Lecture-23

Theory of mechanisms

Roberts-Chebyshev Theorem

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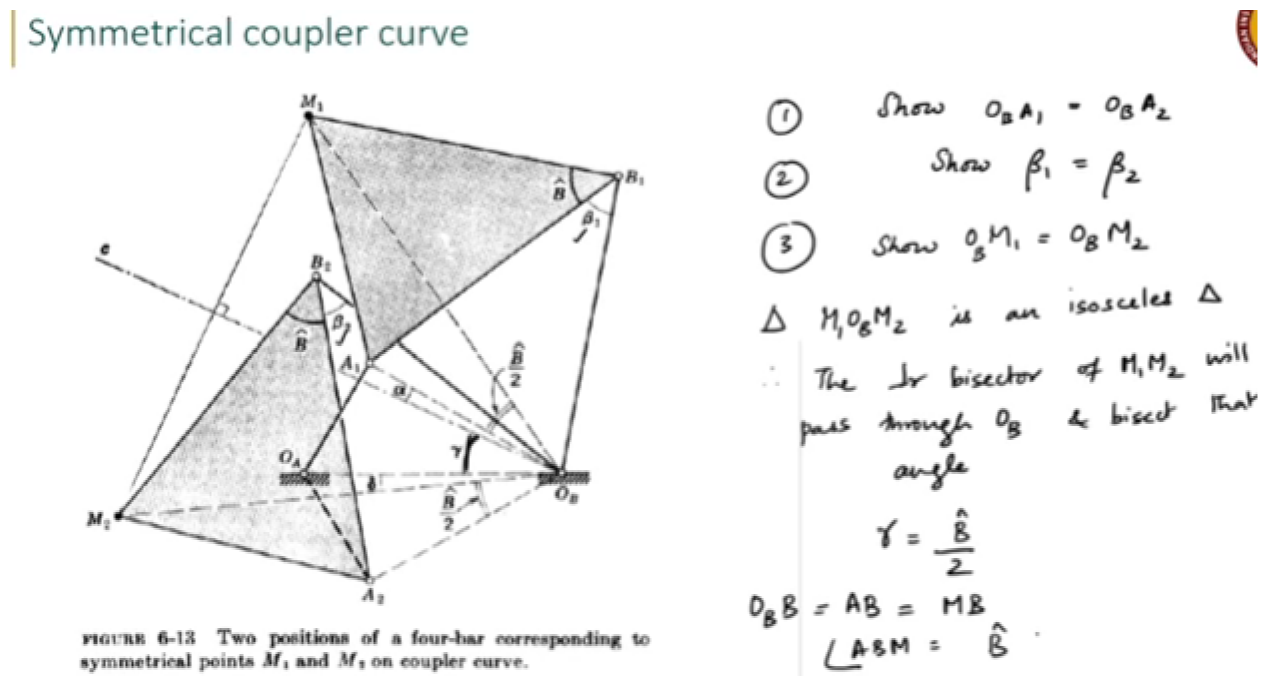

Yesterday we looked at, you know how we can generate, a symmetrical coupler curve. So, we looked at if m1 and m2 are two Positions, of the coupler curve, then we saw that so, first I look at show, I show OB, a1 equal to OB, a2 okay? I take symmetrical positions of the crank, about the frame and I show OB, a1 equal to for the two positions of the couple of points ,I show this then I show beta 1 equal to beta 2 this angle, these two angles are equal, then from there I show, OB, m1 equal to OB, m2. Okay? once I show That, triangle oh sorry, m1, OB, m2 is an isosceles triangle, once I show that therefore ,the perpendicular bisector of m1, m2 will pass through OB, and bisect that angle, will pass through the opposite vertex and bisect that angle, and then we showed that gamma. The angle that that bisector, makes with the frame is a constant that means, that means as the linkage moves, it does not that line of symmetry does not change. Okay? m1 will always be reflected about that line because, that line does not change. So, with the other conditions of you know. So, you're choosing so, our synthesis Conditions, where O,B, b equal to a, B and also equal to M,b and M,b would be at an angle because, the circle passes through all three points, M lies on that circle and makes an angle equal to twice, the angle that the line of symmetry, makes with the frame. Okay? So, angle a,b,m equal to b cap. Okay? So, this is how you synthesize, a symmetrical coupler curve. Okay?

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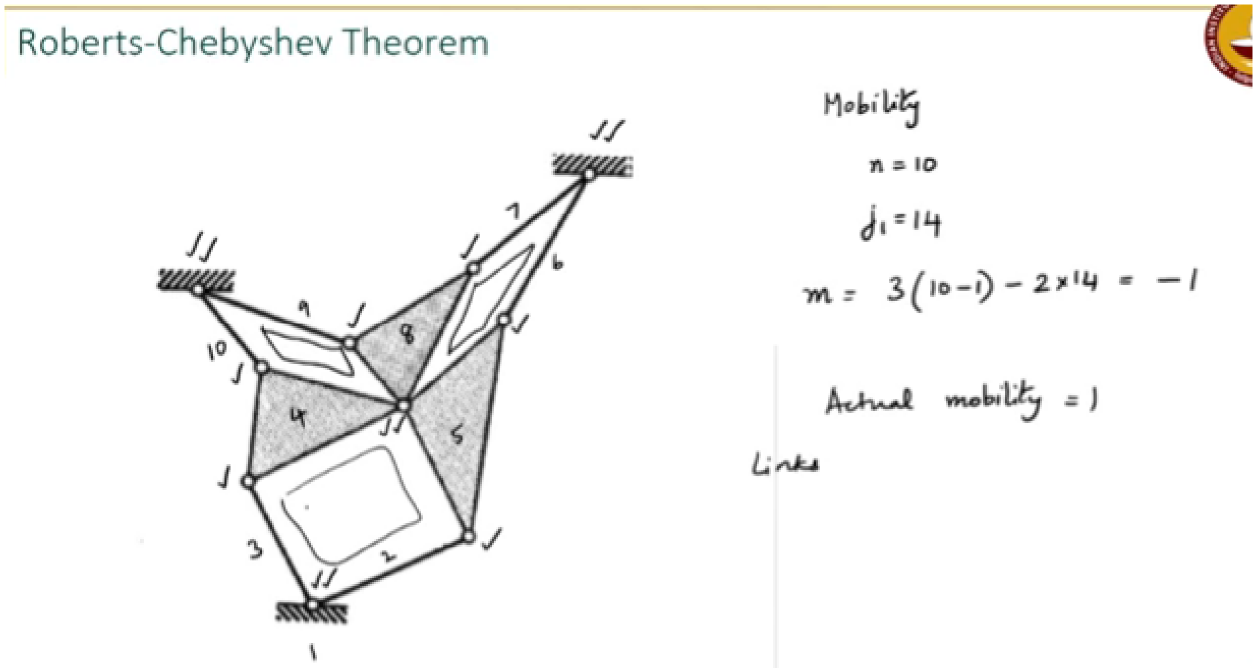

Now, we will move on to one of the most interesting results in coupler curves, which is called the Roberts chebyshev's theorem. So, if you remember I took this diagram from your first tutorial. Okay? I gave you this mechanism, I didn't tell you anything about, anything special about the links, and I asked you, to find the mobility of this mechanism, go ahead and do that now, and tell me what the mobility of this is, No what do you get? What is j1 equal to 14? Yeah. What is M? Minus 1, that means, it's an over constrained linkage .so, you would not expect it to move, right. Okay? But ,there is something special about, this linkage, with certain link lengths. So, if you look at these three you know, this is a parallelogram, this is a parallelogram, this is a parallelogram. So, if the links form these three parallelograms, then you actually have a mobility of 1, for this mechanism, provided those links satisfy that you know. So, links these are parallelograms.

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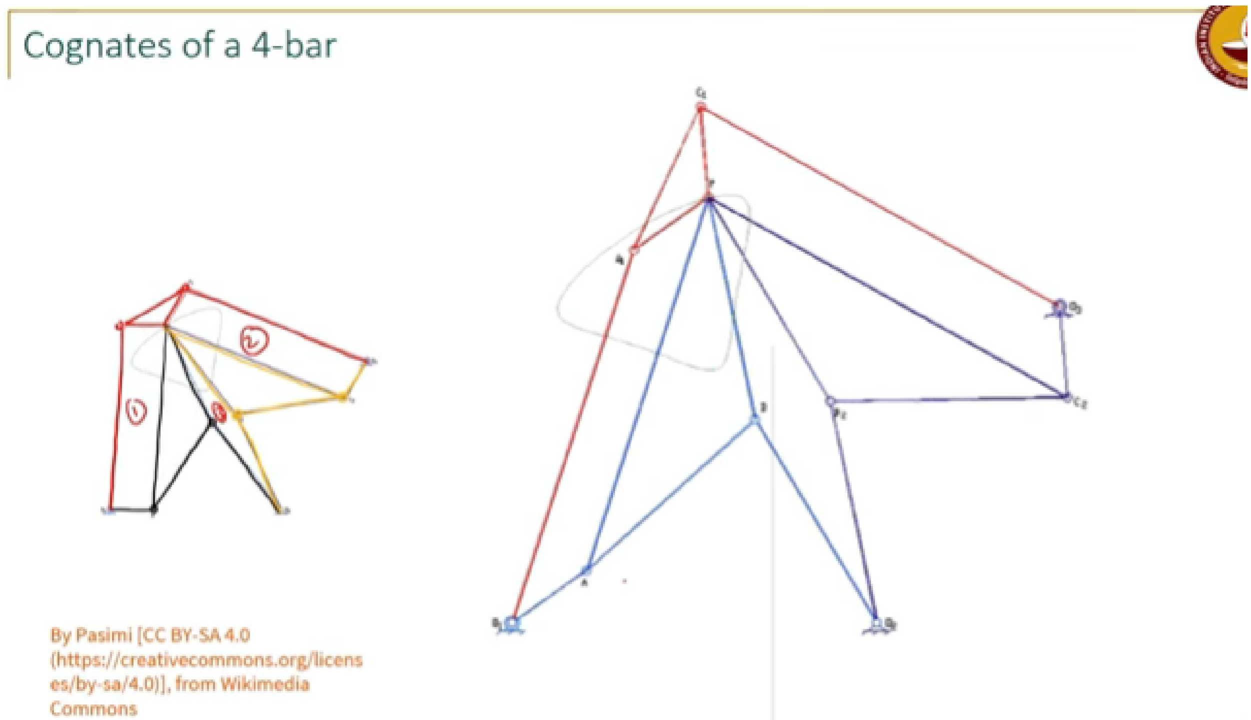

So, here is another configuration so, you see you see here, you have one linkage like this. Okay? And that has this, coupler point. Okay? You have another linkage, another four bar, here this says the coupler point, and then you have the red one. Okay? This is the coupler point and this is the four bar. So, if you look here these three four bars, are assembled such that these form, these parallelograms, right? You have one, two, three, parallelograms. Okay? And you see, what's happening here? So, it is moving, and you can see that, the point P, traces the same curve in all three, I mean 0.3 traces one curve .so, these are just four bars, if I disconnect them. Okay? I will again, get single degree of freedom mechanisms, okay. And that coupler curve, that the curve that point, P traces should be the same, I can disconnect the three linkage, three Linkages, make them separate four bars, and my point. So, this is what Roberts? Say Bishop theorem states, that planar, There Are, three different planar four-bar linkages, which will trace identical coupler curves. So, if you have a particular coupler curve, you want to trace, and you can find two more linkages that will trace the same coupler curve. Okay? Why is that significant because, the space requirements of the other two linkages may be more suitable for your application because, the location of the other pivot, other fixed pivot, may be more suitable for your particular? so, if you have designed, or if you have picked a forward bar, which has a certain coupler curve, you can find two more four bars, using this theorem, that will generate the same coupler curve, for that particular point. Okay? So, that is what this shows? So, I can have the red linkage. So, if I just have the blue linkage for Instance, that point P is going to trace that coupler curve. Now, with a special construction, I can determine the red and the purple linkages that will also generate that same coupler curve .okay? Okay. So, that is the Roberts Chebyshev theorem.

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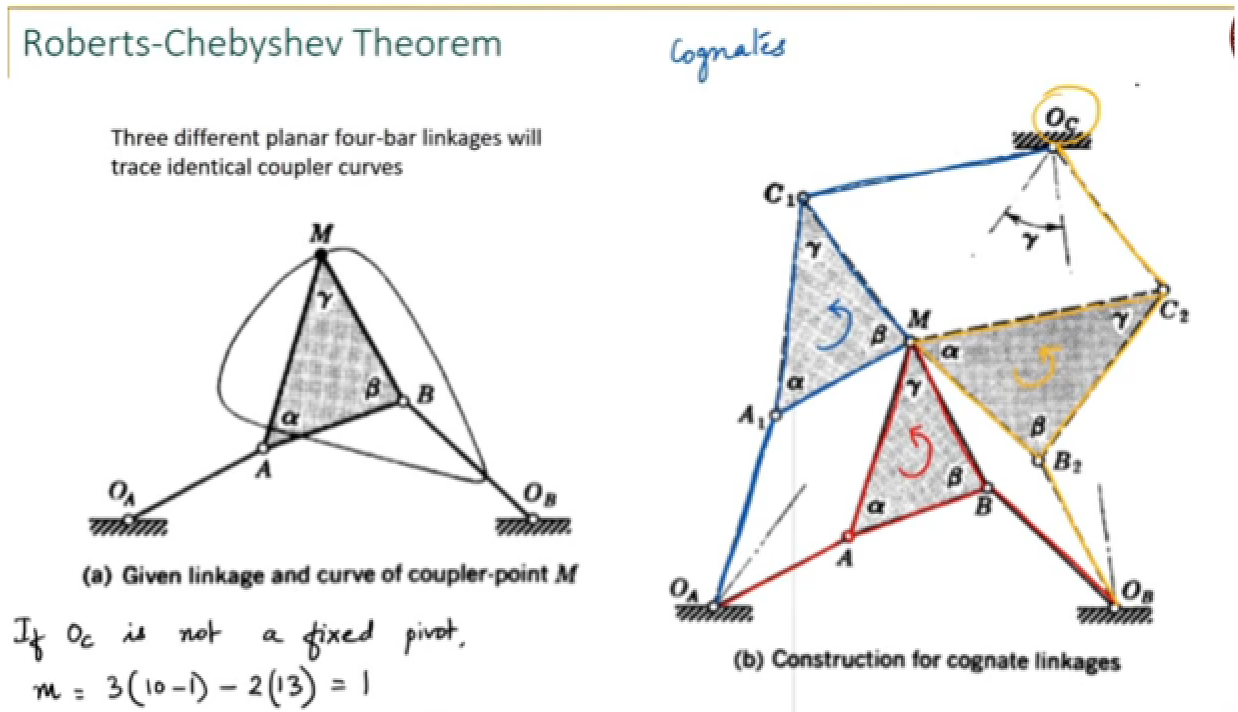

So, let's look at so, I can separate it out so, if this trace is a curve, I have let's see how we construct, the other two linkages. So, if I have so, this is my point p, which is the coupler point. So, here so let's start off with a fourth bar, that generates a certain coupler curve, and that coupler curve is generated by this point M, which is located with respect to the base a, B you know, and this is the coupler triangle, these are the angles that define the coupler triangle, because, we will use that when we look at the construction. So, the first things that you do you have, the original four bar. Okay? Now, from o,a you draw, a line parallel to A,m okay. And then a line parallel to parallel and equal to o,a. okay? Stut form this parallelogram. Okay? Now, using this base. Okay? I construct a triangle that's similar to the coupler triangle A, B, M okay. I have to follow the same order of angles. So, if this okay? From here I construct, this is the base I have I construct at an angle alpha and an angle beta that will give me this point, c1. Okay? So, this order, this is the coupler base A, b for this, this is the eye with a1, m. I construct the same coupler triangle, similar triangle, it's not congruent. Okay? Because, it depends on this base A, M it is actually equal to O, A, A here now, on the right hand side, okay. Here, I have O, B, B and B,M I first construct this parallelogram ,this is parallel to this, this is parallel to this. Okay? Now, I use this as the base for my coupler, m, b2 okay. And I have to construct the same, alpha from here, beta from here, the order is very important. the order which you construct the angles. Okay? if you mess up and say, this is beta and that is alpha you won't get the desired effect. you got it you try it out and see, what will happen you are saying, swap for these two, try it out and see because, there is something. So, this there is a particular condition, for OC right? I mean you have to have, OC as a fixed point. When you construct it right? And that so now, look at finishing up so now, I have m, c2. So, from c1,o, c2 I construct a parallel. That will form the fourth link of my left cognate. So, these linkages, that generate the same coupler curve, are called cognates. so ,for a particular four bar, I will have a you know, here I call this, the left cognate and yellow one the right cognate .okay? so, then this becomes parallel to this, and whatever point they intersect at is what I call? my OC now, imagine OC is not, a fixed pivot. okay ?suppose OC is not a fixed pivot, then in my mobility equation if OC is not a fixed pivot, my mobility equation becomes, 3 into 10 minus 1 -, how many joints go away? one joint, the one between the fixed link, and the so, I'll get 13 J ones ,and I end up with the mobility of one. Okay? So, if I don't fix OC, I know that this linkage can move. Okay? for this space. so, even if I have regardless of any condition on the lengths of the links, or the geometry, if I don't have OC as a fixed point, this linkage will move. Okay? So, what we will show is that for this particular geometry, OC stays stationary, which means I can use that as a fixed pivot, which means I can now, split this into three four-bar linkages. Okay? so, without any conditions on the link links, if OC was not a fixed pivot. this would have a mobility of 1, this linkage would be fine, if I just remove one of these fixed, if I don't fix one of the payments, however you know that point is going to move, what I am going to show is that for this specific geometry, that point OC ,even if it is not fixed, it will essentially remain stationary, as the linkage moves that point OC, will not move. So, if I can show that then that means, I can split this into three linkages at M ,then I can make I can make OC a fixed pivot ,and split this linkage at M, into three separate for bars, and all of them will have the same path for M, which means they are tracing the same Coupler curve. Okay?

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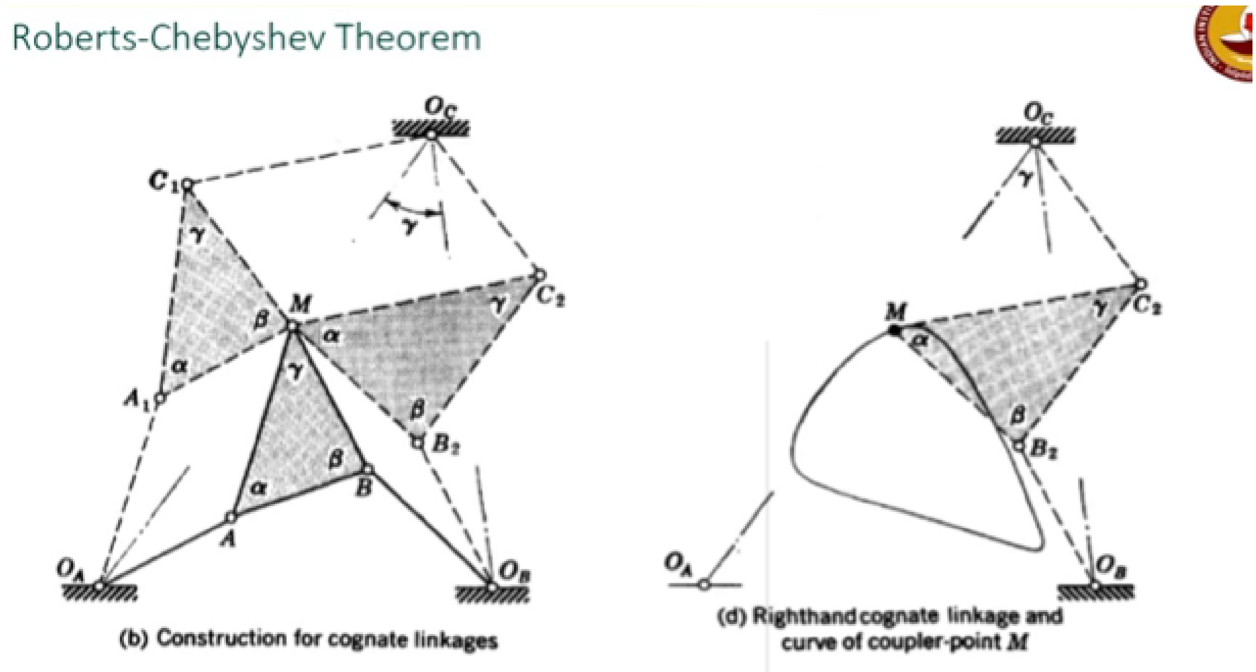

so, here we have it's basically just showing the split links, I have so ,I can have depending on my application ,I can get a linkage that is more suitable, in terms of size and location of one of the fixed pivots. So, I get three, if I solve a path generation problem, I immediately have two more alternatives, by constructing the cognates for that particular coupler curve. Okay? Because, for power generation remember scaling is not, an option for function generation, I can scale the linkage, I can you know play with that for power generation, it's not an option. Okay? The path generation, I can get to other options by strutting, the cognates for the four baths. So, this is the right cognate same coupler curve.

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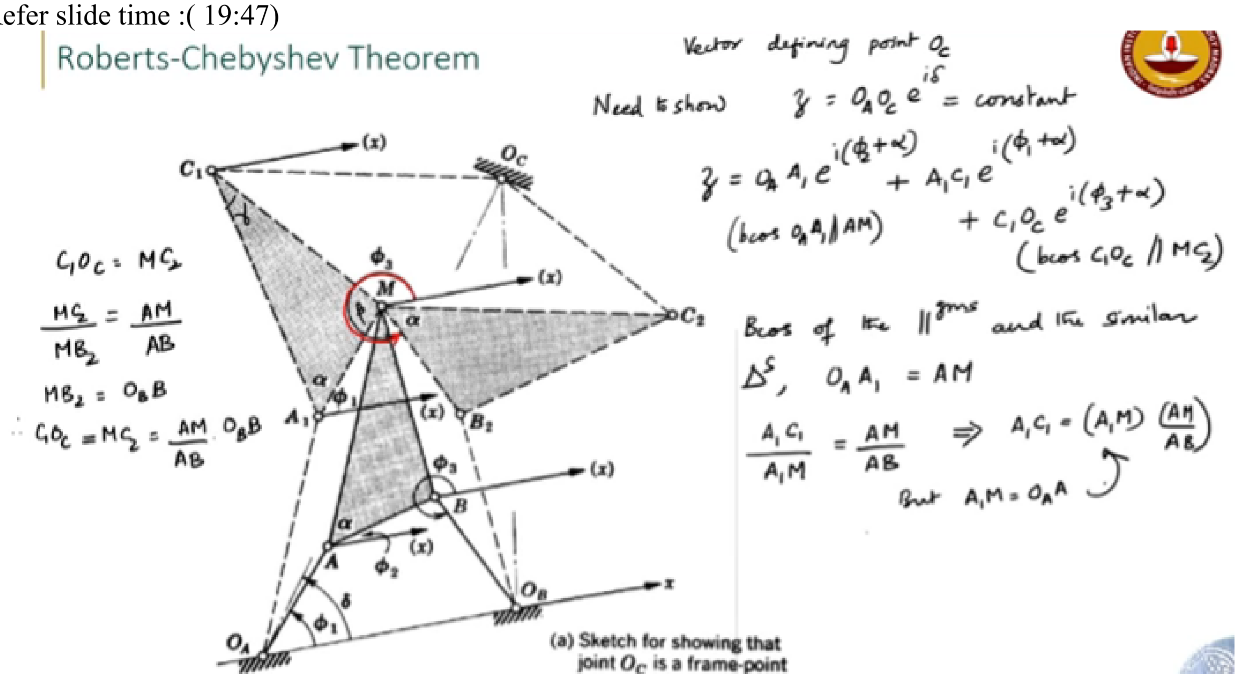

So ,now we are going to show ,how this joint OC can be you know, does not move, for this particular geometry ,for this particular construction ,of the cognates. Okay? so, that's what we are going to show, is the construction clear, construction of the cognates, given a four bar, this is the construction of the cognates, this is the important part, constructing this triangle such that, it is similar with the angles in the same orde,r I mean even if I switch the young it will be the angles, have to be in the same order. Okay? Okay. so ,now let's look at we'll use complex numbers, to show this so, if I denote, this by you know, I say the vector Z is OA, OC, e bar I Delta. So, here it's broken here but essentially, think of Z as the vector of length, oA,OC with respect ,I am taking the coordinate system, with the X aligned with the frame, just for simplicity, and if that makes an angle Delta then, the vector defining point OC, is OA, OC,E bar hider, if I can show that both the length ,and the orientation ,of this vector do not change, as the linkage moves, then I will have ,then I will be then it means OC is a stationary point. Right? So, I have to show that this is equal to a constant. Okay? So, what we'll do is, we will express this Vector, in terms of the dimensions, of the original linkage. So, I have OA, Ab, OB. Okay? is my original linkage, and i need to and m of course so ,that's my original four bar, i want to express the this distance, in terms of the parameters of the original linkage, and they have to be when I do that, ultimately I want something, that is independent of the angles Phi 2, Phi 3 etc because, those are the ones that are going to change as the linkage moves. ok ? Phi 1, Phi 2, Phi 3 that's the notation, that's used here, it should be independent of those three angles, which are the angles of the four bar, our usual theta 2 ,theta 3, theta 4 here the, okay .so, if it's independent which means, as the linkage moves if this vector remains constant ,then you've established that, I might as well make it a fixed pivot. Okay? I can make that a fixed pivot, without any issues. ok ?so, let's look at, this vector Z, I can write this as Z equal to oa, a1 okay? E bar this angle is Phi 1 .okay ?and they are, it's parallel to AM. Okay?so, the angle with respect to the positive x ,of that is hmm, alpha plus yeah. so ,this is e bar, I Phi 2 plus alpha, that's the angle with the x-axis, plus a1, c1 what is its angle? you can see here, with respect to the X it's angle is, Phi 1 plus alpha, and then plus c1, OC, e bar I, it is Phi 3 plus alpha because, it’s parallel to MC 2 ,all the angles are measured positive counterclockwise from the x axis, from the positive x axis, which is taken along the frame in this case. So, this angle, the angle of MC 2 with respect to the x-axis is, Phi 3 plus alpha, this angle is Phi 3 plus alpha. Okay? Because, the in this case OA one is parallel to am, **and here C one** is parallel to mc2. so, because of the parallelograms and the similar triangles, I have the following in terms of the original linkage, I want to express everything in terms of the original linkage, OA, a1 is equal to a AM. Okay? I want to take anything that is not in terms of the original linkage, and express it in terms of that, if I look at these similar triangles, the coupler triangle a1, c1 by this is alpha, this is beta this is gamma. Okay? So, I have a 1, C 1 by a 1, M will be, equal to what am by a B in terms of the original triangle because. they are similar triangles therefore, a 1, C 1 equal to a 1 M into a M by a B. now ,again a 1 M is equal to OAA Therefore, a1,C1 I can write it as, a M by a b. okay? so ,I can basically substitute that into this, let's just then a ones even is that completely in terms of the original four bar. Then for the third, I have c1, OC okay. mc2 is equal to C 1, OC let's C 1 ,O C is equal to M C 2. Okay? In this again comparing it, to this triangle MC 2 by M B 2 is equal to AM by a B, and again M B 2 is equal to OB b Therefore, C 1, OC equal to MC 2 is equal to AM by a B into OB b. okay?

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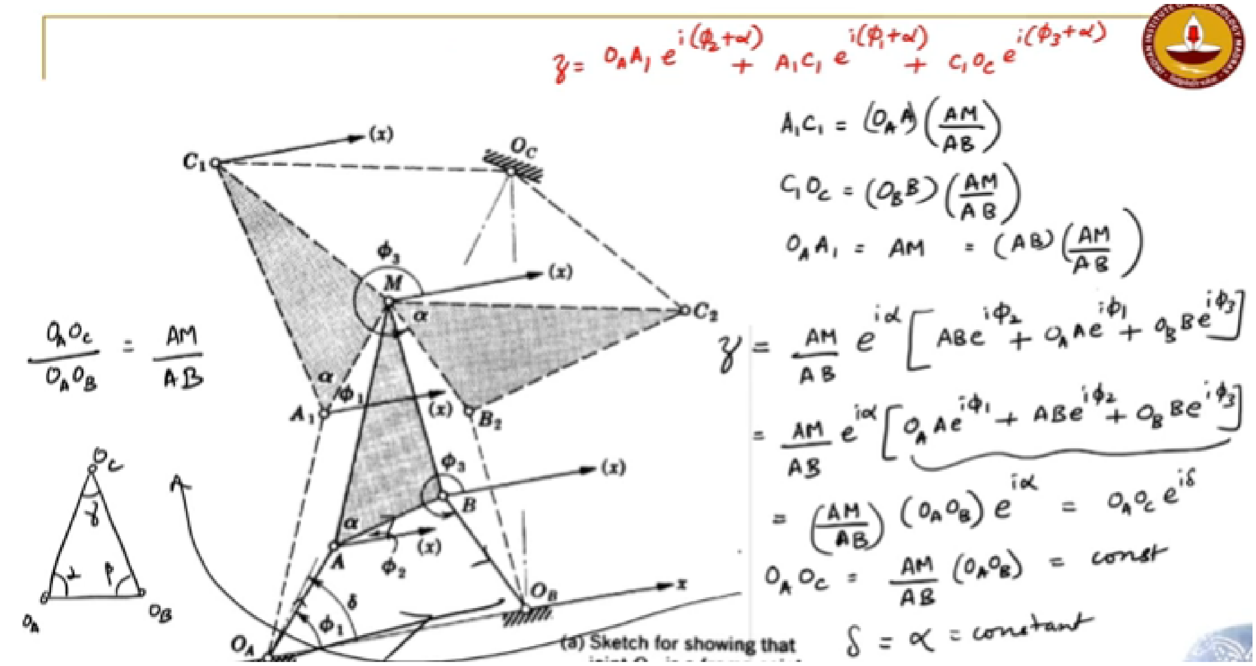

This is what? I have I have shown that a1, c1 equal to OA a into a M by Ab, and c one, Oc is equal to OB b, into am by ab. Okay? Is that I hear this, yeah. And oa, a 1 equals am, right. So, I can also write this as, a B into am by ab. So, my Z I can now remove, what out of the Equation? All of them have a M by a B right? a M by a b, e bar I alpha, I can get that out, and I have for Oa, a 1 I have a b,e bar I, Phi 2 plus for this one I have, Oa a, e bar I, Phi 1 plus for this one I have OBB, E bar I, Phi 3 .okay? I can just rearrange that and write it as, right? if I start from here ,I have OAA, E bar I Phi one I'm just rearranging those terms, plus a b, e bar I, Phi 2 + OB b ,E bar I ,Phi 3 what is this equal to? it's nothing but, from your loop closure this Plus, this Plus, this is equal to this. Ok? It’s just the length OA, OB because; I've taken X along that the angle of that is zero. Right? So, I get this is equal to a M by a B into Oa, OB e bar I alpha, all these lengths are constant am, ab Oa, OB are constant. So, my Oa, OC is equal to am by a B into Oa, OB which is equal to a constant, and my Delta equal to the angle that, that Z makes is equal to alpha, which is also a constant. So, that means that point OC, can be taken as a point on the fixed link. because ,it remains stationary for this particular geometry, if I even if I don't have that as a fixed pivot there, for this particular geometry ,that point will not move. Because, its location as this linkage moves is, remains the same. Okay? So, this is a geometric proof of why these three linkages, why these other two linkages, will also generate the same coupler curve. Okay? So, this tells you now, that you have okay. One more thing, if you look here it says, OA, OC equal to AM by a b into OA,OB. So, from here OA, OC by OA, OB equal to AM by AB, that tells me that, the triangle formed by the three fixed pivots, is similar to the coupler triangle, I've already shown this, angle is alpha. Okay? so ,this will be beta ,this will be Gamma. So, you locate the third fixed pivot, by constructing a triangle with base o a, OB that is similar to the coupler point whose motion you want to write. Okay? So, this is, this is how you got get the location of the third pivot. So, if you have the original four bar ,you can find the location of and if that is more suitable, then you can look at using one of the cognates, if OA,OB one of those is not suitable, then you could get an alternative for bar, which has OCS one of the fixed pivots. Okay? from here, OA,OC I showed here, right ?Z this I took it as, the distance OA, OC e bar I Delta. Okay? So, here I found that, the length of OA, OC equals this. Okay?

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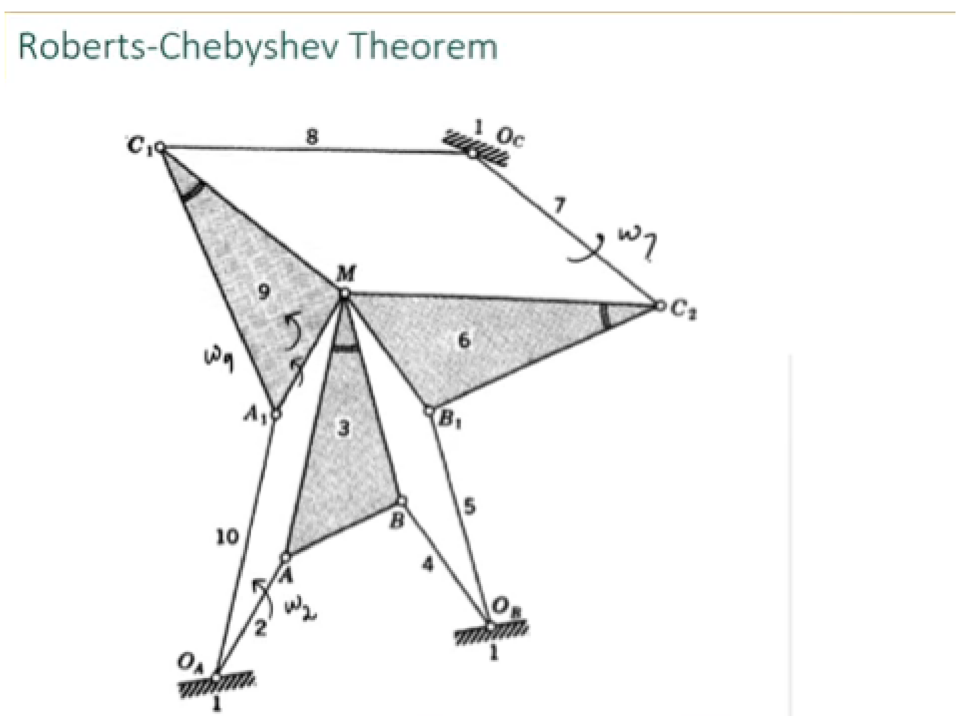

a simpler Construction. so now ,you can find OC oh one more thing. So, if because of this construction, the velocities are also so, if your original linkage, you know, if Omega 2, is your input linkage. Okay? Now, the velocities of these three Linkages, are also related right? because, of the you can think of each of these as a because, these form parallelograms right? So, if this is Omega 2, Omega 2 will be the same as Omega 9, right? So, this is Omega 9 ,and that will be the same as this is parallel to this, Omega 7. Okay?

So, how you say, if the speed at which you are traversing the coupler curve, is important then you have to know, which one you know when you change the linkage, what you give as input, also will matter. Okay? So, there are velocity relationships based on these parallelograms, between the various links and therefore, you will use that to determine how you drive the linkage.

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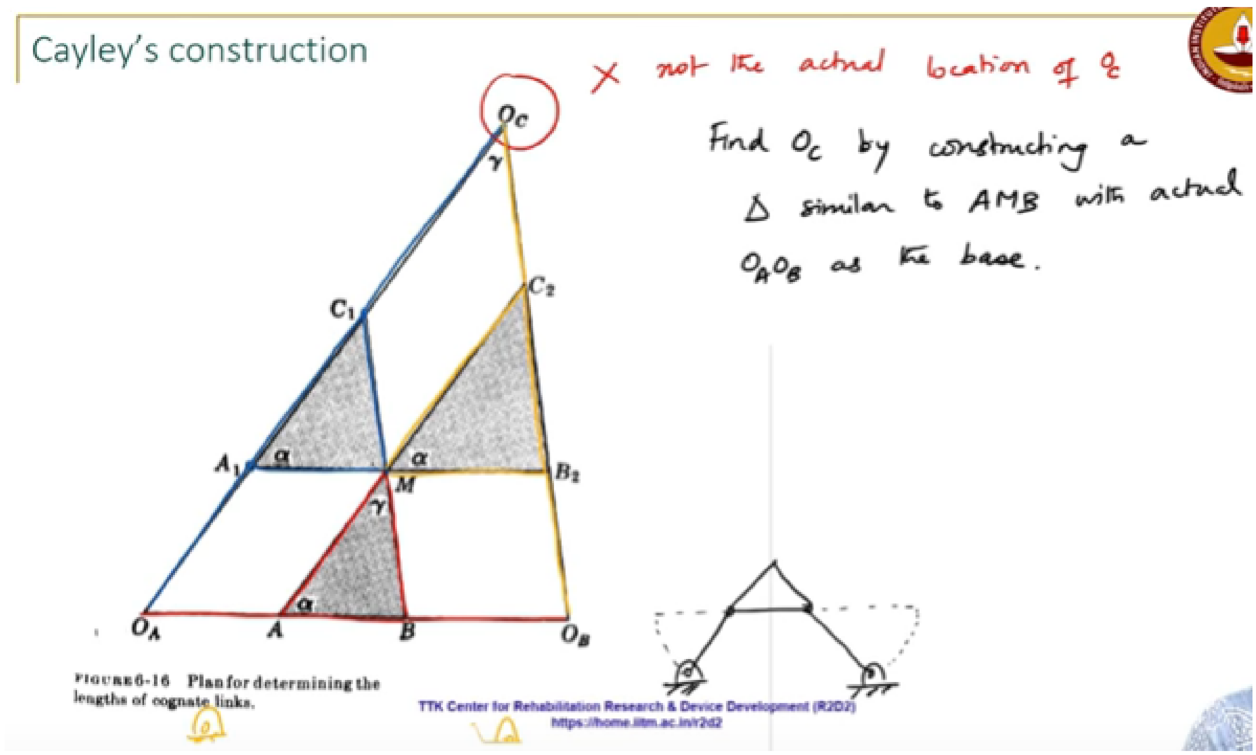

Kelly came up with so, you can look at OC, from Roberts Chebyshev theorem right? By just constructing the triangle, that is similar to the coupler triangle. So, the location of OC is immediately determined, Kelly came up with a different method for determining the link lengths of the other two linkages. So, instead of doing the parallelogram you know, the construction this is a simpler construction to determine the link lengths. So, you think about this original linkage, which was like this okay. Imagine that you disconnect it from here, and you straighten it out, similarly here disconnected from the fixed pivot, for the purposes of construction and you straighten it out. So, that is OA this that is, this configuration here, what you see here? is you have the coupler triangle, and you have the linkage, disconnected from its fixed pivots and flattened out so, that it is flattened out with respect to the base of the coupler, and then you just construct lines parallel ,this is not the actual location of OC, you have to be very clear about that, this only gives you the link lengths for the other but, from here, I can get the link lengths O a, a 1, C 1, OC C 1, M okay? From this construction, from Kelly's construction I get the link lengths of the other two cognates. So, one will be this, okay? And this will be the coupler. So, the link-lengths you get from this construction. But, the actual location, of OC you get by constructing, a triangle that is similar to ABM with OA, Ob as base because, there’s not that these are not the actual locations of Oh a, OB although this triangle is also similar, to the coupler triangle, OA these are not the actual locations, of OA, OB ,o a, OB are somewhere here. Okay? Somewhere out there. Okay? So, you cannot use that for the location of OC, find the location of OC by constructing a triangle similar to AMB with actual O a, OB as the base. so ,you can use this so, suppose you have a coupler point ,that's not away from the coupler base say ,it's on the coupler base right? Essentially this whole thing collapses now, you don't so, M is not, M is on a B. okay? So, it's like this whole triangle just flattens out right? You can still use them, will I'll show you an example tomorrow, you can still use the, similarity the ratio in which M divides a B you can use that to construct the cognates for that.