Lecture - 15

Theory Of Mechanisms

Four-bar Position Analysis Dyad or

Standard Form Synthesis

Okay, so we had for the Four bar,

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$$A\cos\theta_4 + B\sin\theta_4 + C = 0 \qquad \text{———} \qquad (1)$$

A, B, C are in terms of the known quantities, viz. $r_1, r_2, r_3, r_4, \theta_2$

Displacement analysis

Subst. $\cos\theta_4 = \dfrac{1-\tan^2\theta_4/2}{1+\tan^2\theta_4/2}$; $\sin\theta_4 = \dfrac{2\tan\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}}$

Let $\tan\dfrac{\theta_4}{2} = t$

$$A\frac{(1-t^2)}{1+t^2} + \frac{2Bt}{1+t^2} + C = 0$$

$$A(1-t^2) + 2Bt + C(1+t^2) = 0$$

$$(C-A)t^2 + 2Bt + (A+C) = 0 \quad \text{— quadratic eqn in } t$$

$$t = \frac{-2B \pm \sqrt{4B^2 - 4(C^2 - A^2)}}{2(C-A)} = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{(C-A)}$$

we had the displacement equation, in the form A cos, theta4 plus, B sine, theta 4, plus C, equal to 0, Okay? so where ABC are in terms of the known quantities, namely R1, R2, R3, R4 and theta 4. Remember we are doing displacement analysis now, so assuming you have designed a 4 bar, you've synthesized a 4 bar, you want to see how it's going to move. So this is same sort of derivation as for Freudenstein’s equation. Except that the objective is different now. So I can take the easiest way to simplify this, is to substitute cos theta4, you put it as 1 minus tan square, theta 4 by 2, by 1 plus tan square theta 4 by 2, and then sine theta4, similarly is 2 tan, theta 4, by 2, by 1, plus tan square, theta 4, by 2, in terms of the tan half angles. So let me just to make my life easier, let me say this is equal to T. So I get if I substitute back into this equation I get A into 1, minus t square, by 1, plus T square, plus B, 2BT, by 1 plus T square, plus C equal 0, which becomes A into 1 minus T square plus 2b t plus C into 1 plus T square equal to 0. If I group the terms I will get C minus A, into T square, plus 2 BT plus a plus c equal to 0. This is nothing but a quadratic equation in T. So T is minus B plus or minus, so there'll be a 2b and then it will get cancelled, so I will get if I simplify this 4b square, minus 4 into, T Square minus A square by 2 into C minus A, which will simplify, minus B plus or minus, square root of, I take out the four there, B Square minus is worthless by and theta4,

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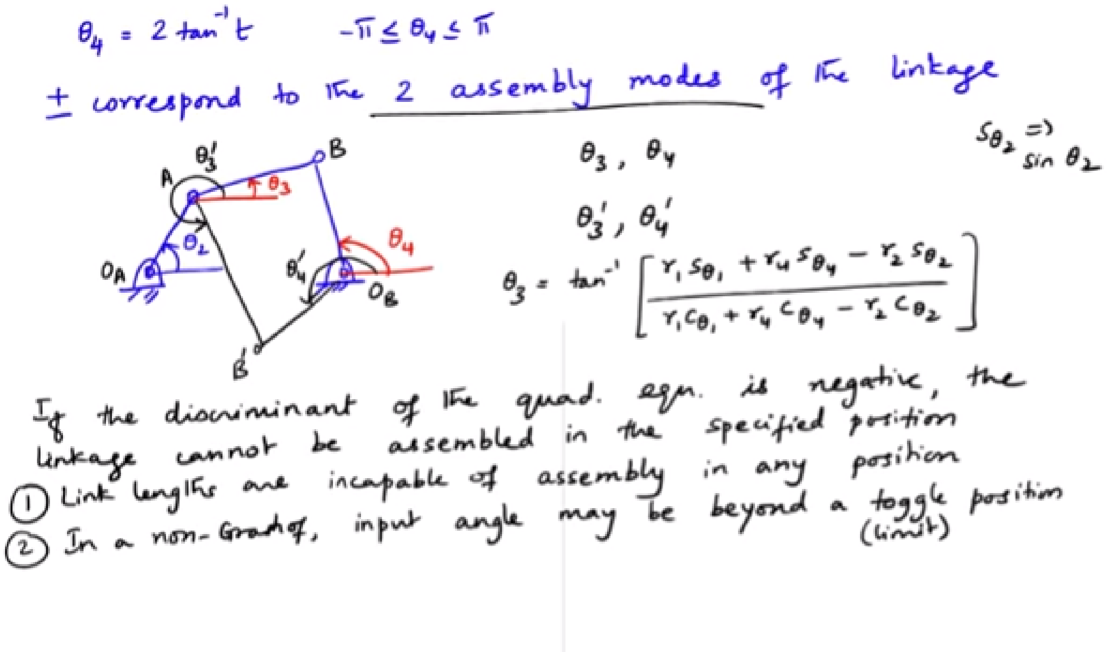

once you find T you'll get two roots of T, theta four will be 2 tan inverse of D, to pi in that range. So the plus and minus, the two roots of T, correspond to the plus and minus, correspond to the two assembly modes, or configurations of the linkage. What that means is for a particular angle theta2, theta 4 and another value theta 4 dash. So theta 2 remains the same, but this will be the second assembly mode. So I have OA, A, B, OB, OAA, B dash, OB. Those will be the two assembly modes for the linkage right, and theta 3 you can find from one of the earlier equations, right, you had where you eliminated theta 3 you go back to that plug in the value, 2 values of theta 4, so you will get two values of theta 3, which correspond to, so for this assembly mode you'll get this, then for the second one this will be theta 3 dash, so you will get a theta 3, theta 4 dash and a theta 3dash, theta 4 dash, corresponding to the two assembly modes of the linkage, okay. So this is how you do the displacement analysis using the loop closure equation which you may have seen before. Okay, so what happens if, so we saw that we had a quadratic equation and their discriminant right, if this is negative, B square minus, C square plus, a square is negative then what does that mean? What does it mean physically? What are A, B, and C functions of? the link lines and the initial angle, so if the discriminant is negative it's possible that the linkage cannot be assembled in the, in that particular configuration. Maybe it cannot be assembled for any value of theta 2, it could be just for a specific, say if it is for specific values then it may be that in a non-grashof it's outside the range of, so it's passed a toggle position in a non-grashof, otherwise it's possible that for that set of link lengths you cannot form a linkage, okay, so that is the, that is what this means. So the mechanism cannot be assembled in the specified position, maybe because one, it could be that the link lengths are incapable of assembly in any position, or in a non-grashof or a grashof type 2, the input angle maybe beyond a toggle position or a limit position. So here the theta 3 from the previous equation you get us an inverse R1, sine theta 1, plus R4 sine theta 4, minus R2 sine theta 2, divided by, so where s theta 2 means sine theta2, it's a short form.

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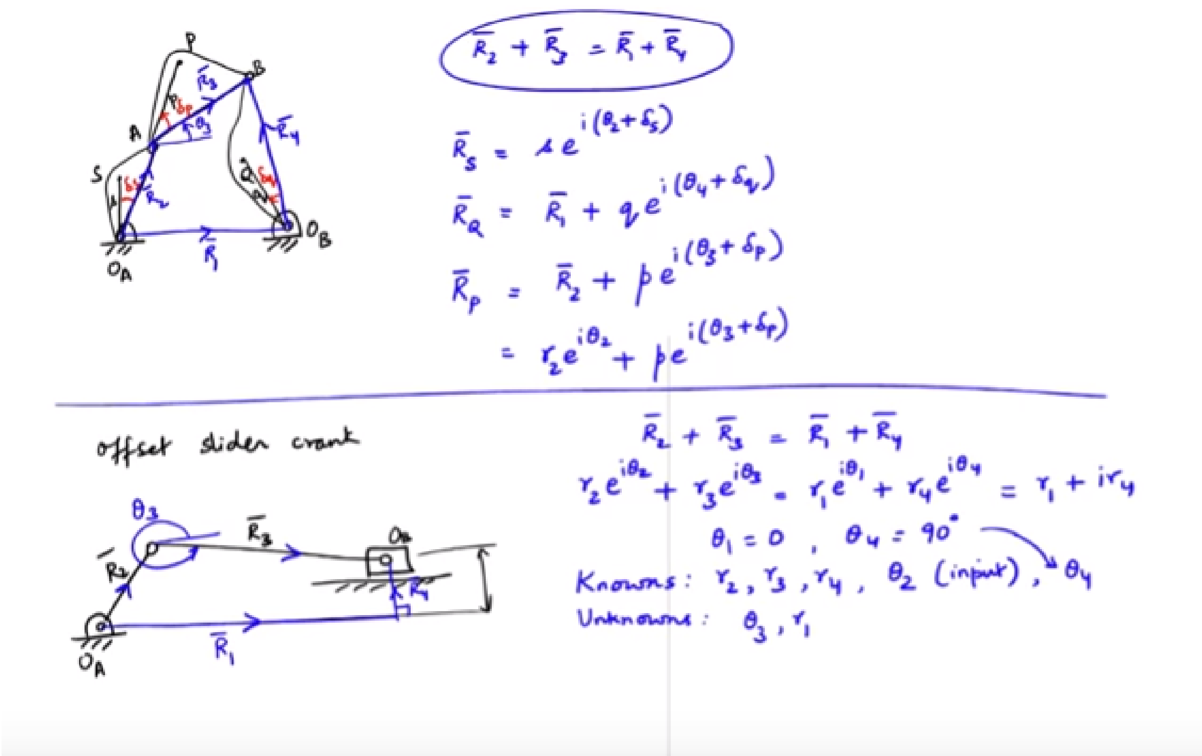

So now suppose I want to find the position of some specific points, so if I was solving a path generation problem right I would want to know whether my point is hitting those specific positions, or even in a motion generation problem I want the position and the orientation of the, so if I want to find, so it could if I'm looking at say a point on this. so this could be part of this link, this could be part of this link, so I may be interested in finding the positions of specific points on various parts of the mechanism, so if I call this, I can, it's part of this rigid body, so let's say I call this distance S, this distance Q, this distance p, small p, and it's easier for me because they're all part of, each one is part of a specific rigid body, it's easier for me to define their location in terms of unknown, so this I could call it Delta 2 or Delta s, to be specific, then I can call this Delta P and I could say with respect to. So now this is my R1, R2, so they should form a closed loop right, so I have R4 and R3. So I have r2 plus r3 that was my loop quotient. So once I have done the position analysis or the displacement analysis okay I know for every configuration I know the relationship between theta2 and theta4. It's not always necessary that theta2 should be the input angle, it could be three two three and your output but it could be theta four, that's also possible or you know in some, it really depends on the application. Typically theta2 is your input; theta four is your output. So here once i do the position analysis for this, you know the kinematic chain, I can find the position of any point on the linkage. So why do I say that once I find theta four okay I can basically specify the location of any point on the linkage okay. So here if I want RS okay, RS is basically s, e power i, theta 2 plus Delta s, okay. So any point on the input link, very straightforward, once I know theta 2, once I know I can find and I know the distance of the point from the origin and I can find that. Similarly RQ is R1 plus Q, e power i, theta 4 plus Delta Q. So any point on the rocker can be determined in this fashion because I know R 1 and depending on the distance from OB, I can find and the orientation of this of the line connecting OB to that point I can find this. Any point on the coupler which is usually what we may be interested in, so if I have an RP it is R2 plus P e power i, theta 3 plus Delta P where theta 3 is separate. So once you do the position analysis for the R1, R2, R3, R4 any point on the linkage is completely determined okay, because s and Delta s will not change, P and Delta P will not change even as the linkage moves, it's relative to that rigid body and P is a point on the rigid body the coupler, so with respect to any line on the coupler it's, or any point on the coupler its position does not change. So this is how you find the location of, so this is R2, e power i, theta 2, plus, P, e power i, of theta 3 plus delta 2. So now if you have synthesized a mechanism for path generation, you can you use these equations after you synthesize, to analyze and make sure it hits the points that you are that you are intending to hit okay, that is. Now you can do the same thing, I will not do the whole thing but I will just show you how to set up the loop closure equations for a slider-crank. Okay, so if I have, so let's take the general case of an offset slider crank where there is an offset between the location of the fixed pivot and the path of motion of the slider. So this is the offset slider crank. What you do for the slider crank is you choose the vectors. You could choose R2, R3 and take R1 from this pivot to this pivot OB but that's not very useful because you, what happens to R if I take that as say R1, its magnitude is going to change, the angle is also going to change, so they're coupled together okay, so it will make my life a little harder. So what I will do is I will choose, so I know that the slider block motion only happens along the direction r1, so I choose one vector parallel to the direction of motion of the slider block and another vector perpendicular to it because if you look at this vector R, for basically specifies the offset of the slider crank therefore it's not going to change as the mechanism moves okay. So I am adding one more vector than necessary but it's a constant vector, so now I have, I can write my route loop closure equation as r2 plus r3 equal to r1 plus r4 in complex form E power I theta 3 equal to r1 t power I theta 1 plus r4 E power I theta 4 now theta 1. So I can always choose my axis such that theta 1 is 0 rights? I choose a parallel to the path so theta 1 equal to 0 and theta 4. I always choose it perpendicular to that path so theta 4 is 90 degrees, so this just becomes R 1 plus. I hire forums okay, now what are the known and unknowns in this equation? Again it's a vector equation okay, so the known’s, so you'll be able to solve it for two unknowns. The known’s here are the link lengths r2 r3 and r4. The case of the 4-bar, all four link lengths were known Here r1 is going to be very, okay, so you have there r2, r3, r4 and your input angle theta, this will be your input okay, and if that is your input sometimes the slider block would be the motion of the slide will block, could be the input in which case r1 is your input. What are your unknowns? What else is known here? Unknowns are theta3 r1, there's one more left, where do I put that? Theta4 is known right, so theta4 is known here. I already took it but, so you are essentially solving for theta 3 and R 1 so theta 3, if I take my vectors like this, this is the angle theta 3, so again you would, I'm not going to do it, you would separate it into the real and imaginary parts and you can solve for theta 3, and are going to do this. Similarly if you want to synthesize, then say you're given inputs r1 corresponding to theta 2 for multiple positions, then similar to Freudenstein's equation that you did for theta 2 and theta 4 here theta 2 and your slider translation would be the inputs and you would try to find the link lengths to achieve that particular input. So for analytical synthesis you would use a form similar to Freudenstein's equation for the, for that. So that is one of your tutorial problems, so I will leave you to do that okay.

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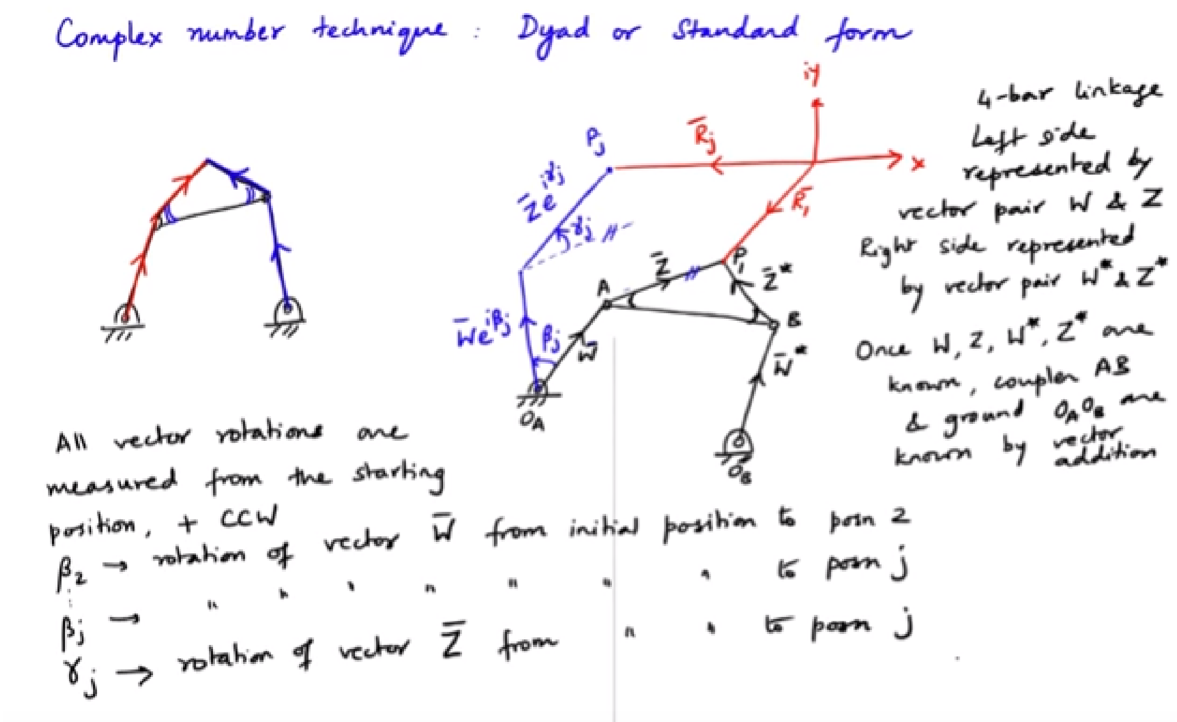

Moving on we'll go to the standard diet or standard form, so using the complex number technique. So these methods that we used were mostly algebraic methods which use the complex number form of the vector loop equation, but there's a more general technique called using what is known as the dyad or the standard form equation. So if you look at a general four bar I can actually look at it as formed by two dyads, so I can say, I have two links, like that okay, such that you know with the condition that this is like this, so I could actually you know connect, so I take two dyads and basically connect these two with a rigid rod. Then I would form a four button right, so this is sort of what we will use to develop this dyad or standard form. So the red yeah, so basically this says that you can synthesize a four bar in terms of these dyads. Our dyad is basically two links okay, so what we do is basically synthesize, this four bar in terms of these two links, so let's just, so let me say that I have a four bar okay. I'll call this vector W and this vector as Z and I'll call this vector as W star and this vector as Z star. A, B, okay, and I have a coordinate system, in which this is defined as, r1 in position 1, so this is point P in position 1 and let's say I give a certain input and this dyad moves to, let me, I'll just look at this dyad first and then I'll say that this moves to, by an angle beta J and the point P moves to this position PJ okay. So this is the angle, so these two lines are parallel okay, so this vector Z, has rotated by gamma J. If this is gamma J, this is the original position; this is the original definition of that vector. I have rotated by gamma J so this is Z, e power i, gamma J, so what is this vector in terms of W? W, e power i, beta J. I have w which has moved to and this here is my vector RJ okay, so here this is a four bar linkage, with the left side represented by the vector pair W and Z, right side w star and Z star. So once I know the two paths, once wz, w star, Z star, are known coupler AB and ground OA,OB, are known by vector addition I can find it from the loop closure. So all vector rotations I've measured from the starting position positive counter clockwise. So here for instance beta 2 is the rotation of vector W 1 and it should position, to position two and so on. Beta J is rotation of vector W from initial position to position J. Similarly gamma J is the rotation of the other vector from initial position to position J.

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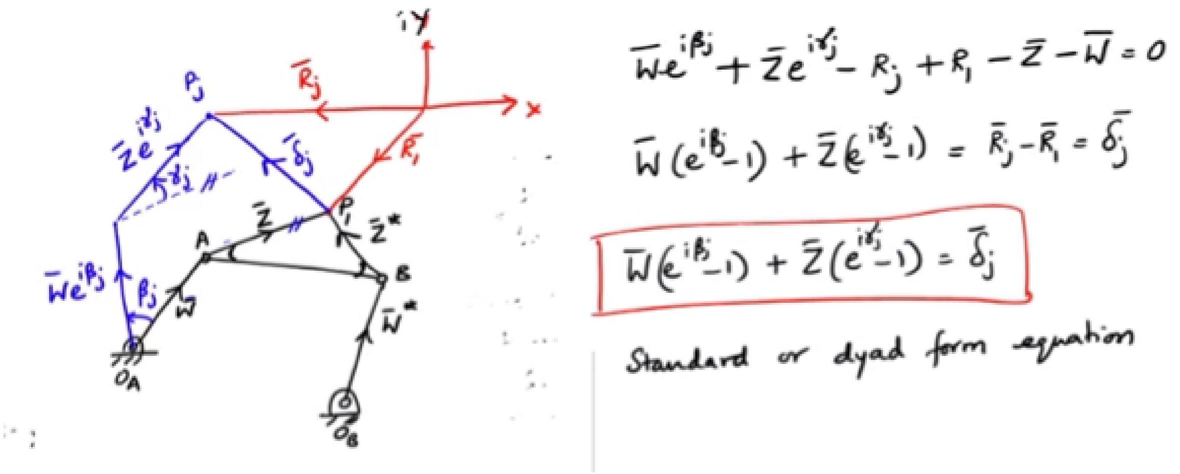

So let's say I want to write the loop closure equation okay. If I look at this I can write this as W e power i beta J, okay, starting from here okay. One more thing, so if I take R1 and RJ what is this vector? Let me call this Delta J okay. So first let me write the loop closure and then, so W e power i beta J, plus Z e power i gamma J, minus RJ because I'm going this way okay. So this is this, this minus RJ okay, then plus R1, okay minus Z, minus W. If I traverse this path I come back to the starting position, I'm back where I started, okay. So I can now write this as, W, e power i beta J, minus 1, plus Z e power i, gamma J minus 1, is equal to R, sorry RJ minus R1, which is equal to Delta J. So what is Delta J? Delta J is the displacement vector of some prescribed trajectory of the point P okay. So this form of the equation, this w e power i beta J minus 1 plus e power gamma J minus 1 equal to Delta J, this equation is called the standard form equation, standard or dyad form equation. Okay, the next class we will see how we can use this to construct for birds and synthesize forward given four different types of functions. Also different types of tasks, motion, path, function, generation, etcetera because I am defining the vector, because see for the point P, the location of the point P, those are not J. So if I define, if I say that the linkage has to hit those points for instance, then I'm giving you the positions of those points in my coordinate system. I am giving you, those are not changing, so here I'm looking at, so I'm moving this dyad from one position to another and trying to define this in terms of the movements of the dyads, the angular movements of the dyads R1 to RJ, are specified vectors okay, They'll also have an angle, they'll have a magnitude and an angle but for this purpose you know the vector completely, what you're going to do with this is to actually synthesize. So knowing beta J and gamma J for instance, you'll want to find out what should be W and Z, to hit these points defined by R1 and RJ okay. Then we'll also look at function generation, you know for W and Z with certain angles, w star, Z star, what the angle should be so that you can do the synthesis of your linkage. Similarly for motion generation we look at, because this displacement vector will be given. This basically shows the displacement of the point P1 to the position PJ okay, and the other thing that you'll be given is, so when you are talking about motion generation you will also be given the orientation of the coupler. You'll be given the location of a point and the orientation of the coupler, so there you will be given Delta J and gamma J, will be given the orientation of the coupler, then you have to synthesize a linkage so we will see all those different forms as we go along okay, but this is called the dyad or the standard form. It's a very elegant method for synthesizing four-bar linkages.