Theory of Mechanisms

Lecture – 16

Dyad Form Synthesis: Multi Loop Linkages

So we've looked at the dyad method of Synthesis, for motion path and function generation and you know we started off with this equation,

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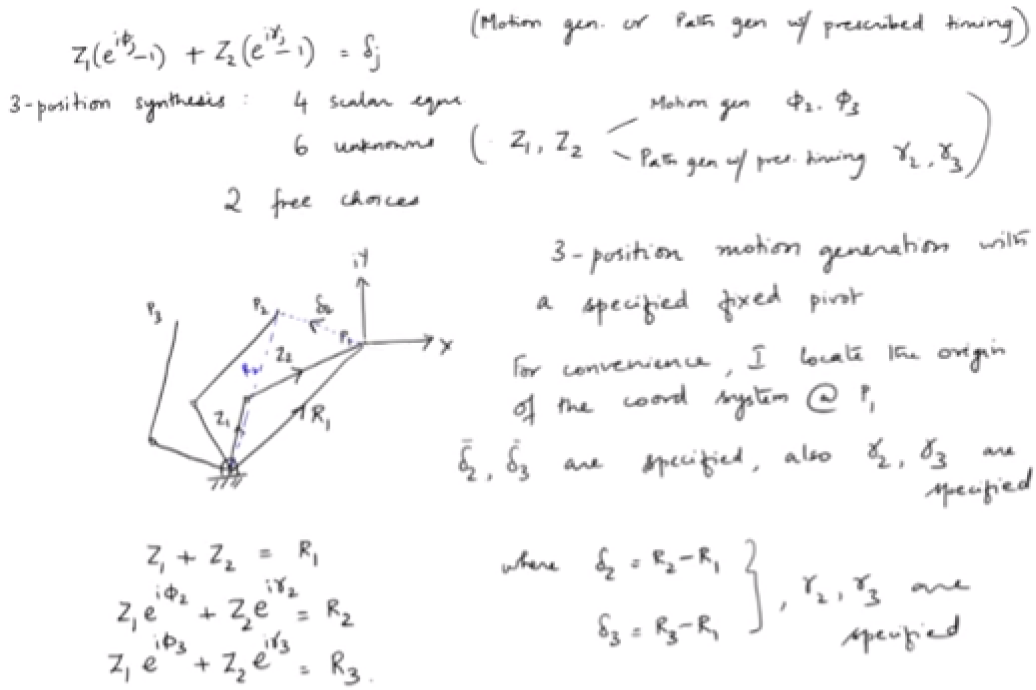

Basically Z1, Phi J minus 1, plus Z2 e power i gamma J, minus 1, equal to Delta J. where if Delta J and gamma J, are specified then you have your motion generation problem or if Delta J and Phi J, are specified then you have your path generation with prescribed timing. So we use so for the three position synthesis if you remember the table or from these you will have how many equations? You'll have how many equations for three position synthesis? You have four scalar equations and so let's we are looking at motion generation or power generation with prescribed timing. Both have the same form. So we have four scalar equations, how many unknowns? will have six unknowns, the unknowns are Z1, Z2, 4 unknowns there and depending on you know if it's motion generation, the unknowns would be Phi 2, Phi 3, Path gen with prescribed timing. The unknowns are going to be gamma 2, gamma 3. Okay so you'll have six unknowns and therefore, you have two free choices, for the 3 position problem, we've seen this before.

Okay so the two free choices, we could either pick, the two angles, make this a system of linear equations and solve for the vectors z1 and z2, or we could pick one of the vectors z1 and z2. so instead what I'm going to do now is in general, it's advantageous to me, if I can specify the fixed pivot, Right when I pick z1 or z2 are not directly specifying the fixed pivot, but I can rewrite this in a form where I can actually specify the fixed pivot, in the plane which you know as a designer may be more advantageous to me, to determine this dyad. So I just write this equal to write these equations in a slightly different form, in order to be able to specify the fixed pivots. if you remember in the three position synthesis graphically, we could do that, we could specify the fixed pivots, find the moving pivot that corresponds to it, so that you can get your mechanism with the chosen fixed pivots. Okay so, here for convenience, so since the form of the equation, is based on the deltas and the gammas, you know my first position is sort of my reference position so for this case I am going to for convenience I'm going to locate my coordinate system, at the first position. So let me take this direct. So I have I'll just first draw the dyad, in three positions and then vary So if this is my p1, p2, p3, this is what I Want, I want to synthesize the dyad, that will go to these positions p1, p2, p3, which say certain orientations gamma 2, gamma 3, with respect to the first position. So I should choose my, I'll choose my origin of the coordinate system at this point, p1, otherwise I'm just going to get you know two known vectors it's so it's not really changing anything. So if I do that then if this is Z1 this is Z2 ok and I call this R1 minus R1, directly specifies the location of my fixed pivot with respect to the coordinate system, okay. So I can so I'm going to write the equations using this form. so we are doing the case of three position motion generation, with a specified fixed pivot. So for convenience, I pick or I locate the origin of the coordinate system at p1. Okay so for three-position motion generation my Delta 2, Delta 3, which are vectors, are specified also gamma 2 and gamma 3, are specified. So now I have, so let I have the two free Choices, so let me write the equations as Z1 plus Z2, equal to R1, first position Okay, then I can write it as Z1 e power i Phi 2, plus Z2 e power i gamma 2, equal to R2, where Delta 2 equal to R2, minus R1 right, So R2 is just Delta 2 plus R 1. I want to specify R1, okay the location of my fixed pivot and then I write, because if you look here if I, if this is R2 then Delta 2 is, this is R2, then Delta 2 is this equal to R2. So this is Delta 2, write p1 to p2 that vector is Delta 2. then for the third position I can write it as Z e power i Phi 3, plus, Z2 e power i gamma 3, equal to R3 and Delta 3, equal to R3 minus R1. So these are my synthesis equations. These and gamma 2, gamma 3, are specified. So here in this set of equations, okay the coefficients of Z2 are known and the R's are also known, okay that's what I'm given.

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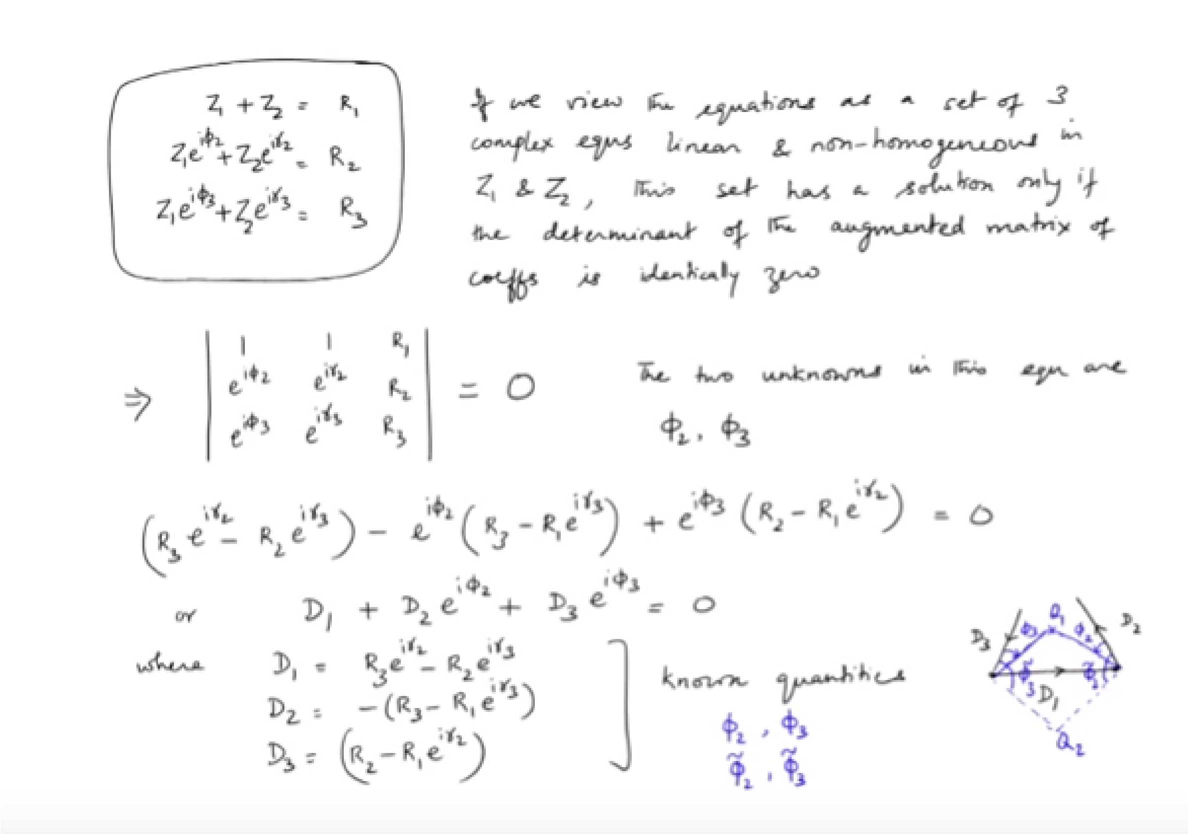

So if you look at these equations, I have Okay, if I look at them as a set of equations within, in z1 and z2, okay, if I look at them with complex coefficients, if I look at them as a set of linear equations with complex coefficients, in z1 and z2, then when will I have a solution, so I have only two unknowns, I have three equations, into one nodes, right, when will this system have, a solution. If I look at it as, let me just write that down, if we view the equations, as a set of three complex equations, that are linear and non-homogeneous, in Z1 and Z2, this set has a solution, only if what condition? you have three equations two unknowns, so yeah so the two of the equations have to be linearly dependent which means the determinant, of the Augmented matrix should be zero. Okay so only if the determinant, of the augmented matrix of coefficients, is identically zero. So that means, if I write the Coefficients, so this becomes a condition that has to be satisfied, in order for this system of equations to have a solution, right? R1, R2, R3, this determinant should be zero, okay. So this represents a complex, so I know gamma 2, gamma 3, R1, R2, R3, in this okay, so I can expand this as a condition on Phi 2 and Phi 3, so not all values of Phi 2 and Phi 3, will be able to satisfy this Condition. okay so if I expand about the two unknowns in this equation are, Phi 2 and Phi 3, if I expand this determinant, so I can expand it, say about the first column because the unknowns are in that so I get e power i, R3, e power i gamma 2, minus R2 e power i gamma 3, minus e power i Phi 2, so I'm expanding about the first column, I Have, R3 minus R1, e power gamma 3, plus e power i Phi 3, into R2 minus R1 e power i gamma 2, this is equal to 0. So this represents a condition that phi2 and phi3, have to satisfy, in order for this system of equations to have a solution, okay.

So my Phi 2 and Phi 3, should be chosen such that, they satisfy this equation in order for me to satisfy find a solution for z1 and z2 that satisfy those three equations. so I can write this in the form D1 plus D2 e power i Phi 2, plus D3, e power i Phi 3, equal to 0, where D1 equals R3, e power i gamma 2, minus R2 e power i gamma 3, D 2 equals, minus of R3 minus R1 e power i gamma 3 and D3 equals, R2 minus R1 e power i gamma 2, ok. Now because R1, R2, R3, all are given and gamma 2, gamma 3, all these are known quantities, okay. D1, D2 and D3, are known vectors, in this compatibility equation. now this equation if you look at it it's rather it's a transcendental equation in phi2 and phi3, so it's not easy to solve that but we can use this to solve it geometrically, because if you look at this, this is like a loop closure equation, you look at Phi 2 and Phi 3 as rotation operators, ok e power i Phi 2, and e power i Phi 3, remember we've been using that as rotation operators all along. so one way to solve this is if you consider D1 vector and assume that D2 and D3 are pinned to this okay, on either ends of this, okay so, say this is D2 sorry, this is D2 and say this is D3, then essentially, what I am trying to find is, how do I through, what angle should I rotate these two? Okay, say this is Phi 2, this is Phi 3, through what angle do I rotate these two so that they meet and form this closed loop? Okay, so if this is D2 this is D2 e power i Phi 2, this is D3 e power i Phi 3. So I can find so in MATLAB or something with the mapping tool box you can basically find you know the lengths of these two vectors D2 and D3, so that'll be one solution. the other solution would be when they meet on this side, I shouldn't call this P2, let me call this Q1 and Q2, ok so this becomes Phi 3, tilde, let me call that this is Phi 2 tilde. So I'll have a Phi 2 and Phi 3, Phi 2 tilde and Phi 3 tilde, 2 solutions such that these will meet and satisfy the compatibility equation.

So when you solve this. So that's why it's easy easier for, so when you're looking at looking at it as a rotation operator, the actual signs will come out when you solve the equation. so when the rotation you know if you do it as you you'll see that Phi 3, will be a negative sign indicating it's a clockwise rotation, okay when you actually solve a problem with this thing don't they I want you to just understand the geometry of this, what we are trying to do here, because it's otherwise rather difficult otherwise you would have to expand it see D1, D2, D3, are also complex, ok so it's not like your loop closure equation for the 4 bar, or something like that where you have real numbers, multiplied by this, so you have complex times complex numbers, so then you would have to then separate out expand this, then separate out the real and imaginary parts, then eliminate one of the angles, using the same thing, you know cos and sine, U Square and that, but it becomes a very tedious procedure, this basically gives you an idea of what we are trying to do, we are trying to find the two intersections, such that this compatibility equation will be solved. One of these of course if you look at the form of the equation itself, one solution becomes obvious, what is that? If Phi2 equal to gamma 2, Phi 3 equal to gamma 3, that is the trivial solution, which will satisfy this compatibility equation. so you have one solution right there. Because you have two columns of the determinant they become equal your determinant is 0, ok. So that becomes the trivial solution. so this is the way that if you specify, the fixed pivot you can now solve, so once you have Phi 2, Phi 3, you can plug that back into, two of these equations, okay you take the first two equations okay and then you can solve for Z1 and Z2, because now you know the Phi 2 and Phi 3, that will satisfy the compatibility conditions, so you cannot assume them as free choices here. You've already assumed your free choice in locating your fixed favourite R1, okay. So now Phi 2 and Phi 3, only if they satisfy a certain condition you can meet all three, you can satisfy all three equations, okay. So that is the synthesis when you have a specified fixed pivot, you specify the fixed pivot by means of so minus R1, is your specified fixed pivot, with respect to your coordinate system. Okay so this is the way you would use the dyad method, to solve for a specified fixed pivot design a 4-bar for it. Ok ok.

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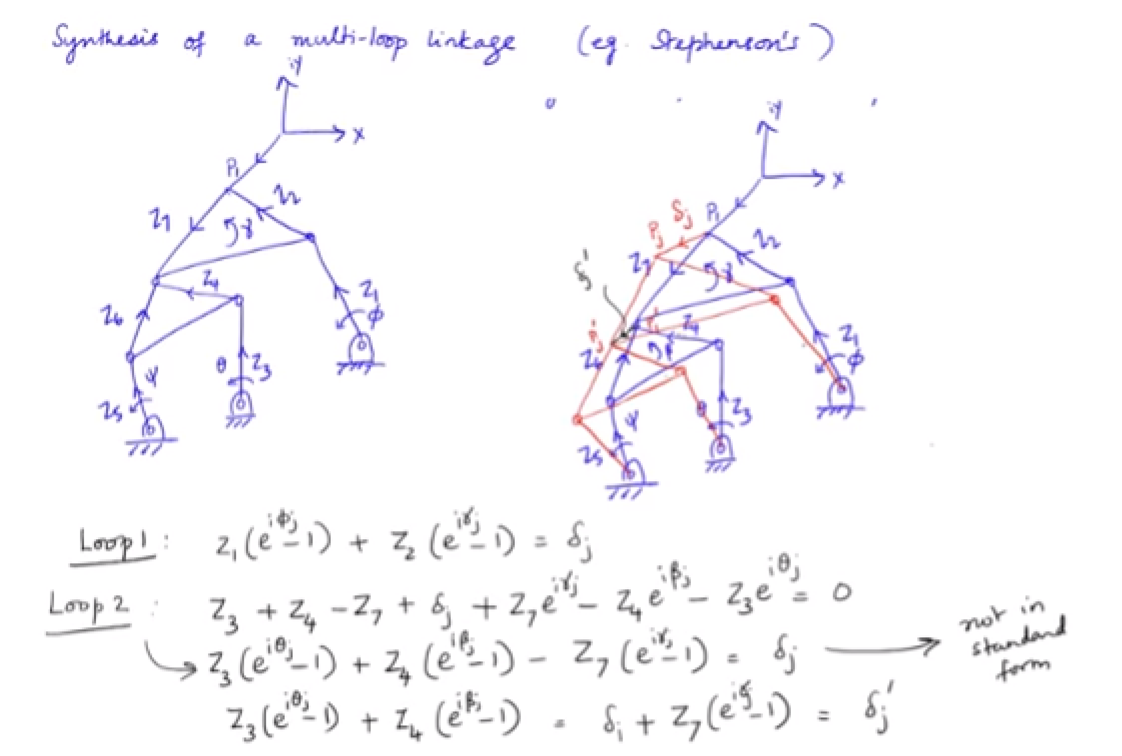

so now this dyad method, can be used also for linkages with multiple loops, obviously you can see that it's fairly easy to extend, you know you're only looking at so if I have a 6 bar, you know I'm going to I can again set up the equations, for multiple loops and then solve for them, for the conditions that I'm specified. We'll just do the case for a Stevenson's mechanism, 4 bar, see what 4 bar is fairly straightforward, you it's a four bar driving a four bar. So it's fairly straightforward. let's look at a Stevenson's instance and see how we would use this dyad method, how you would extend this to multiply loop mechanisms. In many cases a four-bar, may be Sufficient, for your application, but as you have more number of positions, one of the ways is to deal with it, is to go to perhaps a 6 bar, you may have more choices in solutions, you know that with the 4 bar, you kind of hit the wall, once you go to I mean it becomes more difficult and and we will see how they will look at the solution for the four position synthesis, to give you an idea of the solution procedure, analytical procedure, for the four position synthesis, but when you go to the five five position, it, it is even harder to find solutions, because you only have a very limited number of solutions. so in those cases going to a multiple loop linkage, like a six bar, or higher can actually give you more options, for your solutions and that's one of the reasons. So we will look at the case of a Stevenson's linkage. What we call a multi loop, okay. So, so you can see here, how do you know this is a Stephenson's and not a watt?

Binary between the two turns, so this is a ternary link, fixed-length is a ternary link, you have a binary link, between the two ternary links, okay again it's a single degree of freedom linkage and you have this coupler point, driving this other dyad. Okay if depending on which which one is your input or. so let's say this is my point P1, okay so this, that's one dyad, okay, then I'll take Z3, Z4 as another dyad and Z5, Z6 and I'll take this as Z7, okay. And say this is the point of interest, coupler point of interest whose location and say this is, undergoes gamma, we call this angular motion of this one as Phi, this one theta, its minus sign and then if I have, say, this is location of P1 or I can just choose and then from here so in another position, so if I call this point P1 this is P1 this P2, so this is my Delta 2, sorry not 2 bar, they should sorry this is only Delta 2, this is not Delta 2 dash, so I can write the equations. so let me take first loop 1, Z1, Z2, this is I'll take this loop with Z1 and Z2, okay, so it's in my standard form, right if I take Z1, Z2, this is Z1 e power i Phi 2 or Phi J, I should just call it J, Okay.

So let me take the first loop, and I can write this as, I can write it in the standard form as, Z1 e power i Phi J minus 1, plus Z2 e power i gamma J minus 1 equal to Delta J, ok, this is loop one, this is a neat standard form. Now the other two loops, let's say I Write, loop 2, let Z3 and Z4, Z7 will also come into the equation, so I have Z3, okay I can let me just write down the loop ones and then so I have Z 3, plus Z4, minus Z7 ok plus delta j, minus Z7 e power i gamma J, okay this one sorry ,it should be plus, because I'm now this is Z7, here then I need to designate an angle for this one, let me call this, beta, okay for this coupler plane the angle is beta okay. So Z4, Z6 will rotate by the angle, beta. So I have okay Z3 plus Z4, I'm looking at the loop with Z3, Z4, this is minus Z7, ok plus delta j, plus Z7e power i gamma J, minus Z4 e power i beta J, minus Z3 e power i theta J, equal to zero.

That's the loop. Yes? I'm going this way no Delta J, is defined like this so I'm going this Z7 is minus, I'm going along Delta J, the way it's Defined, then this is opposite, sorry this is along the way that Z7 is defined, that's it. so if I take that to the other side I can write this as, Z3 e power i theta, so I take everything else but, Delta j, to the other side then you will get that for okay so Z3 e power i theta J minus 1, plus Z4 e power i beta J minus 1, ok minus Z7, e power i, gamma J, minus one, equal to Delta J, Y is? So Z7 so you will see that because when I will now take it say I can take it in any direction that I want it really doesn't matter, this is just so it gives a nice form for this equation, okay, because I'm defining Z7, from from this point P1, see P1 is specified so what I'm going to do is actually take that as a free choice, so that I can define the shape of the coupler play, I can, because I know P1, see these are things that I need to synthesize, so I can kind of determine the shape of the linkage by varying Z7, so I'm going to take that as a free choice, so for that reason and it will, so when I move this to the other side, for this slope I get, Z3 e power i theta J minus 1, plus Z4 e power i beta J, minus 1, equal to Delta J, plus Z7 e power i gamma J, minus 1. What is this? Delta J plus z7e power if I look at this vector, if I look at this loop, okay I can call P1 dash to PJ dash, as Delta J dash. - okay this in this direction. Minus Z7 ok plus delta j, plus z7 e power i gamma j, this is nothing but delta j dash in this. See that? It's the displacement of that point, p1 dash. Okay so now what happens to my equation? if I pick z7 okay I know gamma J so if it's a motion generation problem, say I'm given, I know the locations of P1 or the Delta J's and I know the gamma J's, the orientation of this thing, okay now if I choose z7, as a free choice, this becomes known, this this equation is not, this is not in standard form, up to this point, okay this loop the loop equation for loop two, is not in standard form however if I choose Z7, is a free choice, okay then this becomes in the standard form, because the displacement of this becomes defined. Okay now this is just a four bar with Delta J dash- as my displacement vector, so for loop three, similarly I can now write it as,

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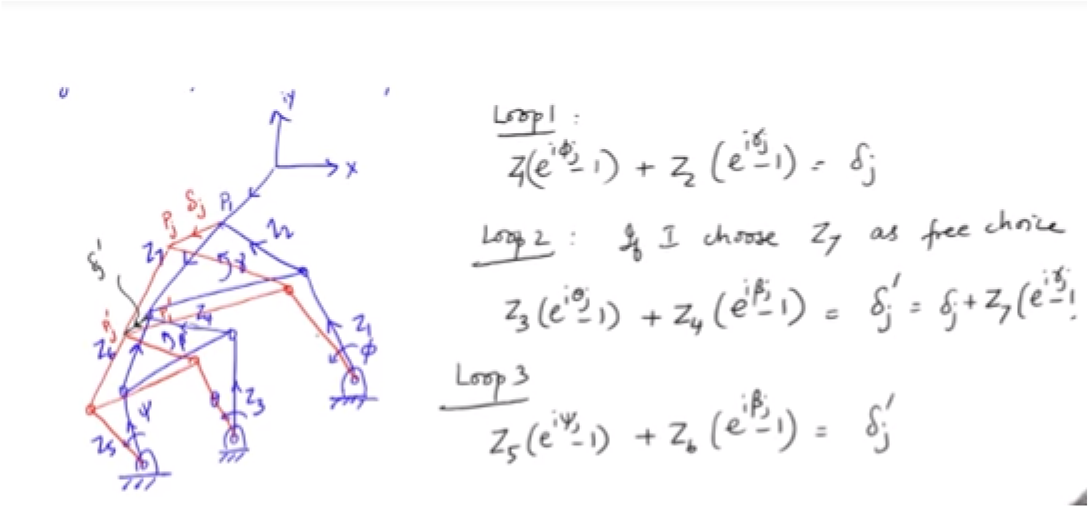

let's do that, so I have loop one, I have the first equation, so I have loop 1, Z1 e power i Phi J, sorry not, minus 1, plus Z2 into e power i gamma J, minus 1, equal to Delta J. Loop 2 now, if I choose Z7 as a free Choice, okay if I choose Z7, then I have Z3 e power I theta J minus 1, plus Z4, e power i, beta J minus 1, equal to Delta J dash. Where Delta J dash is Delta J plus Z7 e power i, gamma J, minus 1, ok. So this again comes and then loop 3 in turn becomes Z Phi e power i, Shi J minus 1, plus Z6 e power i, again beta J minus 1, equal to Delta J dash. All in standard for, choosing Z7, gives me lots of options now, okay then I can after I choose Z7 I get three these three dyad equations and then I have three choices in them depending on the number of positions. so you can see that my number of infinities of solution starts expanding, okay. The other advantage is I can now synthesize simultaneously for function generation and motion generation. so I can do this dyad for motion generation and also link theta J and Shi J, for instance, okay so I get more options, even more important see we never talked about motion generation with prescribed timing, although you would think that is important right, see we talked about path generation with prescribed timing where the path of the point and you can coordinate that with the input motion, but with motion generation we didn't really talk about that, because already you have another angle that needs to be specified. So a problem becomes too constrained. Here I can do that, I can link the motion generation, to the input timing of one of the angles, so I can actually do motion generation with prescribed timing. So I can reach a point and a specific orientation in the plane and coordinate that with the input angle, which I couldn't do with a, with just a 4 bar. So going to a six bar offers Me, more options and you can see that I can extend the dyad synthesis, I can extend it to other multi loop mechanisms, not just a six bar I can go higher, okay it gives me options for more numbers of solutions, of course the trade office you have a more complex mechanism, it's going to take up more space it's not as simple as a four bar, always trade-offs in design, but you can do more with a six bar, that's what I wanted to convey to you and you can now all three equations for the three loops are in standard form, once you pick Z7 and there were choices that you have for Z7 gives you varieties of solutions for this, because delta J dash, is not specified that's something now you can play with in the equations, ok. So that that is how you extend the dyad form of synthesis to multi loop equations.