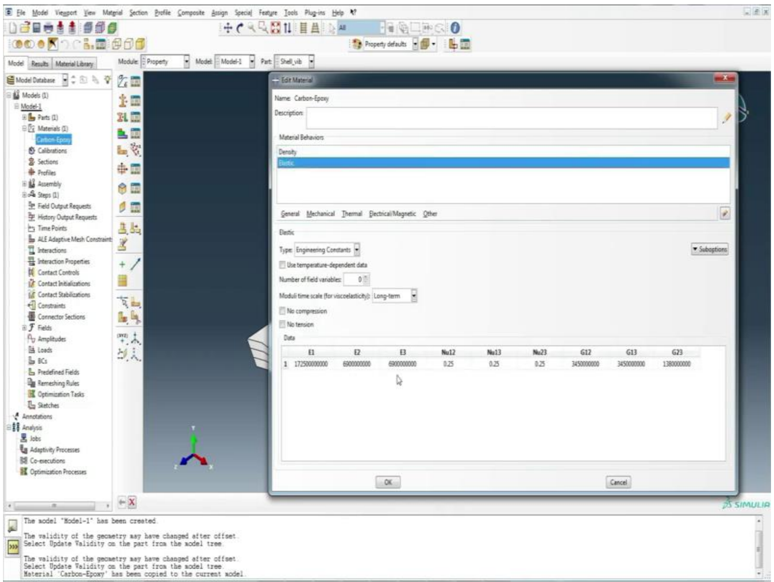

The material properties are those shown in the problem statement earlier.

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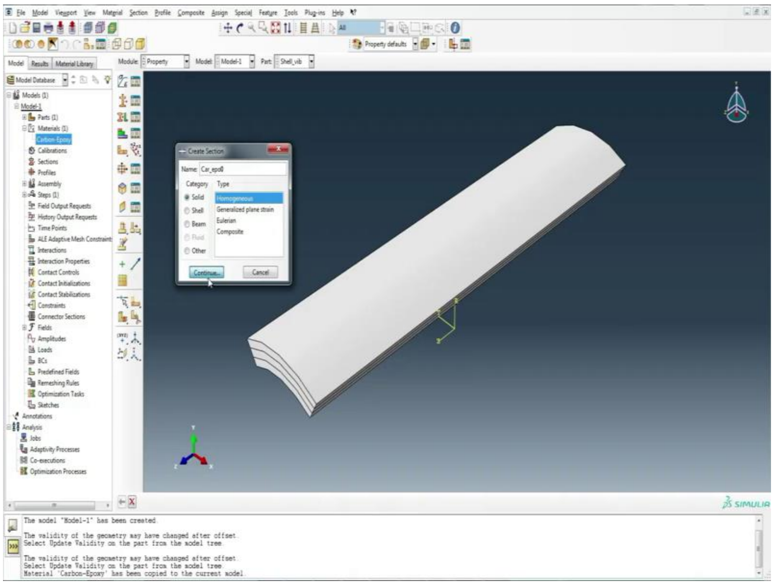

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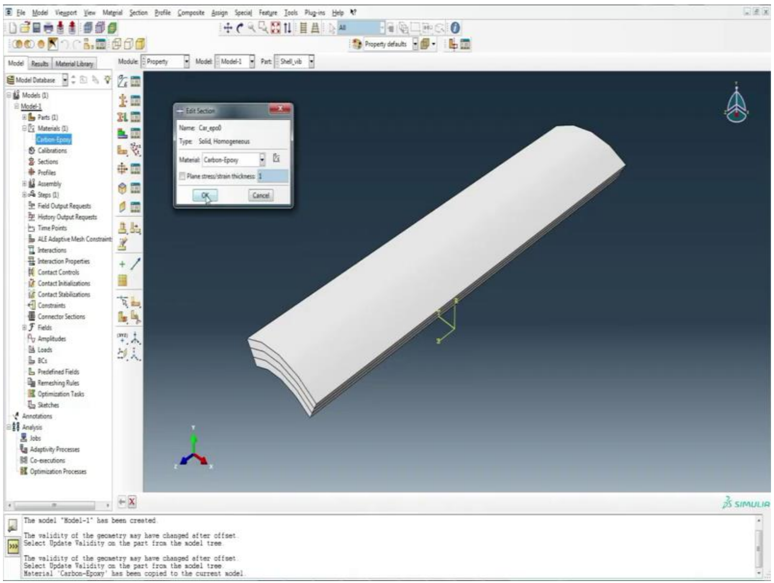

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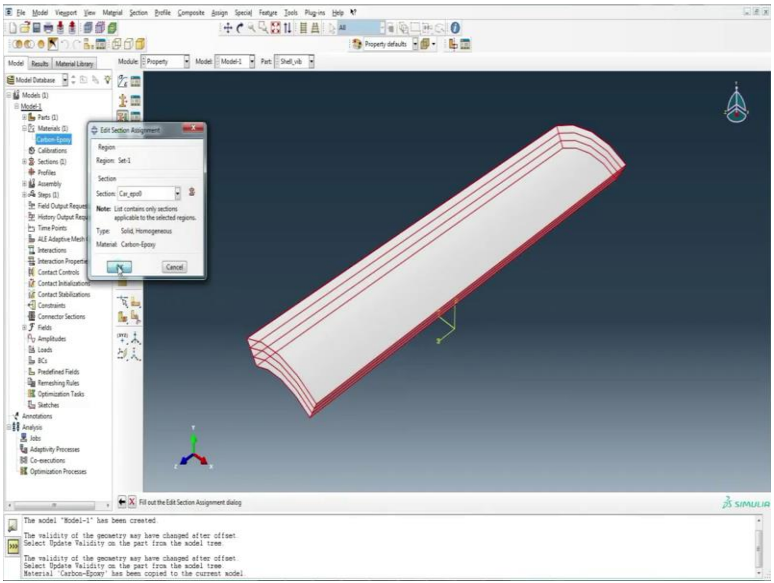

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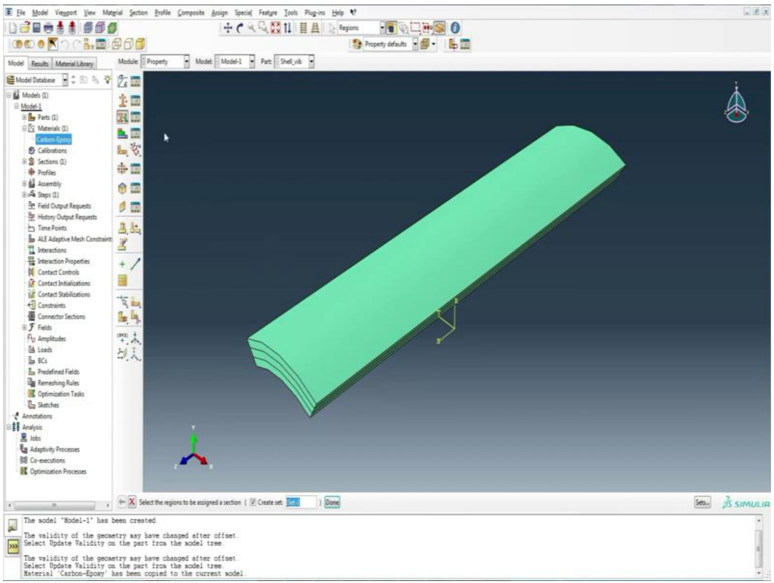

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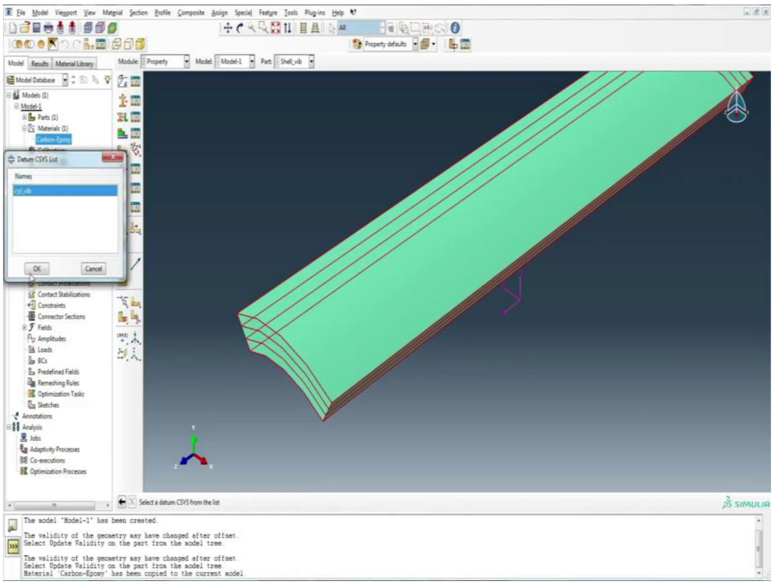

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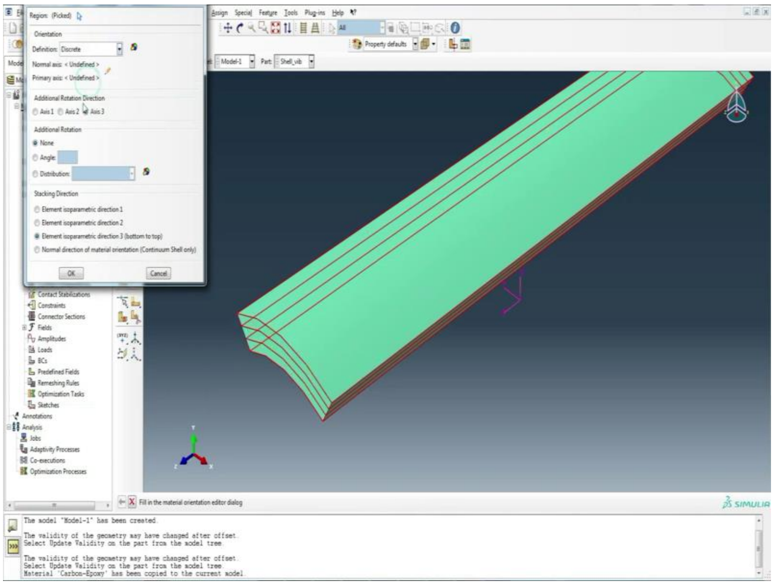

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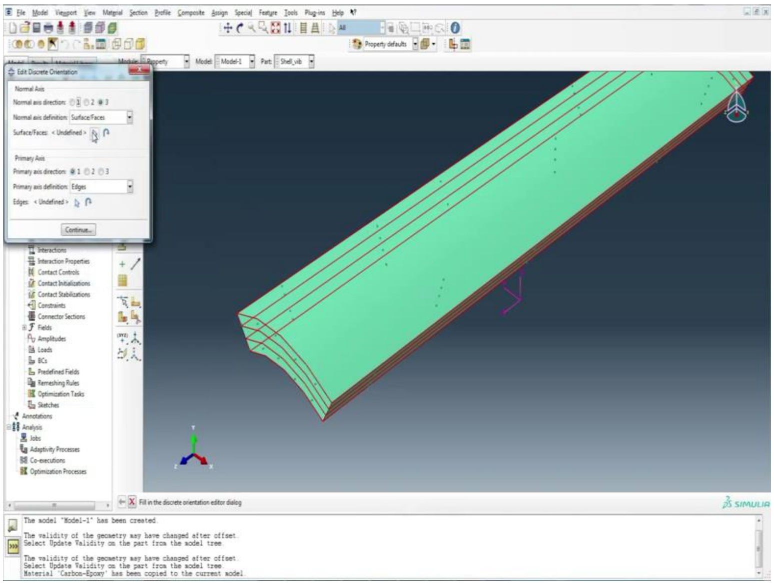

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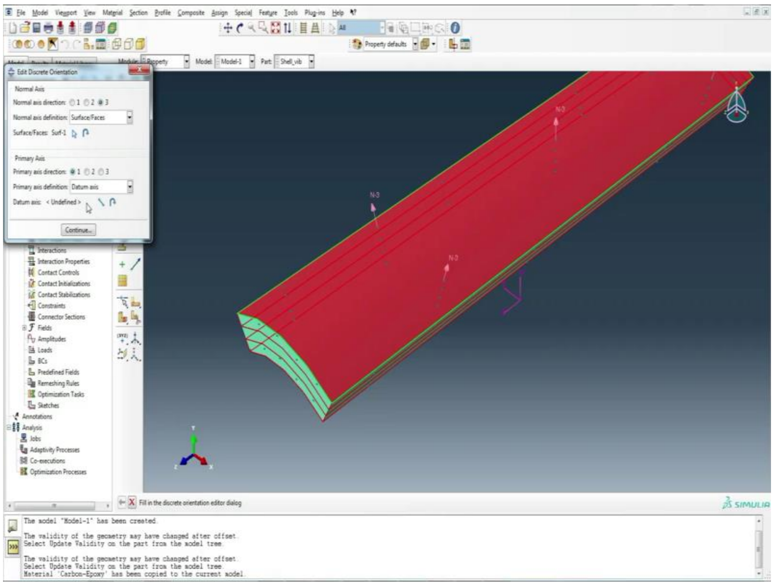

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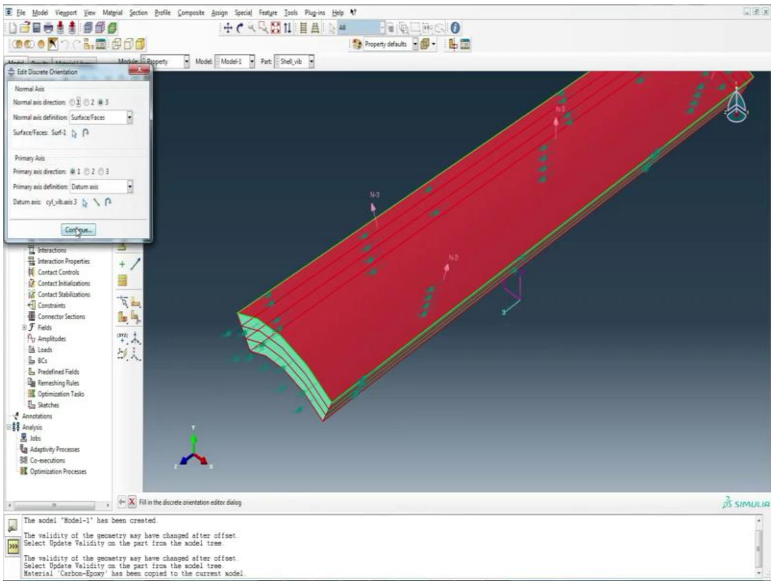

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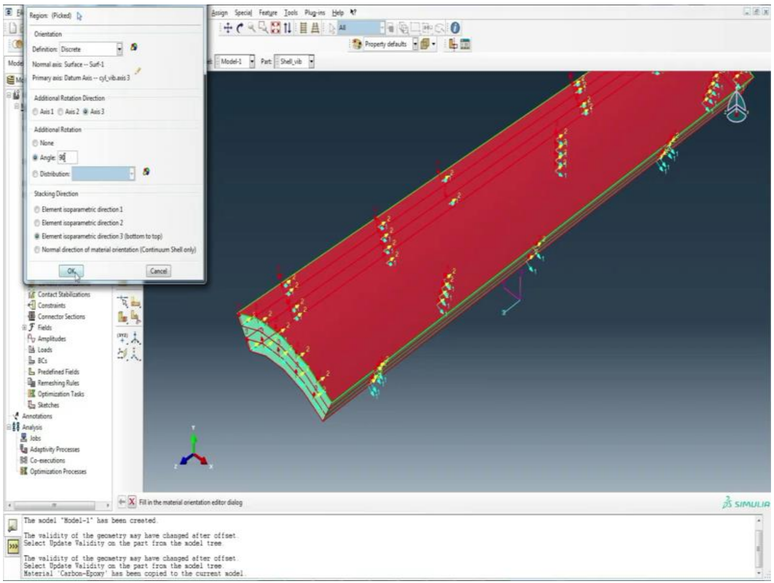

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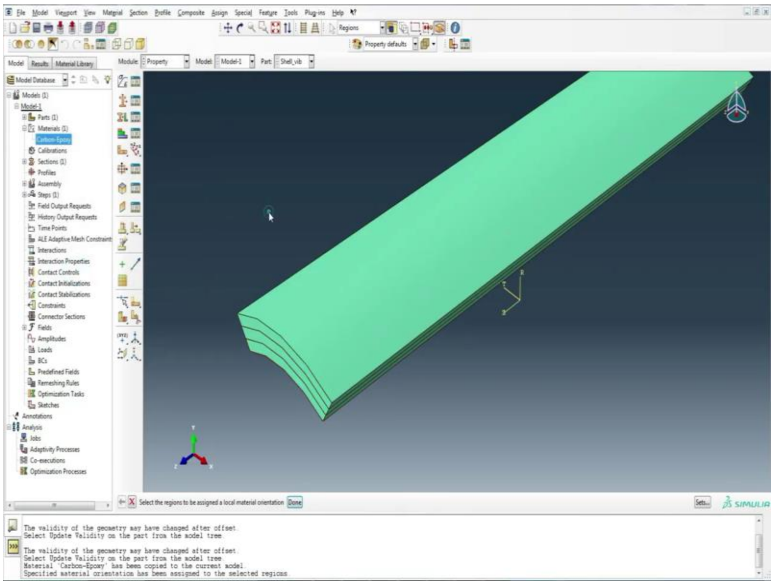

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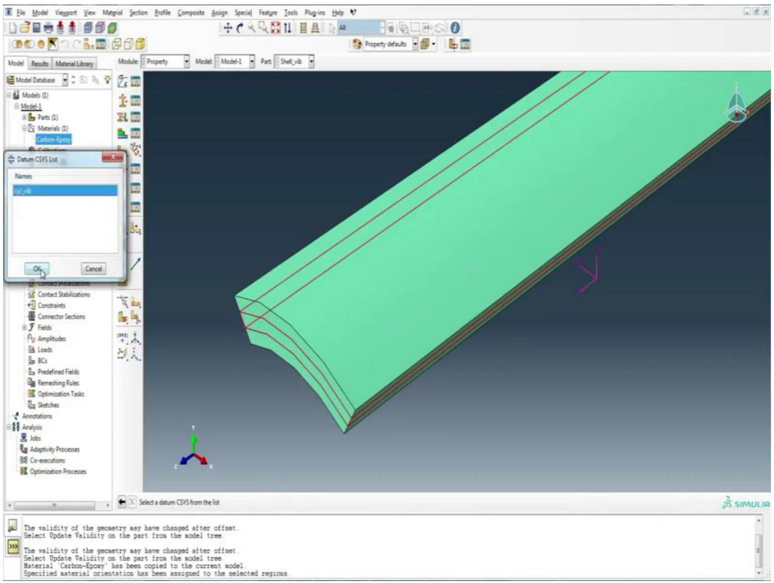

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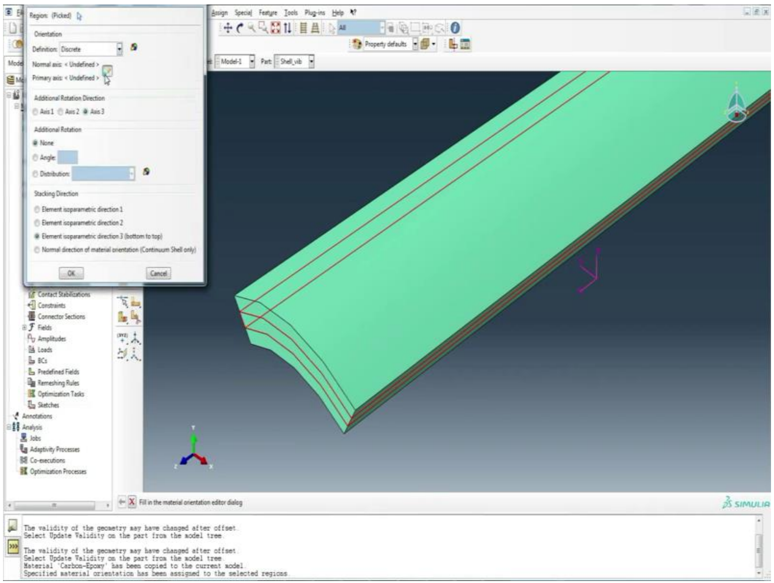

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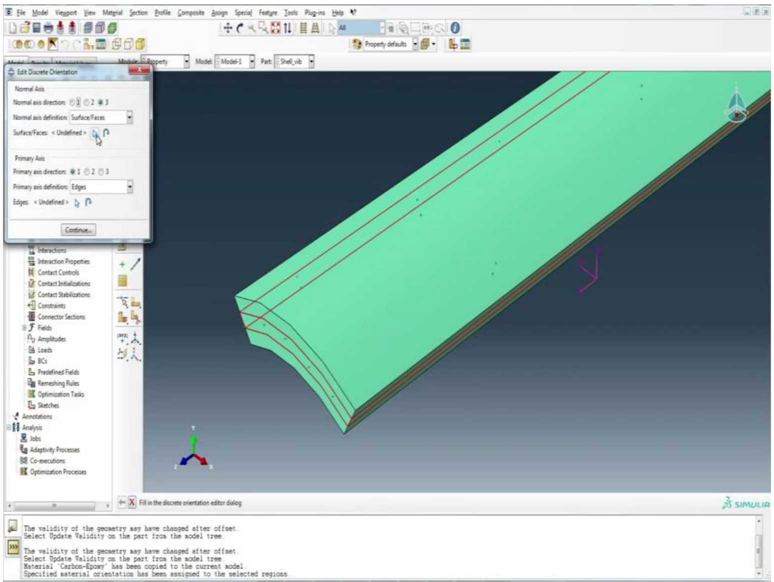

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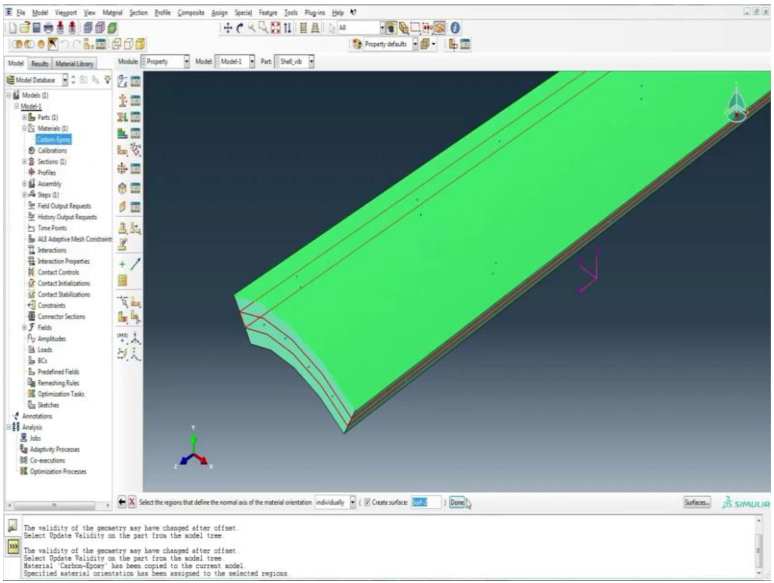

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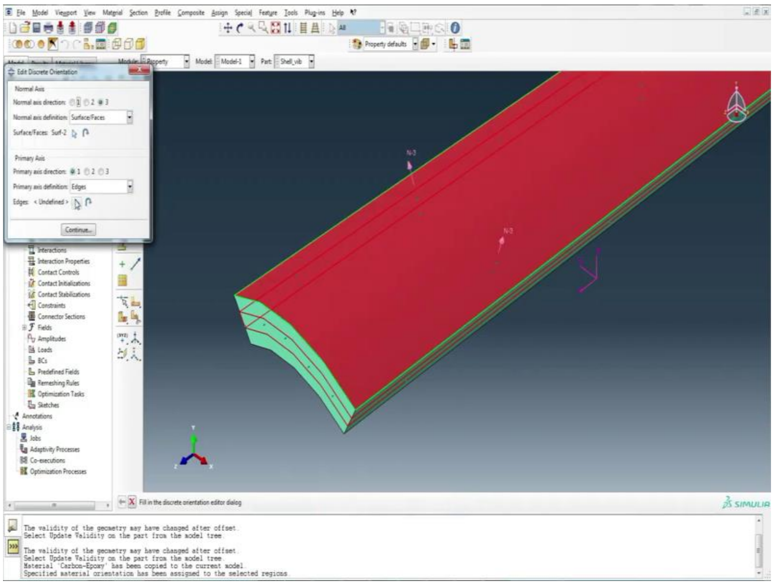

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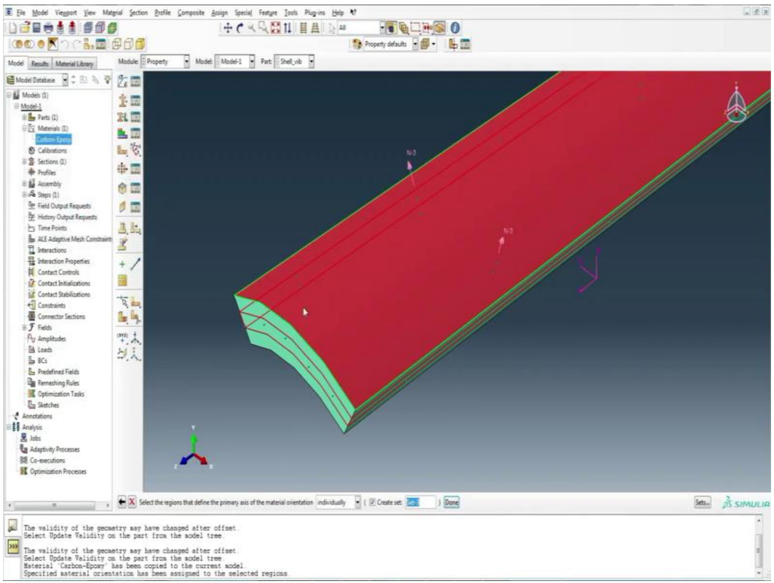

The laminate scheme is taken to be 0/90/0.

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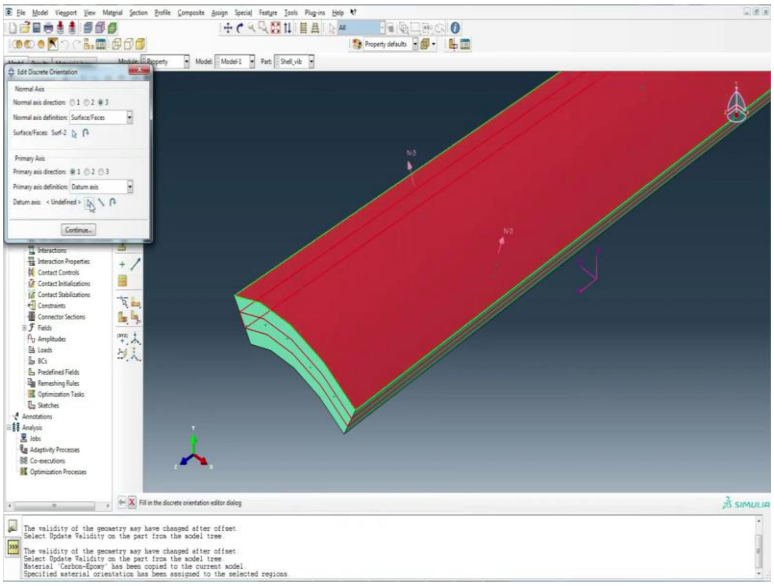

The outer two layers are at 0 degree.

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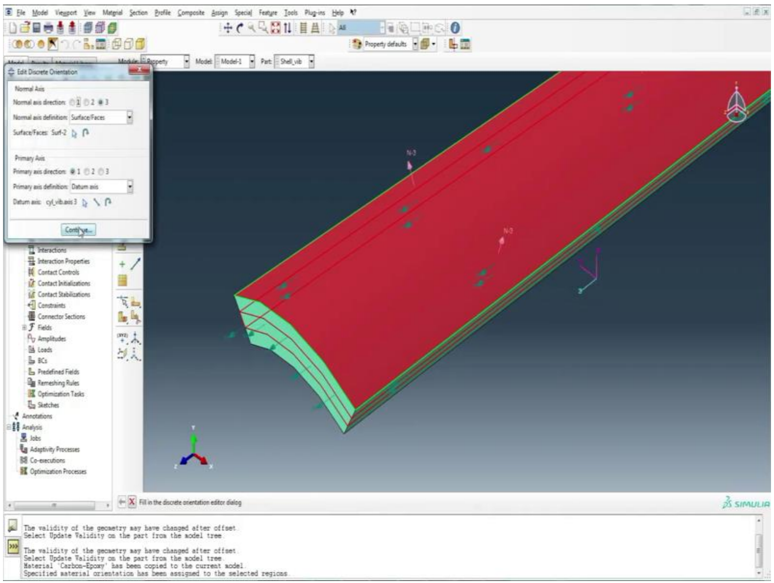

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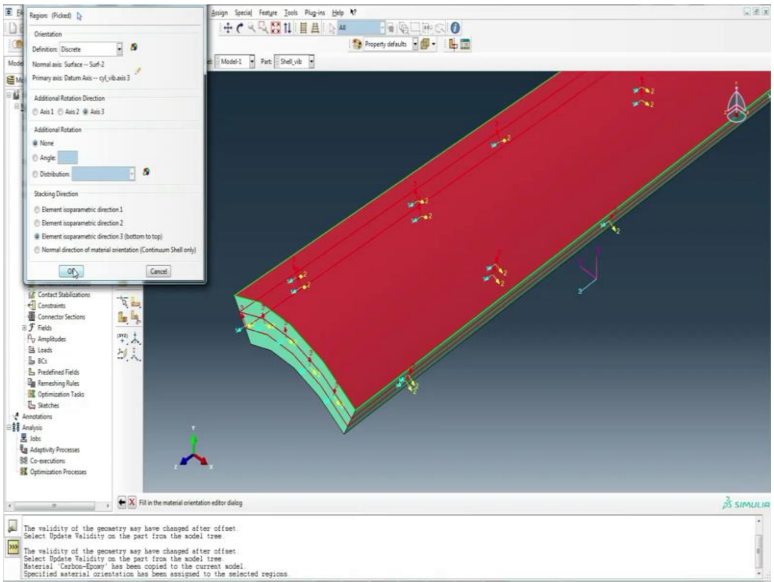

And the middle layer is at 90˚ orientation.

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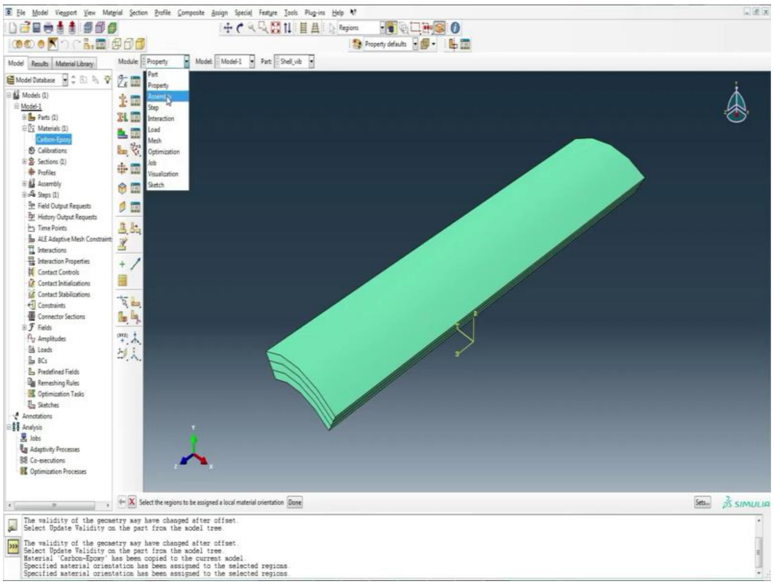

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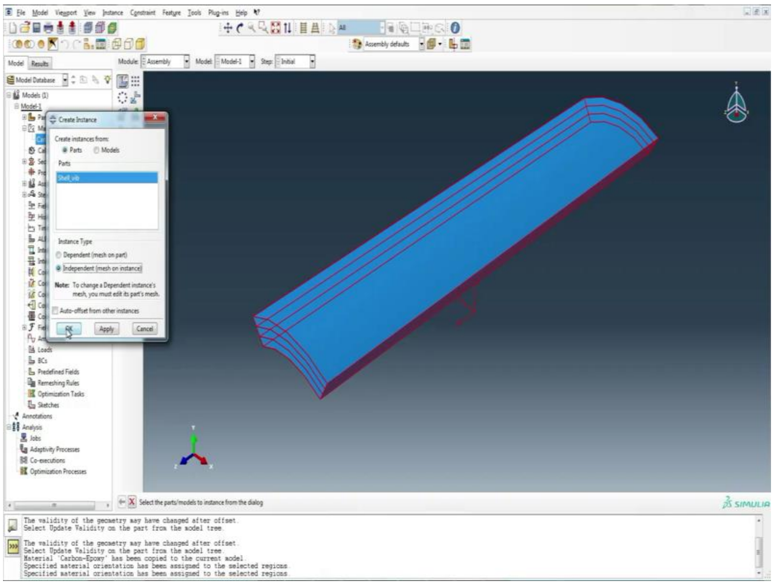

Thereafter, we go for the assembly of the shell.

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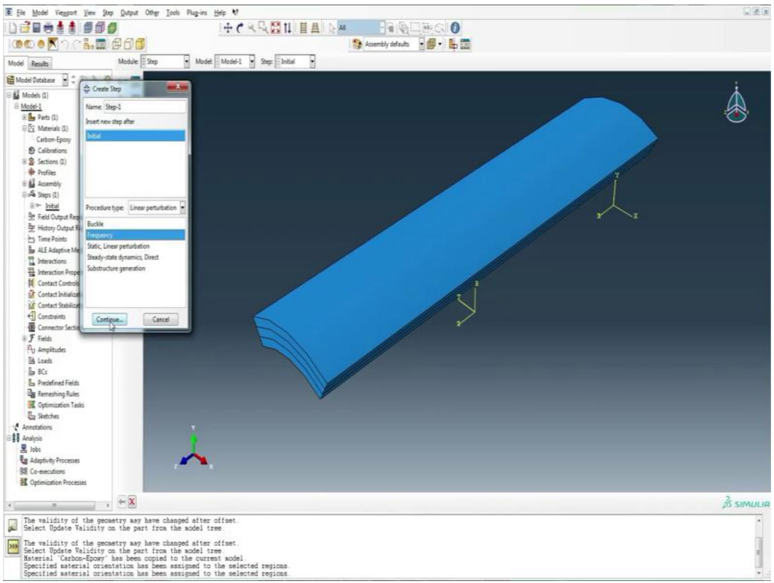

And in the step module, this time we need to do the free vibration problem. We go further linear perturbation type and go for frequency.

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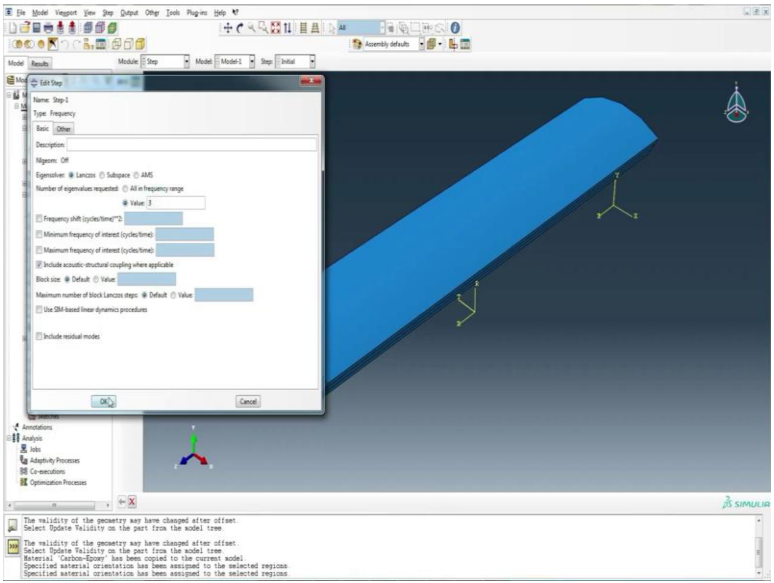

Using the solver of Lanczos the number of mode shapes that we want to calculate can be given in the values field.

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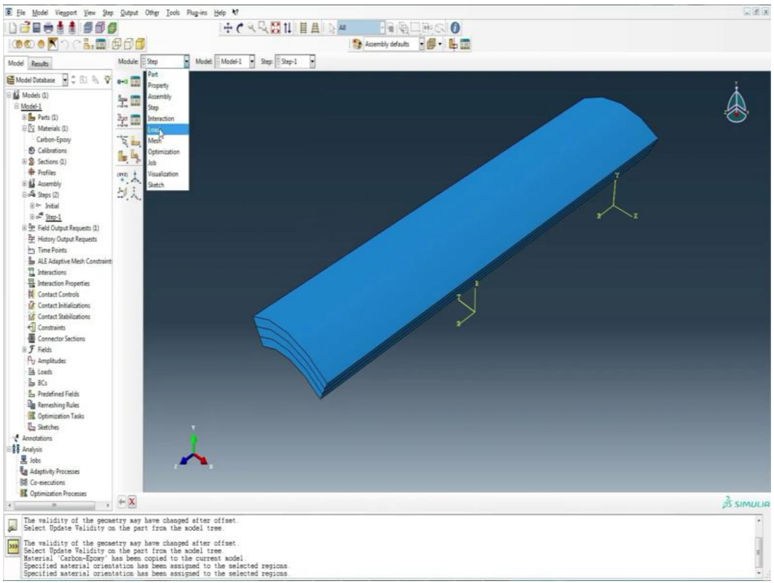

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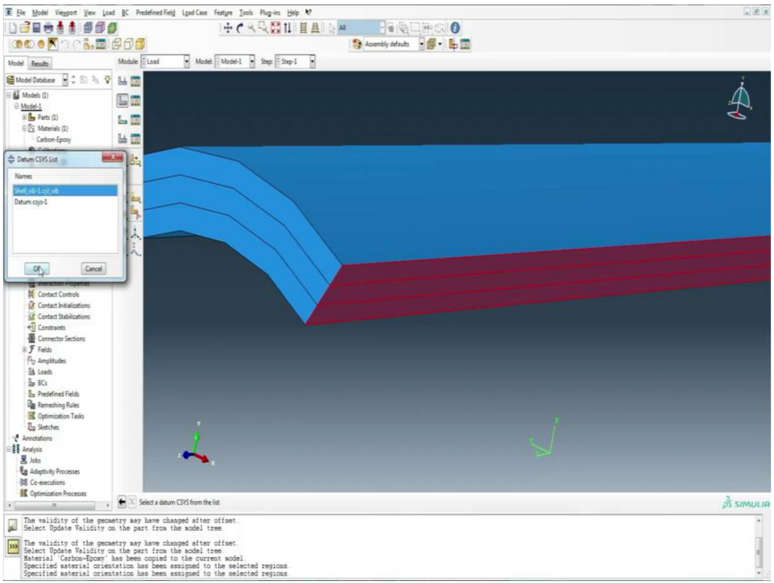

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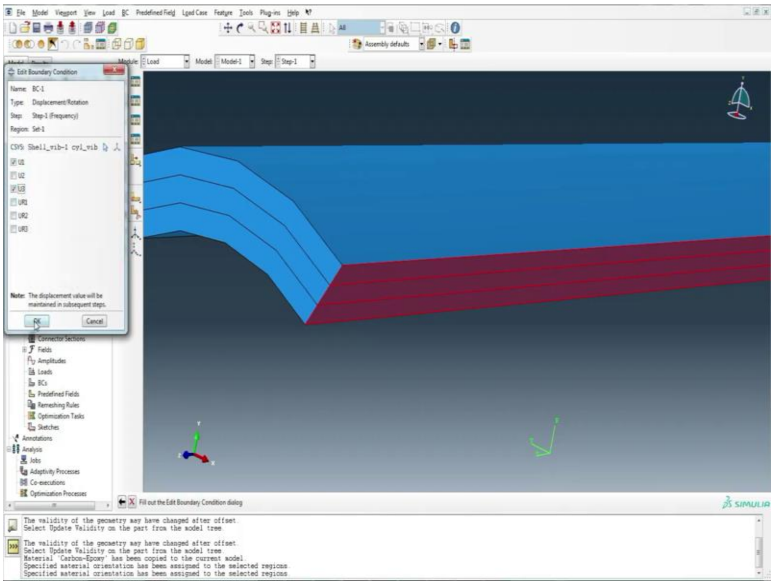

Then, we set the boundary conditions which is simply supported.

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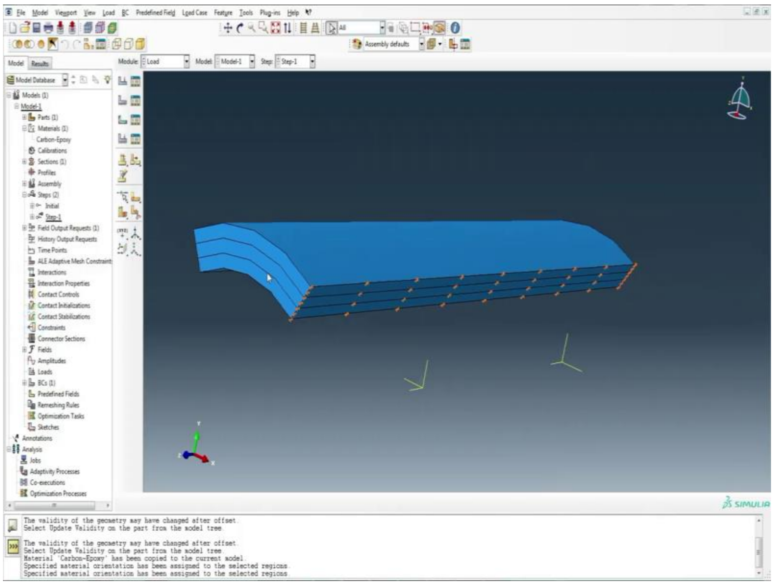

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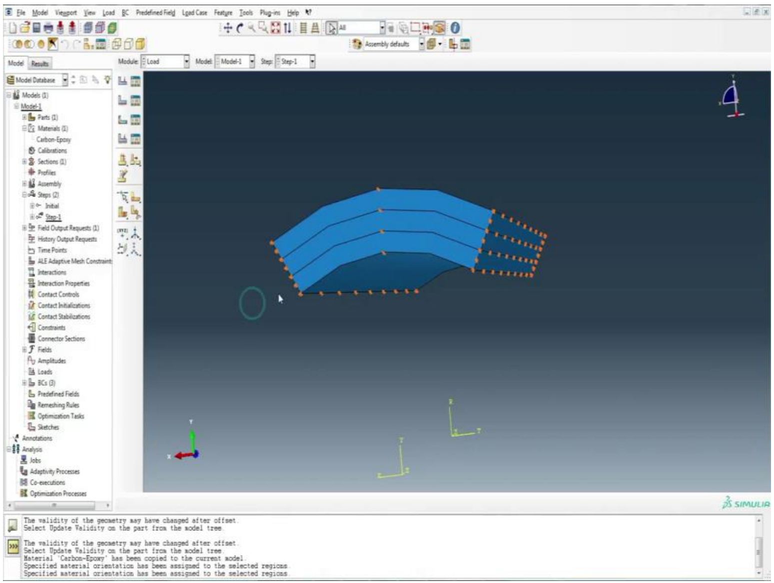

There is no load as it is a free vibration analysis.

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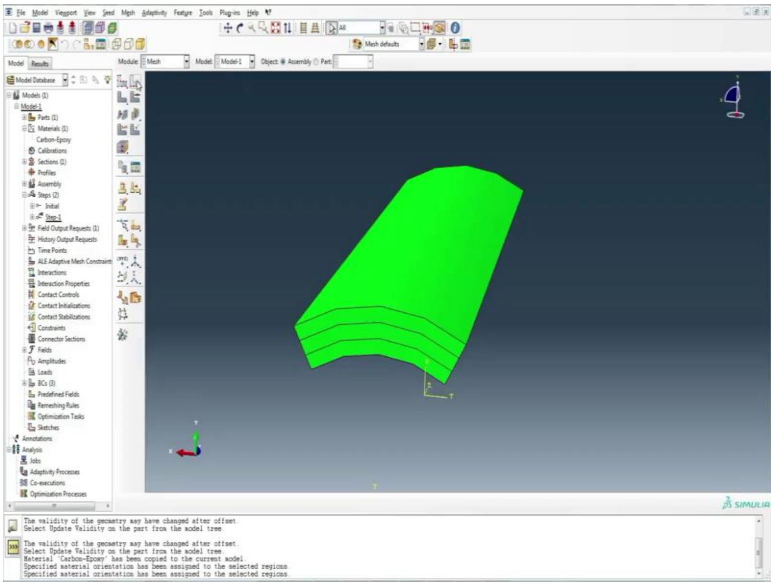

Then the meshing is done after a convergence study.

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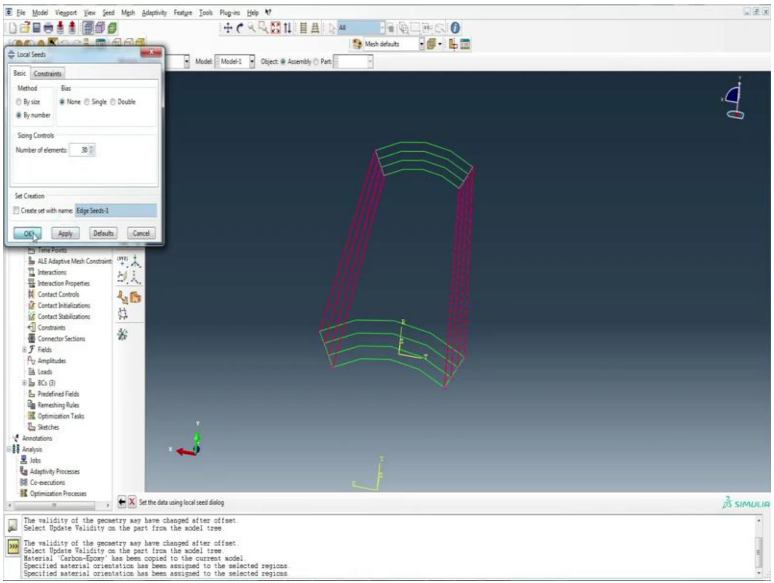

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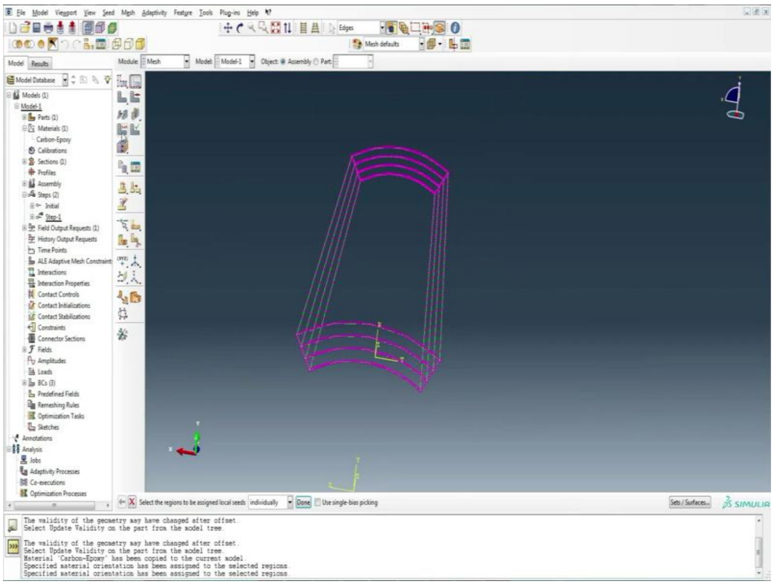

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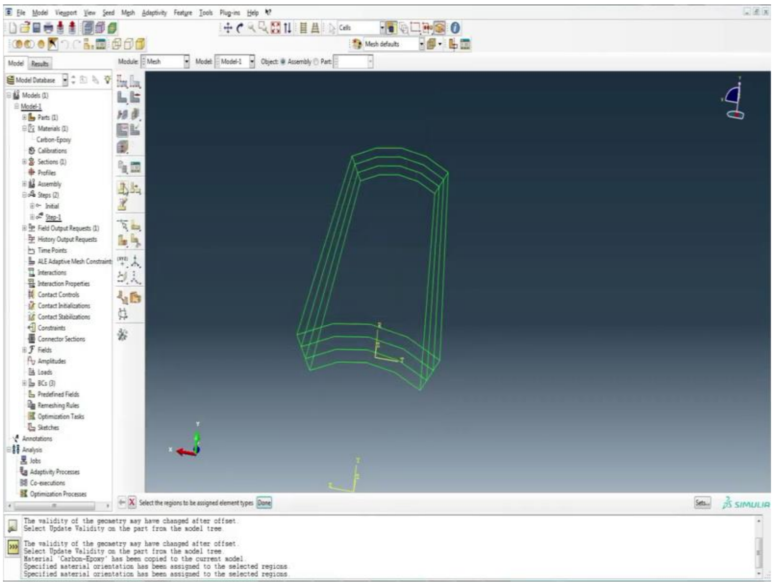

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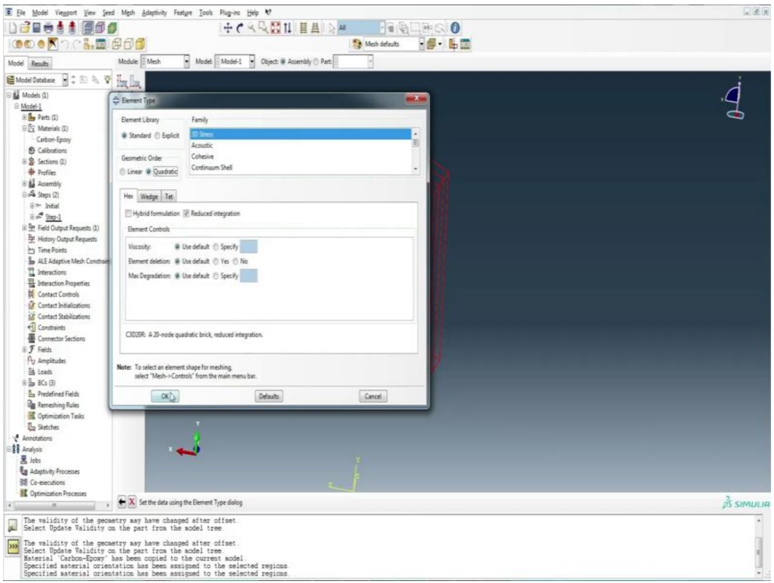

We take the 3D type element.

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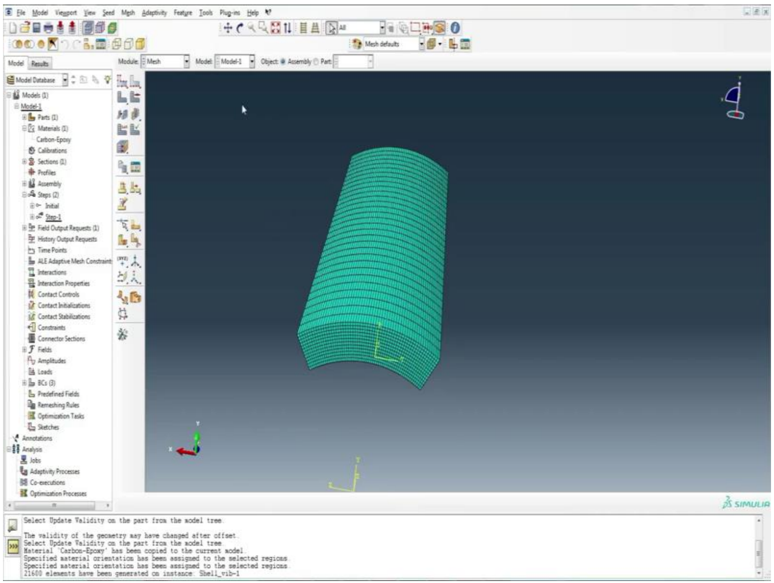

Finally, the job is submitted for analysis.

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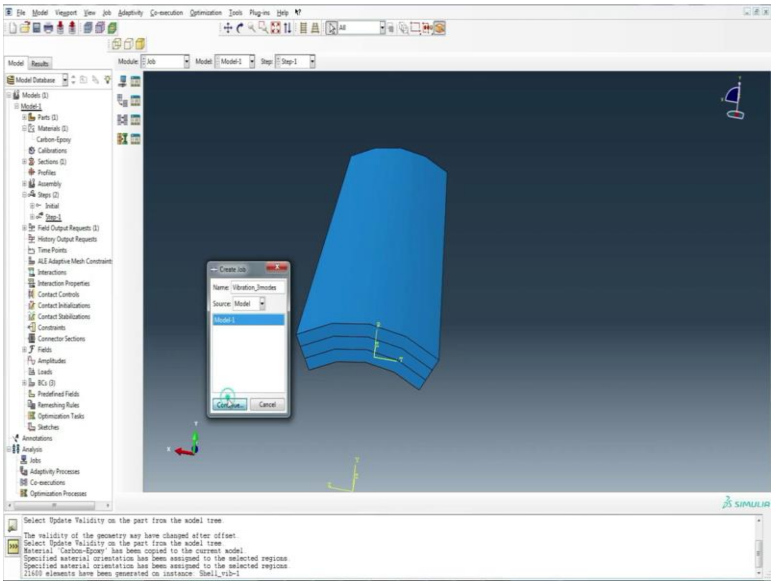

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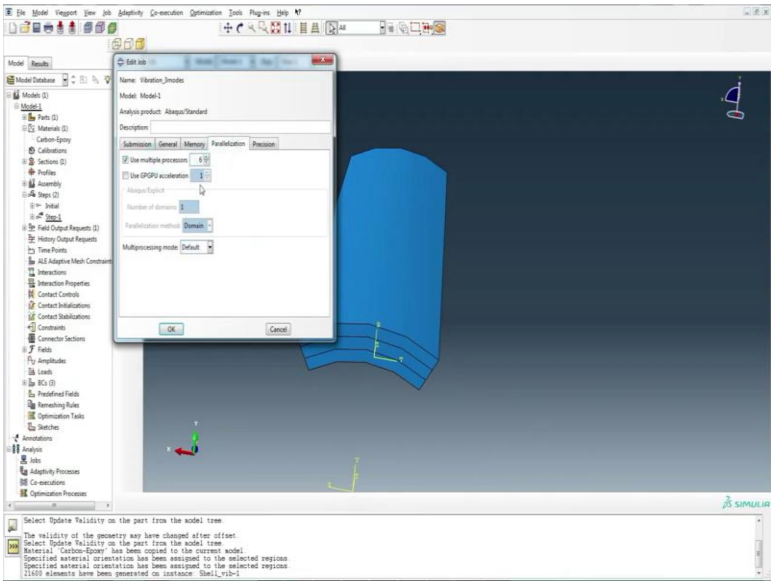

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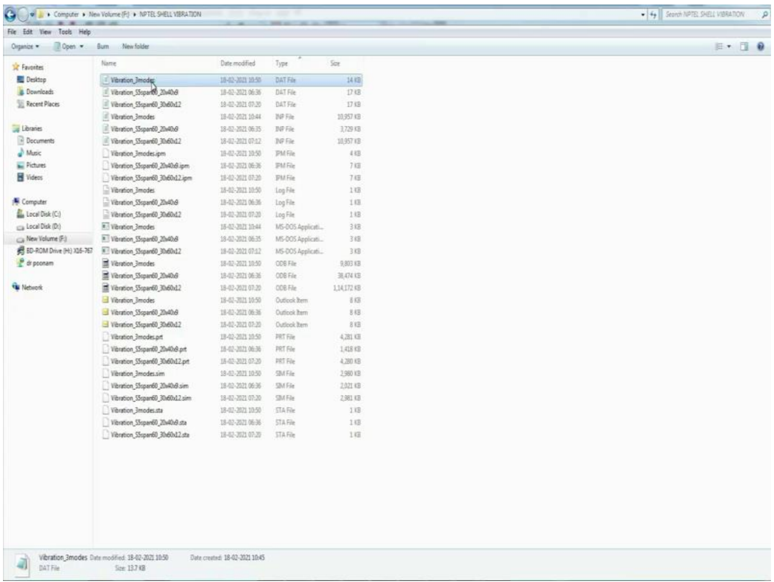

Once the analysis has been done, the data file for the analysis can be seen in the working directory as a DAT file.

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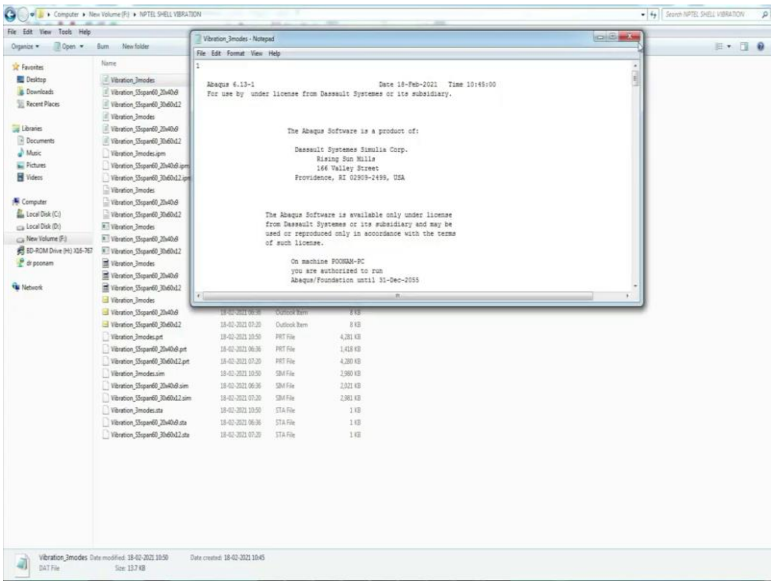

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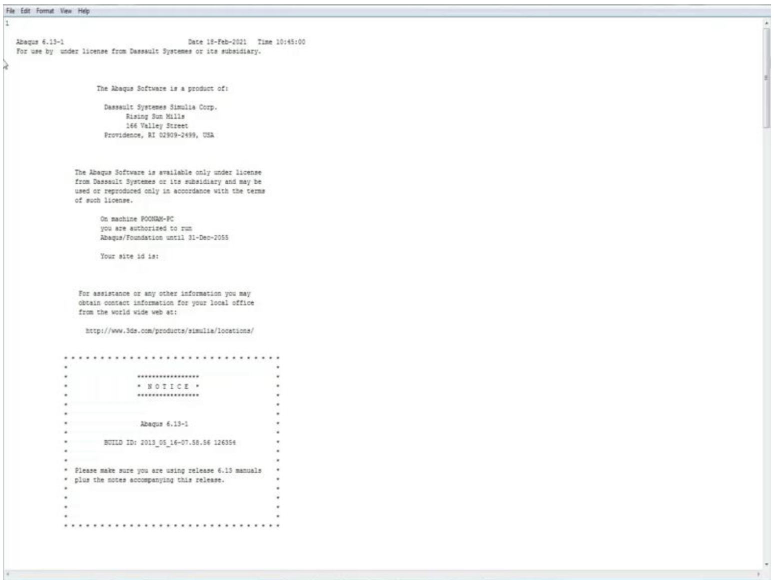

In the DAT file, we can find the calculated frequencies corresponding to the mode shapes.

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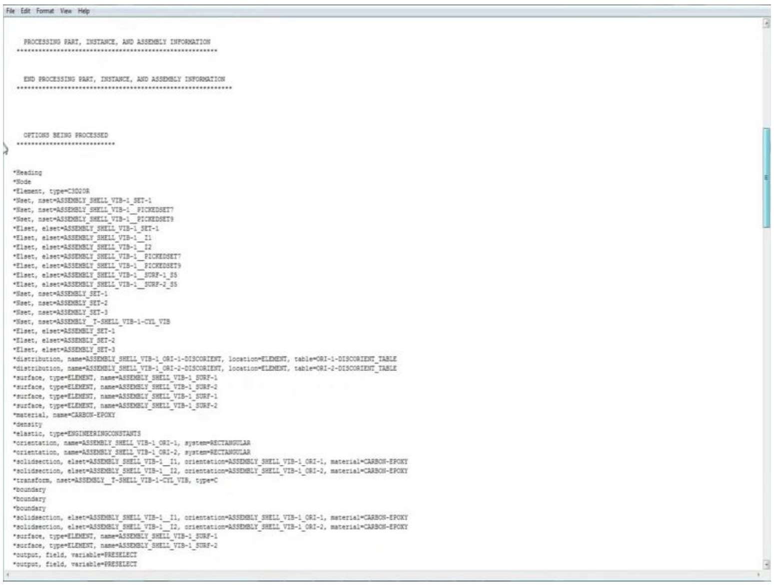

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*Elset, elset=ASSEMBLY_SHELL_VIB-1_SET-1
*Elset, elset=ASSEMBLY_SHELL_VIB-1__I1
*Elset, elset=ASSEMBLY_SHELL_VIB-1__I2
*Elset, elset=ASSEMBLY_SHELL_VIB-1__PICKEDSET7
*Elset, elset=ASSEMBLY_SHELL_VIB-1__PICKEDSET9
*Elset, elset=ASSEMBLY_SHELL_VIB-1__SURF-1_S5
*Elset, elset=ASSEMBLY_SHELL_VIB-1__SURF-2_S5
*Nset, nset=ASSEMBLY_SET-1
*Nset, nset=ASSEMBLY_SET-2
*Nset, nset=ASSEMBLY_SET-3
*Nset, nset=ASSEMBLY__T-SHELL_VIB-1-CYL_VIB
*Elset, elset=ASSEMBLY_SET-1
*Elset, elset=ASSEMBLY_SET-2
*Elset, elset=ASSEMBLY_SET-3
*distribution, name=ASSEMBLY_SHELL_VIB-1_ORI-1-DISCORIENT, location=ELEMENT, table=ORI-1-DISCORIENT_TABLE
*distribution, name=ASSEMBLY_SHELL_VIB-1_ORI-2-DISCORIENT, location=ELEMENT, table=ORI-2-DISCORIENT_TABLE
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-1
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-2
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-1
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-2
*material, name=CARBON-EPOXY
*density
*elastic, type=ENGINEERINGCONSTANTS
*orientation, name=ASSEMBLY_SHELL_VIB-1_ORI-1, system=RECTANGULAR
*orientation, name=ASSEMBLY_SHELL_VIB-1_ORI-2, system=RECTANGULAR
*solidsection, elset=ASSEMBLY_SHELL_VIB-1__I1, orientation=ASSEMBLY_SHELL_VIB-1_ORI-1, material=CARBON-EPOXY
*solidsection, elset=ASSEMBLY_SHELL_VIB-1__I2, orientation=ASSEMBLY_SHELL_VIB-1_ORI-2, material=CARBON-EPOXY
*transform, nset=ASSEMBLY__T-SHELL_VIB-1-CYL_VIB, type=C
*boundary
*boundary
*boundary
*solidsection, elset=ASSEMBLY_SHELL_VIB-1__I1, orientation=ASSEMBLY_SHELL_VIB-1_ORI-1, material=CARBON-EPOXY
*solidsection, elset=ASSEMBLY_SHELL_VIB-1__I2, orientation=ASSEMBLY_SHELL_VIB-1_ORI-2, material=CARBON-EPOXY
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-1
*surface, type=ELEMENT, name=ASSEMBLY_SHELL_VIB-1_SURF-2
*output, field, variable=PRESELECT
*output, field, variable=PRESELECT
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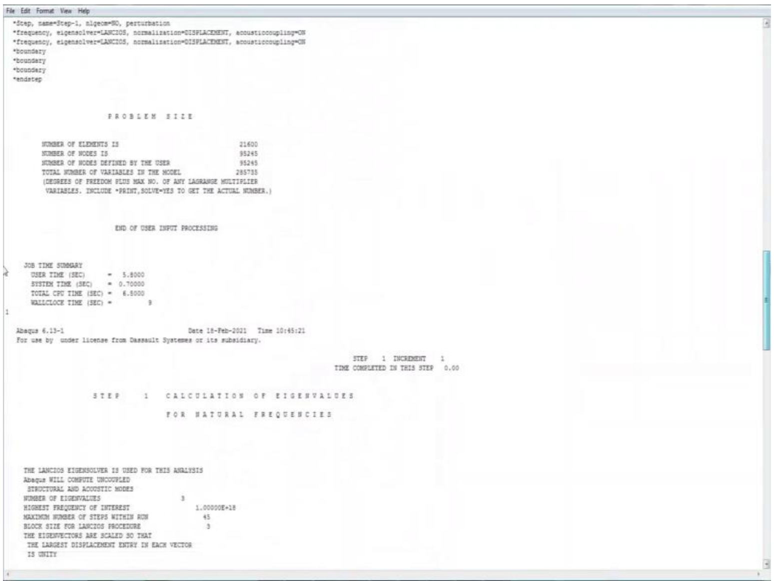
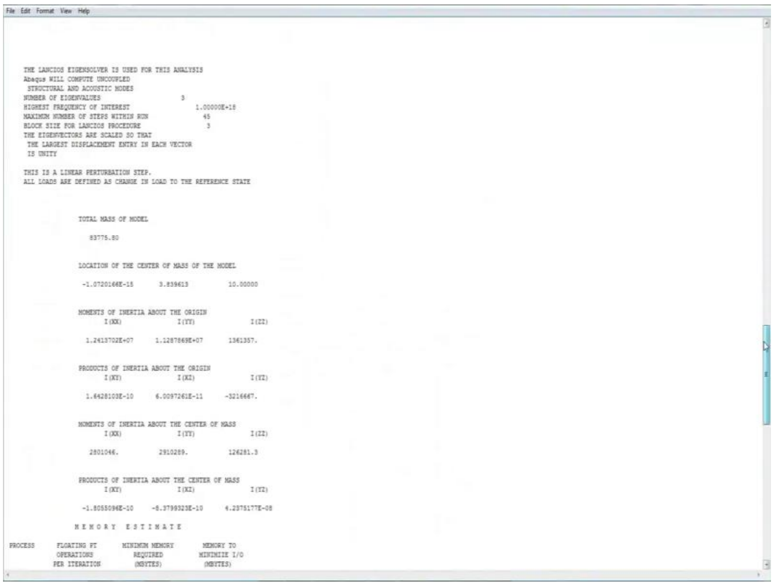
(Refer Slide Time: 10:32)



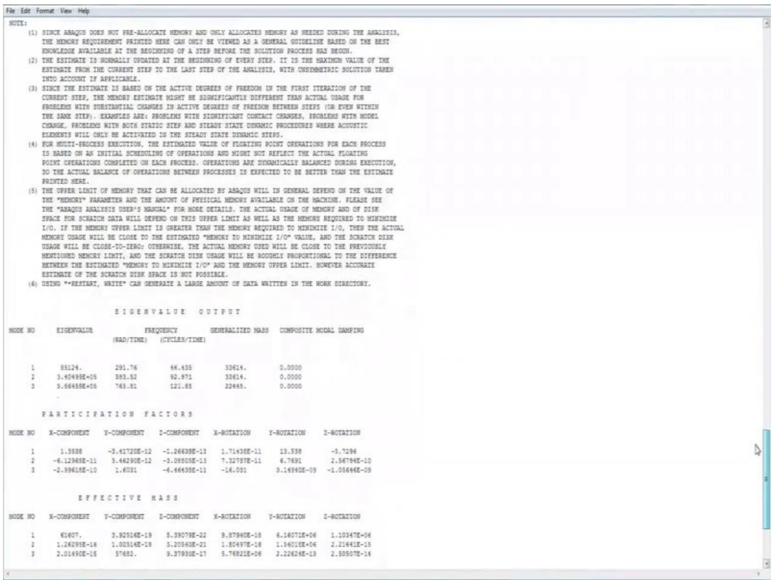
```
File Edit Format View Help
*Step, name=Step-1, nlgeom=NO, perturbation
*frequency, eigensolver=LANCZOS, normalization=DISPLACEMENT, acousticcoupling=ON
*frequency, eigensolver=LANCZOS, normalization=DISPLACEMENT, acousticcoupling=ON
*boundary
*boundary
*boundary
*endstep

                  P R O B L E M   S I Z E

       NUMBER OF ELEMENTS IS                              21600
       NUMBER OF NODES IS                                 95245
       NUMBER OF NODES DEFINED BY THE USER                95245
       TOTAL NUMBER OF VARIABLES IN THE MODEL            285735
       (DEGREES OF FREEDOM PLUS MAX NO. OF ANY LAGRANGE MULTIPLIER
        VARIABLES. INCLUDE *PRINT,SOLVE=YES TO GET THE ACTUAL NUMBER.)

                   END OF USER INPUT PROCESSING

   JOB TIME SUMMARY
     USER TIME (SEC)      =   5.8000
     SYSTEM TIME (SEC)    =  0.70000
     TOTAL CPU TIME (SEC) =   6.5000
     WALLCLOCK TIME (SEC) =          9
1

  Abaqus 6.13-1                          Date 18-Feb-2021   Time 10:45:21
  For use by  under license from Dassault Systemes or its subsidiary.

                                                          STEP    1  INCREMENT    1
                                                     TIME COMPLETED IN THIS STEP   0.00

             S T E P     1    C A L C U L A T I O N   O F   E I G E N V A L U E S

                          F O R   N A T U R A L   F R E Q U E N C I E S

   THE LANCZOS EIGENSOLVER IS USED FOR THIS ANALYSIS
   Abaqus WILL COMPUTE UNCOUPLED
    STRUCTURAL AND ACOUSTIC MODES
   NUMBER OF EIGENVALUES                       3
   HIGHEST FREQUENCY OF INTEREST                 1.00000E+18
   MAXIMUM NUMBER OF STEPS WITHIN RUN              45
   BLOCK SIZE FOR LANCZOS PROCEDURE                 3
   THE EIGENVECTORS ARE SCALED SO THAT
    THE LARGEST DISPLACEMENT ENTRY IN EACH VECTOR
    IS UNITY
```

(Refer Slide Time: 10:35)



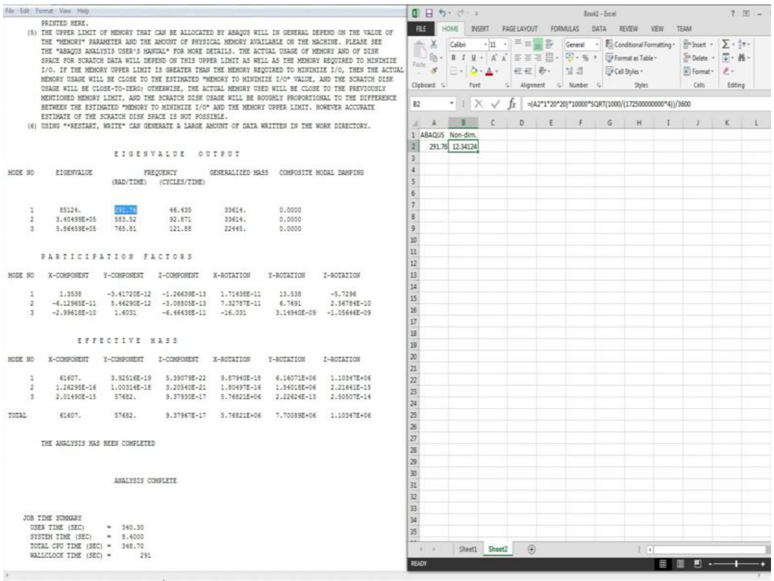
```
THE LANCZOS EIGENSOLVER IS USED FOR THIS ANALYSIS
Abaqus WILL COMPUTE UNCOUPLED
 STRUCTURAL AND ACOUSTIC MODES
NUMBER OF EIGENVALUES                    3
HIGHEST FREQUENCY OF INTEREST                1.00000E+18
MAXIMUM NUMBER OF STEPS WITHIN RUN             45
BLOCK SIZE FOR LANCZOS PROCEDURE                3
THE EIGENVECTORS ARE SCALED SO THAT
 THE LARGEST DISPLACEMENT ENTRY IN EACH VECTOR
 IS UNITY

THIS IS A LINEAR PERTURBATION STEP.
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE


          TOTAL MASS OF MODEL

            83775.80

          LOCATION OF THE CENTER OF MASS OF THE MODEL

           -1.0720166E-15      3.839613         10.00000

          MOMENTS OF INERTIA ABOUT THE ORIGIN
                I(XX)               I(YY)              I(ZZ)

            1.2413702E+07     1.1287869E+07      1361357.

          PRODUCTS OF INERTIA ABOUT THE ORIGIN
                I(XY)               I(XZ)              I(YZ)

            1.6428103E-10     6.0097261E-11     -3216667.

          MOMENTS OF INERTIA ABOUT THE CENTER OF MASS
                I(XX)               I(YY)              I(ZZ)

             2801046.           2910289.          126281.3

          PRODUCTS OF INERTIA ABOUT THE CENTER OF MASS
                I(XY)               I(XZ)              I(YZ)

           -1.8055096E-10    -8.3799323E-10     4.2375177E-08

         M E M O R Y   E S T I M A T E

PROCESS      FLOATING PT       MINIMUM MEMORY       MEMORY TO
             OPERATIONS          REQUIRED          MINIMIZE I/O
            PER ITERATION        (MBYTES)            (MBYTES)
```

(Refer Slide Time: 10:35)



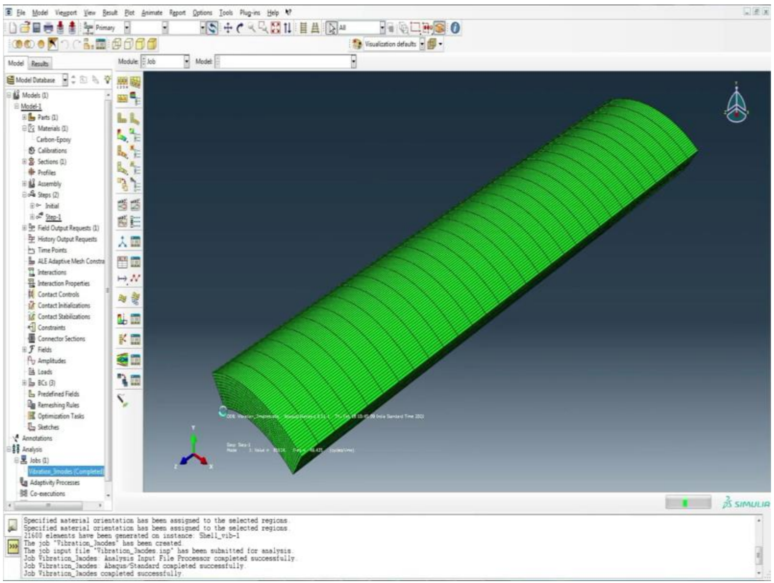
```
NOTE:
     (1) SINCE ABAQUS DOES NOT PRE-ALLOCATE MEMORY AND ONLY ALLOCATES MEMORY AS NEEDED DURING THE ANALYSIS,
         THE MEMORY REQUIREMENT PRINTED HERE CAN ONLY BE VIEWED AS A GENERAL GUIDELINE BASED ON THE BEST
         KNOWLEDGE AVAILABLE AT THE BEGINNING OF A STEP BEFORE THE SOLUTION PROCESS HAS BEGUN.
     (2) THE ESTIMATE IS NORMALLY UPDATED AT THE BEGINNING OF EVERY STEP. IT IS THE MAXIMUM VALUE OF THE
         ESTIMATE FROM THE CURRENT STEP TO THE LAST STEP OF THE ANALYSIS, WITH UNSYMMETRIC SOLUTION TAKEN
         INTO ACCOUNT IF APPLICABLE.
     (3) SINCE THE ESTIMATE IS BASED ON THE ACTIVE DEGREES OF FREEDOM IN THE FIRST ITERATION OF THE
         CURRENT STEP, THE MEMORY ESTIMATE MIGHT BE SIGNIFICANTLY DIFFERENT THAN ACTUAL USAGE FOR
         PROBLEMS WITH SUBSTANTIAL CHANGES IN ACTIVE DEGREES OF FREEDOM BETWEEN STEPS (OR EVEN WITHIN
         THE SAME STEP). EXAMPLES ARE: PROBLEMS WITH SIGNIFICANT CONTACT CHANGES, PROBLEMS WITH MODEL
         CHANGE, PROBLEMS WITH BOTH STATIC STEP AND STEADY STATE DYNAMIC PROCEDURES WHERE ACOUSTIC
         ELEMENTS WILL ONLY BE ACTIVATED IN THE STEADY STATE DYNAMIC STEPS.
     (4) FOR MULTI-PROCESS EXECUTION, THE ESTIMATED VALUE OF FLOATING POINT OPERATIONS FOR EACH PROCESS
         IS BASED ON AN INITIAL SCHEDULING OF OPERATIONS AND MIGHT NOT REFLECT THE ACTUAL FLOATING
         POINT OPERATIONS COMPLETED ON EACH PROCESS. OPERATIONS ARE DYNAMICALLY BALANCED DURING EXECUTION,
         SO THE ACTUAL BALANCE OF OPERATIONS BETWEEN PROCESSES IS EXPECTED TO BE BETTER THAN THE ESTIMATE
         PRINTED HERE.
     (5) THE UPPER LIMIT OF MEMORY THAT CAN BE ALLOCATED BY ABAQUS WILL IN GENERAL DEPEND ON THE VALUE OF
         THE "MEMORY" PARAMETER AND THE AMOUNT OF PHYSICAL MEMORY AVAILABLE ON THE MACHINE. PLEASE SEE
         THE "ABAQUS ANALYSIS USER'S MANUAL" FOR MORE DETAILS. THE ACTUAL USAGE OF MEMORY AND OF DISK
         SPACE FOR SCRATCH DATA WILL DEPEND ON THIS UPPER LIMIT AS WELL AS THE MEMORY REQUIRED TO MINIMIZE
         I/O. IF THE MEMORY UPPER LIMIT IS GREATER THAN THE MEMORY REQUIRED TO MINIMIZE I/O, THEN THE ACTUAL
         MEMORY USAGE WILL BE CLOSE TO THE ESTIMATED "MEMORY TO MINIMIZE I/O" VALUE, AND THE SCRATCH DISK
         USAGE WILL BE CLOSE-TO-ZERO; OTHERWISE, THE ACTUAL MEMORY USED WILL BE CLOSE TO THE PREVIOUSLY
         MENTIONED MEMORY LIMIT, AND THE SCRATCH DISK USAGE WILL BE ROUGHLY PROPORTIONAL TO THE DIFFERENCE
         BETWEEN THE ESTIMATED "MEMORY TO MINIMIZE I/O" AND THE MEMORY UPPER LIMIT. HOWEVER ACCURATE
         ESTIMATE OF THE SCRATCH DISK SPACE IS NOT POSSIBLE.
     (6) USING "*RESTART, WRITE" CAN GENERATE A LARGE AMOUNT OF DATA WRITTEN IN THE WORK DIRECTORY.


                    E I G E N V A L U E   O U T P U T

MODE NO     EIGENVALUE             FREQUENCY         GENERALIZED MASS   COMPOSITE MODAL DAMPING
                             (RAD/TIME)   (CYCLES/TIME)


      1       85124.        291.76        46.435        33614.        0.0000
      2     3.40499E+05     583.52        92.871        33614.        0.0000
      3     5.86459E+05     765.81        121.85        22445.        0.0000


        P A R T I C I P A T I O N   F A C T O R S

MODE NO    X-COMPONENT    Y-COMPONENT    Z-COMPONENT    X-ROTATION     Y-ROTATION     Z-ROTATION

      1       1.3538      -3.41720E-12   -1.26639E-13   1.71438E-11     13.538        -5.7296
      2    -6.12365E-11    5.46290E-12   -3.08505E-13   7.32757E-11     6.7691        2.56794E-10
      3    -2.99618E-10     1.6031       -6.46435E-11   -16.031        3.14940E-09   -1.05646E-09


              E F F E C T I V E   M A S S

MODE NO    X-COMPONENT    Y-COMPONENT    Z-COMPONENT    X-ROTATION     Y-ROTATION     Z-ROTATION

      1       61607.       3.92516E-19    5.39079E-22   9.87940E-18   6.16071E+06   1.10347E+06
      2     1.26295E-16    1.00514E-19    5.20540E-21   1.80497E-18   1.54018E+06   2.21641E-15
      3     2.01490E-15     57682.        9.37930E-17   5.76821E+06   2.22624E-13   2.50507E-14
```

(Refer Slide Time: 10:39)

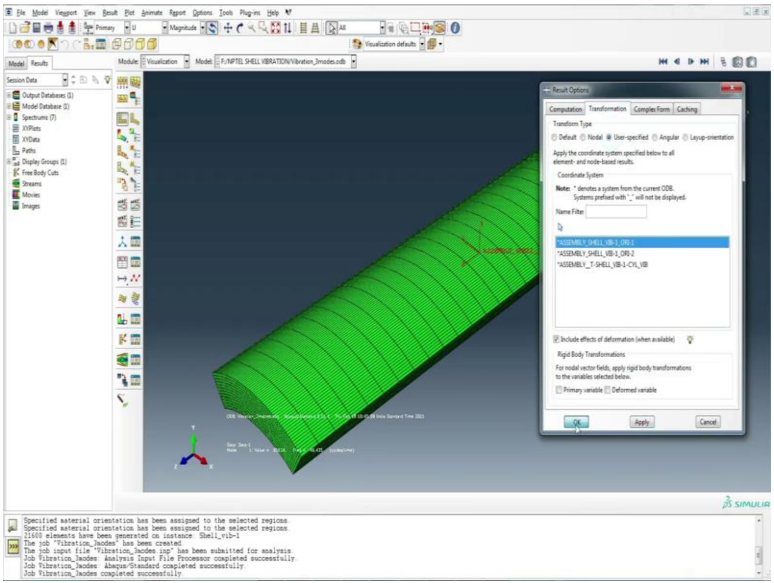


We can see the eigenvalues listed here and on comparing the results after non-dimensionalizing, it nearly matches with the results presented.

(Refer Slide Time: 11:29)

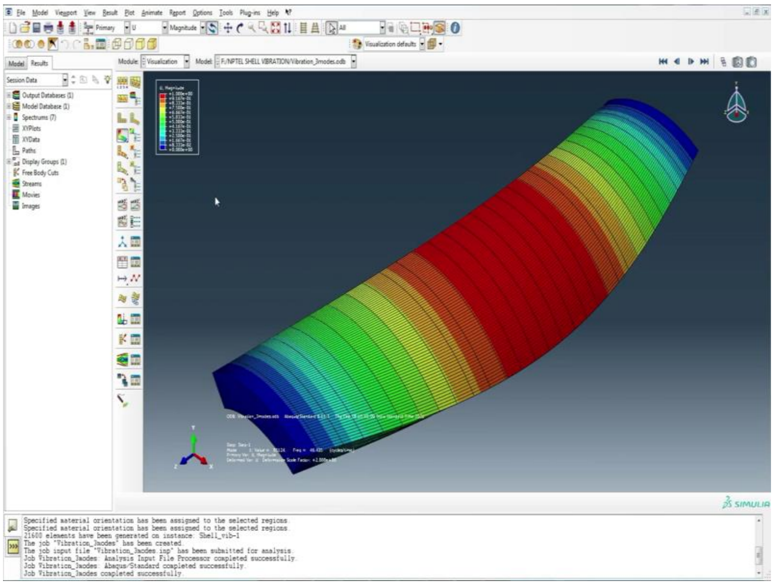


(Refer Slide Time: 11:35)

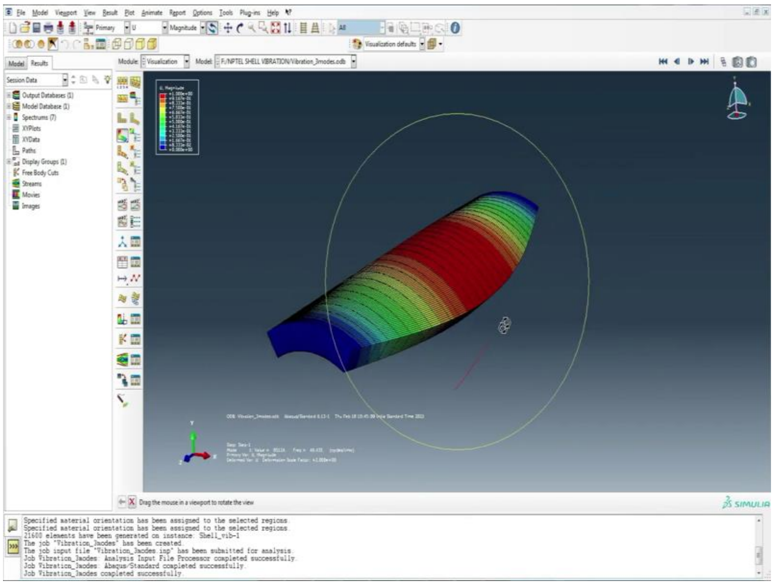


We can also visualize the mode shapes.

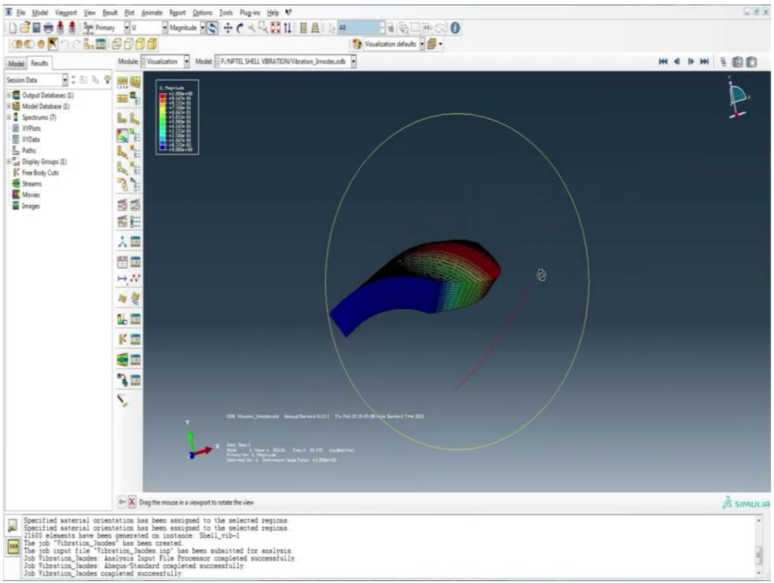
(Refer Slide Time: 11:47)



(Refer Slide Time: 11:50)



(Refer Slide Time: 11:50)



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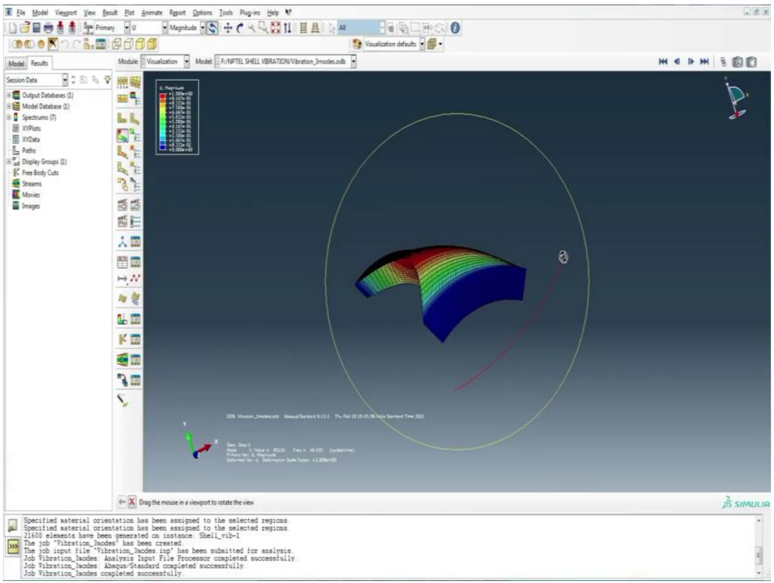
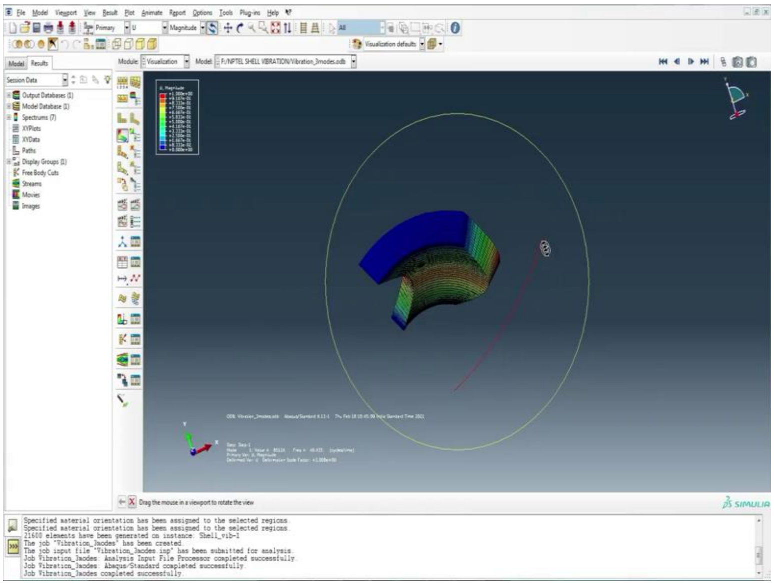
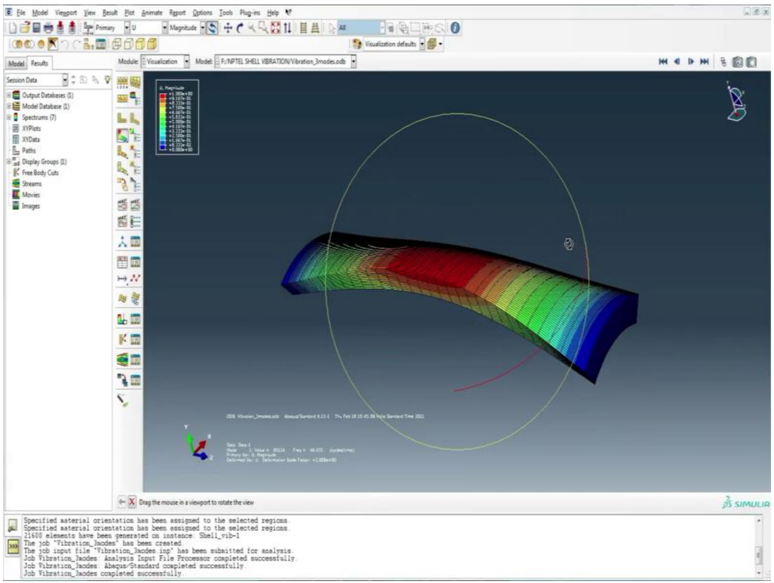


(Refer Slide Time: 11:53)



(Refer Slide Time: 11:54)



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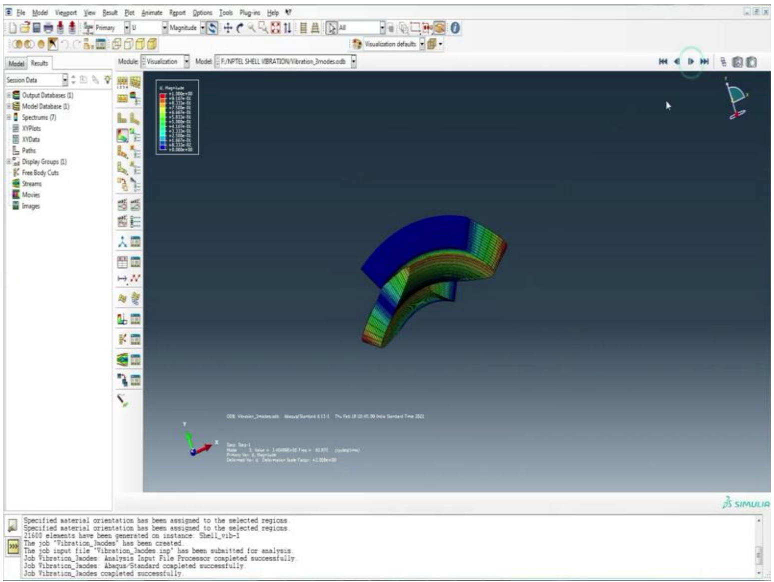

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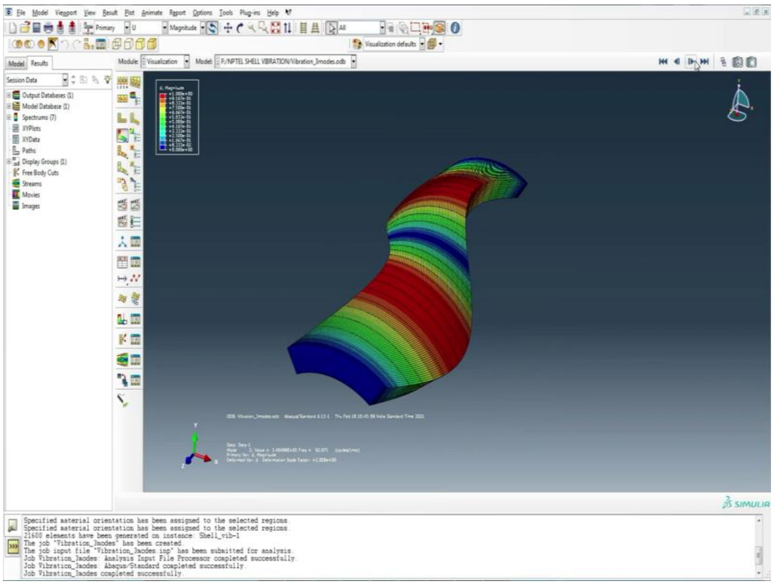
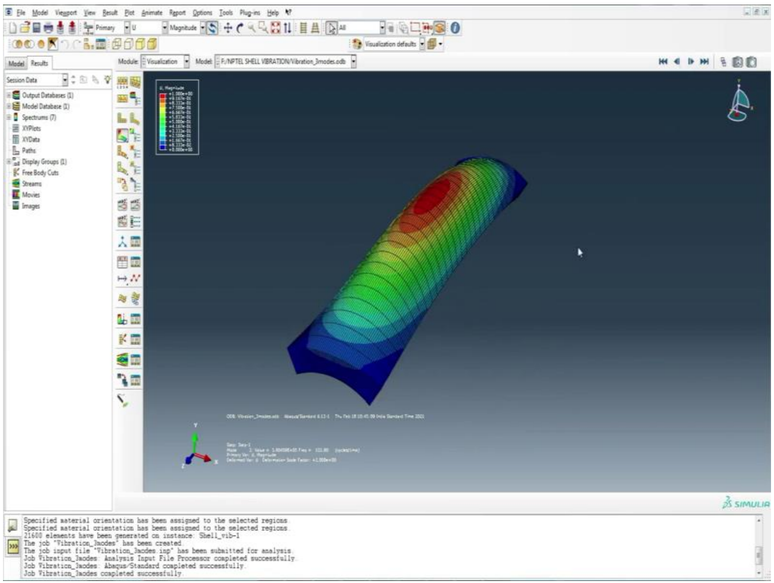


(Refer Slide Time: 12:03)



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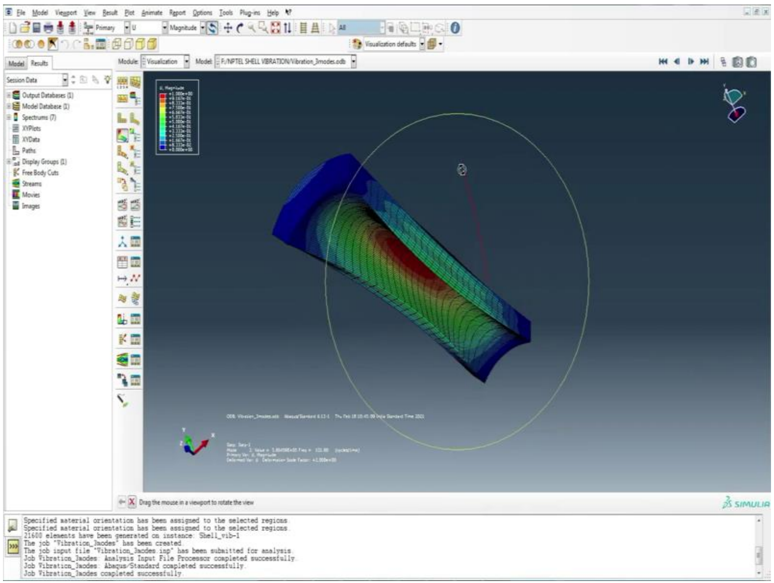

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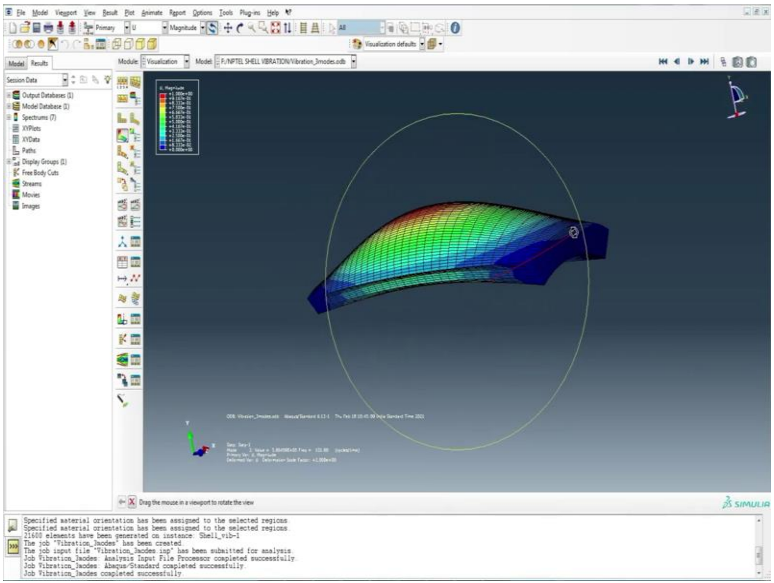

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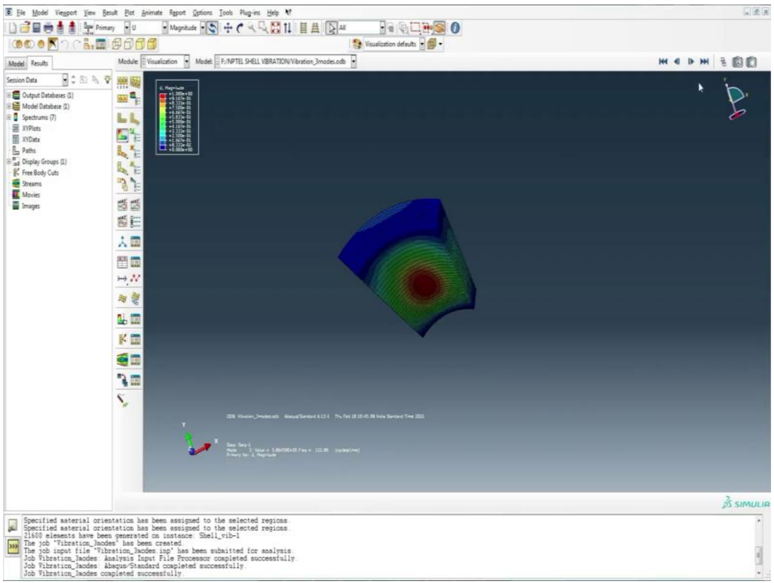

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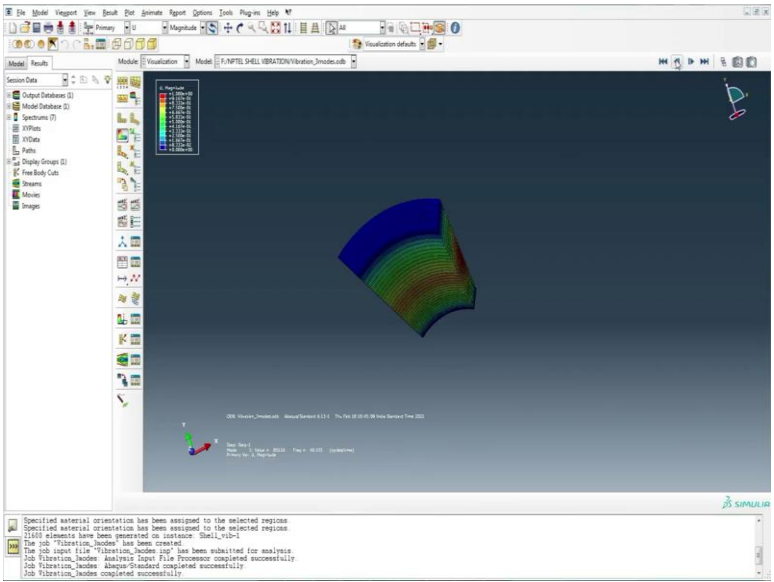

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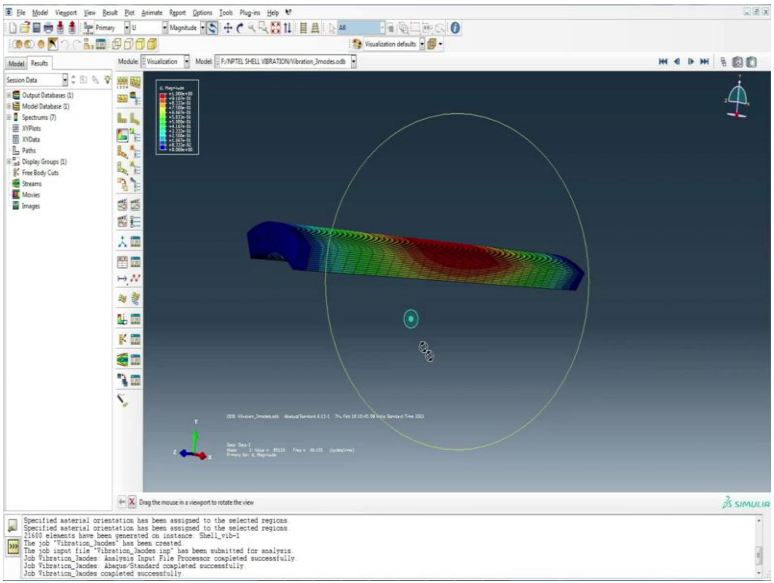

(Refer Slide Time: 12:18)