Theory of Composite Shells

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Week - 02

Lecture - 01

Basic Theorems of differential geometry

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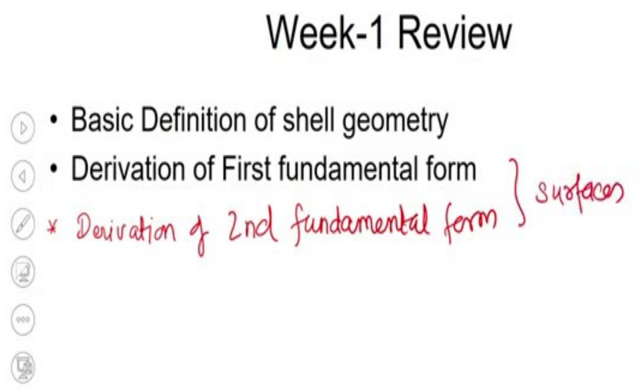

Dear learners welcome to the first lecture of the second week. In the 1st week, we have done the basic definition of shell geometry, derivation of first fundamental forms, and derivation of the second fundamental form of surfaces.

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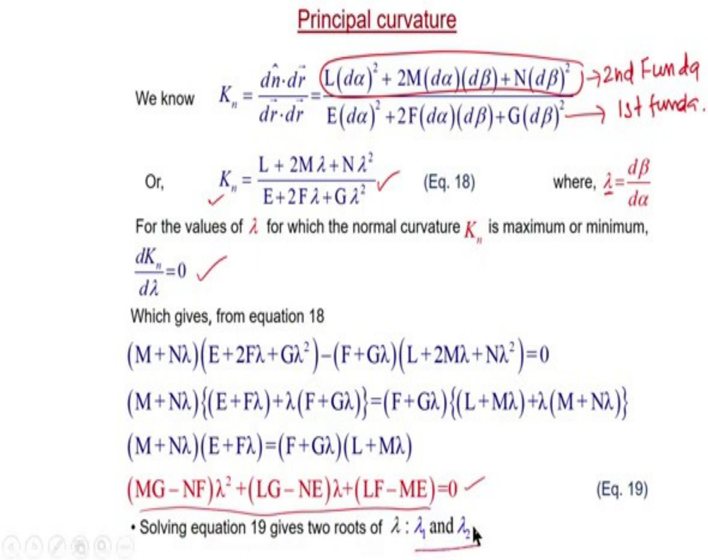

In the first week, we have derived the definition of curvature, normal curvature K_n can be written as $K_n = \frac{d\hat{n}.dr}{dr.dr} = \frac{L(d\alpha)^2 + 2M(d\alpha)(d\beta) + N(d\beta)^2}{E(d\alpha)^2 + 2F(d\alpha)(d\beta) + G(d\beta)^2}$

This numerator term is known as the second fundamental form and the denominator is the first fundamental form of surfaces. If we divide it by $d\alpha$, top and bottom, then it reduces to equation (18)

$$K_n = \frac{L + 2M\lambda + N\lambda^2}{E + 2F\lambda + G\lambda^2} \qquad \text{where, } \lambda = \frac{d\beta}{d\alpha}$$

If we differentiate equation (18) with respect to λ and equate to 0; $\frac{dK_n}{d\lambda} = 0$,

$$(M + N\lambda)(E + 2F\lambda + G\lambda^2) - (F + G\lambda)(L + 2M\lambda + N\lambda^2) = 0,$$

ultimately it reduces to a quadratic equation

$$(MG - NF)\lambda^2 + (LG - NE)\lambda + (LF - ME) = 0$$

and the solution of the quadratic equation gives the two roots. $\lambda : \lambda_1 and \lambda_2$

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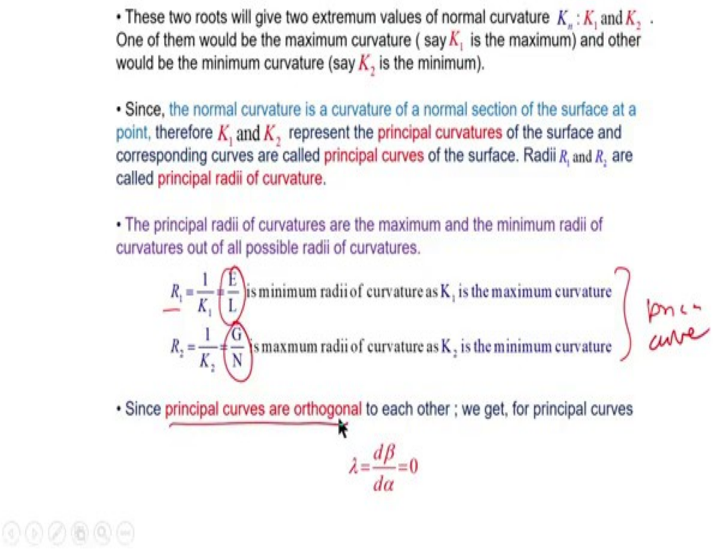

These two roots are extremum values of normal curvature K_n : K_1 and K_2, and these are known as principal curvatures. If you are interested to find the principal radii then we can say that $R_1 = \frac{E}{L} = \frac{1}{K_1}$ and $R_2 = \frac{G}{N} = \frac{1}{K_2}$. But the formula written here is valid when the parametric or principal curves are orthogonal to each other

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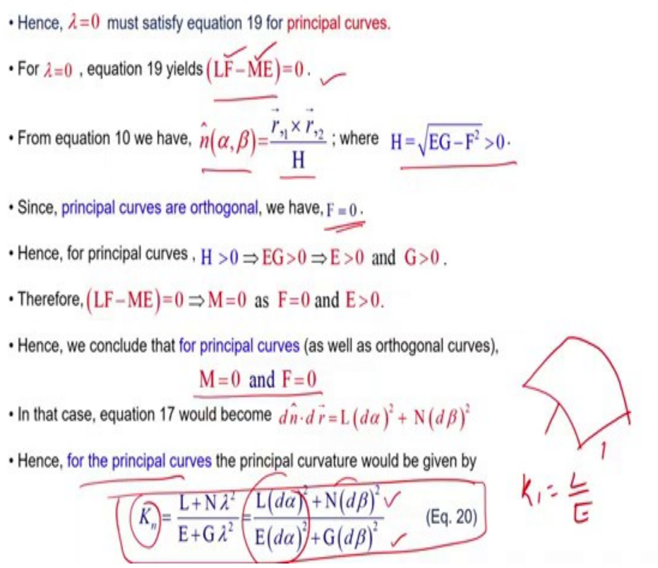

How to reduce this? In case, λ is 0, it must satisfy equation (19)

$\left(MG-NF\right)\lambda^2+\left(LG-NE\right)\lambda+\left(LF-ME\right)=0$ for the principal curves.

If λ equals to 0, this equation (19) becomes 0. And from the equation of the normal, we get this equation where H $>$ 0. If principal curves are orthogonal, then F = 0 and M = 0 and this will give you nothing. Ultimately for the principal curves, we conclude that F & M are equal to 0 and it reduces to equation (20)

$$K_n=\frac{L+N\lambda^2}{E+G\lambda^2}=\frac{L\left(d\alpha\right)^2+N\left(d\beta\right)^2}{E\left(d\alpha\right)^2+G\left(d\beta\right)^2}.$$

We cannot reduce from the previous equations, by just putting $d\beta=0$ or $d\alpha=0$. And now from this equation, we can say that the curves where either β is constant or α is constant; for constant β - lines $d\beta=0$, if $d\beta=0$, then $K_1=\frac{L}{E}$.

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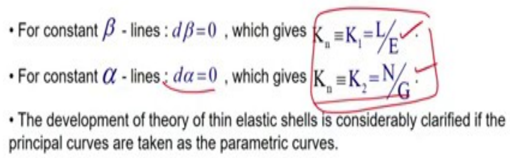

For a constant, α - lines $d\alpha = 0$, because there is no change in α for a constant α -line which gives you $K_2 = \frac{N}{G}$. So, through this assumption, where principal curvatures are orthogonal to each other, we get $\frac{L}{E}$ and $\frac{N}{G}$.

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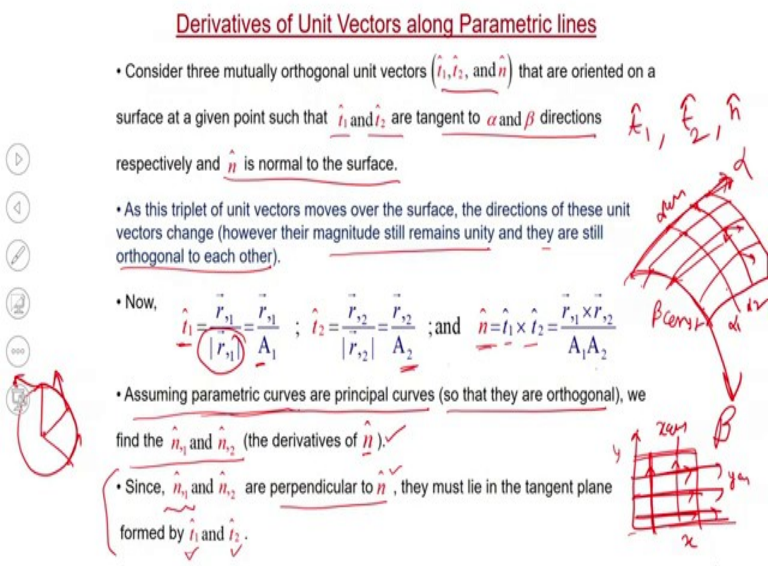

Now, we are going to find out the derivative of the unit vector along its parametric lines, which is very important. These derivatives help us to frame up some more theorems which are ultimately used to develop the fundamental equations for the surfaces.

Consider three orthogonal unit vectors, if I say this is surface (s), you may assume in any direction α and β. So, take β is increasing here. At a particular line here β is constant and α is increasing here, and at a particular line here α is constant.

There is no need to worry about x and y if we have a grid system. What is a Grid system? A line here x values remain constant. Over here x is constant if we draw like this over here y value is constant. In the same way, we can say that here on which line it is going, for perpendicular lines β value will be constant and here α will be constant, let us say α_1, α_2, and so on.

We consider three mutually orthogonal unit vectors, $\hat{t}_1, \hat{t}_2$, and $\hat{n}$, where $\hat{t}_1$ is a tangent vector along line 1, and $\hat{t}_2$ is a tangent vector along line 2, and $\hat{n}$ is the surface normal. These are oriented on a surface at a given point such that $\hat{t}_1$ and $\hat{t}_2$ are tangent to α and β directions (first line and second line of curvatures) and $\hat{n}$ is normal to the surface. This triplet of unit vectors moves over the surface because if we move from one point to another point some different tangents can be defined.

Now, the direction of these unit vectors changes. However, their magnitude remains unity and they are still orthogonal to each other. If you remember initially, I said that in a circle, if you change from here to here, its tangent vector changes. So, it's sign changes.

$\hat{t}_1$ can be defined as $\hat{t}_1 = \frac{r_{,1}}{|r_{,1}|}$; $|r_{,1}| = A_1$, A_1 is lame's parameter. So $\hat{t}_1 = \frac{r_{,1}}{A_1}$. Same way

$\hat{t}_2 = \frac{r_{,2}}{|r_{,2}|}$, and $|r_{,2}| = A_2$. So, $\hat{t}_2 = \frac{r_{,2}}{A_2}$. A_2 is lame's parameter

then $\hat{n} = \hat{t}_1 X \hat{t}_2$ and it can be written as $\frac{r_{,1} X r_{,2}}{A_1 A_2}$.

Assuming the parametric curves are principal curves and they are orthogonal. We can find the derivatives of surface normal and derivatives of tangents. Since, $\hat{n}_{,1}$ and $\hat{n}_{,2}$ are perpendicular to $\hat{n}$, they must lie in the tangent plane formed by $\hat{t}_1$ and $\hat{t}_2$. So, this is a very important frame-up that $\hat{n}_{,1}$ and $\hat{n}_{,2}$, will be perpendicular to $\hat{n}$ and they will lie in the tangent plane $\hat{t}_1$ and $\hat{t}_2$.

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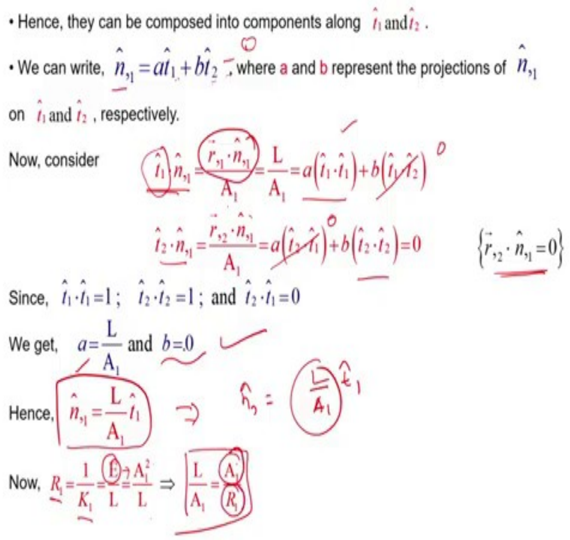

If this is the condition, then we can write that $\hat{n}_{,1} = a\hat{t}_1 + b\hat{t}_2$ because they lie in the plane of tangent plane, where a, b represents the projection of $\hat{n}_{,1}$ on the plane $\hat{t}_1$ and $\hat{t}_2$.

Now, multiply this equation with $\hat{t}_1$. If you multiply with this equation with $\hat{t}_1$; then, $\hat{t}_1.\hat{t}_1 = 1$ and $\hat{t}_2.\hat{t}_1 = 0$. So, basically, this term will go to 0 and it remains 'a'. From here we can say $\hat{t}_1 \cdot \hat{n}_{,1} = \frac{r_{,1}.\hat{n}_{,1}}{A_1}$ ($\hat{t}_1$ is $r_{,1}$ upon A_1). So, this is the definition of $\frac{L}{A_1}$ and $a = \frac{L}{A_1}$.

Same way if we multiply $\hat{n}_{,1}$ with the $\hat{t}_2$, then $\hat{t}_2.\hat{n}_{,1} = \frac{r_{,2}.\hat{n}_{,1}}{A_1}$ and from here this $\hat{t}_1 \cdot \hat{t}_2$ =0. So, this term is going to be 0. For an orthogonal system, it is going to be 0. From these two equations, b = 0 and a = $\frac{L}{A_1}$. We can say that $\hat{n}_{,1} = \frac{L}{A_1}\hat{t}_1$. Now, we will further simplify it, if $R_1 = \frac{1}{K_1}$ (radius of curvature) and it can be written as $\frac{E}{L}$ and E = A_1^2 lame's parameter.

$\frac{L}{A_1}$ from here R_1 is here and A_1 goes from here, so $\frac{A_1}{R_1} = \frac{L}{A_1}$. Hence, $\frac{L}{A_1}$ can be written as $\frac{A_1}{R_1}$ in terms of lame's parameter and radius of the curve. In this way we will get the equation here, $\hat{n}_{,1} = \frac{A_1}{R_1}\hat{t}_1$.

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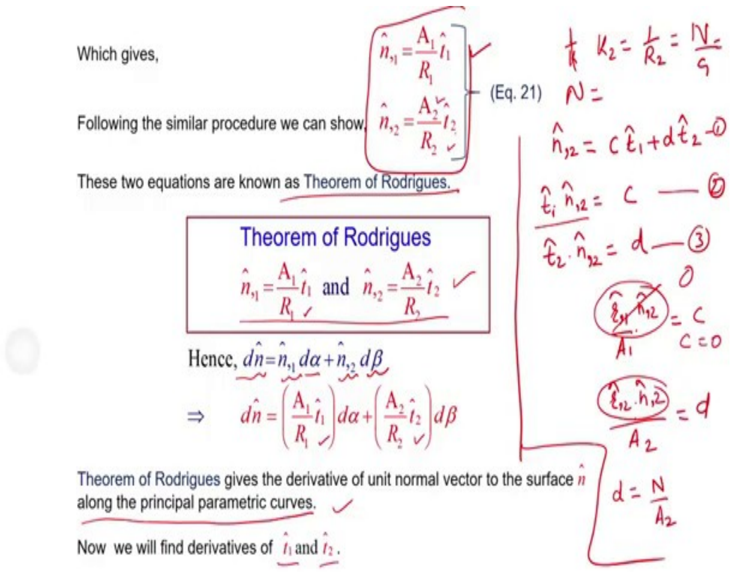

By following the same procedure, I am just going to derive it further, let us say $\hat{n}_{,2}$ = c $\hat{t}_1$ + d $\hat{t}_2$. Now, if you multiply this equation with $\hat{t}_1$ it will give you c. If you multiply this equation with the help of $\hat{t}_2$ that gives you d, let us say (2) and (3). Because we know that $\hat{t}_1$ and $\hat{t}_2$ are perpendicular to each other then, $\frac{r_{,1} . \hat{n}_{,2}}{A_1}$ = c.

And $\frac{r,_2 . \hat{n},_2}{A_1}$ = d. These are orthogonal to each other (curvilinear parameters), this is going to be 0, so c becomes 0. From here we can say that $d = \frac{N}{A_2}$.

Now, using the concept of $K_2 = \frac{1}{R_2} = \frac{N}{G}$ and from there again $N = \frac{F}{G}$. F becomes $\frac{A_2^2}{\hat{n}}$

Using these concepts we can derive that $\hat{n},_1 = \frac{A_1}{R_1}\hat{t}_1$ and $\hat{n},_2 = \frac{A_2}{R_2}\hat{t}_2$. The surface normal derivative along one direction can be written as $\frac{A_1}{R_1}\hat{t}_1$. A_1 & A_2 are the lame's parameters, these are very easy to calculate, and R_1 & R_2 are the radii of curvature.

These two differential equations are known as the theorem of Rodrigues. These are very useful in developing the theorem for surfaces. Now, we can derive some more relations.

Let us say $d\hat{n} = \hat{n},_1 d\alpha + \hat{n},_2 d\beta \quad \Rightarrow d\hat{n} = \left(\frac{A_1}{R_1}\hat{t}_1\right)d\alpha + \left(\frac{A_2}{R_2}\hat{t}_2\right)d\beta$

$\hat{n},_1$ and $\hat{n},_2$ can be replaced with the help of the theorem of Rodrigues. It gives the derivative of the unit normal vector surface along the principal parametric curves.

The very first theorem is the theorem of Rodrigues. The starting one is derivative of the surface normal will lie in the plane of $\hat{t}_1$ and $\hat{t}_2$, using the simple mathematics multiplying with $\hat{t}_1$ and $\hat{t}_2$ we can find the basic relations. Now, we will find the derivative of $\hat{t}_1$ and $\hat{t}_2$.

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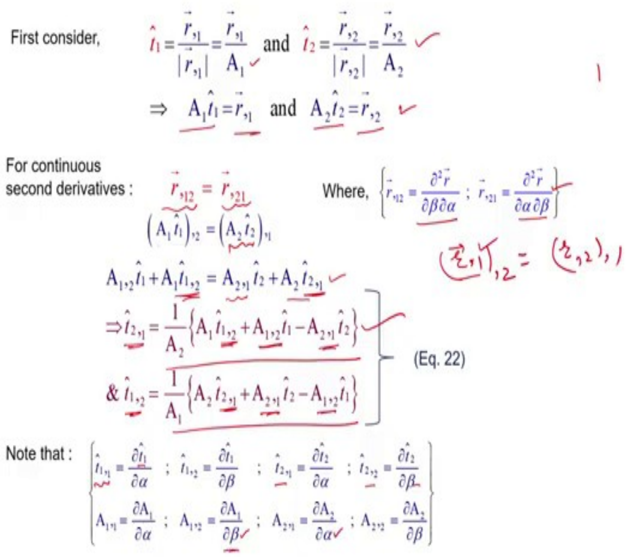

First, we will consider that $\hat{t}_1 = \frac{r_{,1}}{|r_{,1}|} = \frac{r_{,1}}{A_1}$ and $\hat{t}_2 = \frac{r_{,2}}{|r_{,2}|} = \frac{r_{,2}}{A_2}$

Now, $A_1\hat{t}_1 = r_{,1}$ $\hat{t}_2$ and $A_2\hat{t}_2 = r_{,2}$. So, we can say that $r_{,12} = r_{,21}$ which is the mixed derivative of $d\beta$ and $d\alpha$, if you change the order, it does not affect the position vector.

We can write that $(r_{,1})_{,2} = (r_{,2})_{,1} \Rightarrow (A_1\hat{t}_1)_{,2} = (A_2\hat{t}_2)_{,1}$

$r_{,1}$ can be written as $A_1\hat{t}_1$ and $r_{,2}$ can be written as $A_2\hat{t}_2$.

The differentiation of the first function will be $A_{1,2}\hat{t}_1 + A_1\hat{t}_{1,2} = A_{2,1}\hat{t}_2 + A_2\hat{t}_{2,1}$.

From this equation, we can say that $\hat{t}_{2,1}$ and $\hat{t}_{1,2}$ can be written as:

$\hat{t}_{2,1} = \frac{1}{A_2}\left(A_1\hat{t}_{1,2} + A_{1,2}\hat{t}_1 - A_{2,1}\hat{t}_2 \right)$ and $\hat{t}_{1,2} = \frac{1}{A_1}\left(A_2\hat{t}_{2,1} + A_{2,1}\hat{t}_1 - A_{1,2}\hat{t}_1 \right)$

We know that $\hat{t}_{1,1} = \frac{\partial \hat{t}_1}{\partial \alpha}$; $\hat{t}_{1,2} = \frac{\partial \hat{t}_1}{\partial \beta}$; $\hat{t}_{2,1} = \frac{\partial \hat{t}_2}{\partial \alpha}$ and $\hat{t}_{2,2} = \frac{\partial \hat{t}_2}{\partial \beta}$.

Similarly, $A_{1,1} = \frac{\partial A_1}{\partial \alpha}$; $A_{1,2} = \frac{\partial A_1}{\partial \beta}$; $A_{2,1} = \frac{\partial A_2}{\partial \alpha}$; $A_{2,2} = \frac{\partial A_2}{\partial \beta}$ and so on.

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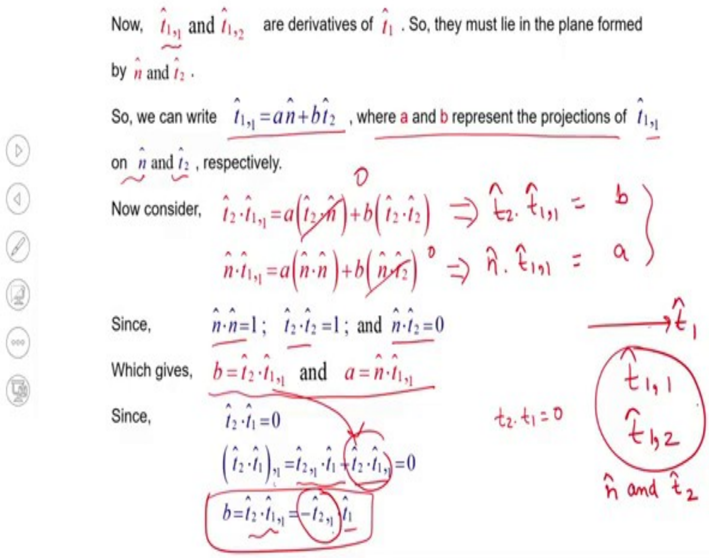

Let us say vector $\hat{t}_1$ and its derivative $\hat{t}_{1,1}$ and $\hat{t}_{1,2}$ are perpendicular to this and will lie on the plane of $\hat{n}$ and $\hat{t}_2$. This is the starting. We frame up the equation again

$\hat{t}_{1,1} = a\hat{n} + b\hat{t}_2$,

where a and b represent the projections of $\hat{t}_{1,1}$ on $\hat{n}$ and $\hat{t}_2$ respectively.

If you multiply this equation with $\hat{t}_2$, that gives you

$\hat{t}_2.\hat{t}_{1,1} = a\left(\hat{t}_2.\hat{n}\right) + b\left(\hat{t}_2.\hat{t}_2\right)$ ultimately, it gives you b.

If you multiply with $\hat{n}$ it will be

$\hat{n}.\hat{t}_{1,1} = a\left(\hat{n}.\hat{n}\right) + b\left(\hat{n}.\hat{t}_2\right)$ ultimately, it gives you a. Because $\hat{n}.\hat{t}_2$ is 0.

Now, $\hat{t}_1$ and $\hat{t}_2$ are perpendicular to each other, their product will be 0. From here we set up another set of equations that $\left(\hat{t}_2.\hat{t}_1\right)_{,1} = \hat{t}_{2,1}.\hat{t}_1 + \hat{t}_2.\hat{t}_{1,1} = 0$

This term which we are using here can be written as $b = \hat{t}_2.\hat{t}_{1,1} = -\hat{t}_{2,1}.\hat{t}_1$

Now, we will find $\hat{t}_{2,1}$.

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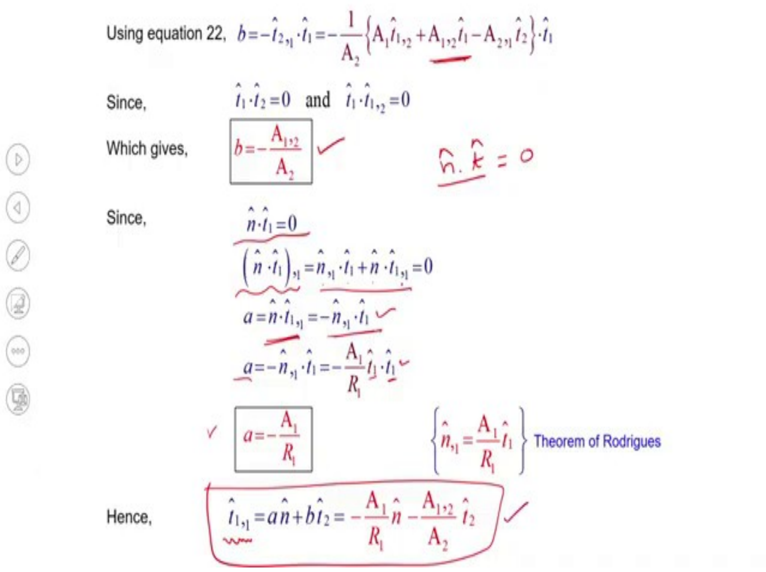

We will use this equation (22) here and multiply with $\hat{t}_1$:

$$b = \hat{t}_2.\hat{t}_{1,1} = -\hat{t}_{2,1}.\hat{t}_1 = -\frac{1}{A_2}\left[A_1\hat{t}_{1,2} + A_{1,2}\hat{t}_1 + A_{2,1}\hat{t}_2\right].\hat{t}_1$$

$\hat{t}_1 \cdot \hat{t}_1$ will give you 1, this term will vanish and again $\hat{t}_{1,2}$ is also perpendicular to $\hat{t}_1$. It only contributes to this term. $b = -\frac{A_{1,2}}{A_2}$

Again, $\hat{n}.\hat{t}_1 = 0$; because surface normal $\hat{n}$ and $\hat{t}_1$ are perpendicular, so their dot product is going to be 0.

Taking differentiation with respect to 1 gives you this equation

$$\left(\hat{n}.\hat{t}_1\right)_{,1} = \hat{n}_{,1}.\hat{t}_1 + \hat{n}.\hat{t}_{1,1} = 0\,.$$

$\hat{n}.\hat{t}_{1,1} = -\hat{n}_{,1}.\hat{t}_1$; Hence, a $= -\hat{n}_{,1}.\hat{t}_1$

What is the substitute of that? $\hat{n}.\hat{t}_{1,1}$ substitute is $-\hat{n}_{,1}.\hat{t}_1$; from the theorem of Rodrigues

$$-\hat{n}_{,1}.\hat{t}_1 = -\frac{A_1}{R_1}\hat{t}_1.\hat{t}_1 = 1; \text{ hence, } a = -\frac{A_1}{R_1}$$

If you substitute all values then $\hat{t}_{1,1}$ can be written as:

$$\hat{t}_{1,1} = a\hat{n} + b\hat{t}_2 = -\frac{A_1}{R_1}\hat{n} - \frac{A_{1,2}}{A_2}\hat{t}_2$$

The derivative of the tangent vector along the first direction can be represented like this.

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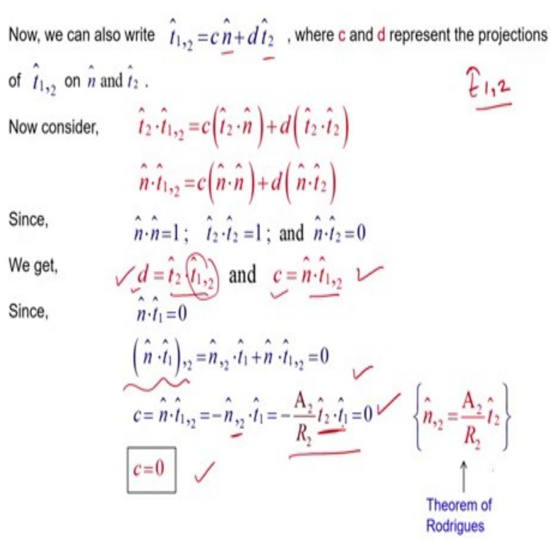

The same way we can derive the derivative of the tangent vector along the second directions. Those will also perpendicular to $\hat{t}_1$ and they will lie in the plane of $\hat{n}$ and $\hat{t}_2$. Now, you multiply with $\hat{t}_2$ and $\hat{n}$. From here, you again get equations d and c and using the concept of $\hat{n}.\hat{t}_1$, now this time taking derivative with respect to 2:

$$\left(\hat{n}.\hat{t}_1\right)_{,2} = \hat{n}_{,2}.\hat{t}_1 + \hat{n}.\hat{t}_{1,2} = 0.$$

For deriving the previous equation, we have taken the derivative with respect to 1.

Again, $c = \hat{n}.\hat{t}_{1,2}$ and $\hat{n}_{,2} = \frac{A_2}{R_2}\hat{t}_2$ using the theorem of Rodrigues.

Hence, c = $-\frac{A_2}{R_2}\hat{t}_2.\hat{t}_1$

Now, you see that $\hat{t}_2.\hat{t}_1$ are perpendicular to each other, it is going to vanish. c is going to give you 0. Now, we are interested to find d. We have to multiply $\hat{t}_{1,2}$ with $\hat{t}_2$ that will give you the value of d:

$$d = \hat{t}_{1,2}.\hat{t}_2 = \frac{1}{A_1}\left[A_2\hat{t}_{2,1} + A_{2,1}\hat{t}_2 + A_{1,2}\hat{t}_1\right].\hat{t}_2.$$

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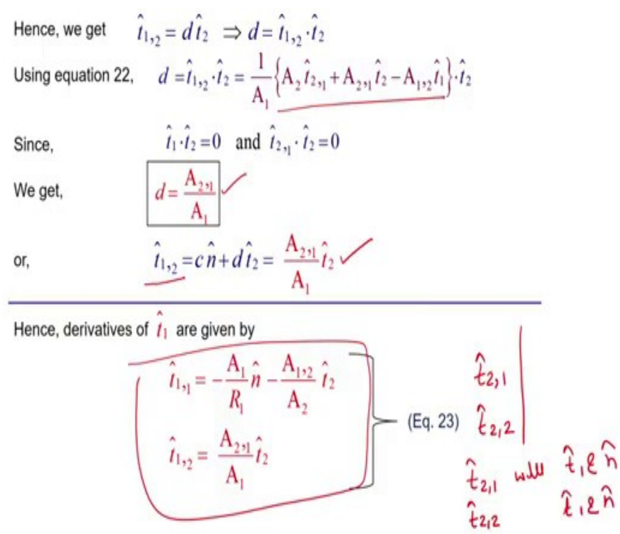

Here, $d = \frac{A_{2,1}}{A_1}$.

$\hat{t}_{1,2} = c\hat{n} + d\hat{t}_2$.

In this way, we get these two equations: -

$\hat{t}_{1,1} = -\frac{A_1}{R_1}\hat{n} - \frac{A_{1,2}}{A_2}\hat{t}_2$ and $\hat{t}_{2,2} = \frac{A_{1,2}}{A_2}\hat{t}_1$

Similarly, following the same procedure, we can get $\hat{t}_{2,1}$ and $\hat{t}_{2,2}$. Again, let us say $\hat{t}_{2,1}$ will lie here plane of $\hat{t}_1$ and $\hat{n}$. Following the similar procedure; the same way $\hat{t}_{2,2}$ also will lie on the same plane.

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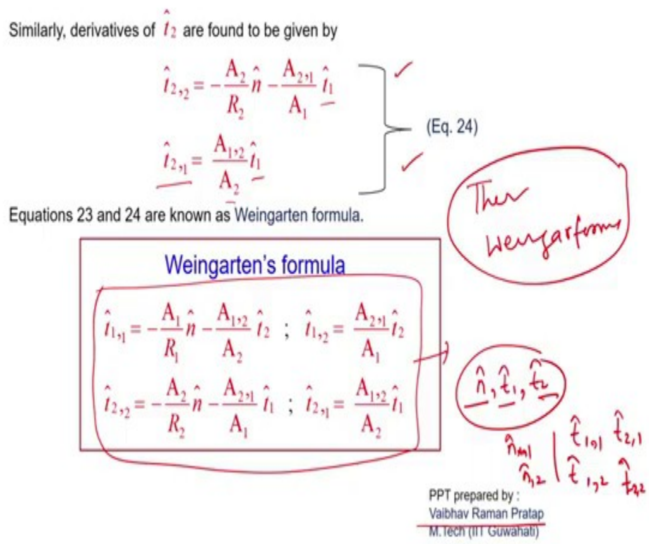

Similarly, we will get two more equations which are written here:

$$\hat{t}_{2,2} = -\frac{A_2}{R_2}\hat{n} - \frac{A_{2,1}}{A_1}\hat{t}_1 \quad \text{and} \quad \hat{t}_{2,1} = \frac{A_{1,2}}{A_2}\hat{t}_1$$

These four differential equations are giving you the relation of derivative of tangent vectors along the parametric curves 1 and 2, which can be represented in terms of lame's parameters and radius of curvatures. These four formulas are known as Weingarten formulas. And these PPTs were prepared by our M. Tech student Vaibhav Raman.

Now, we have the theorem of Rodrigues and Weingarten formulas. Why we have derived this? What is the use? These formulas will help us to derive the theorem of surfaces. These are very much required to develop the differential equation for the theorem of surfaces. In that process we need these formulas, these are standard formulas.

Once you know, these 3 vectors $\hat{n}$, $\hat{t}_1$, and $\hat{t}_2$ and using these relations we can find derivative of $\hat{n}_{1,1}$; derivative of $\hat{n}_{1,2}$. Similarly, derivative of $\hat{t}_{1,1}$; derivative of $\hat{t}_{1,2}$; derivative of $\hat{t}_{2,1}$, derivative of $\hat{t}_{2,2}$.

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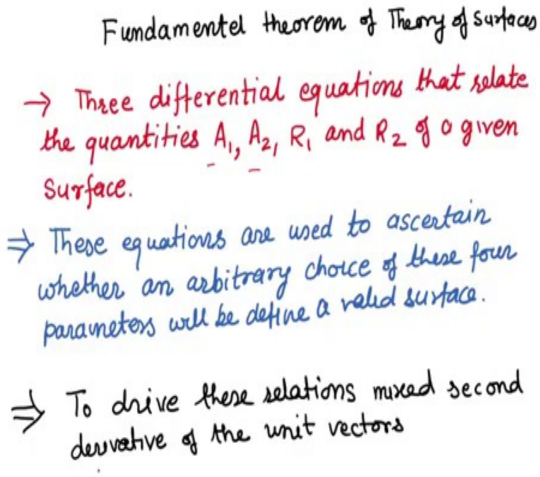

Fundamental theorem of the theory of surfaces, we are interested to find three differential equations that relate the quantities A_1, A_2, R_1, R_2 of a given surface. These equations are used to ascertain whether an arbitrary choice of these four parameters will define a valid surface or a real surface. So, to derive these relations we will be going to use the second mixed derivative of unit vectors.

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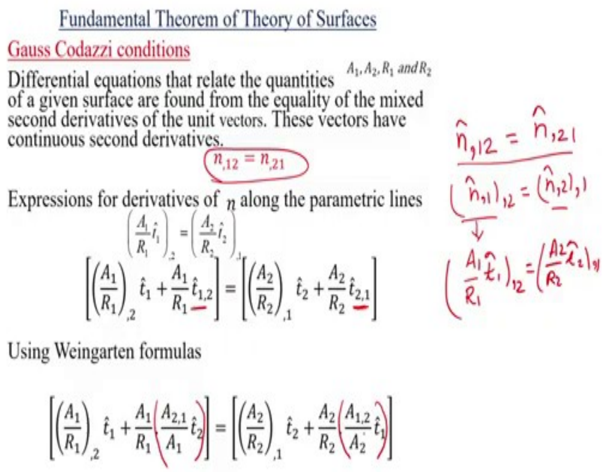

To derive the Codazzi equations we are going to use this relation the mixed derivative $\hat{n}_{,12} = \hat{n}_{,21}$.

We can say $\left(\hat{n}_{,1}\right)_{,2} = \left(\hat{n}_{,2}\right)_{,1}$.

Now, using the formula of Rodrigues we can write like this:

$$\left(\frac{A_1}{R_1}\hat{t}_1\right)_{,2} = \left(\frac{A_2}{R_2}\hat{t}_2\right)_{,1}$$

And then differentiate with respect to 2:

$$\left[\left(\frac{A_1}{R_1}\right)_{,2}\hat{t}_1 + \frac{A_1}{R_1}.\frac{A_{2,1}}{A_1}\hat{t}_2\right] = \left[\left(\frac{A_2}{R_2}\right)_{,1}\hat{t}_2 + \frac{A_2}{R_2}.\frac{A_{1,2}}{A_2}\hat{t}_1\right]$$

Using the chain rule of differential that first term as it is the differentiation of the second term.

Now, we know that what is $\hat{t}_{1,2}$ and $\hat{t}_{2,1}$:

$$\hat{t}_{2,1} = \frac{A_{1,2}}{A_2}\hat{t}_1 \text{ and } \hat{t}_{1,2} = \frac{A_{2,1}}{A_1}\hat{t}_2$$

If we substitute: $\hat{t}_1\left[\left(\frac{A_1}{R_1}\right)_{,2} - \frac{A_{1,2}}{R_2}\right] = \hat{t}_2\left[\left(\frac{A_2}{R_2}\right)_{,1} - \frac{A_{2,1}}{R_1}\right]$

and now collecting the coefficients of $\hat{t}_1$ and $\hat{t}_2$.

If you arrange from the left-hand side or right-hand side then it is going to be 0.

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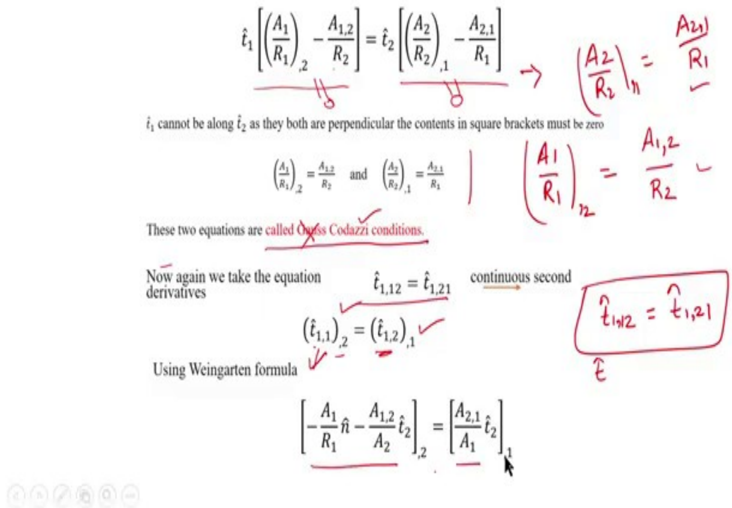

Now, the question is that $\hat{t}_1$ cannot be along the line $\hat{t}_2$, they are perpendicular to each other. The relation will be valid only if their coefficients vanish. If we are saying that it is going to be 0, then, $\left(\frac{A_1}{R_1}\right)_{,2} = \frac{A_{1,2}}{R_2}$.

From this we get $\left(\frac{A_2}{R_2}\right)_{,1} = \frac{A_{2,1}}{R_1}$s ; these two differential equations give you the relation between A_1, A_2, R_1, R_2 and these are known as Codazzi equations or Gauss-Codazzi equation. Generally, these are known as Codazzi equations. Later on, the Gauss equations will come.

Now, we are going to use the mixed derivative of tangential vectors $\hat{t}_{1,12} = \hat{t}_{1,21}$. Same way $\left(\hat{t}_{1,1}\right)_{,2} = \left(\hat{t}_{1,2}\right)_{,1}$.

Now, using the formula of Weingarten:

$$\left(\hat{t}_{1,1}\right)_{,2} = \left[-\frac{A_1}{R_1}\hat{n} - \frac{A_{1,2}}{A_2}\hat{t}_2\right]_{,2} \text{ and } \left(\hat{t}_{1,2}\right)_{,1} = \left[\frac{A_{2,1}}{A_1}\hat{t}_2\right]_{,1}$$

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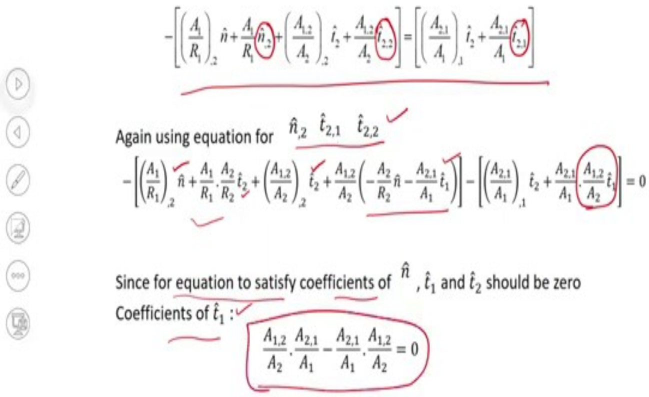

And it will give you this big equation

$$-\left[\left(\frac{A_1}{R_1}\right)_{,2}\hat{n} + \frac{A_1}{R_1}\hat{n}_{,2} + \left(\frac{A_{1,2}}{A_2}\right)_{,2}\hat{t}_2 + \frac{A_{1,2}}{A_2}\hat{t}_{2,2}\right] = \left[\left(\frac{A_{2,1}}{A_1}\right)_{,1}\hat{t}_2 + \frac{A_{2,1}}{A_1}\hat{t}_{2,1}\right]$$ in which we have coefficients of $\hat{n}_{,2}$, $\hat{t}_{2,1}$ and $\hat{t}_{2,2}$. If we use $\hat{n}_{,2}$ using the theorem of Rodrigues and theorem of Weingarten here. If we substitute these values:

$\hat{n}_{,2} = \dfrac{A_2}{R_2}\hat{t}_2$; $\hat{t}_{2,1} = \dfrac{A_{1,2}}{A_2}\hat{t}_2$; and $\hat{t}_{2,2} = \dfrac{A_{2,1}}{A_1}\hat{t}_2$.

$$-\left[\left(\frac{A_1}{R_1}\right)_{,2}\hat{n} + \frac{A_1}{R_1}.\frac{A_2}{R_2}\hat{t}_2 + \left(\frac{A_{1,2}}{A_2}\right)_{,2}\hat{t}_2 + \frac{A_{1,2}}{A_2}\left(-\frac{A_2}{R_2}\hat{n} - \frac{A_{2,1}}{A_1}\right)\hat{t}_1\right] - \left[\left(\frac{A_{2,1}}{A_1}\right)_{,1}\hat{t}_2 + \frac{A_{2,1}}{A_1}.\frac{A_{1,2}}{A_2}\hat{t}_1\right] = 0$$

This is a big equation and all things are to be taken as the left-hand side and equated to 0. Here you see that equation to satisfy the coefficient of $\hat{n}$, $\hat{t}_1$, and $\hat{t}_2$ all are perpendicular to each other, this equation will be valid only when its coefficients will vanish. From saying that coefficients of $\hat{t}_1$ will give you this equation

$$\frac{A_{1,2}}{A_2}.\frac{A_{2,1}}{A_1} - \frac{A_{2,1}}{A_1}.\frac{A_{1,2}}{A_2} = 0$$

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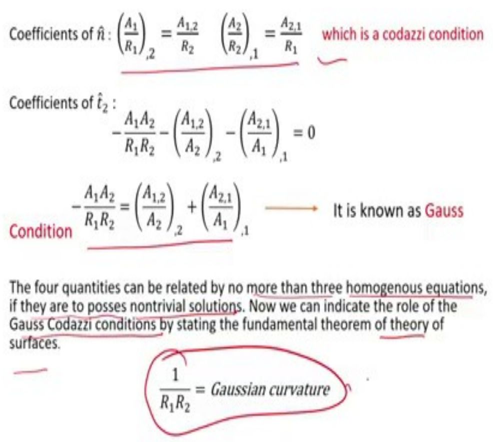

Then coefficient of $\hat{n}$ will gives you:

$$\hat{n} : \left(\frac{A_1}{R_1}\right)_{,2} = \frac{A_{1,2}}{R_2}; \left(\frac{A_2}{R_2}\right)_{,1} = \frac{A_{2,1}}{R_1}$$ Codazzi equations

And coefficients of $\hat{t}_2$ gives you:

$$\hat{t}_2 : -\frac{A_1 A_2}{R_1 R_2} = \left(\frac{A_{1,2}}{A_2}\right)_{,2} + \left(\frac{A_{2,1}}{A_1}\right)_{,1} \text{ the Gauss conditions}$$

These four quantities can be related by not more than these three homogenous equations, and if they are to possess nontrivial solutions, then we can say that the role of Gauss-Codazzi conditions gives you the fundamental theory of surfaces, and this term is known as Gaussian curvature.

And based on these Gaussian curvatures we classify the shell surfaces whether it will be an anti-last stake, class stake or it will be a regular surface or developable surface based on this Gaussian curvature.

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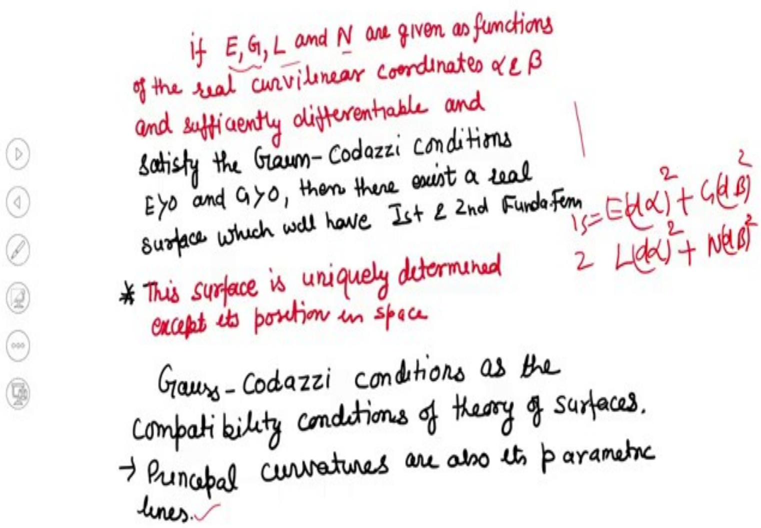

Now, we have known that these Gauss Codazzi conditions are very important. Now, if E, G, L, and N; this E and G comes from the first fundamental form, L and N come from second fundamental forms. If they are given as functions of the real curvilinear coordinate system α and β, and sufficiently differentiable, and satisfy the Gauss-Codazzi conditions.

Then we can definitely say that there exists a real surface which will have a first and second fundamental form like this; $E(d\alpha)^2 + G(d\beta)^2$ 1st form and 2nd form will be $L(d\alpha)^2 + N(d\beta)^2$. This surface is uniquely determined except its position in space. So, if they satisfy all the conditions, our surfaces are uniquely determined.

Next, Gauss-Codazzi conditions are also known as compatibility conditions of the theory of surfaces. If the surface is not satisfying those conditions if there may be a crack. So, principal curvatures are also its parametric lines if they satisfy the Gauss-Codazzi conditions and they are orthogonal to each other.

With this, I end lecture 01. In lecture 02, we will study different surfaces; the Classification of Shell Surfaces.

Thank you