Theory of Composite Shells

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Week - 02

Lecture - 02

Classification of shells

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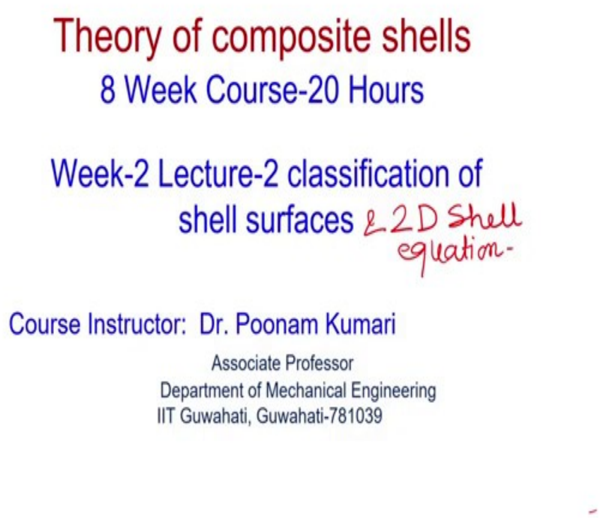

Dear learners welcome to lecture- 02 of the second week. This lecture will cover the Classification of shell surfaces and starting of the 2-dimensional shell equations.

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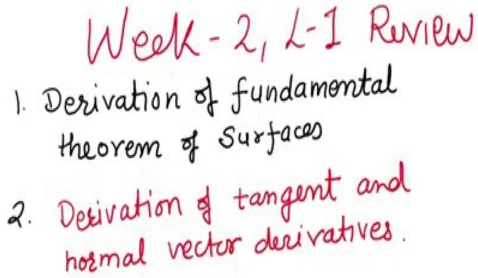

In week 2, lecture- 01, we have covered the basic derivations of fundamental theorems of surfaces, then I derived the derivatives of a tangent, the theorem of Rodrigues and Weingarten formulas, then I derived the Gauss theorem.

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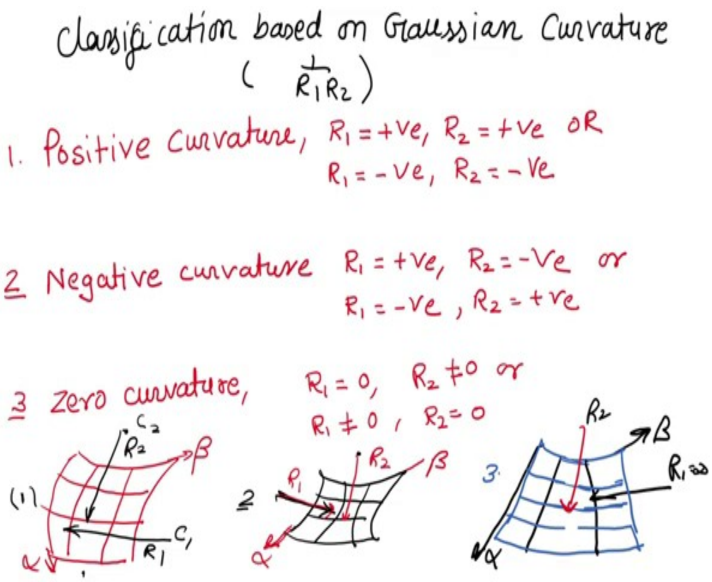

In the last lecture, the term $\frac{1}{R_1 R_2}$ is known as Gaussian curvature. In the doubly curved surfaces, there will be two radii R_1 and R_2; if we take the product of that is known as Gaussian curvature. Based on the Gaussian curvature, we can classify the shell surfaces. If R_1 and R_2 are positive or negative, If R_1 and R_2 are positive, we call it is a positive curvature and the shell will be defined as that positive curvature shell.

Then the negative curvature; if any one of them either R_1 or R_2 is negative then this product will be negative. We can say that in one direction it is negative and, in another direction, it is positive then it will be a negative curvature shell. If R_1 or R_2 is 0, then it will have a zero curvature.

Generally, the zero curvature is in one direction curved surface shells like a cylinder or a cone. When there is a curvature in one direction and the other direction is straight, then this type of shell surface will have zero curvature and here you can see that the centre of curvature lies on the same side. It is a zero-curvature shell. When the centre of curvature is lying on the bottom side of a surface, then it will be a negative curvature.

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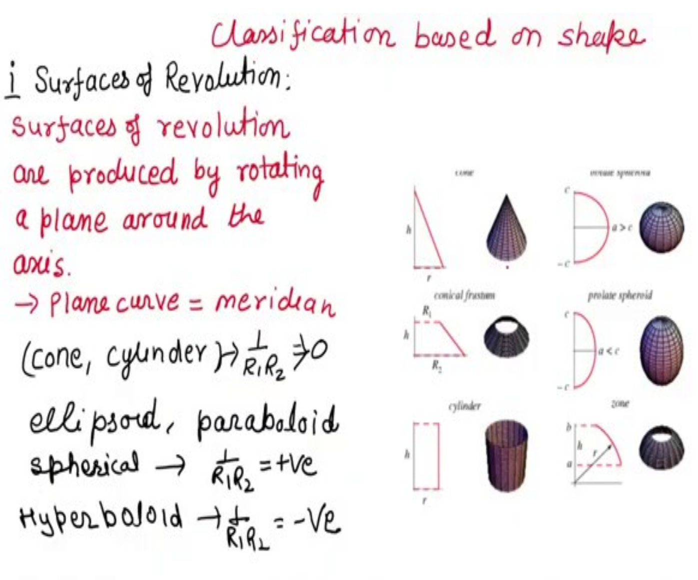

The second classification is based on shape. There are three categories; one is the surface of revolutions. Here are a number of shells that can be generated by revolving the plane curves around the axis. For example, if we have this straight line and if we revolve it around this axis, then a conical shell will be formed. If we have a line like this then a frustum will be formed, if a straight line is revolved around this centre, then a whole cylinder will be formed.

Similarly, a prolate spheroid will be formed. In the case of the plane curve, this line, sometimes it is called a generator line or a meridian line. In the case of cone and cylinder, the Gaussian curvature is zero, in the case of ellipsoid it is positive, and in the case of hyperboloid, it is negative.

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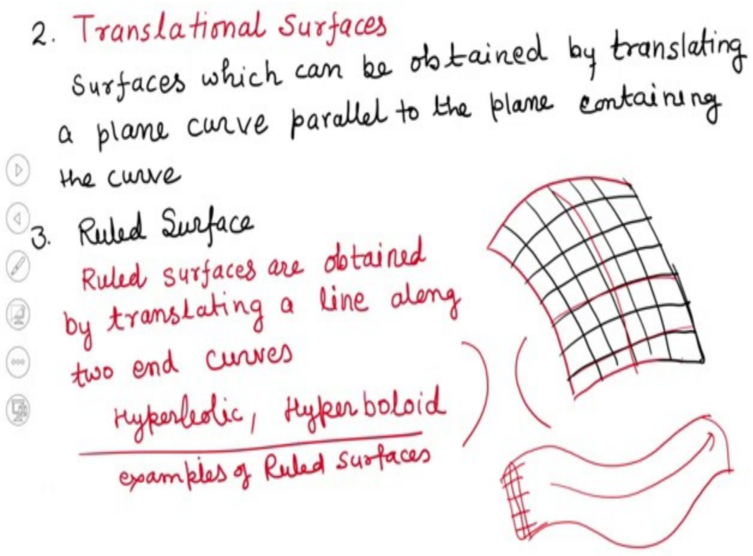

Some surfaces can be generated by translating a plane curve over the curve parallel to the plane containing the curve that moving these lines over the surface, we can generate the curve like this. This type of surface can be generated just by translating the curve line over the curve surface line. So, these types of surfaces are known as translational surfaces.

Then comes the ruled surface. In the case of translational surfaces, there may be a curved line parallel to the plane, but in the case of ruled surfaces; they are obtained by translating a line along two end curves like hyperbolic or hyperboloid surfaces, these are known as ruled surfaces.

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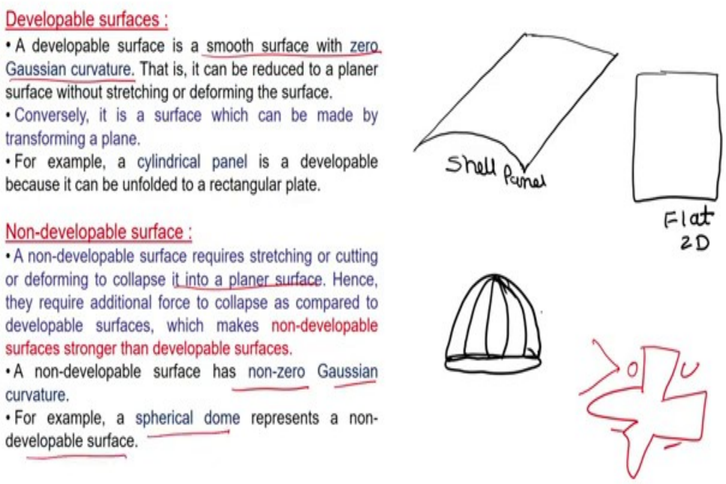

Then, we can classify the surfaces based on their developability. Generally, a surface is said to be developable if it has zero Gaussian curvature. It means that the surface can be reduced to a planer surface without stretching or deforming the surface.

If we can do that like in engineering drawing courses, we used to teach the developable surfaces. Singly curved surfaces are generally developable surfaces. For example, if you take a cylinder and you open it up, it reduces to a flat plate. There is no distortion in that. That surface is known as developable surfaces generally cones, prisms, etc. So, all these are developable surfaces.

A non-developable surface is a surface that requires stretching, cutting, deforming, or which can collapse into planer surfaces. Suppose, if you want to make a 2-D, then there will be a crack in that surface or it may require stretching and it will not join smoothly. They maybe have some flower-like pattern kind of thing i.e., there will be some gaps if you open it.

These surfaces are non-developable. Non-developable surfaces have non-zero Gaussian curvatures. So, it may be positive or negative, but it will not be zero. For example, spherical domes, represent a non-developable surface.

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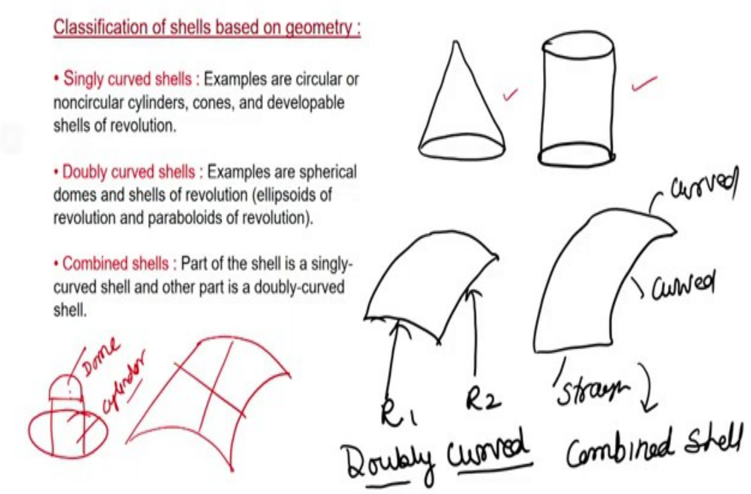

Already we have discussed that shells can be further classified based on geometry like singly curved shells. The singly curved surfaces have zero Gaussian curvatures, these have developability features and these have one curvature.

In a doubly curved shell, we have curvature in both directions, and spherical shells or shells of revolutions like ellipsoid or paraboloids are known as doubly curved shells. There may be some shell surfaces that may be combined, if you want to take a patch in one direction it is a straight line, but the other three directions may have curved lines like a roof of a building.

In that case, it will be a mixed kind of thing that some part of it is doubly curved and some part of it is singly curved, if you talk about a cylinder and above that there is a spherical dome.

Up to here, it is a singly curved surface and the dome is a doubly curved surface. In this way, it will be a combined shell. If you want to study, you have to consider these things.

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$\frac{1}{R_1 R_2}$ / Classification	Positive	Negative	Zero
Surface	Doubly curved or Synelastic	Doubly curved or Antielastic	Singly curved and Flat surfaces
Developability	Non Developable Paraboloid, Sphere	Non Developable Eg: Hyperboloid revolution	Developable
Existence of straight line	None	Two sets of straight line exists	One set of straight line exists

Now, in a tabular form, we have classified shell surfaces based on the surfaces, developability, the existence of straight line, and Gaussian curvature (positive negative and zero). A surface that has positive Gaussian curvature is doubly curved and it is also known as synelastic surface.

And the same way, if it has a negative Gaussian curvature, then they will also have doubly curved surfaces, but these are known as antielastic surfaces. If Gaussian curvature is zero, then it is known as a singly curved or flat surface. Flat surfaces also have zero curvature like a circular plate, they have a Gaussian curvature zero.

The positive one is non-developable negative one is also non-developable, but zero Gaussian curvature has the developable feature. Then the existence of a straight line has transformability. In the positive surface, they don't have any concept of the straight line.

In a negative surface, they will have two sets of straight lines. And in zero curvature, there is only one set of straight lines like this. In this, there will be a straight line, but in the other direction, it will have the curved one.

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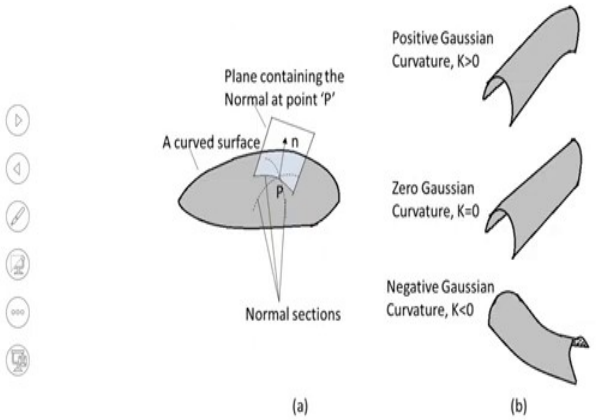

(a) Principal curves. (b) Surfaces with positive, zero, and negative Gaussian curvatures.

There are some examples of what how the surfaces look like. For a normal surface, we take a subsection, for the positive one it looks like this, for zero and the negative one it is just turning up from that side.

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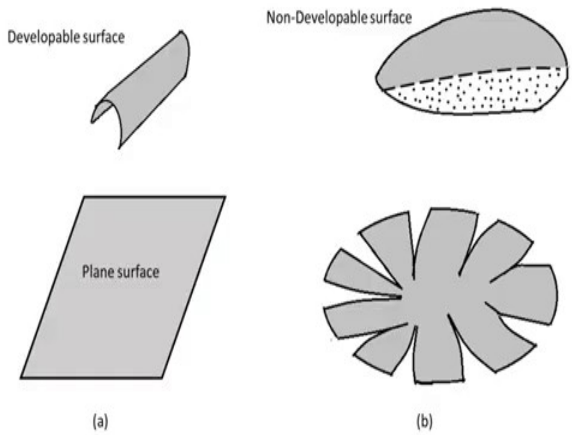

(a) Developable surface. (b) Nondevelopable surface.

And for the case of developable and non-developable surfaces; for non- developable surfaces, already I have discussed that this figure may have some openings. Here it will not satisfy the compatibility equations because if you try to develop this, the cracks will appear. These are non-developable surfaces.

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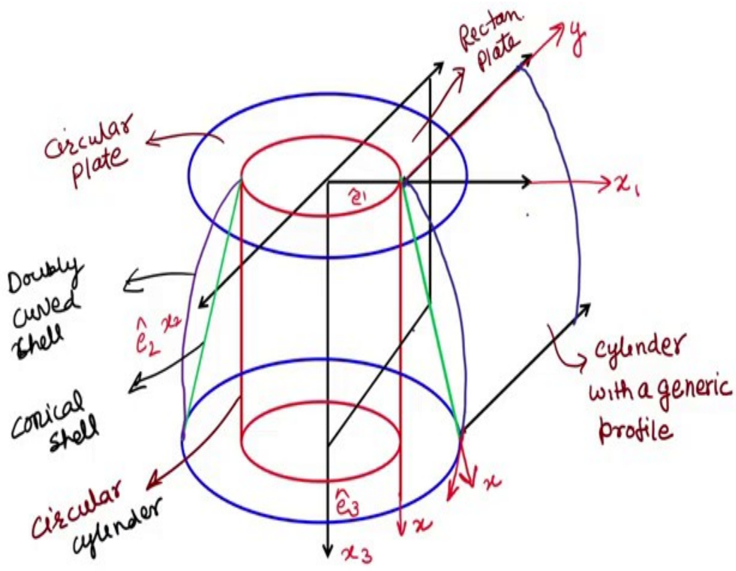

Then the surface of revolutions; through the generator, and this is a very important figure which you will find in most of the textbooks of shells. With the same concept of $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$, x_1, x_2 and x_3, we can generate different surfaces of the shell. If we want to generate a cylinder, then this red one becomes a cylinder and this surface reduces to a circular plate.

If we connect this inner cylinder to the outer cylinder, then it will become a conical shell. Instead of a straight line if we make this cylinder from a curved line, then it will be a doubly curved shell and again over this line, if you take a straight line then it will be a rectangular plate and over this surface, if you extend, this will be a cylinder with the generic profile.

The profile may be elliptical or of any kind of shape, we can generate. In this figure, it is shown that the most important part is the development of position vector and generation of the shell just ah by proper creation of the generators.

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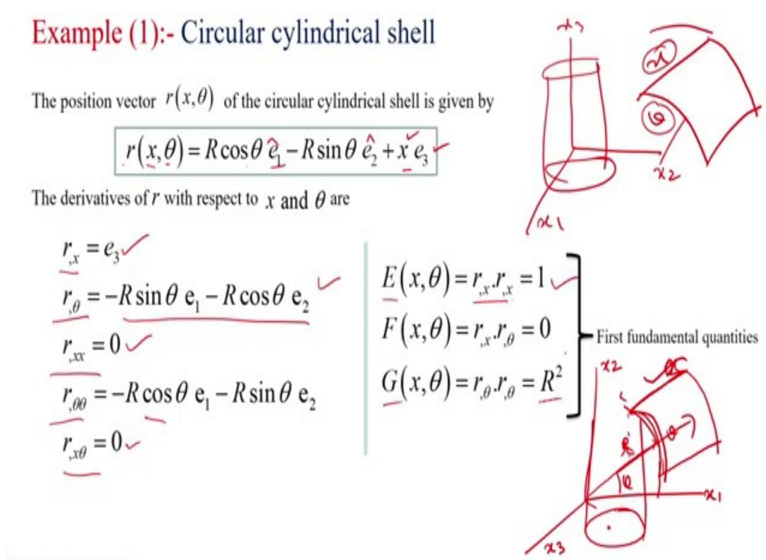

Now, I will give just 1 or 2 examples. The very important part is that first, we have to find the position vector. In differential geometry; it is slightly different from the cylindrical coordinate system or spherical coordinate system when we convert to a cylindrical coordinate system or in rectangle coordinate system.

Let us say, this is our system x_1, x_2, and x_3 and you are having a cylindrical surface in the third direction which is the longitudinal direction. If you want to have a surface over this or if you want to take one patch and this patch will be like this also.

In terms of this r and if this makes an angle theta on this direction and this is our third direction (if we talk in terms of x_1, x_2 and x_3). The third direction is our longitudinal direction, then x_1 can be represented as $R\cos\theta$. So, there will be two parameters, one will be this line and another will be this line. There will be two parameters one along θ and one along r.

From there, x and θ or sometimes it is known as x. First, we will have to take a patch, x and θ, these are two curvilinear parameters and from that, we have to find the position vectors. If this is the radius of a cylinder, then, $r(x,\theta)$ will be: -

$$r(x,\theta) = R\cos\theta\hat{e}_1 - R\sin\theta\hat{e}_2 + x\hat{e}_3$$

Here, x is the length along the third direction $\hat{e}_3$. This is the position vector.

Most of the time, the position vector is the most important or a critical thing. For a given shell, first, you have to find a patch and decide the curvilinear parameters and then accordingly find this position vector. Our aim here is not to find the position vector. Suppose position vector is given to you, can you find the first fundamental form basis, E, Fand G lame's parameters and then L, M and N and then the Gaussian curvature.

We have to do a small exercise, let us say, $r_{,x} = \hat{e}_3$

$r_{,\theta} = -R\sin\theta\hat{e}_1 - R\cos\theta\hat{e}_2$

$r_{,xx} = 0$

$r_{,\theta\theta} = -R\cos\theta\hat{e}_1 - R\sin\theta\hat{e}_2$

$r_{,\theta x} = 0$

$E(x,\theta) = r_{,x}.r_{,x}$, it gives you 1

$F(x,\theta) = r_{,x}.r_{,\theta}$, it gives you 0

$G(x,\theta)$ is $r_{,\theta}.r_{,\theta}$ it gives you R^2.

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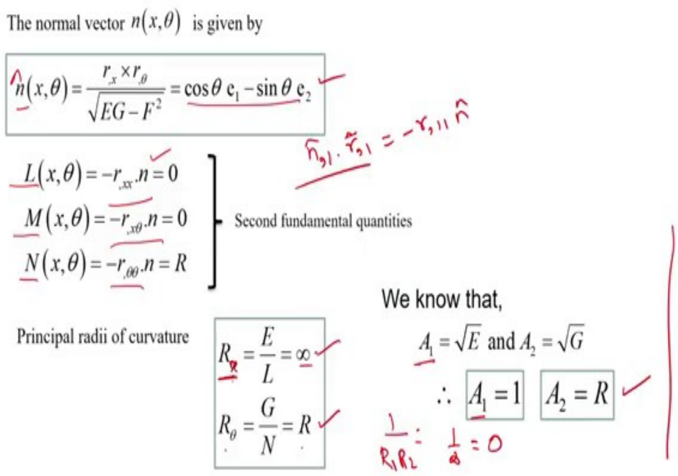

Then normal vector $\hat{n}$ can be found by : $\hat{n}(x,\theta) = \frac{r_{,x}\, x\, r_{,\theta}}{\sqrt{EF - F^2}} = \cos\theta \hat{e}_1 - \sin\theta \hat{e}_2$

Then we have to find L, M and N.

$L = \hat{n}_{,1} . r_{,1}$

If you remember I told you that $L(x,\theta) = -r_{,xx} .\hat{n} = 0$

Similarly, $M(x,\theta) = -r_{,x\theta} .\hat{n} = 0$

$N(x,\theta) = -r_{,\theta\theta} .\hat{n} = R$

L, M, and N we get and then the radius of curvature, along ϕ direction will be $\frac{E}{L}$. And

$R_2 = \frac{E}{L} = \infty$;

$R_\theta = \frac{G}{N} = R$

It becomes 1 and then lame's parameters; A_1 = 1 and A_2 = R. In this way, the Gaussian curvature $\frac{1}{R_1 R_2}$ = 0, because $\frac{1}{\infty}$ = 0. It is also clear that in a circular cylinder the Gaussian curvature is 0, lame's parameters are 1 and R, and the radius in ϕ direction or we can say in x-direction is ∞ and in θ direction is R.

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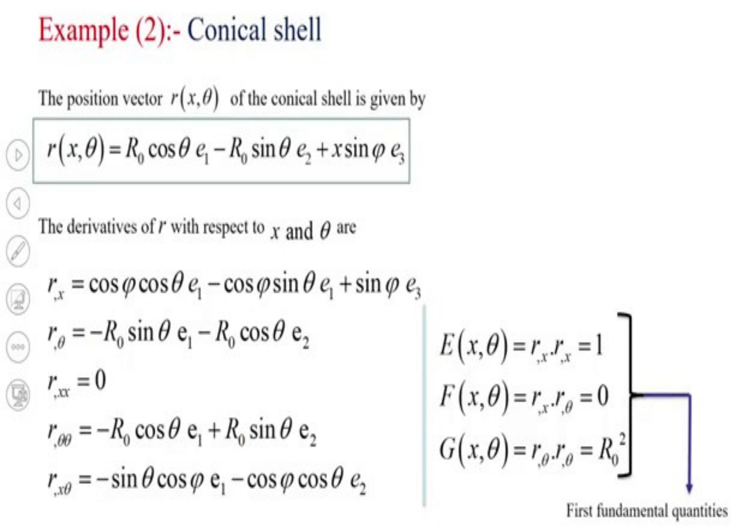

Same way for the case of the conical shell, the position vector changes slightly instead of just x.

Now, $r(x,\theta) = R\cos\theta\hat{e}_1 - R\sin\theta\hat{e}_2 + x\sin\varphi\hat{e}_3$.

Here, φ is another angle. We can find again the derivatives, mixed derivative, and double derivative and obtain E, F, and G.

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The normal vector $n(x,\theta)$ is given by

$$n(x,\theta)=\frac{r_x\times r_\theta}{\sqrt{EG-F^2}}=\sin\varphi\cos\theta\ e_1-\sin\varphi\sin\theta\ e_2-\cos\varphi e_3$$

$$\left.\begin{aligned}L(x,\theta)&=-r_{,xx}.n=0\\M(x,\theta)&=-r_{,x\theta}.n=0\\N(x,\theta)&=-r_{,\theta\theta}.n=R_0\sin\varphi\end{aligned}\right\}\text{ Second fundamental quantities}$$

Principal radii of curvature

$$R_\varphi=\frac{E}{L}=\infty$$

$$R_\theta=\frac{G}{N}=\frac{R_0}{\sin\varphi}=\frac{R_b+x\cos\varphi}{\sin\varphi}$$

We know that,

$$A_1=\sqrt{E}\text{ and }A_2=\sqrt{G}$$

$$\therefore\ A_1=1\quad A_2=R_0$$

Then L, M, N; and from here we can find the radius of curvature. Lame's parameters are almost the same, but the radius of curvature is slightly different, herein the case of circular shell it was just star, but now :

$$R_\theta=\frac{R_b+x\cos\varphi}{\sin\varphi}\text{ and }R_\varphi=\frac{E}{L}=\infty$$

The purpose to discuss these things here is if position vector is given to you and you can find the lame parameters and R_φ and R_θ. These are very important for developing the shell theory. We must be aware of what is R_φ and R_θ, in terms of the curvilinear coordinate system and what is lame's parameters.

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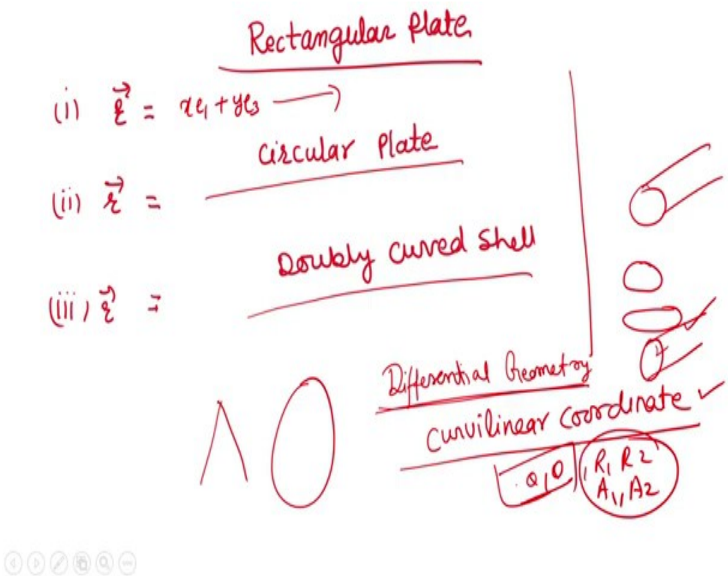

In the case of a rectangular plate, if you want to write position vector, let us say,

$$r = x\hat{e}_1 + y\hat{e}_3$$

You can develop it for a rectangular plate. Similarly, one can develop for a circular plate and doubly curved surface shell. From an assessment point of view, that position vector is given to you and then one can develop it.

This differential geometry itself is a big subject of mathematics and there are a lot of rules to find the position vector in differential geometry coordinate system or I would like to say the curvilinear coordinate system. It is slightly different but our aim is not to explore thoroughly the curvilinear coordinate system. We aim to just derive the basic theorems required for developing the shell theories.

In that direction, I am moving. First, I defined the curvilinear parameters, first fundamental form of surfaces, second fundamental form of surfaces, and various theorems like Rodrigues theorem, Weingarten theorems, and Gauss Codazzi equations. We aim to use those equations, but if somebody is interested to go beyond this; definitely, they can go. They can read the books on differential geometry, differential calculus or curvilinear coordinate systems, there is a number of books available.

I think, infinite kind of system of shells like elliptic, even for the case of a cylinder; let us say, it is circular, elliptical, and oval. This surface may be of any kind though, the other one will be a straight line. Same way, in the case of cone and sphere also.

A variety of surfaces are there. Position vectors can be developed and these position vectors are easily available for giving the curvilinear system means coordinates will be given α and θ whatever they will be. Based on that you can develop the surfaces.

For developing the shell theories, one needs to have a proper position vector system means one must be aware or can take from the book or develop themselves and then find R_1 and R_2, A_1 and A_2; for particular shell these four parameters are required. If we know these things, then definitely a two-dimensional shell theory or a three-dimensional shell theory can be developed properly.

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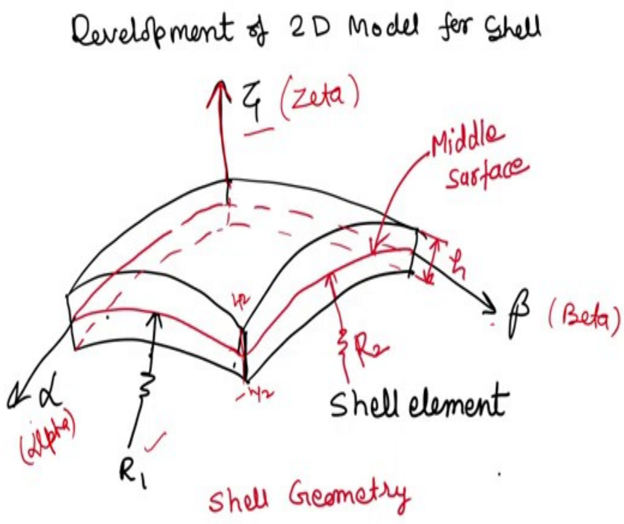

Now, in this direction very first step is the development of a two-dimensional model for a shell. Let us take a shell element and already in the very starting lecture, I have said that the middle surface will govern the shell. If we take plus half and bottom half and join together, then it will make a complete shell and its middle surface governs all the property of the shell. So, we will try to develop our basic governing equations in the middle surface form.

We have two curvilinear coordinate systems α and β and this red one is our middle surface and R_1 is the radius in this direction and R_2 is the radius in this direction and along the thickness or third direction which you call a normal that is zeta (ς). We call it zeta (ς), this is β, and this is α.

And the total thickness is h and from the middle surface, this may be minus h by 2 and plus h by 2. This is a doubly curved shell element. We have taken a small element; first, we will develop the basic equations like lame's parameters or distortion parameters, and then we use it for developing the strain displacement relations.

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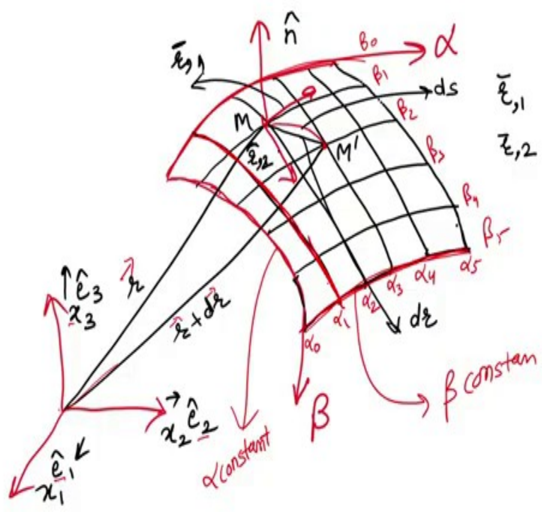

We made a grid, here a middle surface is taken and curvilinear parameters α and β are taken like this and then parametric lines are drawn parallel to α and β. In this geometry, I am sorry that I could not draw in a proper sense, but if you draw through a program or a scale, then definitely they will be aligned to α and β.

These are the lines where α is changing i.e., $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and α_0. These are the lines where β is changing, let us say, $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$. These lines are called α constant and these lines are called β constant. You can say, over this line β_5, β is constant here and α is changing.

But over this line α is constant and β is changing from β_0 to β_5. This is the rectangular coordinate system, x_1, x_2, x_3 and the position vectors are $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$. From this curvilinear coordinate system, let us say, a point 'M' whose position vector is r. After some time or after applying the force and subjected to the boundary condition this material point moves to this point; which is this M to s and now the position vector will be r plus dr.

Over this point, tangent to this will give you $r_{,1}$ and tangent along this line will give you $r_{,2}$ and normal to this surface will give you $\hat{n}$ surface normal because this is our tangent plane this is our surface normal. We are moving from point M to M', then a straight line will be dr and a curved line will be ds. We aim to find the ds for this system.

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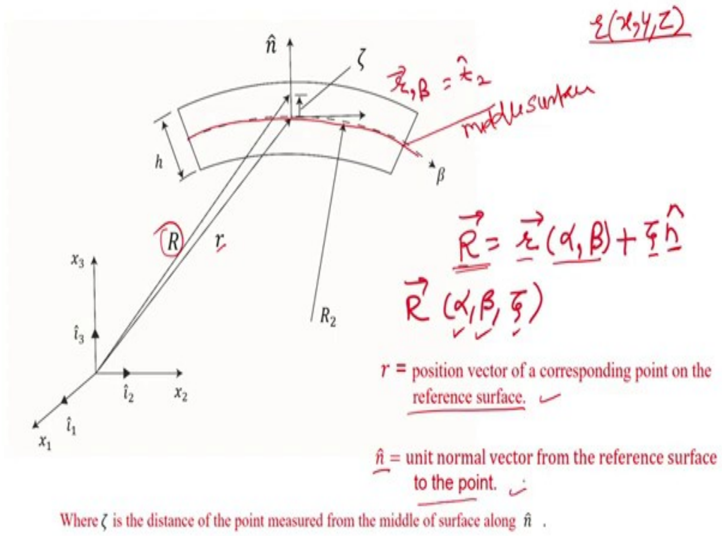

Now, let us say, this is our reference plane if you go back because we are talking about this kind of system. At the reference plane, the position vector r is just slightly above that, and let us say, this is R_1 with the thickness and the length is ς. So, the position vector will be R and we are interested to develop in that sense.

$\hat{n}$ in that zeta moved and this dotted line is our middle surface line, one view is shown here. This is r and this will be R. Now, we know that R can be represented as r which is a function of α and β plus $\varsigma\,\hat{n}$, in that direction the unit vector is $\hat{n}$.

Now, R is a function of α, β and ς; ς is the thickness coordinate, like we used to say some position vector r in rectangular coordinate system x, y, and z. r is the position vector of a corresponding point on the reference surface or the middle surface. Already, I have discussed $\hat{n}$ is the unit normal vector from the reference surface to the point.

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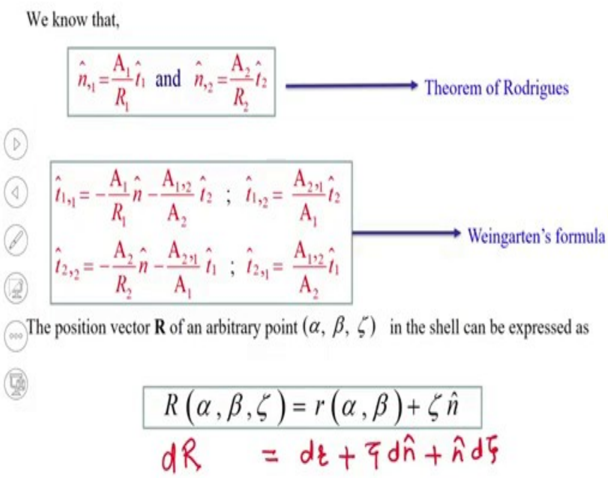

Now, we will find the dR. Before that, I would like to recall the basic theorems that we are going to use here. The first two theorems are the theorem of Rodrigues;

where, $\hat{n}_{,1} = \frac{A_1}{R_1}\hat{t}_1$ and $\hat{n}_{,2} = \frac{A_2}{R_2}\hat{t}_2$

Then the Weingarten formula where we find $\hat{t}_{1,1}$; $\hat{t}_{1,2}$; $\hat{t}_{2,1}$ and $\hat{t}_{2,2}$.

R is written here. Change in dR will be: $dR = dr + \varsigma d\hat{n} + \hat{n} d\varsigma$

In a curvilinear coordinate system, each unit vector changes with respect to α and β; they are not constant. Change in the unit vector will also take place.

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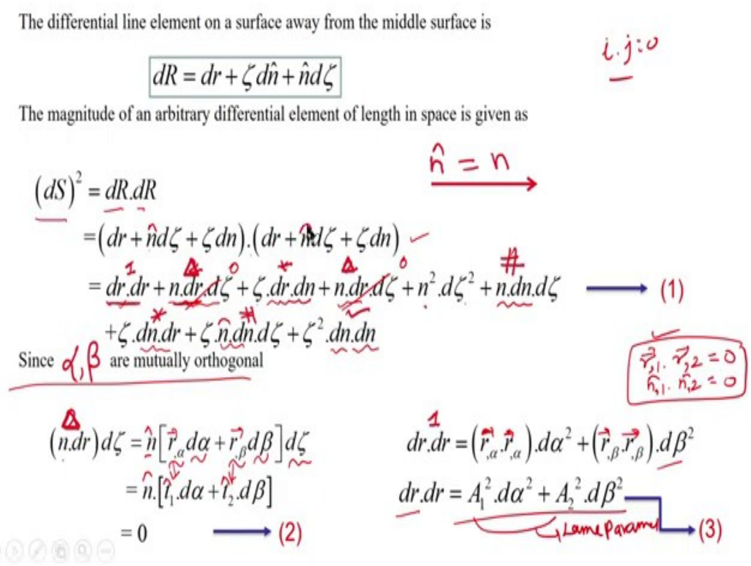

Now, $(dS)^2 = dR.dR$.

We aim to find dS, at some height from the reference surface what will be the ds because of the movement in three dimensional. If you multiply dot product of $dR.dR$:

$$(dS)^2 = dr.dr + \hat{n}.dr.d\varsigma + \varsigma dr.d\hat{n} + \hat{n}.dr.d\varsigma + \hat{n}^2.d\varsigma^2 + n.d.d\varsigma + \varsigma.d\hat{n}.dr + \varsigma.\hat{n}.d\hat{n}.d\varsigma + \varsigma^2.d\hat{n}.d\hat{n}$$

We got this equation (1).

But in this equation (1), we are aware that, $dr.dr$ can be converted into a lame's equations first fundamental form, but what about others?

We have to explicitly write here α and β are mutually orthogonal. It means that their $r_{,1} . r_{,2} = 0$. Same way, $\hat{n}_{,1} . \hat{n}_{,2} = 0$. If it is an orthogonal curvilinear system, then they are other derivatives means dot product of two tangent vectors, which are not same is going to be 0; for example, if in rectangular coordinate system i. j = 0.

The same way here $\hat{n}.dr.d\varsigma = \hat{n}\left[r_{,\alpha}d\alpha + r_{,\beta}d\beta\right]d\varsigma$

$r_{,\alpha} = \hat{t}_1$ and $r_{,\beta} = \hat{t}_2$

We can write $\hat{n}.dr.d\varsigma = \hat{n}.\left[\hat{t}_1.d\alpha + \hat{t}_2.d\beta\right]$.

I would like to say, there will be $\hat{n}$ here and there will be $\hat{t}$ because initially, we started from there. Just removing all these things, but they are the same there is no change.

There is some arrowhead for a vector, these are all vectors. $\hat{n}$ is perpendicular to $\hat{t}_1$, $\hat{n}.\hat{t}_1 = 0$ and $\hat{n}$ is perpendicular to $\hat{t}_2$ also. This delta term will give you nothing.

It is straightforward going to be 0.

Then, $dr.dr = \left(r_{,\alpha}.r_{,\alpha}\right).d\alpha^2 + \left(r_{,\beta}.r_{,\beta}\right).d\beta^2$

Already, I said that $r_{,\alpha}$ and $r_{,\beta}$ is going to be 0. It reduces in this form:

$dr.dr = A_1^2.d\alpha^2 + A_2^2.d\beta^2$

Here, A_1 and A_2 are known as lame parameters.

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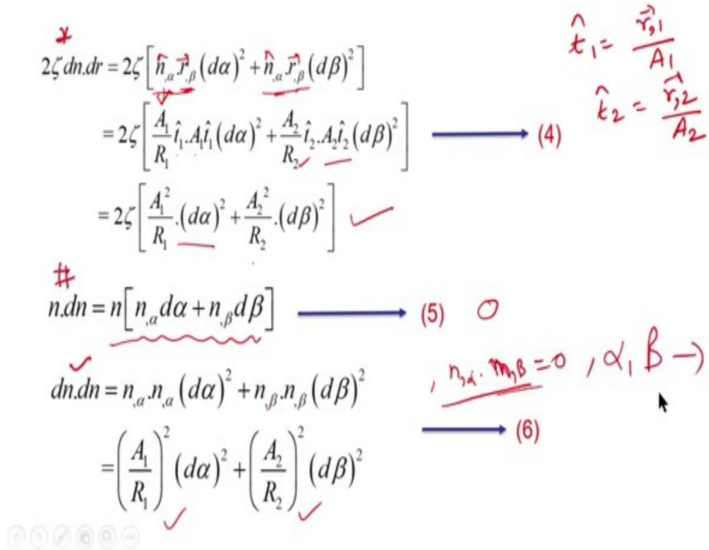

Then, $2\varsigma dr.d\hat{n} = 2\varsigma\left[\hat{n}_{,\alpha}.r_{,\beta}\left(d\alpha\right)^2 + \hat{n}_{,\alpha}.r_{,\beta}\left(d\beta\right)^2\right]$

Using the Rodrigues theorem, it can be represented as $2\varsigma dr.d\hat{n}$

$$= 2\varsigma\left[\frac{A_1}{R_1}\hat{t}_1.A_1\hat{t}_1\left(d\alpha\right)^2 + \frac{A_2}{R_2}\hat{t}_2.A_2\hat{t}_2\left(d\beta\right)^2\right]$$

we can write in terms of that because if you remember, I said that $\hat{t}_1 = \frac{r_{,1}}{A_1}$ and $\hat{t}_2 = \frac{r_{,2}}{A_2}$

$\hat{t}_1 \cdot \hat{t}_1 = 1$ and $\hat{t}_2 \cdot \hat{t}_2 = 1$.

$$2\varsigma dr.d\hat{n} = 2\varsigma\left[\frac{A_1^{\,2}}{R_1}.\left(d\alpha\right)^2 + \frac{A_2^{\,2}}{R_2}.\left(d\beta\right)^2\right]$$

Now, $\hat{n}.d\hat{n} = \hat{n}\left(\hat{n}_{,\alpha}d\alpha + \hat{n}_{,\beta}d\beta\right)$ will give you 0 because $\hat{n}_{,\alpha}$ is perpendicular to $\hat{n}$

similarly $\hat{n}_{,\beta}$ is perpendicular to $\hat{n}$.

Then $d\hat{n}.d\hat{n} = \hat{n}_{,\alpha}.\hat{n}_{,\alpha}\left(d\alpha\right)^2 + \hat{n}_{,\beta}.\hat{n}_{,\beta}\left(d\beta\right)^2$

This will give you $\left(\frac{A_1}{R_1}\right)^2 (d\alpha)^2 + \left(\frac{A_2}{R_2}\right)^2 (d\beta)^2$

The mixed product of $\hat{n}_{,\alpha} \cdot \hat{n}_{,\beta}$ = 0, we have assumed that α and β are orthogonal to each other.

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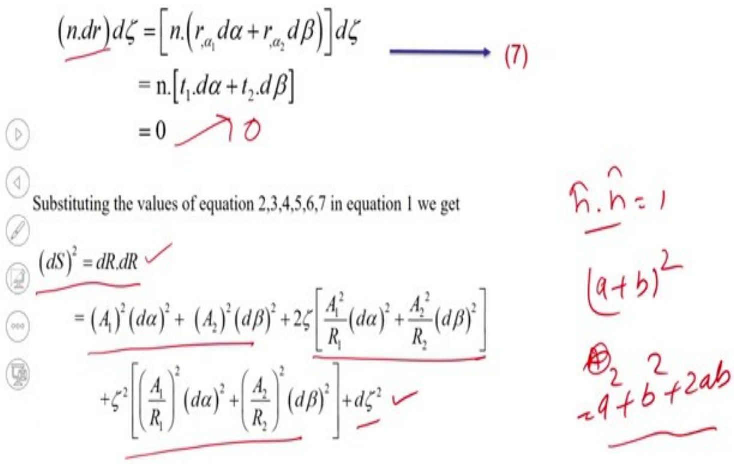

Then, $\left(\hat{n}.dr\right) d\varsigma = \hat{n}.\left(r_{,\alpha_1} d\alpha + r_{,\alpha_2} d\beta\right) d\varsigma = \hat{n}.\left(\hat{t}_1.d\alpha + \hat{t}_2.d\beta\right) d\varsigma = 0$

Now, we can replace equation (1) with the values of equation (2),(3),(4),(5),(6) and (7).

$$(dS)^2 = dR.dR$$

$$(A_1)^2 (d\alpha)^2 + (A_2)^2 (d\beta)^2 + 2\varsigma \left[\frac{A_1^{\,2}}{R_1}.(d\alpha)^2 + \frac{A_2^{\,2}}{R_2}.(d\beta)^2\right] + \varsigma^2 \left[\left(\frac{A_1}{R_1}\right)^2 (d\alpha)^2 + \left(\frac{A_2}{R_2}\right)^2 (d\beta)^2\right] + d\varsigma^2$$

Now, further mathematically simplifying in the form of $(a+b)^2 = a^2 + b^2 + 2ab$.

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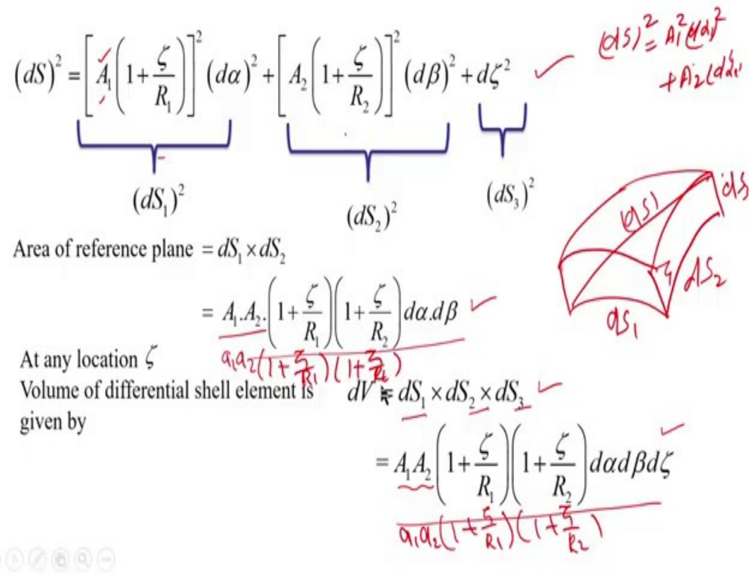

These are mathematically simplified and can be written as:

$$(dS)^2 = \left[A_1\left(1+\frac{\varsigma}{R_1}\right)\right]^2 (d\alpha)^2 + \left[A_2\left(1+\frac{\varsigma}{R_2}\right)\right]^2 (d\beta)^2 + d\varsigma^2 ; dS_1 x dS_2 = A_1.A_2\left(1+\frac{\varsigma}{R_1}\right)\left(1+\frac{\varsigma}{R_2}\right) d\alpha.d\beta$$

$$(dS)^2 = (dS_1)^2 + (dS_2)^2 + (dS_3)^2$$

Curve length along this and this and we have a third curvature also. Let us say, ς times. We find out that moving from one position to another position that dS will become dS_1, dS_2, and dS_3; in one direction, two directions and three directions.

The area of a reference plane will be $dS_1 \text{ x } dS_2$. From here we can get it and the volume dV = dS_1 x dS_2 x dS_3

Here, point to be noted that if you want to correlate with the previous system, where we have obtained in the case of first fundamental form.

$(dS)^2 = A_1^2 (d\alpha_1)^2 + A_2^2 (d\alpha_2)^2$ in the 2d case.

We are going to replace this A_1 with the a_1 . So that there will be no confusion in future, otherwise, we will call this A_1 whole product and this is also A_1, then there will be confusion.

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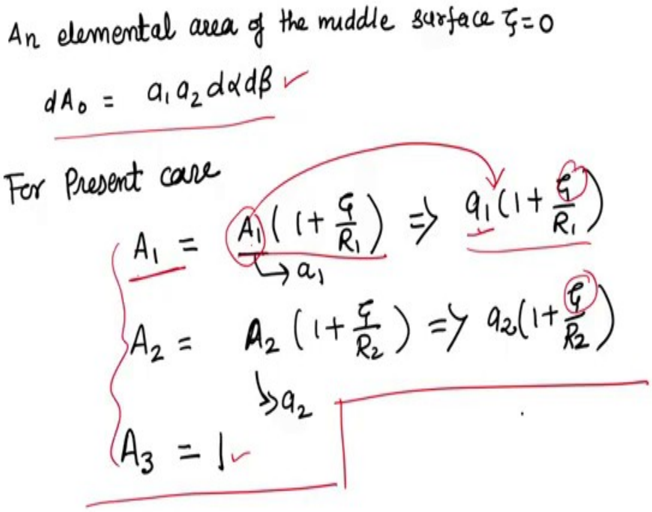

In the present case, A_1 is this a_1 and in some of the books this A_1 is written as this A_1 and also this a_1, that creates a problem. Instead of A_1, we are going to replace it small a_1, where this is the reference plane. From now onwards:

$$A_1 = a_1\left(1+\frac{\varsigma}{R_1}\right)$$

$$A_2 = a_2\left(1+\frac{\varsigma}{R_2}\right)$$

And $A_3 = 1$

The elemental area of a middle surface where $\varsigma = 0$.

You can put ς as 0 and we can find small a_1, a_2, $d\alpha$ and $d\beta$. After that, once we know, A_1, A_2 and A_3; already we have expressed that the volume, dS_1, dS_2, dS_3. Here,

A_1 and A_2 we will be replaced by $a_1\left(1+\frac{\varsigma}{R_1}\right)$ and $a_2\left(1+\frac{\varsigma}{R_2}\right)$.

With this I end this lecture 02. In lecture 03 of week 2, we will start the strain displacement relations in the curvilinear coordinate system, and using this A_1 and A_2, we will simplify the strain displacement relations and we will use this for developing the shell theories.

Thank you very much.