Lecture – 25

Theory Of Mechanisms

Velocity Analysis: Review of Velocity Polygons

So today we will, begin with, the analysis, of mechanisms.

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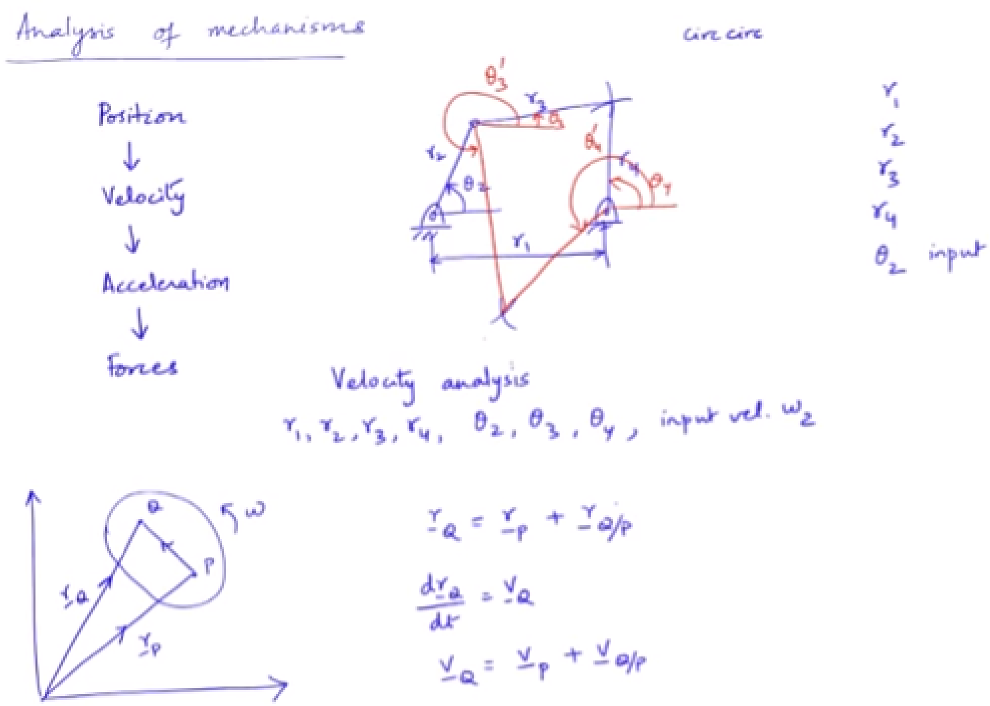

So far, in the course, we've looked at synthesis. Basically creating the mechanism, meaning finding the link lengths, to achieve a certain performance, be motion generation, function generation, dead center synthesis, with a particular time ratio, or path generation. Path generation, we also looked at, you know, just hitting specific points or path generation, in terms of, certain types of coupler curves, that you want for the mechanism. So synthesis, is creating the mechanism, analysis is once you have created the mechanism, figuring out, whether it performs, the way you, intended it to. Okay? And in terms of the analysis, so even though most of the synthesis tasks were related to position, they were related to position and displacement. In the case of analysis, in a real life application, you ultimately want to create, a real life, you know, if you want to create a mechanism, then you need to know, what kind of strengths, that the links in the mechanism, need to have. Which means, you need to know, the forces that are acting, on the mechanism? Which means, you need to know the accelerations and you can't find accelerations, without finding velocities, you can't find velocities without knowing the position. So analysis usually follows, you have, position analysis, then, which will lead to velocity analysis, which will lead to acceleration analysis and then typically, forces, because you need to find inertia forces, which, are dependent on the accelerations, of the links. So you can, you cannot violate, this order. Okay? You cannot try to do an acceleration analysis, without first, doing the position and velocity analysis; you have to follow that order.

So we will look at, some techniques, for position velocity and acceleration analysis. You would have already seen, a few of these techniques, in the undergraduate course. I will quickly run through, some of those, just to refresh your memory. And then we will also look at, a couple more techniques, that don't all, that will work for, some complex mechanisms, for which the, the techniques that you have looked at so far, will not work. Okay? So we will do that. So typically when your, the position analysis, if you do it analytically, it is typically non-linear. You've seen the equations, for even a four bar. Okay? Lots of COS theta, sine theta, it's, it's a very non-linear equation, it's quite complex, in that sense, but graphically it's a lot easier to do. And so if you take a four bar, for instance and you know the link lengths. So for analysis, typically, you know the link lengths. If you want to do a position analysis and it's a mechanism, with one degree of freedom, you will need one input variable, related to position. So typically, it's an angle. So you may be given an angle, theta 2. So if I know the link lengths, say r1, r2, r3, r4, then position analysis, of a four bar, is quite easy, so I take this, and I am given, input theta 2. Okay? So for a particular theta 2, so this is my R 2, at this theta 2. Okay? Then I know R 1, so that also I can mark off, so this, is R 1. Okay? And then I take R 3, R 4, an arc, from here. That gives me, the location of, so that gives me the position analysis, for this four bar. So you know that, so MATLAB has this, Circ, command. Right? which basically finds the intersection of, these two circles. One with radius R 3, one with radius, you give it the center and the radius. Okay? And similarly for the other circle and then it will find the two intersections. So the other intersection basically will give you, the other configuration, for this four bar.

So the other configuration will be, so for the same input angle, these are my, two configurations. Okay? So one would be, theta 4, the other would be theta 4 dash, one is theta 3, the other would be theta 3 dash. Okay? That gives me the, two configurations. So position analysis graphically is, easy to do, analytically, it's a little bit more involved, but that's where MATLAB helps, with this circ circ command it, you don't have to program all the equations, for the, for constructing the, 4 bar. So we'll look at velocity analysis, today. Velocity analysis really looks at, we look at the concept of relative motion. Okay? So the relative motion, when you, so when you talk about velocities, again, if it's a one degree of freedom mechanism, you need one input velocity. Okay? So the position analysis needs to be complete. You need to know exactly, what the configuration, is going to be, in all the positions, in which you want to analyze, do the velocity analysis. Because you are going to use, in the velocity equations, all these, you know, you'll need the configuration, of the mechanism. Okay? So if it's analytical, you need theta 2, theta 3, theta 4, that is, that you get from the position analysis that will be there in the velocity equations. So in the case of velocity analysis, your inputs, so what you'll need to know is, R 1, R 2, R 3, R 4, in addition, you'll need to Know, theta 2, theta 3, theta 4. Okay? for an analytical this thing or basically the configuration. And you need, one input velocity, say, Omega 2, typically. Because this is for a, 4 bar, it's a one degree of freedom mechanism, you need, one angular velocity, as input. So let's look at, how we would do this, velocity analysis and it's based on this concept of relative motion. So let me take, this simple case. You have a rigid body. Okay? And say, you have a point P and a point Q. P and Q are two points, on a rigid body. Okay? That is moving with some angular velocity, Omega. Okay?

So if I, say this is Rp, and say this is RQ, I can write RQ as, RP plus, R of Q, relative to P. Okay? Now I differentiate this, so, with respect to time. So this is the velocity of Q. So I can write the velocity of Q, as, the velocity of P, plus the velocity of Q, relative to P. Okay? How does that vector change, with respect to time?

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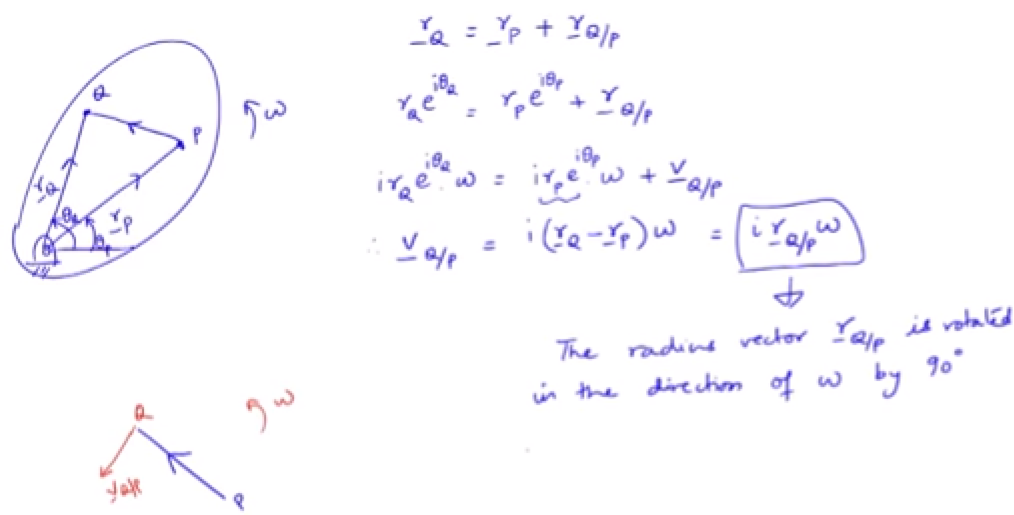

Let's take a body, that's pivoted, about a point and have two points on this. Okay? So let's say, this is moving with Omega. RP is, RQ is RP, keep doing this, R of Q related to B. Okay? So, if I take, differentiate this, I get velocity of Q and I can write R of Q as, theta Q, say theta P, this is RQ, E power i, theta Q, equal to, RP, e power I, theta P, plus R of, Q/P. So if I differentiate this. Okay? In this case, when it's rotating about a fixed pivot, the distances, RP and RQ, are constant. Right? So if I differentiate this, I get I, RQ, e power I, theta Q, into Omega, equals I, R P, E power I, theta P, into Omega, plus RQ. Therefore, sorry, this will be the velocity of Q, relative to P. Therefore the velocity of Q, relative to P is, i I can write this as, RQ, minus RP, into Omega, which is i R Q relative to P omega. Okay? I am clubbing this as R P, this is R Q, because that is. Okay? So what this tells me is, if the two points, are on the same rigid body, then the relative velocity, sometimes some books call this, the velocity difference, if there are two points on the same rigid body, relative velocity where, the velocity of Q, relative to P. okay? Is given by this. It's the radius vector, scaled by Omega and rotated in the direction of, Omega by 90 degrees. That's my velocity vector. Okay? So this implies the radius vector, the relative position vector, R Q relative to P, is rotated, in the direction of Omega. Okay? If Omega is positive, it will be a Counter clock wise rotation, if Omega is Negative, it will be a clock wise rotation, is rotated. in the direction of Omega, by 90 degrees. Okay? This is what we will use, when we look at, the velocity diagram. So what this, what this means? Okay, so this means that, the velocity vector, so if this is Q, this is P. Okay? The velocity, so Omega is counter clock wise like this, in which case, this is the direction of, velocity of Q, relative to P. Okay?

What this tells you is, see these are two points on a rigid body, they cannot move closer to each other, the only sort of motion they can have, is one, rotating about the other, that's what this is, establishing. Right? Because there can be no component of velocity, along the radius vector, because that would mean, that you are violating the rigid body principle.

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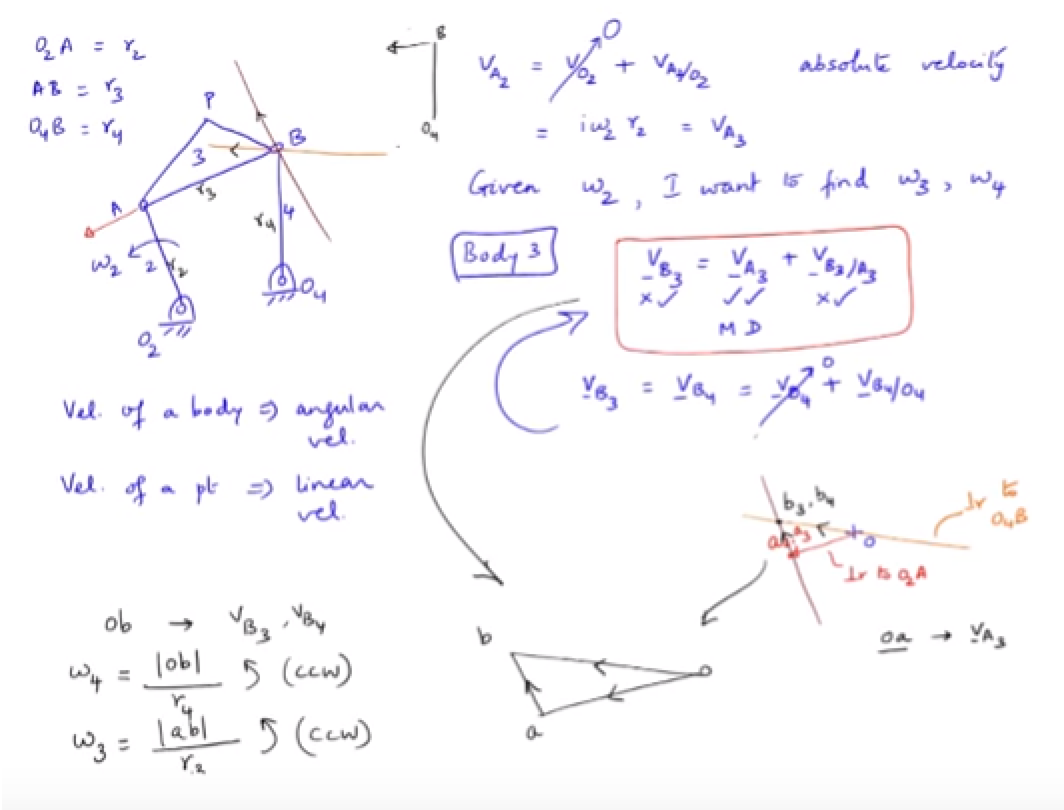

So now, we will look at, using this relative velocity concept, to analyze, a 4-bar mechanism. So let's do, the graphical method, we'll do a few different methods. So let's start with the, this you should be familiar with, but just to, refresh your memory and do this. So you have A, B and you have point P. In fact from now on I will refer to these two as, O2 and O4. So if this is linked to 2, 3, 4, O2 is the pivot, fixed pivot for link 2, O4 is the fixed pivot for link 4. Okay. We will also denote, so when I talk about a particular point, I will also indicate, which body it's on. Because now you have, multiple, interconnected bodies. Okay? So when I talk about the velocity of A, I will say, I'll talk about the velocity of A on body 2, that will be the notation, that I use. Okay? So let's say the input, angular velocity, is omega 2. That's the given angular velocity. And your task is to find out, the velocities of, all the other links. So when we say velocities of other links, when, when I say velocity of a rigid body, I am talking about angular velocity. Then I say velocity of a point, I am talking about a linear velocity. Okay?

So velocity of a body, a point does not have, an angular velocity, essentially. Velocity of a body implies the angular Velocity, velocity of a point, linear velocity. You cannot talk about the, linear velocity of a body, unless it's in translation. Because different points, on the body, if it's in general plane motion, different points, will have different linear velocities. If I am able to talk about the linear velocity, of a body, the only cases, when all the points in the body, have the same velocity, which means, the body is in translational. Okay? So, velocity of the body. Okay. So let's say, so, if I look at 2 as my rigid body, from what we saw earlier, the velocity of A on body 2, equals the velocity of O, on body 2, plus, the relative velocity, I'm going to leave out the vector notation, because, you know, you should know by now, that these are vector equations, we are writing. Okay? VO2 plus v a2, relative to O2. Now what is the velocity of O2? It is zero. So in this case, this relative velocity is actually, an absolute velocity. When the velocity is expressed, with respect to the ground reference frame, we talk about absolute velocities. So the velocity of A on body 2, we saw that this is equal to I Omega 2 and R 2, R 2 is my, length of link 2. Okay? So here I have, O2A, length is R 2, AB, equal to r3 and O4b equal to r4. So this is also equal to, now if I look at, body 3, the velocity of A on body 2, because A is common to body 2 and body 3. So this is also the, absolute velocity of, A on, body 3. So the only thing I know, the things that I need to find out, in my velocity analysis, given, Omega 2. I want to find, omega3 and omega4. Okay? We'll see whether, that'll be enough to, find the velocity, essentially I want to find the velocity of, any point on the mechanism. But let me, you know, let's say we want to find omega3 and omega4 and whether that will be enough for us to find, everything else. Okay? Given Omega 2, this is what I want to find. Now if I look at, body 3. Okay? I know the velocity of A on body 3. So let me see if I can relate it, to the velocity of B, on body 3. So I can write the velocity of B, on body 3, as, VB 3, equals VA 3, plus V, b3, with respect to A3. See the notation, it's like, A3 cancels out, so. Okay?

So, now in this equation, right now I don't know the velocity of B on body 3. Okay? Here let me just retain the vector notation for now, okay. Okay? So velocity of B on body 3, I don't know anything about it right now. Velocity of A on body 3, I know completely. Okay? I know both, magnitude and direction. Because that's the same as, that's the based on, the input velocity and this one, B 3 relative to A3. Okay, what do I know about this? These are two points, on the same rigid body. I don't know the magnitude, because, that depends on Omega 3, which is still unknown. Okay? So I don't know the magnitude, but, what do I know about the direction? I know it has to be perpendicular to a B. So I know the direction. So this is a vector equation, which means have two scalar equations. If I have one more, if I know one more thing, I should be able to solve this vector equation, for two one's. Okay? Let's see if, I can find something about, the velocity of B, on body 3. If I look at this, B, the velocity of B on body 3 is also equal to the velocity of B on body 4. And body 4, is again pivoting about a, fixed pivot. Okay? So I do know the direction. Okay? So if this is rotating, this is in pure rotation, then B, the velocity of B, is going to be perpendicular to, O4B, so I know that. So I do know, if I go back to this, I don't know the magnitude, because I don't know Omega 4, but I do know the direction. So now this becomes a vector equation that I can, solve, for two unknowns. Okay? So let's do that graphically, so let's take in the velocity diagram, all the points with velocity zero, are represented, as O. Okay? Now here, this is the vector equation, I am trying to solve. This is the one, that's completely known, velocity of A on body 3 and I know that it's an absolute velocity, because it's with respect to, one of the fixed pivots. Okay?

So I know that, it's with, so it will start from O, the velocity vector. So I know that velocity of a, a 3, is going to be perpendicular, to, O2A. And it will be of magnitude, Omega 2, into R 2. Okay? So I know that vector completely. So I transfer that that is my, vector here. So this, so in the velocity diagram, the points are indicated by lowercase. So a will be and you can put a on body 2, a on body 3. Okay? Now, velocity of B on body 3, again B which is equal to the velocity of B on body 4, which is equal to velocity of O4, plus, velocity of B 4, with respect to O4, relative to O4. Because this is zero, again the relative velocity is the absolute velocity, which means I draw this, from the point O, because it's with reference to O4, which also has velocity zero. So if I look at, I know that this perpendicular to O4B. Okay? But I don't know, in which direction, because Omega 4 could be positive or negative. Okay? But I know that it lies, along that line and it is an absolute velocity, which means, it has to pass through O. So I transfer that line, over here. So this is a line that is perpendicular to O4B. This here, the input, this is perpendicular to, O2A, as I've shown them. Then the third one is, so I have to solve this equation. I need velocity of A on body 3, plus a vector, this relative vector, relative velocity vector, should be equal to, O4, sorry, velocity of B on body 3. So if I do that, now, this is a relative velocity, for which I know the direction, again I don't know the, magnitude. So if I look at AB, I know that, this should be, perpendicular to AB. Again I don't know in which way. Okay? So now and this will start from where? From O or from A? It has to start from A, because it's relative to A.

So now, now I transfer this, to this point. So the intersection of these two, gives me, the point, B, B on body 3, which is also equal to B on body 4. Okay? So in my velocity diagram, now I have I basically solved this vector equation. Okay? so now directions. I have so OA, in the velocity diagram, represents, velocity of, A. Okay? Or in this equation, velocity of A on body 3, this direction. Velocity of, so this, plus this, if I put the arrow like that. Okay? This plus, this, should give me this. That is my equation here. Okay? So I'll just draw that bigger. So I have O, A and B, this is my, this is, just this magnified, so this is OAB, So the direction of this, in order to satisfy this equation. Right? should be, this, plus this, equal to this vector addition. Okay? So what does this tell me now? Now, from here, OB is the, velocity of B on body 3, which is equal to, the velocity of B, on body 4. Therefore my Omega 4, equals, the magnitude of OB / O4B or R4. Okay? And what would be the direction? Look here, here, from this vector equation, I found that the direction is like this. Which means, what will be the direction of Omega? Omega is counter clock wise; it's like this, so it is counter clock wise. Okay? Because, O4B, has been rotated in this Direction, to give counter clock wise to give you this vector. Similarly if you look at Omega 3, from this diagram, I have, mod a B, by R 3, if this is R 3, R2, R4. Okay? And what would be the direction of this? So a B here, it shows, it's like this. Right? So B relative to a, is in this direction, which means, again Omega 3, is also counter clock wise. Now suppose, so in many cases, so for Instance, if this was the solution that I got for a path generation problem and I was really interested in the velocity of P. Okay? Why not take P straight away? P is also a point on link 3. Right? So instead of going through B, what if I had taken, so if P is what I want, if the velocity of P is what I want, why not take that directly?

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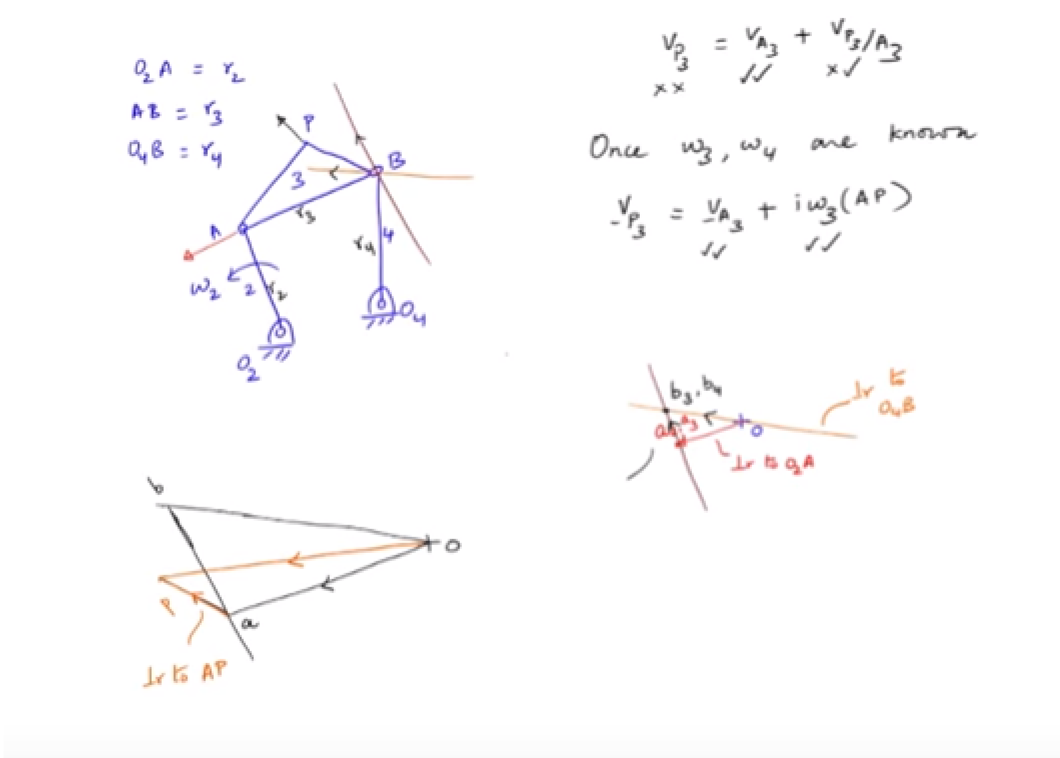

Suppose I had written that as velocity of p, on body 3, equal to velocity of a on body 3, plus, velocity of P on body 3, relative to a 3, Perfectly valid equation. I could have tried to solve this, directly. Again, I know both magnitude and direction, of this. Here again, I don't know the magnitude, but I know the direction, of both our points, on the same rigid body. In this case, with P, I neither know the magnitude, nor do I know the direction. So when I am looking at, this, I have to look at another point, on the same link, about which something is known, in order to be able to, solve this. So you cannot do that directly. But now, once I determine omega3 and omega4, what is the, once I determine, once Omega 3, Omega 4 are known, then it's easy for me to find, the velocity of P. Because I just use the same equation, plus I, Omega 3, into ap, that's it. So I know this completely, I know this completely.

So if I go back, to my diagram, let me just copy this over. So here, I should probably; I'll rescale this and draw, because it's too small. So I have O, because I can choose any scale for my velocity diagram, as long as I know, what that, scale is Okay, okay. So this is my B, this is my a. Okay? So now, I have, if I want to find the velocity of P, I have the velocity of a, plus, what would be the direction of this one? This will be Omega times AP and I found that Omega 3 is counterclockwise, so this would be the and it would be a relative velocity. Right? So from a, I would lay off this vector, well doesn't look like it. AB, sort of perpendicular, and that's because this one is, not very, so this vector, so this denotes point P. Okay? So now I have the absolute velocity of point P, will be this vector, this is P relative to A. And so, similarly, so once I know the angular velocity, of any link and I know the velocity of, one point on that link, I can find the velocity of every other point that is on the link. So therefore, once I know the angular velocities, of the 3 links, because I know, because these are fixed pivots, from there I can find the velocity of, any point, on the mechanism. So this is perpendicular to AP. We get the directions, from the vector equations, for the omegas, so that is done.

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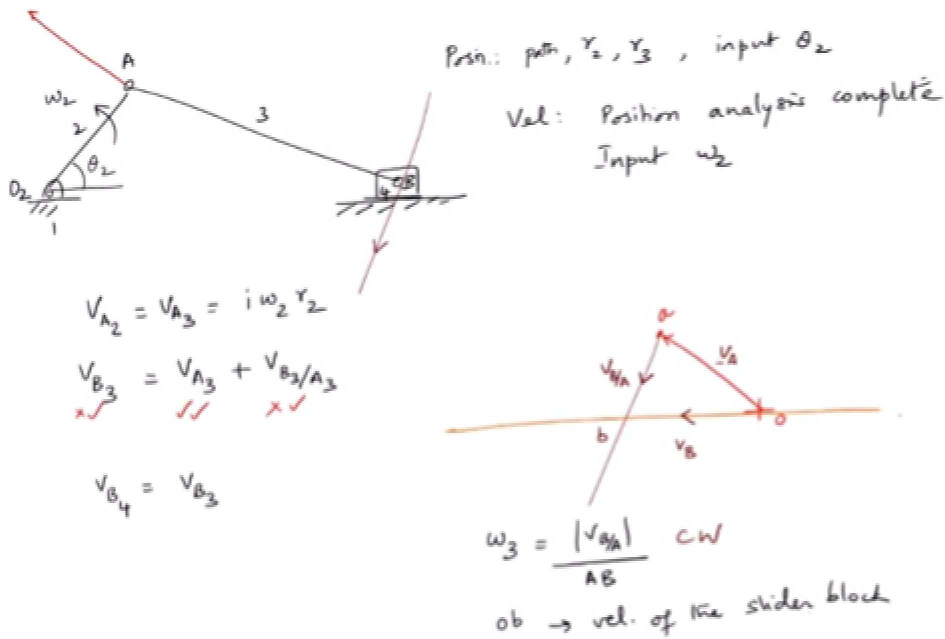

So we can do the same thing with a slider crank. So I have O2, AB 1, 2, 3 and 4. Okay? Again position analysis is easy, graphically. If I know theta 2, I know the path. Right? So I can and I know that, I know R 3, so I can indicate, where on the Path, it will intersect. Okay? So here, I know R 2, R 3, input theta 2, for the position analysis, I know the path direction also and then for the velocity analysis, position analysis, complete input maybe, Omega 2, it could also be the velocity of this slider, as in the case of an IC engine, for instance. Okay? The input, you could be driving it, from the, slider, so your input is Omega 2. So let's do the velocity analysis for this. Again. I look at, this link, I have velocity of a, on body 2, is the same as the velocity of a, on body 3 and it equals, i Omega 2, into R 2. Okay? because it is, an absolute velocity, with respect to O 2, which has velocity zero. Now if I consider body 3, I consider point B and a. So I have velocity of B on body 3, equals velocity of a, on body 3, plus, velocity of a 3. Okay? So in my velocity diagram, I have, O, which represents, O2, fixed-point, with zero velocity. Then I have the known velocity, of point, this one, so if I look at this vector equation, velocity of a, on body 3, completely known, because that's my based on my input. B 3relative to a 3, magnitude is not known, direction is known. And B on body 3, what do I know? I know the direction, because it is restricted to move, in that direction, so I don't know the magnitude, but I know the direction. So these are the unknowns, the unknowns that I'll solve for, are the magnitude of the slider velocity, because here it's not the angular velocity, it's not, it's moving linearly. Okay? And the omega 3, for this connecting rod, these are the two things that I will solve for. So let's see. I have from here, because Omega 2 is counter clock wise, this will be my vector.

Let me just take a bigger scale, so that I get. So I will have, this will be point A, on the velocity diagram, then I know that, velocity of B, I don't know the magnitude, I know the direction, is along. Okay? P lies, because, it can only move along that direction. Then B relative to a, is going to be, perpendicular to a b. Okay? B can only move, in a circular arc, about a, which means, the velocity will be perpendicular to, a B. So if I do that and that has to come from A, because it's a relative velocity. So this becomes point B, intersection of these two. So now if
this is the direction of the velocity of a, this is velocity of a, this will be the depth velocity of B, relative to A, which means, this is the velocity of P. Where velocity of B, on body 4, is the
same as the velocity of B on body 3. Okay? So now from this, from the velocity of B on body a, my omega 3 is, magnitude of that, divided by a B, a distance aB. And what would be the direction of omega3, from this, diagram? So it is this. Right? So I found out, that this is the direction, which means, Omega 3 is clockwise. Case of Omega 4, sorry, there's no Omega 4, the OB, is the velocity of the slider block, linear velocity of the slider block.