Theory of mechanisms

Lecture 12

Chebyshev Spacing

So, last class we looked at, the structural error and a way to tackle and minimize that error, is to use, Chebyshev Spacing, to find your precision points. So we saw that, you can assume a linear relationship, between the input angle, of the four bar and your input variable, then look at, then transform the output angle of the four bar, to the output of the variable. So let's see how we would use this, to determine the parameters or the input and output angles, for the four bar, that we have to design. Okay? Use of, so given a certain function, we will look at, how we will apply, a Chebyshev Spacing. This was the geometric construction for Chebyshev spacing and we will now do, an example. Okay?

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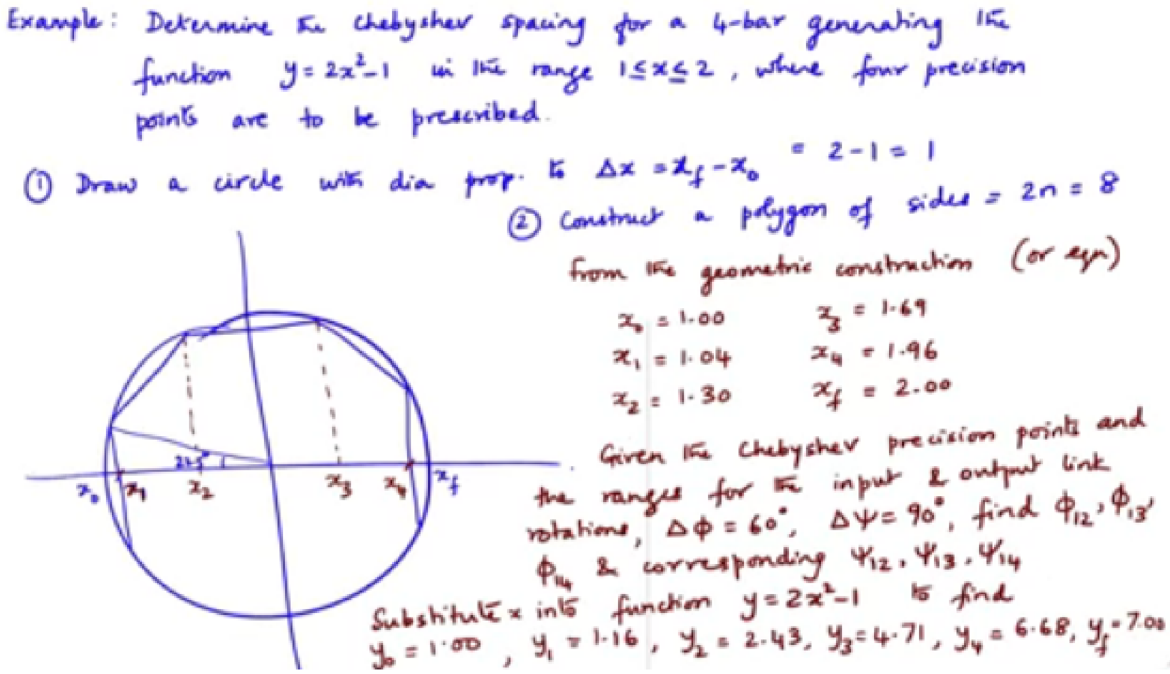

So let's say, determined, Chebyshev Spacing meaning, find the precision points, for the design of a four bar, which should generate the function, y equal to two x squared, minus one, in the range, 1 & 2, between 1 & 2, Where 4 precision points, are to be prescribed. Okay? So what would you do? You need for precision points, so you would have to again draw the, inscribe, so first draw a circle. What would be the diameter of the circle? What would it be proportional to? Yeah, the diameter proportional to Delta X, equal to XF, minus X naught, which is equal to 1, 2 minus 1, 1. So I draw the circle, such that, the diameter, this is, X naught, this is X F. Okay? Then, I have to construct a polygon, outside of shi , 8, 2n equal to 8. And you have to construct it, we saw that, the angle, that you're looking at, for the, is, this angle would be twenty two point five degrees. So this would be the shi of the polygon, that would be the. Then using that, I can construct the rest of the polygon. I need to construct only half of it. And so from the diagram, I can read off, this will be x1, I draw the vertical lines, from these are the vertices, so this would be X 2, this would be X 3 and this would be X 4. Or I can use the equation. Right? I can compute it that way. So from this, from the geometric construction, I get, or equation, you could use that as well. I have X naught, equal to 1, X 1, 1 point 1.04, X 2, 1.3, even if you don't remember the equation, you know how to, from the geometry of this you can find, X

1, X 2, X 3. So X 3, equal to 1, 1.69, X 4, equal to 1.96, because you may not be able to measure this off, that accurately, other ways. So you can use the geometric construction to, then set up the equation to find these values. Okay? And XF is 2. Because you are trying to, look at error here, you want to have sufficient number of significant figures, you don't want a roundoff. Ideally if you can have more significant figures, that's better. Okay? So once I have the values for X, I am given. One more thing I need to be given. What would be the range of rotation, for the four bar? So I'm given Delta Phi, the input angle and Delta shi is, ninety degrees. Okay? So okay. So the first part is, the finding the precision points for this relationship.

So now given the Chebyshev precision points, for the function, in so in terms of X and the, because I could design multiple linkages for this. Right? Depending on the range that I should choose, for the input and output links. And the ranges for the input and output link rotations, rotations, are given as Delta Phi, equal to 60 degrees, Delta shi, equal to 90 degrees, I want to find, Phi 12, Phi 13, Phi 14. Right? From the first position, how much do they. So Phi 1, Phi 2, Phi 3, Phi 4, will give me this and correspondingly Si 12, Si 13, Si 14. Okay? So far you know how to design, if you're given two, Phi 12, Phi 13 and say, Shi 12, Shi 13, but theoretically it is possible to design for, more number of precision points. You may need other methods to do so, but it is possible, Okay? So to do that, I first need to find, so I'm given, I found the X values, I need to find the corresponding Y values, to relate. Then I will relate, X to Phi and Y to shi. Okay? So how will I find the Y values? I am given the function. Y equal to 2x square minus 1. So substitute the values of X. Substitute into function, y equal to 2x square minus 1, to find, substitute X values into this function, to find y naught is 1, y 1, is 1.16, 2.43, 6.68 and at X equal to 2Y is, 7. Okay?

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$$\Delta x = 1, \quad \Delta y = 6, \quad \Delta\phi = 60^\circ, \quad \Delta\psi = 90^\circ$$

$$R_\phi = \frac{\Delta\phi}{\Delta x} = \frac{60}{1}$$

$$R_\psi = \frac{\Delta\psi}{\Delta y} = \frac{90}{6}$$

$$\phi_1 = R_\phi(x_1 - x_0) + \phi_0$$

$$\phi_2 = R_\phi(x_2 - x_1) + \phi_1$$

$$\phi_3 = R_\phi(x_3 - x_1) + \phi_1$$

$$\phi_4 = R_\phi(x_4 - x_1) + \phi_1$$

If I choose $\phi_0 = 0$

$$\phi_1 = 60(1.04 - 1) + 0 = 2.4^\circ$$

$$\phi_2 = 18.6^\circ$$

$$\phi_3 = 41.4^\circ$$

$$\phi_4 = 57.6^\circ$$

If I chose $\phi_1 = 0$

$$\phi_0 = -2.4^\circ$$

$$\phi_1 = 0$$

$$\phi_2 = 16.2^\circ$$

$$\phi_3 = 39.0^\circ$$

$$\phi_4 = 55.2^\circ$$

Find ψ using R_ψ, y

So now, my, I know my Delta X equal to 1, my Delta y equal to 6 and I'm told that Delta Phi should be 60 delta Phi should be, 90. So I can find my scale factors. I'm assuming a linear relationship, between X and Phi, similarly between Y and Shi, so I can find my scale factors. Okay? So I have, R Phi equal to, Delta Phi, by Delta X, 60/1, R Shi, equal to, Delta Phi, by Delta Y, which is 90/6. So you're asking, if the function has some other, a maximum in that interval? Okay. So we are looking at a linear relationship. Right? Between X and so, that would not hold. First we would have to see whether the Chebyshev Spacing holds for that kind of. You would probably, split it and do it, but now, then we need to know the conditions, basically he's asking, what are the conditions, in which you can use the Chebyshev Spacing.

Because if, a maxima like that occurs, in between the interval, we don't really talk about, we, we are talking about a linear relationship, so the assumption is that, but the linear relationship is between the, Delta X, the angle and the, I'll have to get back to you, on what the what conditions the function should satisfy, in order for the Chebyshev Spacing, to be valued. Okay? So when you have this, now I can find, my Phi 1, as R5, into X 1, minus X naught, plus phi, Right? because of the linear relationship, between X and Phi, So, there are two ways to go about this. because they are interested in, the change Right? from position 2, to position 1, Phi 12, Phi 13, Phi 14. You could choose Phi naught, such that Phi 1 will be 0. Or you can choose Phi 1 to be 0 and everything will just be shifted, by a certain angle, because you are interested in the difference, from the first position, the angular displacements, from the first position, is what we are interested in. So you could either choose Phi naught to be 0, without any loss of generality or you can choose Phi naught, such that Phi 1 will be 0. Then whatever you get as, Phi 2, Phi 3, Phi 4, will actually be Phi 1 2, Phi 1 3 and Phi 1 4, because you've chosen Phi 1 to be 0. Okay? So that is possible. So the scale basically, just gets shifted, the range does not change. So I will just, so when you do this, or I can just compute, I can choose Phi naught, to be 0. Okay? Let's say, let Phi naught, equal to 0, then Phi 2 will be, R 5, x2 minus x1, plus Phi 1, Phi 3 is r5. Okay? So if I say, if I choose, Phi naught equal to zero, I get Phi 1 equal to 60, into 1.04 minus 1, plus 0, so I get Phi 1 equal to 2.4 degrees in this case and get similarly I can evaluate, Phi 2, Phi 3 and Phi 4. Okay? If I choose, Phi 1 equal to zero, then, my Phi naught is basically, minus 2.4 degrees, So let's just shifted the scale. So I will get, Phi 1 equal to 0, Phi 2 equal to 16.2 degrees, Phi 3 equal to 39, Phi 4 equal to 55.2 degrees, the deltas don't change. Okay? So it's essentially the same linkage, starting at a different position. You would synthesize essentially, the same linkage, in both cases. Okay? And then you can find the, corresponding size in a similar manner, in order to synthesize the linkage. This gives you the phi; you have to then synthesize the shi. Using the relationship between R Shi and Y. Okay? So this is how you use Chebyshev spacing, for function generation, in order to determine the precision points.

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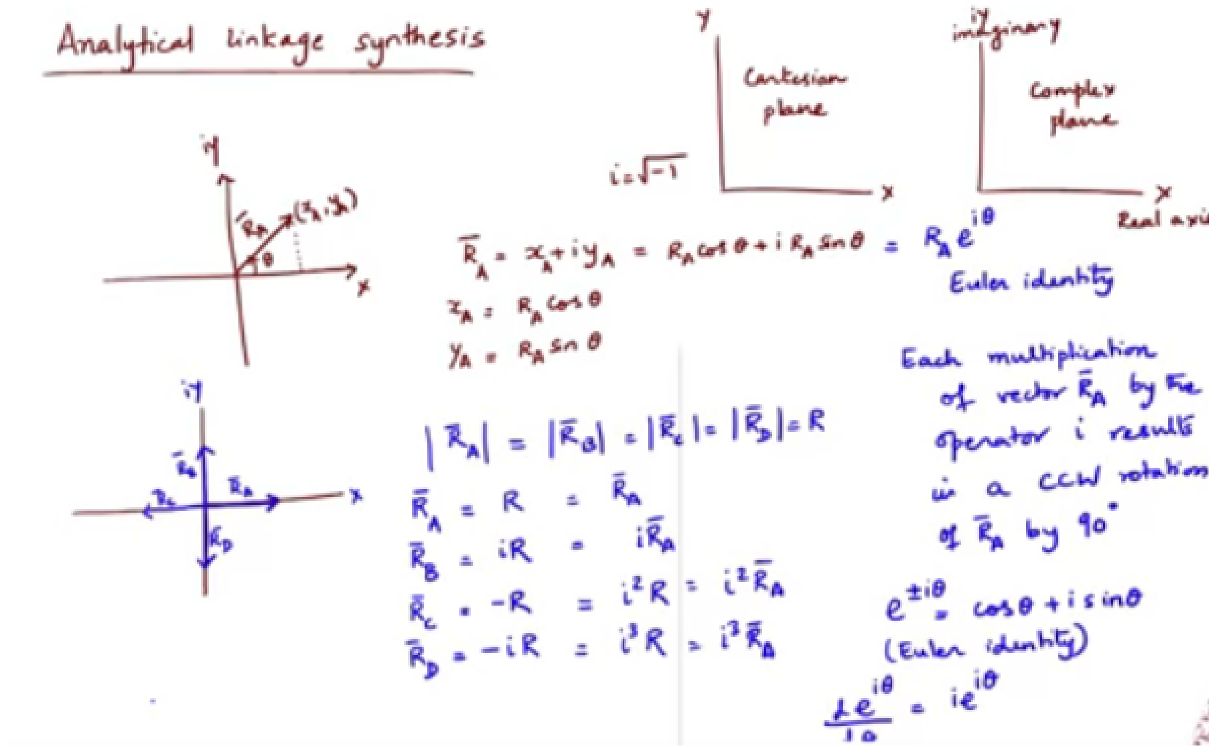

Step so, in analytical linkage synthesis, okay, so far we've looked at graphical methods, for synthesis, which as I've said before, are intuitive and easier to do, easier to understand, easier to set up. Analytical methods, give you more power. So graphical methods are essentially based, the design that you do, the synthesis that you do, is only based on position. So your design requirements, if it's based on position and orientation, based on displacement, basically, because we have not really looked at graphical methods, where you involve time derivatives, of the displacement. Say I want a linkage, where when the input link has a certain velocity, angular velocity, I want the output link, to have a certain angular velocity, we didn't, that's not something we were able to do with the graphical synthesis. We were able to say, okay, if it attains a certain position, I want the output link to attain a certain other position. So it was based more on displacements, but in many cases, you may want to have a combination, of displacement velocity acceleration, only velocity acceleration, you're involving time derivatives. You may want to involve the transmission angle in the design. You want a linkage that satisfies certain requirements for the transmission outlet. So you may have other conditions that you impose, on the design that cannot usually be dealt with, in graphical synthesis. Graphical synthesis you would have to, you know, if you have other conditions like that, the only way to do it is, you synthesize a linkage, you look at it, you analyze it, see if it meets the requirements, if not, go back. So it's a trial and error process. Analytical methods are more difficult to set up, but they give you more power, more control, over the design variables. So that is what we will be looking at. So when you want to have like an optimal 4 bar, for a particular application, based on these, you know, based on specifying the link length ratios, or the transmission angle, as I mentioned. Then you want to explore, analytical methods for synthesis, because they can help you cycle through or they can help you put constraints, on some of the design variables, more easily. Okay?
So the instantaneous kinematic condition is also, a key reason, why you might want to look at, analytical linkage synthesis. So the Chebyshev spacing, what we looked at, would be sort of like the first step. If you know, you know to determine your precision points, if you want to design a linkage, you would first analytically determine and it is for a case, where it satisfies, the conditions for using Chebyshev spacing that would be there like the starting point, for your design. You would determine

the precision points, using that, if it's a function generation, of course, it gets more complex, if it's a motion generation or a path generation problem. But for motion generation and path generation, in many cases, the application dictates. You know, it's not so much the structural error, you are looking at specific points, in the plane, that you want to attain, specific points in orientations, for your particular application and then you design the linkage for that. You're not looking at the behaviour over a range. Okay? The Chebyshev spacing, applies when you want to minimize the structural error, over a range. But if your application only demands that you have you know, that there are set positions that you want to attain, then you don't really need to go through that process. Okay? So the basis that we will use, in many cases, we are, one of the elegant ways of doing analytical linkage synthesis, is through the use of complex numbers. Okay? I'm sure all of you are familiar, with using complex numbers, in linkages. Okay, so I will briefly show you, how complex numbers, can be used instead of vectors, when you are looking at linkages, you can use. So for instance, instead of looking at this plane, as the Cartesian XY plane, I can look at it, as the complex plane, where I have the real axis and the imaginary axis. So this is my complex plane, it's just a different notation; it has certain advantages, when it comes to working with linkages. Okay?
So if I look at a vector, in this complex plane, I can write this vector R as X, so if it has, XRYR or let me just call this point P, let's just call it A. Okay? So if, if I wanted RA is XA, plus I, Y A. Okay? So I have the real part, which is the part, along the x axis, so if this makes an angle theta, with the x axis, X a is nothing but, R a, where r a without the bar, is the magnitude of that vector, into cos theta. And the imaginary part, is the, is RA sine theta. So this is RA, cos theta, plus I, R a, sine theta, where I has always as, root of -1. But here, we will use I, as more of an operator. We're not really looking at it, as a value, but you will see that, we will use it as an operator, for this application. So if I look at, so let's say I have a vector, so let's say, I have four vectors of equal length. Okay? We call this, RA, RB, RC and RD. So RA, so all these vectors have the same length and let me call that R. Okay? So I can write vector RA, as just R. Okay? Vector RB, would be IR, vector R C is minus R and R D is minus I R. Okay? So I can also write this as, minus R, I can write it as I square R and minus I R, I can write it as, I cube R. So if I look at this, multiplication by the operator I, is rotating my vector, by 90 degrees in the, counter clock wise direction. So the operator I, in this notation, so each multiplication, so this, I can even write it in, off, sorry, does, I, RA, I square RA, IQ, RA. So each multiplication of vector, R a, by the operator I, results in a counter clock wise rotation, of RA by 90 degrees. The major advantage of, using this notation is, because of the Euler identity. So I can write, cos theta, plus I sine theta, is E power I theta, using the Euler identity, because I have, e power, plus minus, I theta, equal to cos theta, plus I sine theta. So when I differentiate this or integrate it, you know if I express it, in the form E power i theta, d e theta, E power I theta by D theta, is I , E power I theta and similarly integrating it also, gives me, e power I theta, by I. So differentiating and integration, differentiation integration become easy, when I express it, in this complex form and we will see that, when we do the kinematic, when we go to velocities and accelerations, it simplifies the math. And also it you can relate it to the directions. It's easier to correlate, with the vector directions, comparing displacement with velocity etcetera, becomes easier. So this is the notation, we will use, for both analysis and for synthesis.

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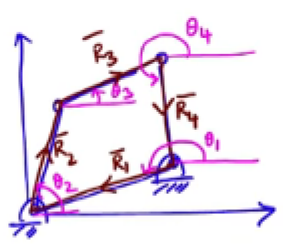

$$\bar{R}_1 + \bar{R}_2 + \bar{R}_3 + \bar{R}_4 = 0$$

$$\bar{R}_1 = r_1 e^{i\theta_1} \qquad \bar{R}_3 = r_3 e^{i\theta_3}$$

$$\bar{R}_2 = r_2 e^{i\theta_2} \qquad \bar{R}_4 = r_4 e^{i\theta_4}$$

Vector loop equation for a 4-bar

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} + r_3 e^{i\theta_3} + r_4 e^{i\theta_4} = 0$$

$$\bar{R}_1 = \bar{R}_2 + \bar{R}_3 + \bar{R}_4$$

Because if you look at a four bar, I can express it, as a vector loop equation, Okay? I can say, from this pivot, to this pivot, is one vector, this to this. Okay, I can do that or keep this, will do this, just to show you, how the angles are measured. So I can express, a four bar linkage, with a loop like this. Okay? I could potentially take r1 in this direction also, in which case. If you look at this, as the linkage moves. Okay? It always holds true. So for the directions that I have taken, as the linkage moves, r1 plus r2 plus r3 plus r4, will be zero. Okay? You close the loop the loop. Because that's how you've assembled the, that's your kinematic chain, it's essentially saying it's a closed kinematic chain. Now as the linkage moves, the values of R 2, R 3, R 4, will change.

The vectors R 2, R 3, R 4, will change. Because the angles that, so in this case, how you measure the angles is, you will measure it from the root, of the vector. So this would be the angle of vector R 1. R 2 will be this angle, say theta 2, R 3 theta 3, you will measure it like this and for R4, so with the positive x-axis, counter clock wise, that is how you will describe the vector. Okay? R1 will be, whatever its magnitude, maybe I'll use small letters for, keep e power i theta 1, in complex form, r2 will be, R2, E power I theta 2, R 3 is R 3, E power I, theta 3, R 4 is theta 4. This would be the form. So make sure, you pay attention to, how we measure these angles, always measured with the positive x or real axis, counter clock wise and measured at the root, of the vector, at the tail of the vector, not the head. So this is called the, ‘Vector Loop Equation’, for a 4 bar. So if I express it, at as these vectors, in complex form, I have, E power I theta, 3 equal to, sorry, plus R4, e power I theta 4, equal to 0. I don't necessarily have to take them in these directions, so if I had r1, like I had it before, in this case, r1 equals r2, plus r3, plus r4. This is also, a valid way of, so depending on the directions of the vectors, you would write the, vector loop equation. So this is the vector loop equation for a four-bar. And we will use this equation, for some methods of analytical synthesis. Let's look at one such method.