Lecture – 21

Theory of Mechanisms

Coupler Curves – II, Fixed and

Moving Centrodes

So last class

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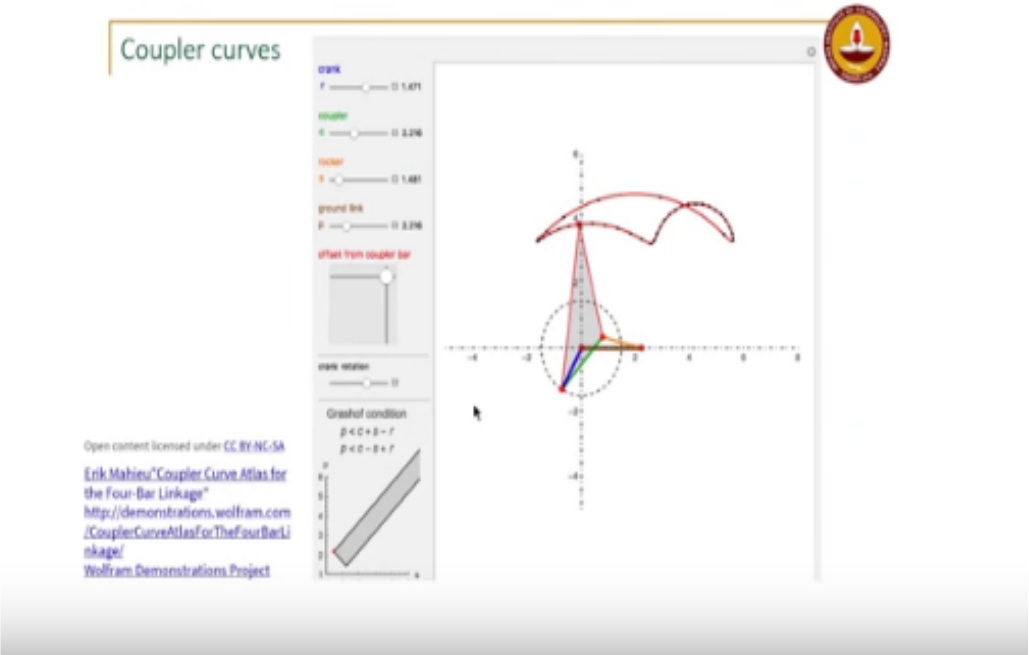

We were looking at; I had some trouble with, making this work. Let's hope it plays now. So this is your, coupler curve. This is a nice demonstration, from the Wolfram demonstrations project. So you can you can play with, the various links and you can see here that the dots on the coupler curve, denote equal amounts of crank rotation and you can generate, different shapes, of the coupler curve, by, you can see, you can see that, it as it traverses the upper portion of the curve, it moves a lot faster, so you can see. And so, the dots denote, equal amounts of crank rotation, that space between, two blue, dots denotes equal amounts of crank rotation. This is the Atlas. Again you can, that shows, I can change, the location of the point and you can see that, I can get very different curves, very fancy curves. Right? So this is a very nice demonstration for the, coupler curve atlas. The original coupler curve atlas basically has, a bunch of these, you know? It denotes the points and then but you could essentially pick something from the coupler curve atlas and animate in this. And also try to find crank rockers, which will give you, your desired coupler curve. Okay? The other branch of the 4-bar, yes, yeah, because the point will be, you know, it's, it's a different motion. Right? For the coupler the coupler motion, will be different, so. Because if you look at, that's something, we'll look at today. But if you look at the motion of the coupler, with respect to the fixed link, you can describe it, as a pure rolling, of the moving centrode, on the fixed centrode. Anybody remember what a centrode is? You know the instant center of rotation, for a four bar. Right? So if you look at, it's going to do that today, but since you asked, we'll, okay, we'll get to that.

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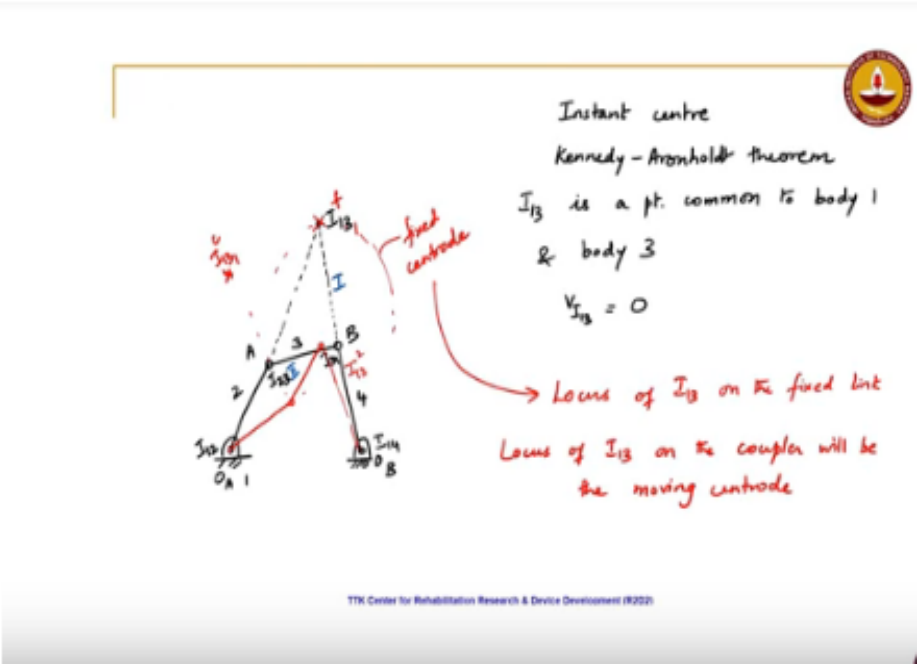

So if you look at, a 4 bar. Okay? Now, you remember the concept of instance centers? An instant Center is a point, that is common to two links and it has the, same velocity, in both the links. If you consider it, has a point on, say links 1 and 3. Okay? It would have the same velocity, as the bodies move, as the linkage moves, that point will have the same velocity, whether you consider it, as a point on link 3 or as a point on link 1, if we are talking about I 1 3. Okay? So that is the definition of the instant Center. And we know from what theorem? Anybody remember? The Kennedy Aaron hold theorem? That if you have three bodies, moving with respect to each other in a plane, then, they're instant centers of rotation, lie on the same line. So if I take bodies, 1 & 2, this would be i 1 2, this is i 2 3, this is i 3 4 and this is i 1 4. Okay? Because these are called fixed, sorry, these are called permanent instant centers. Because when you have a pivot, those two bodies are always going to rotate about each other, with respect to that point and that point whether you consider. So for instance i2 3, whether I consider it as a point on body 2 or body 3, it has the same velocity, which goes back to the definition of the instant center. Okay? So now if I want to find the instant center of i 1 3, I know that ,if I extend, i 1 2, i 2 3 and then this, this gives me, i1 3. Okay? So from the Kennedy Aron Hold theorem, I can locate, i 13, on the intersection of the, lines through, i12, i23, and i14 I 3 4. Okay? If I consider bodies, 1 2 3, it has to lie on this, if I consider bodies 1 3 4, it has to lie on this line. Okay? So now, i 1 3 is a point, that's common to body 1 and body 3. Okay? And it has as the same velocity, on both the bodies, at that particular instant. Say because as the linkage moves, the location of i 1 3, is going to change, it's not going to stay the same. So if it has the same velocity, at this instant if it has the same velocity, whether you consider it as part of body 1 or body 3, what is the velocity of i 1 3 at this instant? Why? Because if it is a part of body 1 its fixed. So the velocity of any point on body 1, is 0. Therefore, I can look at, the motion of body 3, at this particular instant. So I can imagine, body 3 and body 1 are pinned, at i 1 3. So at this particular instant, the motion of body 3, can be described as a pure rotation about, i 1 3. Okay? So that is the concept of the instant Center and extending it, you what you can do is, if I plot this i 1 3, as this body moves, so i 1 3 at this position, it is this now when this

moves to another position. Okay? From A2, B2. Okay? Now the instant center is going to be, somewhere here. Okay?

This is i13, at this position, i 13 at this position is this, i1 3 at this position, is this. Okay? Now this, when it moves to this, so I have i 13, i 13, if I plot i 13, in position 1, on the coupler, it would have moved to, I can just locate it, as a couple at point. Right? So it would have moved to, say some point here, this is i 13, on the coupler. The one that was in position 1, if I plot it as part of the coupler, when it moves to this point, this point will coincide with this. Okay? So similarly for the entire instance Centers, I can plot, so at, when that is the instance center, the two points will coincide. Okay? There'll be different points. On the on the coupler, that will coincide with a point. So if I just plot the instant Center, on the fixed link, say, this is the instant Center, on for the position 1, on the fixed link and this is the one on the coupler. Okay? So as the coupler moves, this point will move, on the fixed link it stays the same. So if I plot all the instant centers, on the fixed link, I'll get what is known as my, 'Fixed Centrode', the fixed Centrode, is the, locus of i 13. Okay? On the fixed link. The same points, if I plot on the coupler. Okay? I'll get another curve and that curve will be called the, 'Moving Centrode'. So the locus of i 13, on the coupler, will be the moving Centrode. I'll show you a couple of videos, that I have, that will make this clearer. Okay? But you see how, in this point, this is the location of i 13, when the coupler moves, that's going to take some other position, this particular point. Something else maybe the instant center, but I'm talking about, the locus of, you know what the instant Center was, and how it moves on the coupler. Yeah? Yes. There is only one i13, at a particular position. So, the reason you have a different curve for the moving center, is because that point keeps changing. See, look at, look at this, look at this, four bar here. Okay? If I plot, i13, Okay? At various positions. Okay? Just move, link 3. Okay?

I'm moving, OA, OB, is my fixed link and I plot i13, this is what I will get, this fixed centrode, this is the centrode, that I will get, this is my fixed Centrode. Now instead, I fix link three and I plot the instant Center now? Then I would get the moving centrode. Remember by inversion, that's the same as saying that, this point, so at this particular instant, this is coincident, these two points are coincident. When the coupler moves, this point is going to a different point; it's going to move to a different location, on the coupler. If I take the coupler as a body and I mark this point, okay, at this instance, this is the instant Center, then as the coupler moves, that's going to, go to a different location, the instant Center for that, for the new thing, will be something else, this is just a point on the coupler, I'm marking, for position one is i13. Okay? So the locus of all those points, on the coupler, will, so what happens is, the motion of the four bar, is essentially, the rolling of this fixed centrode, on the moving centrode. So I can remove all the other links. And if I have something that is shaped, like the fixed centrode and something that's shaped like the moving Centrode, then the motion of the coupler, with respect to the fixed link, is the same as the, rolling of the moving Centrode, over the fixed Centrode. I'll show you. So look at this. The blue curve, okay, this, you can see the point there, the intersection, so you see the IC. Can I write on this? Okay, so you see the IC? The problem with this is, it's a jiff, so I can't stop it at any point. But you can see that, you see the point there, the blue, the, the green is rolling on the blue curve and you see the point of contact, at each instant, is the instant center. Okay? I'll show you a couple more. So you have the fixed Centrode, which is in blue here, you can see the instant Center, as that yellow point, that you see and the motion of the 4 bar, can be replicated, by the rolling of the moving Centrode, on the fixed Centrode. It's a, it's an idea, that takes a little while, to wrap your head around. Okay? But this is so, look at this one. You see, the instant Center there, so the blue one stays Stationary that is you're fixed Centrode. You can see that the

yellow point is always on the blue curve. Okay? But it comes into contact, with different points on the green curve.

The green curve, is your moving centrode, at different instances, different points, on the moving centrode, come into contact, with the fixed Centrode, at the location of the, instant Center. Because it's rolling, because at that point, what is the velocity of that point, at that instant? It's zero. So it's pure rolling, of the moving Centrode, on the fixed Centrode. That replicates the motion of the coupler. So you can remove those other two links, links two and four and you can replicate, the motion of the coupler, as a pure rolling of the, moving Centrode, on the fixed Centrode. Okay? It's the instant in instant Center, yes, correct. And that's why it's changing. No-no-no, instant Center, the blue curve is the locus of the instant Center. That's why you see, you see, the look at the instant center of the linkage, given by point, that yellow point. It's always on that blue curve, that's a specific, it's not a coupler curve, we are talking about here, we are not talking about a coupler curve here, and we're not talking about a coupler point, here. We are talking about the instant Center. Okay? You can see at the intersection of these two links. Right? And that point, so this is the locus, the blue is the locus, of the instant Center, as the linkage moves and the blue is attached, to the fixed link. So you see, it's not changing. Okay? That same point, you transform onto the coupler. So when the coupler moves to some other location, that point is going to be somewhere else. Okay? And only at a particular instant, it will coincide with the fixed center. Okay? So that is. So, yes, if you have a different configuration, your instant centers are going to be different, so the motion of the coupler will be different. Yes? What's happening to them, because, the instant Center is, look, look, at what's happening to the instant, they're becoming almost parallel, so you can't show them, you, you see here the two lines, the dotted lines? They're going to infinity, they need not be closed, no, no, they need not be closed.

The reason for this is, we'll see in a minute, but anyway. So, the green curve that, that was what I was trying to explain here. So the green, okay I should not draw this, probably as. The green curve is obtained by, so I have i 1 3. Okay? Which is located on the coupler, I can locate it with respect to a and B? When it's, at. So I have i 13 here, I put a thumbtack there, on the coupler. Okay? Now when it has moved to some other location, something else is the instant Center for that, but this point still remains, this thumbtack. So for each of the instant centers, I put thumbtacks, on the coupler, that's going to move with the coupler. So here i 132, on the fixed Centrode, it will be at this location. But if the coupler moves to some and with respect to a and B, it will always be at this location, if I put a thumbtack here and the coupler is now moving, that will be another point, on the coupler. So the locus of, all these points, all these thumbtacks, on the coupler, if I join them, that gives me my moving centrode. And, no, it's not a coupler curve. A coupler curve is the curve, traced by one point, on the coupler, as the linkage moves. Coupler curve, is the point, curve traced by, one point on the coupler, as the linkage moves. Here I am plotting, how the various instance, centers. So when, this is the instant Center, for the linkage, this thumbtack and that point will coincide. The point, so the point on the fixed Centrode, that corresponds to the instant Center and this thumb tack that I put on the, coupler will coincide. At some other thing, it will be some other thumbtack and a point on the fixed center. So if I connect all these thumbtacks, which give me my moving centrode. I13, at this instant is coincident. Okay, i13, whether you consider it, as a point on link 3 or is a point on link 1, which is the ground, so the absolute location of i13, is always going to be that. Exactly, yes, that's exactly, what, that's exactly how I am getting the moving centrode. No, but it's not the same, the instant Center keeps changing. The instant Center at one orientation of the coupler is not the same, as what it's going to be at some other orientation of the coupler. Whereas the coupler curve, that point, does not change. So here it is this. Okay? So this is one point, on the moving Centrode, this is

another point, on the moving centrode. What would be the location of this point on the coupler, when the coupler is in this position? I would have to, so in the first position if I locate, see, I have AB, this is my point, so, this would be somewhere here, or this would be i 13 of 2, in the first position. Right? Because, it is, below A B. Okay?

So this would also be a point, on the moving Centrode. Because in the first position, when it comes to, okay, this is position 1 and this is position 2, in position 1, these two will coincide. Okay? i1, in position 2, i132, will coincide with, this one, because it'll, but this is the, this is in position 1, that I'm plotting, i 132, I am plotting in, position 1, it would be this triangle. Right? Which would be similar, congruent, to this triangle, I am plotting the relative location. What's that? It's not a coupler curve. No it's off the position, of this point. See its different points, on the coupler. So i1, if you look at i13, as the linkage moves. Okay? So like I said, you have, you know, I locate i13, I put a mark, on the coupler, I put a mark on the fixed link, I put a mark on the coupler, with a pen. Okay? Some other position, something else is my instant center, I put a mark on the fixed link, I put a mark on the coupler, again, these are different points, but they exist, say as the coupler moves, once I have a location, once I have all these points marked, on both the fixed link and the coupler. Okay? Corresponding points will coincide, depending on, wherever it is, the instant Center, whichever is the instant Center. But if I look at it separately, I'm going to get a curve, which is all the points on the coupler, all the points on the fixed link, the fixed link, all the points will be the fixed Centrode, points and the coupler, will be the moving Centrode. The reason I wanted to talk about the centrode, was because, it relates to the location of the cusp, in the coupler curve. We talked about these double points or cusps, where, let me see. Okay, take a simple case, of this.

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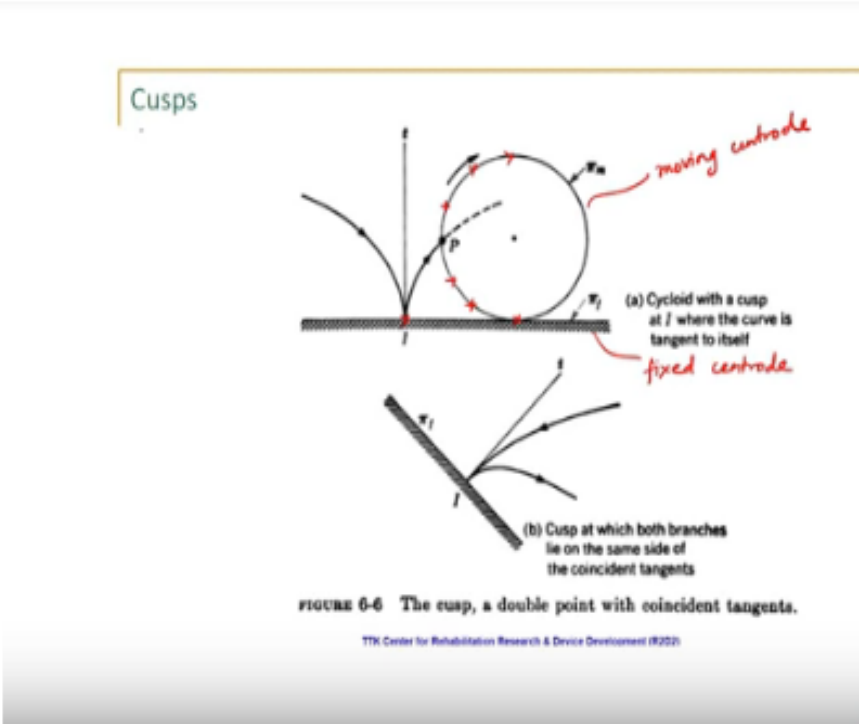

Cylinder rolling on the, on a plane, okay? Now if I look at this, so you know that the instant Center, for rolling, is the point of contact, for pure rolling, that is your instant Center, that's the point with, velocity equal to zero. So it's the point, that's common to both, the rolling disc and the surface and has zero velocity. Now if I look at, So this is a way to, so, the instant center, on the fixed plane, lies on this Line, it's always on this line, so the point of contact ,is always on this line. But if you look at the disk, as the disk rolls, essentially, the perimeter of this is your moving Centrode. Because this point was in contact, this point was in contact, and then as it rolls, some other point comes in contact, but with respect to the cylinder, this is the location of, the point that was on contact. So for this, this is my moving centrode and this is my fixed Centrode. Okay? Same thing we apply for the 4-bar, except that, the curves that you get, for the moving centrode and fixed Centrode, are more complex. Okay? So, but the same principle applies, you are looking at plotting the same instance center, one on the fixed plane and then how it moves on the, moving plane, gives you the, moving centrode and the fixed centrode. So here if you see, if you look at, the locus of, a particular point, now we are coming back to, one particular point, on the moving centrode. Okay? So if I look at, this curve here, shows me the, this is a cycloid and it's the locus of this point P, as this rolls, I have this point P, which makes contact, then reverses direction, as it rolls away. Okay? and this it, so the velocity smoothly goes to zero and then reverses direction. This is an example of a, Cusp. Okay? And we use the same analogy, for the 4 bar. So if I have a coupler point, that is located on the moving centrode, then, where the move, where it coincides, with the instant Center, on the fixed centrode, that point, will be a cusp, similar to this example here. So, see this is another one. You see the, blue? Always at the intersection, so the points Are, blue is the fixed centrode and the moving centrode, rolls, on the fixed centrode and at that point they are coincident, the point of contact, between the moving centrode and the fixed centrode, is the instant Center.

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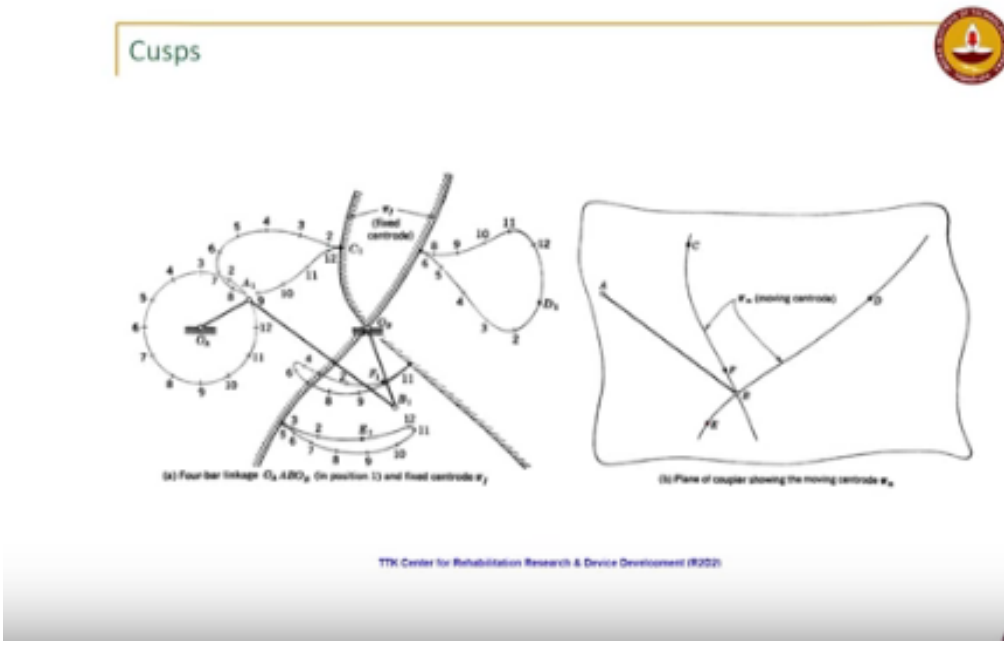

So here if I take a portion of, so I have the coupler base, A B and I take the, you know moving centrode and I take points on the, moving centrode, C D E and F. Okay? So with respect to a b, c is located like this, so this is c1. So at the instant, that this point, this coupler point, coincides with intersects the fixed centrode that point is going to be a cusp. Similar to, because at that instant, it's going to roll, with respect to that point, it's rolling, that becomes the point of contact, as the moving centrode, rolls with respect to the, fixed centrode. And that gives you a cusp, which is very useful for, reversal of motion, without any shock, because, the velocity goes to zero and then reverses direction, so that's a very useful feature, for using, ‘Coupler Curves’.