Theory of Mechanisms

Lecture 17

Dyad Form Synthesis: Path and Function Generation

Let's just do the, the comparison between the graphical and the analytical for the two position synthesis.

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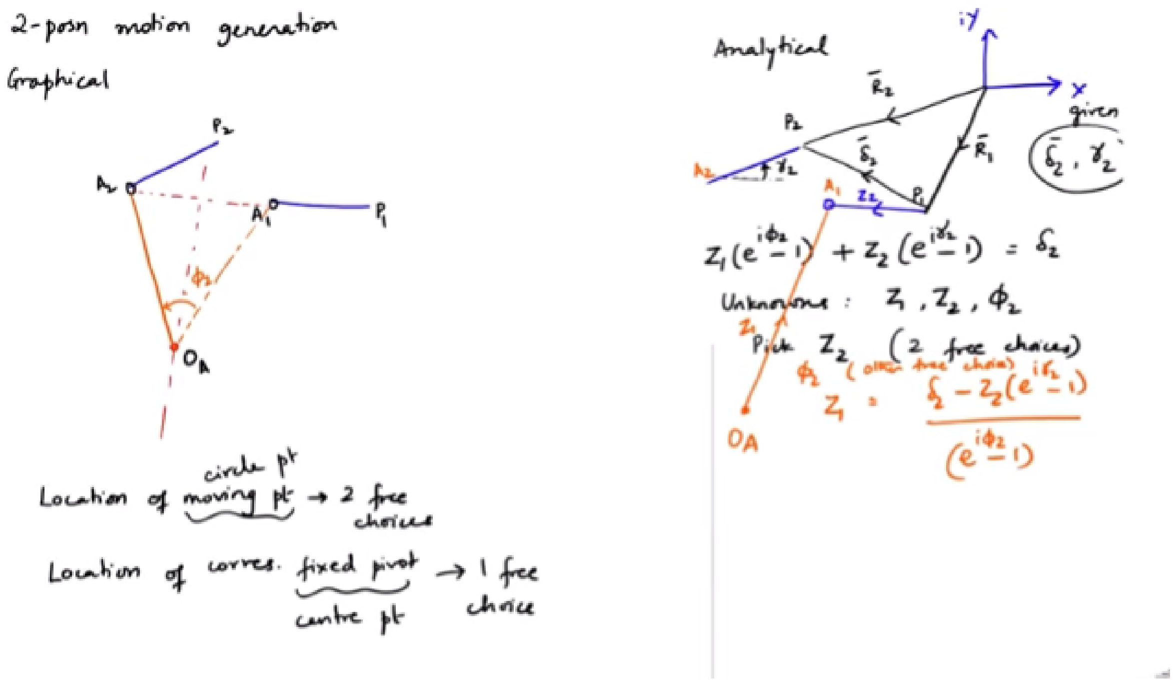

So say I have these two given okay? So let me just, and similarly for, so I have the same problem, specified analytically we just look at what we do for the two cases. So if I have okay, so in the analytical case okay, for a motion generation problem I would be specified. These two positions, so in the graphical case, say this is a2, a2, a1, b1, so I am given these two positions, okay? The same thing would be specified as the two position motion generation. This is graphical; this is analytically, so in this case, I would be given Delta 2 and R2 or R1, R2 and gamma 2 okay? This is given in the graphical skills. I would be given these two locations as this, okay? So the equation that I have here between the vectors I want to find is Z power i, Phi 2 minus 1, plus Z2, e power i, gamma 2 minus 1, equal to Delta 2, position. Problem, this is the only better equation I have okay, and I know from this table here that I have three free choices for the solution, right? I have to position problem, I have, these are my unknowns, Z1, Z2 and a number of free choices is 3, so the difference is, so here what I, the way I exercise my free choices is, I pick A1 to be my moving pivot okay? If I pick A1 to be I can pick anything else. Also so I exercise two free choices there okay?

Now A1 I know in the second location has moved to A2 ok, So then I construct a. I know that the corresponding center point or fixed pivot lies on the perpendicular bisector of A1, A2 okay? So if I construct the perpendicular bisector of a1, a2, then my third free choice is locating a point on this perpendicular bisector okay? So location of A, of moving pivot, 2 free choices, I use up there, and then location of the corresponding fixed pivot okay? So I could. This is typically called the circle point right, and their corresponding center point, so this gives me another free choice, okay? Same prop, so this becomes OA and that's how I get this dyad. So now my dyad is OA, A2 and I already have, I know P2 so this ROA, A1, and that. So these are the two things that define my, define 1 dyad, right? OA, A2 corresponds to Z1, A2, P2 corresponds to Z2, in the unlit, if I look at it as a dyad, okay? So that's how I determine the solution for one half of the 4 bar in the graphical method. We know this in this case so essentially let me say that my unknowns are z1, z2. I also don't know Phi 2, those are my unknowns, here What I do is I solve for, say I pick z2 that becomes two free choices. scalars. okay? Two free choices, then so what is Z2 here? Z2 is essentially this vector here, right? So if I pick Z2, I am saying I am locating this circle point or moving it when I pick Z2, that's what I am doing here, which is the same as my picking A1 in this case, and then I solve. I also have to pick Phi 2, so that I can solve for Z1. That is different from the graphical method because here I cannot say that that's like picking this angle. So this is Z1 right? So from, from this position okay, to this position, this rotates by an angle Phi2 okay? That is what I am picking in this case as my free choice, then I can find the corresponding z1.

What is that vector Z1 that rotates through this angle Phi 2 so that A1, moves to A2? So I am trying to locate this point away by that, means I'm saying where should OA be located ,such that this z1 okay? Then I can from this equation I solve for Z1 as Delta 2 minus, Z2 e power i gamma 2. So this is less intuitive, right? It's easy for the equation. This is how I solve it, right? So Phi 2 is the other free choice, but phi2 then, then this helps me locate Z1 when i solve for Z1, I get this. So then I am now able to locate OA from this and it is such that it will load rotate by Phi2 okay? Is that clear? So the number of free choices do not change, what you pick as your free choice changes, okay, and that determines how you solve for the other unknown vector, okay? So then you get, this becomes Z1, once I pick Phi 2, I can solve this equation. Here similarly you can, you know I do this comparison for three position also, but I will, you know you, you get the idea this is basically to illustrate that the, this is much harder to do here you know, trying to find something that will rotate through a certain angle. Of course here you can again consider it will still involve the same kind of construction, but I could essentially get the same solutions from both, the analytical method and the graphical method, okay, and the same numbers of solutions are also possible.

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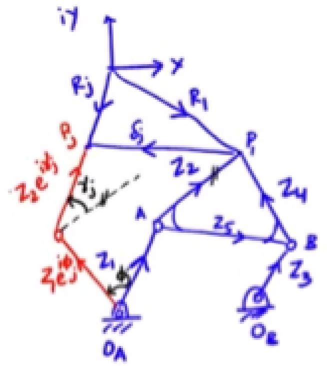
Path generation with prescribed timing



$$z_1(e^{i\phi_j}-1)+z_2(e^{i\gamma_j}-1)=\delta_j \qquad j\geq 2$$

Given ϕ_j, δ_j

So we have Z1, e power i PI 2 minus 1, e power i, gamma 2 minus 1, equal to Delta 2. The dyad form, the standard form, so here or more generally I can write it as Phi J minus 1, gamma J minus 1, right, between J greater than or equal to 2, sorry, okay? Now if I want to solve a path generation problem, so let us say we are looking at path generation with prescribed timing. What does that mean? Prescribed timing means, for a certain input angle, okay? You want the coupler point to be at a certain location.

Path generation means you're only worried about the location of this coupler point P, at successive positions, you're not really, you don't really care about the position or the orientation of the coupler, you only want, care about where P is located, with prescribed input timing, means if the input angle given a specified given Phi J, you have a corresponding then touch okay? Every time the input link moves through a certain angle the path point should move through certain, should undergo a certain displacement.

That's your, so here essentially you will be given Phi J and Delta J. If you look at the form of this equation for the motion generation we were given Delta J and gamma J Delta J, Gamma J, given is a motion generation problem. If you are given Phi J and Delta J it's a path generation with prescribed timing. If I'm given only Delta J, then it's just a simple path generation problem, you don't have any conditions on the input, you know coordinating it with the input motion, but if you look at this, it's essentially identical because of the form of the equation. This problem is essentially identical to the motion generation situation. So what do you expect?

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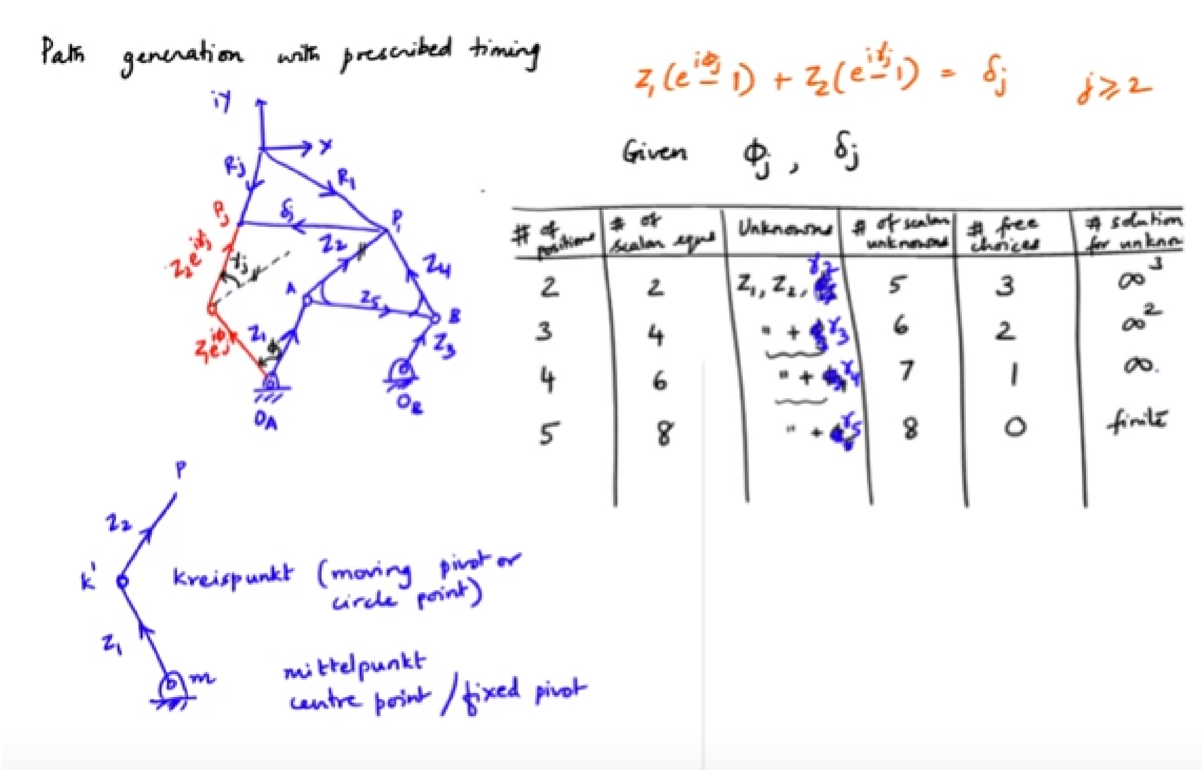

So whatever you got here you remember that table that we had, okay? This table, okay, I could essentially reproduce this so if I do this then what changes is that, okay, so the only thing that changes for the path generation problem is now this is not an unknown. This will be gamma to the unknowns, will still be Z1, Z2, gamma 2, here it will be Z1, z1, z2, gamma 2, plus gamma 3, gamma 4, everything else remains the same. Question: which one this is not a function generation? Input angle 2 and output angle, yeah, input angle 2, output angle that is your function generation. See every 4 bar has all these relationships, there's going to be a coupler point which for the classification is based on your requirement. Ultimately you're getting a four bar, the four bar, you know in the fourth bar you decide that the input, output angle relationship is what you want, then you call it function generation. You call it a function generator, if in the four bar you're looking at the coupler, its position and orientation, then you call it a motion generator, same for but depends on what you wanted to do and you synthesize accordingly, okay?

The four bar still retains all these relationships, there will be points you know on the coupler which will follow certain paths. The coupler will have certain orientations input and an output links will have certain angular relationships that remain. The classification is entirely based on what you as a designer want. Still a four bar okay, so here the form remains the same, so the number of solutions, everything for the dyad, will be the same. So I use the same, so I could essentially use the same code, for motion generation as well as path generation, with prescribed timing. I'm just giving in a different angle in this okay? Our output is still the dyad that I won okay and that again up to three positions you will have a linear system of equations. Beyond three positions it becomes a nonlinear system, so it's more difficult to solve okay? If I didn't have Phi J, if I only gave Delta J right, then that's your regular path generation problem. Just path generation, if I don't coordinate it with Phi J okay, I only care about the coupler point being at certain positions, I don't care how it, in what relation it is to the input link, then will I have more or less number of solutions for each case?

Am i constraining the problem more or less? Less, so what do you expect you would have More solutions, because again phi j then becomes a free choice okay? Phi's a becomes an unknown, it's not known, so in the unknowns here this goes up okay? So you can work that out and see how many positions you can go for a path generation problem where you don't care about the, how it is coordinated with the input angle okay? So that's now. Let's look at the next problem. We look at, is function generation and that's slightly different, so this is for one dyad, this infinity cube say for the two position problem is for one dyad, so one set of circle and center points, see, because essentially when you design the dyad that's what you're doing. So this is if this is P right so you're essentially minus z2 locates the circle point from there minus z1 locates the corresponding center point okay, and these are usually denoted as, okay, circle points. Remember even in the, when we did the slider-crank synthesis and we drew the locus of the locations, you know I designated the circles as K okay? That again comes from the, this is the circle points, are denoted by K, they are called Kreispunkt, this is moving pivot for circle point, and this is M. This is easier to remember because it's mittle point, so the middle point, that's how I, so it's the center point fixed okay? So that's essentially what you are doing when you find Z1 and Z2 okay? You're finding when you find these two vectors you're locating the circle point and the corresponding center point so this stable, when I do it for only one dyad, it's for one side of the 4-bar, again for the other side I have the same table, so if I combine the two then I have to multiply, so it's infinity to the power six, for infinity square, The number of choices that I have okay, for the two dyads that form the fourth power okay?

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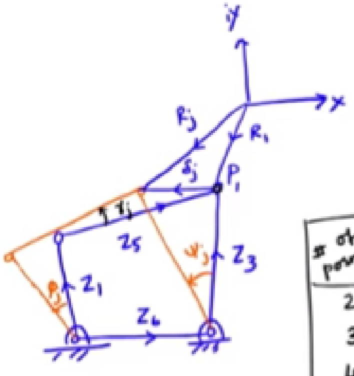
Function generation – correlate prescribed rotations of the input link ϕ_j with rotations of the output link ψ_j

Given: ψ_j, ϕ_j, $j = 1, 2$



$$z_1 + z_5 - z_3 = z_6 \quad \text{— ①}$$

$$z_1 e^{i\phi_j} + z_5 e^{i\gamma_j} - z_3 e^{i\psi_j} = z_6 \quad \text{— ②}$$

Subtract ① from ②

$$z_1 (e^{i\phi_j} - 1) + z_5 (e^{i\gamma_j} - 1) - z_3 (e^{i\psi_j} - 1) = 0$$

# of points	# scalar eqns	Unknowns	# scalar unknowns	# of free choices	# of solutions
2	2	z_1, z_5, z_3, γ_2	7	5	∞^5
3	4	" $+\gamma_3$	8	4	∞^4
4	6	" $+\gamma_4$	9	3	∞^3
5	8	" $+\gamma_5$	10	2	∞^2
6	10	" $+\gamma_6$	11	1	∞
7	12	" $+\gamma_7$	12	0	Finite

So now let's look at the function generation problem, using this dyad synthesis okay? So function generation what we are doing is we want to correlate prescribed rotations of the input link Phi J with rotations of the output link say to. So here I don't really care about a coupler point because I am only looking at coordinating D, so if I go here, so I could essentially take, this is my point P, this is also on the coupler okay, be in the position one and then see how I can set up the. So if this is R1, Rj, delta j there's magic okay? The angle between these two will be gamma j, so this I take it as z1, take this as Z3, and in my original thing this was z5, and this fixed link is Z6. So I can write my loop closure as just for the 4-bar. I can write it as Z1 plus Z5 minus Z3 equal to Z6. Then I can write it as Z1, e power i, Phi J, plus Z Phi, e power i gamma J, minus Z3, e power i, Shi J, equal to Z6 does not rotate, it's my fixed link. so if I subtract one from two I get z1 e power i Phi J, minus 1 plus, Z Phi e par i gamma J minus 1, minus Z3 e power i Shi J minus 1 equal to 0.

So this is not in standard form, is not in the standard form. Here what would be given the function generation, the given quantities are Phi J and Shi j, right? I know how they are supposed to baby number of positions, equations. What are my unknowns? Number of scalar unknowns, number of free choices, their full number of solutions okay? Do positions, how many equations will I have? I have only two scalar equations okay? J will be 2, so what are my unknowns? Practically everything, so I have Z1, z5 Z 3, and gamma 2 plus Phi 2 and Shi2 are given. That's the relationship I am designing for it, so the number of scalar unknowns is 2, 4, 6, plus 1, 7, all the Z's are vectors, so I have 7 scalar unknowns, so my number of free choices is 5, so I have order of infinity to the power 5 okay? You should remember that this infinite, it's, it's just to indicate you know, what the different infinity are. To the power five infinity is the largest number possible, so infinity, it's not really, it's rather meaningless, but the idea is to indicate how many great choices you have okay? so num, number of solutions because each of those infinities corresponds to a different quantity. Essentially that's what indicates, they go to a three proposition number of scalar equations now becomes four unknowns, are all these okay? So I'll just put a ditto, plus gamma 3 so number of scalar unknowns is eight. It goes up by one so the number of free choices goes down by one, you have infinity to the power four, four positions, you now have six equations. All these are unknown plus gamma four 9, 3, okay? So five. How far can you go? before you hit no free choices? Five, so each time this goes up by two, but the number of unknowns only goes up by one, gamma five, so ten, 2, six number of equations, ten, this plus gamma 6, 11, and 7, you have 12, all this plus gamma 7, 12 you end up with zero free choices and you may get a maximum of a finite number of solutions, there may be no solution. That's always possible but maximum you could get a finite number of solutions okay, but this again is for the whole mechanism. You're looking at for the function generation; you don't split it into the two dyads okay? You're looking at the mechanism as a whole okay? You can also think of the function generation problem, as a special case of the power generation problem, where the paths point is the point P1, here on this output link. What is the path of this point? It's a circuit, it's a circular path so it's a special case of the path generation problem where this point P, is common to the coupler and the output link and it follows a circular path, okay? So this is for the function generation problem.

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$$Z_1(e^{i\phi_j}-1) + Z_5(e^{i\gamma_j}-1) - Z_3(e^{i\psi_j}-1) = 0$$

$$Z_1(e^{i\phi_j}-1) + Z_5(e^{i\gamma_j}-1) = Z_3(e^{i\psi_j}-1)$$

Picking Z_3 as an arbitrary choice converts the eqn to the standard form

$$Z_1(e^{i\phi_j}-1) + Z_5(e^{i\gamma_j}-1) = \delta_j = Z_3(e^{i\psi_j}-1)$$

This is now equivalent to the motion/path generation task

Choosing Z_3 (or Z_1) specifies the scale & orientation of the function generator: changing this does not change the input/output relationship

Path & motion generators — prescribed path will not be achieved if the scale & orientation of the linkage are changed.

Now I can rewrite this equation. I can put it in a different form. So I have, so I have the Z1 e power i Phi J minus 1, plus Z5 e power i gamma, J minus 1, minus Z3 e power i, Shi J minus 1, equal to 0, okay? I can rewrite this as plus Z Phi e power i gamma J minus 1, equal to minus 1. Okay now if I pick z3, if I choose z3 as two of my three choices okay, they use up two of my three choices this is off the form okay, and side a is known because it's a function generation problem. I'm given this so this right side becomes of the form Delta j, if I choose okay? Picking standard form Z1, Z5, e power i gamma J minus one equal to Delta J where Delta J is z3 e power Shi j minus one okay? So this is now equivalent to the motion generation task or the path generation task so what this, what choosing z3 does is you're basically picking a scale and an orientation for the linkage. Remember a function generation because the output, the input and output of the function generation mechanism are internal to the system, internal to the chain kinematic chain okay? So you we have seen earlier that's scaling it or changing its orientation will not change the input output relationship, that's what you're doing when you pick z3. When I pick one of the vectors, so when I pick this, when I specify that I'm essentially specifying the scale of the linkage and its orientation okay? So I can always change that and it's not going to give me a new mechanism. The input-output relation it, I mean it will be new in terms of the dimension but the input-output relationship will not change okay? So that's what we are doing here when we pick z3 and then convert this to the standard form of the dyad equation okay? So this is now equivalent to the motion or path generation Task. So choosing, I could have chosen Z1 also doesn't matter, one of the two because I know Phi and I know Shi, so if I choose one of these it gets back to the standard form okay? So choosing Z3 or Z1 specifies the scale and orientation off the function generator.

So once it is synthesized for function generation, it can be scaled up and down or oriented differently and you will not change the input-output relationship. This does not work with motion or path generation, because in motion generation and path generation the output is external to the system, the output is specified external to the system with respect to some other coordinate system right? It's not internal to the system as it is in the case of function generation so part and motion generators, this is important. The prescribed path will not be achieved, if the scale, scale and orientation of the linkage are changed okay? So here changing this does not change the input-output relationship. So that's an important difference between function generators and patent motion generators okay? So we will stop here.