Theory of Mechanisms

Lecture 26

Velocity Analysis

Velocity Polygons (contd.)

And Instant Centers

So yesterday we looked, at using the velocity difference between two points on a rigid body, to analyze the velocities in a four-bar linkage. So, you saw that if there are two points on a rigid body.

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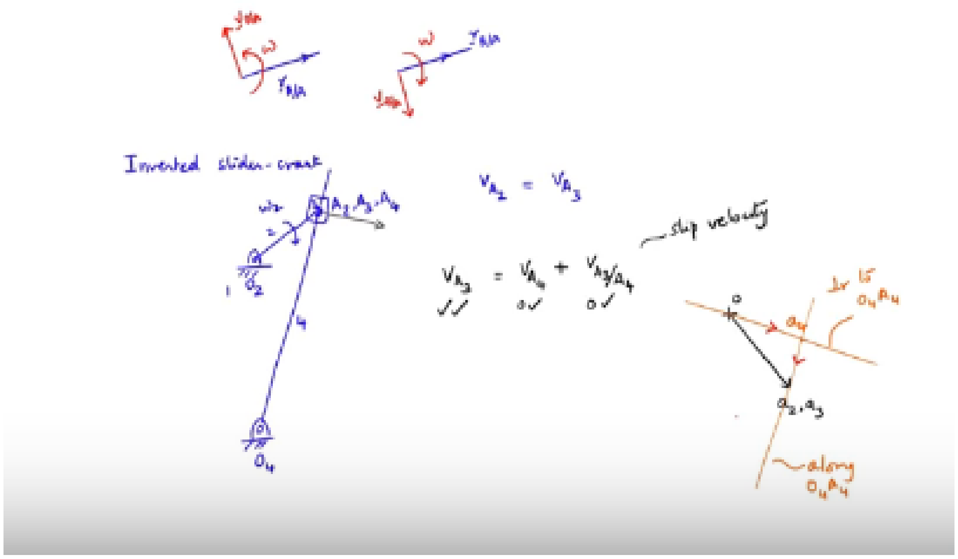

that are described by, a vector R, B relative to a, then the velocity of B with respect to a, if Omega is counterclockwise, then this will be velocity of B, relative to a. Otherwise, it will be, if omega is like this, then this will be the velocity of B, relative to a. Okay? So this is how the direction, or from the direction of the velocity, you can determine the direction of Omega so for planar problems Omega, will be positive counter clockwise, it's typically taken positive counterclockwise and negative clockwise. Okay? So, we also looked at, the slider crank mechanism, so we looked at two points on the same rigid body, in some cases for the graphical analysis, we'll have to consider points, that are momentarily coincident, but on different bodies. So take for instance, an inverted slider-crank like this. Okay? So you have a block, which is sliding, so you have linked 1, 2, 3 is the block, which is pivoting with 2 and then I have link 4, so this is o4, this is O 2. And let's call this point a. Okay? Now at a particular instant. Okay? So here, I know the velocity of a, on body 2, so say this is my, let's just say I take it this way, just for a change. Okay? So let's say, I take omega 2, as clockwise ,so I know the velocity, so here if I actually look at a, I can say there is an a, on body 2, there's an a, on body 3 right, the pin and I can also find a momentarily coincident point a on body 4. Okay? So that's what we are going to Consider, so I'm going to consider coincident points ,on body 3 and body 4, body 2 and body 3, it's, it's permanently together, so they're always going to be coincidence.

Coincident, so the velocity of a on body 2, will be equal to the velocity of a, on body 3. Okay? So if I draw my and if Omega 2, is this and I'm given R 2, so I can say, that it will be, draw my velocity diagram. Okay? It will be an absolute velocity, because it's rotating about a fixed pivot and this will be, point a, on body 2, which is also the same as a, on body 3. Okay? Because those two are, now if I look, at the motion, so I want to relate, if I sit on four. Okay? If I sit on link four, this point a, three, is going to move along the link. Okay? So I can write, so I know the direction of this, remember we have to, know something about the other point that we take, in order to relate two velocities. Right, so I need to know something in, in the case of the four bar, I picked B, which was coincident with the body, that was rotating about a fixed pivot, so that I would know the direction of that velocity, here, I know the direction of this relative velocity, so I can relate these two velocities as, velocity of a, on body three, is the absolute velocity of a ,on body four, plus this relative velocity. Okay? So in this I know completely the velocity of a, on body three, velocity of a, on body for the body four, its absolute movement is pivoting about o four. Okay? So, if I take a point that is fixed body four. Okay? I know the direction of that Velocity, I don't know the magnitude, because, I don't know Omega four. Okay? And in the case of this velocity, how the two are slipping, with respect to each other, the block and the path, I know the direction, but I don't know the magnitude of that slip velocity. Okay? That velocity is called the slip velocity, so I use this, to solve the velocity polygon and I get, so I have this is velocity of a, on three, then I have velocity of a on body four, would be perpendicular to this.

Right, it could be perpendicular to this, so and it's an absolute velocity, so I again start from O, so this is perpendicular to o4, a on body four and then this the slip velocity, I know is along the direction of this, so it's along this, so I don't know which direction, so but I know, that it's and so also perpendicular to this one, so this is along O, four, a, four. So this intersection here, gives me a, the velocity, the point a, on body four, so now I have, this velocity of a, on body three, my equation is equals this, Plus this. Okay? So my slip velocity, so three, is moving in this direction, from a4, a on body three, is moving this direction from a, on body four, which as you can see, because of the clockwise rotation of the crank makes sense, so it's moving away, from that coincident point in this direction. Okay? So this is how you do the graphical velocity analysis, when you have and this velocity will become important, when we do the acceleration analysis, because when you have a sliding joint on a rotating path. Okay? Earlier when we looked at the slider crank, the path remains fixed. Okay? But, when you are looking at inverted slider cranks, you're looking at a path, which is also rotating and in those cases, the Carioles acceleration will come into play and that Carioles acceleration is, directly related to the slip velocity, that you find from the velocity diagram. Okay? So, this is the case of the inverted slider crank.

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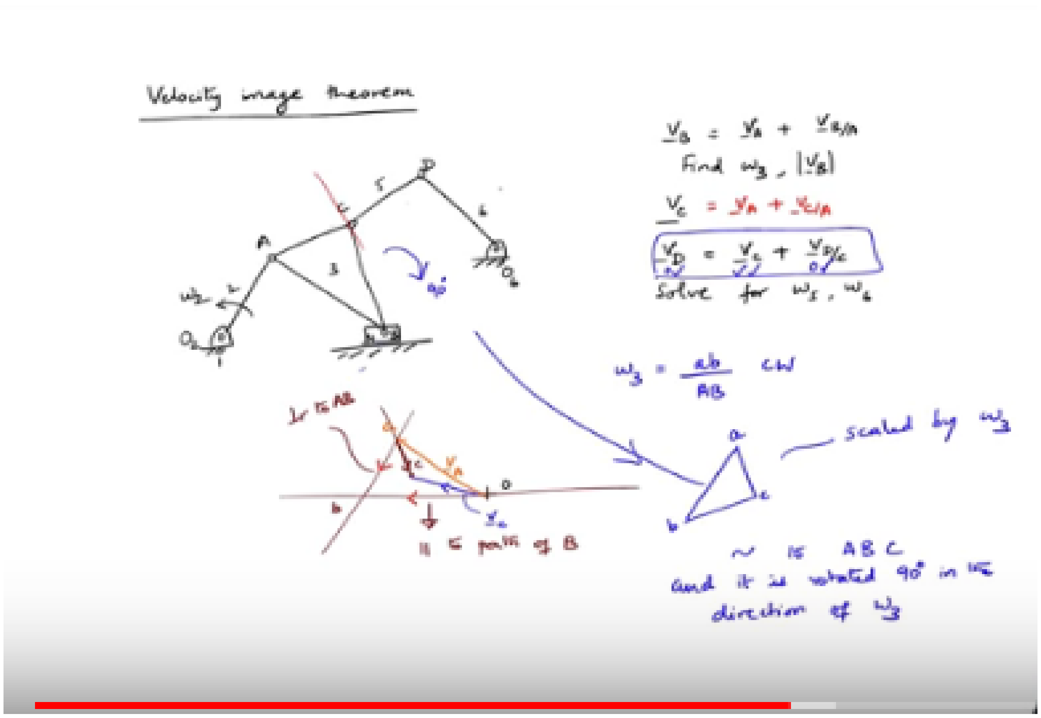

Now, we will look at the velocity image theorem, which is for, So, when you are doing graphical analysis, of making mechanisms other than a four bar, so for instance .If you have a mechanism like this, a six bar, we're another diet is driven by a coupler point of the. Okay? In this case it's a slider-crank, but you could I could, technically have a four bar also, I can just, instead of putting a slider here, I can just put a four bar. And then, so it's basically a 6 bar. Okay? So, in mechanisms like this, once you analyze, so you have multiple loops, in the mechanism, so this part of it, is essentially a slider crank. Right, so if I have o, to a, B, C, D and let's, so let's say, 1, 2, 3, 4, 5, 6, so this is o 6. Okay? So I can do the velocity analysis, for, 0, 2, a, B, which is simply a slider crank, so say this is my input link. Okay? Then, my, the output that I'm interested in, is omega-6, say that's the output that I'm interested in, because, otherwise there's really no reason, if the only output I was interested in this. So, if four, is the output, then I don't really need, the rest of this mechanism. Right, so because the motion of this part is going to be independent of this, it's this coupler Point C, which is driving this diet and I am interested in say omega-6, as the output of this particular mechanism configuration, so, an easy way, so once I do the analysis of this 4 bar, or 4 bar equivalent ,the slider crank, I can easily find the velocity of this point C, so essentially I have to apply the relative velocity equation ,sequentially in order to compute omega-6. Right, so I have to, say I keep writing, so just as I do velocity of B ,equals velocity of a, plus velocity of B, relative to a, then once I find ,so I do that to find, Omega 3 and the velocity of B, the magnitude of the velocity of B, then after that, once I find Omega 3, I'll find velocity of C, then I will find velocity of B equals, velocity of C, plus velocity of D, relative to c. Okay? Because, each time I have to go sequentially, I have to keep looking for two points, which are on the same link to do this and then once, I do this I can now solve for, with this equation, I can solve for Omega 5 and Omega 6, when I lit B and C.

Right, then I can solve for Omega 5 and Omega 6, because once I find Omega 3, I can find the velocity of Point C, so let me just do that. Okay? So, once I find Omega 3, right I know the velocity of a, so velocity of C, will just be the velocity of a ,plus the relative velocity, C with respect to a, which I can find out, once I know Omega 3, then I relate C and B, and once they relate C and D ,that say time I can solve for Omega 5 and Omega 6, so I know all the angular velocities in this mechanism, and with velocity, if I know the velocity, of any one point on each link, then I know the, absolute velocities of any point on the linkage, the velocity image theorem will help you find the velocities of say Point C, we saw that yesterday, when so if I just do the velocity analysis of this, I have o, I have, velocity of a. Okay? Which is perpendicular to o to a. And then, I know the absolute velocity of B, is along this path, I know B relative to a, from here is along this path, so this is perpendicular to a, B, this is parallel to path of B, so this gives me B. Okay? So from here, I have, from this equation I have velocity of B, equals this velocity of B, equals velocity of a, plus B, relative to A. Okay? From here, I know omega-3. Right, omega-3 is a, B, by the length a, B, so if you look at this triangle. Okay? Then, now if I want to find the velocity of C, I have velocity of C, is velocity of a, plus velocity of C, relative to a. Okay? Now when I do that, I already have velocity of a. Okay? And velocity of C, relative to a, is going to be along that.

Right, so I would have this, so this, also I know the magnitude now, once I know omega-3, so I would just construct that vector, like this. Okay? To give me Point C, yeah, so that then gives me, the absolute velocity of C, the blue one is the absolute velocity of point C. Okay? Which I need for the next step, for this step, because then this becomes the input, which is completely known, this one magnitude is not known, Direction is known, again this one, magnitude is not known, direction is known. So again, I can solve for, Omega five, Omega six, but we see is what I need for that, now if you look at, so first what is, the Omega 3, let's just determine Omega 3 is, from the velocity diagram, it's this length a, B by a, B. Okay? And what would be the direction, from this diagram, so it's B, what would in the direction, b, relative to a and it is in this direction, which means, Omega 3 is clockwise. Okay? Omega 3 is clockwise. Now look at the triangle, small A, B, C, in the velocity diagram, I have A. Okay? So, I found Omega 3 is clockwise and if you look at each of the sides, of this triangle, a, B is proportional to, capital a ,B ,A,C is proportional to capital A, C and B,C is proportional to capital B,C, because they, the whole thing is scaled by what number? Omega 3, it's scaled by Omega 3, this one; I'm just taking this outside. Okay? So all the relative velocities on that body are scaled by Omega 3, which means this triangle, in the velocity diagram, is similar, to the original triangle A, B, C and it is rotated how? And it is rotated 90 degrees, in the direction of Omega, Omega 3, in this case, because all these points belong to length 3. Okay? So I don't really need to, so once and it and I could have other points, on this, that I'm interested in and all I have to do is construct, in the velocity diagram directly, instead of having to compute, each of these velocities in this manner, if I construct, something similar to the shape of that particular link, rotated by that ninety degrees, then I can directly read off the velocities of those points ,that's what the, so the velocity diagram is an image, of the position diagram, of the linkage. Okay? With all the relative velocity scaled by ,Omega of the link and rotated by 90 degrees, in the direction of omega 3, so you can see here, see the order a, so this is, if you take this and rotate it by 90, this is what you would get and scale it, you can see C, has moved down. Okay? So you should be careful, when you construct in the velocity diagram, because if you put C on the other side, then that would be wrong, because that would mean that it has rotated counterclockwise, so that is important, the direction in which Omega acts, so once you find Omega for that link, then you can construct the image for the velocities. So let me just write a couple of things.

So, triangle A, B, C, in the velocity diagram This applies not just to triangles, because any shape you can, any polygon for instance, you can break it up into triangles, so it applies to any shape of the body, so the shape in the velocity diagram, would be proportional to and rotated by, 90 degrees in the direction of

Omega. Okay? So this is the, so once you solve the first one here, you can directly then solve the second equation, without actually, you can construct the image of A,B,C, on the velocity diagram and then use that to solve for the next two velocities. Okay? So that is the sorry, that is the velocity image theorem. Now, what's another method for finding velocities that you have seen. Okay? A graphical method, another graphical method, see with the velocity difference method, I have to go through the whole, I can't jump to finding omega-6, just by knowing Omega 2, I have to go through, I have to go through each of the motion transfer points, so to speak to connect them and find all the intervening angular velocities, before I get to the angular velocity that I am really interested in. Okay? So I can't skip, finding any of the intermediate angular velocities here. Okay? But I may just be interested in Omega 2, and Omega 6, for instance. Okay? So in that case, we can use the method of instant centers. Okay? So the method of instant centers, so the instant Center is.

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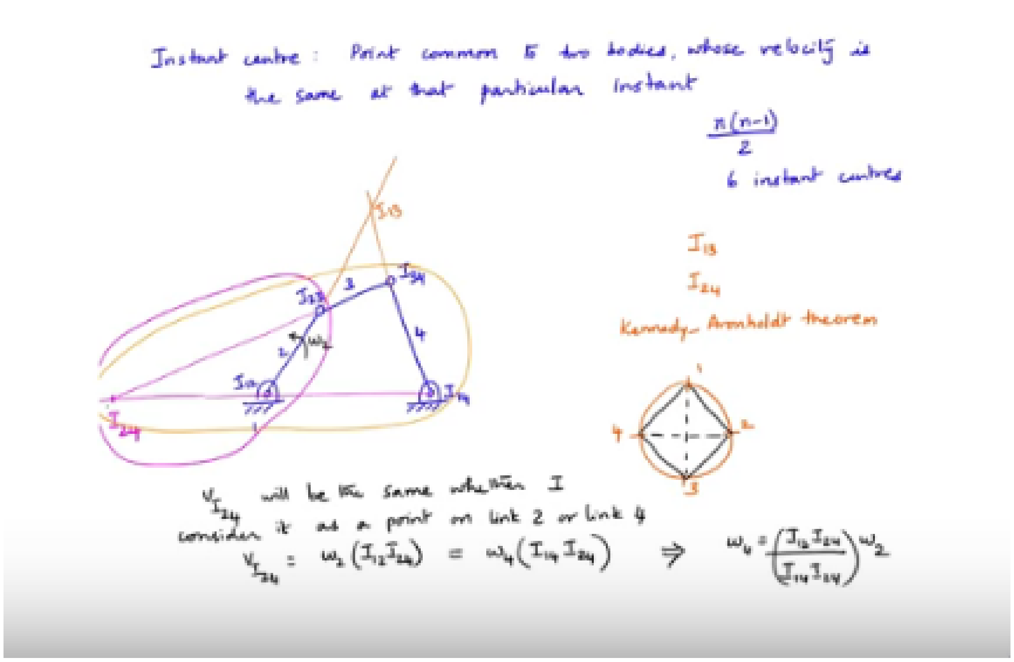

What is the definition of the instant Center? It is a point Okay? That is one definition, so relative to one, the point velocity is, is zero with respect, it's only in some cases, the links are rigid bodies anyway, so yeah relative to the, see what you said initially was correct, you know, at that point, the relative velocity is zero, which means, you can imagine that one body is rotating, in pure rotation about the other body, at that particular instant, but for the purposes of velocity analysis, what is the basic definition of the instant center of velocity, it's a point common to both bodies. Okay? What else, there could be many points coming to both bodies, what is special about this point. Okay? Which means what? So that point acts like a hinge, which means in this big scheme of things, what's happening to, what is the velocity of that point, in both the bodies, so it is a point common to two bodies. Okay? Whose velocity is the same, at that particular instant. Okay? So let's, let’s start off with the, our friend the four bar and look at, so if you have, how many instant centers, will you have, when you have four links. Why? The number of instance centers is n into, n minus 1 by 2, n, c2. Okay? So for a 4 bar, you will have 6 instant centers.

Okay? So I have I, 1, 2, I, 2, 3, so if you look at the pivots, they're automatically instant centers, because they are common to both the links. Okay? And they're going to have them, physically they are common points to both the links, the pivots and they're also called permanent instant centers for that reason. Because, it's always that location, in the in the two bodies ,it's always going to be that location, which is the instant Center for those two bodies, so this is I, 2, 3, then I have, I, 3, 4 and I ¼. Okay? Now we have to Find, a couple more instance centers ,the ones that are missing are, for bodies that are not directly connected to one another .Right, so here, the missing instance centers are I, 1, 3 and I, 2, 4. So, to do that, we use make use of the, Kennedy Arenholt theorem, which states That, if you have three bodies, moving in a plane, then they in the three instant centers they lie on a on the same line. Right ,so and, and we use this, what we call the circle diagram, to help us find the instant Center, so here I know R, 1, 2 2, 3, 1 ,2, 3, 4, denote the four links, I know 3/4 and 1/4. Okay? Now I want to find 1, 3. Okay? So, if I look at this diagram 1, 3, is a common side, 2, 1, 2 and 2, 3 similarly for, 1, 4 & 3, 4. Okay? So what that means, is I, 1, 3 lies on, the line joining 1, 2, 3 and the line joining 1, 4, 3, 4, so this gives me I, 1, 3. Okay? We've seen I, 1, 3 before, when we looked at Centrioles.

Right fixed and so I, 1, 3 is special because, it belongs to body 1, as well, which means from the definition of the instant Center, its velocity is always zero, so in certain cases ,the velocity of the instant Center will be zero. Okay? The velocity of this point I, 1, 3, is zero, because, it is common to both link 1 and Link 3 and it has the same velocity in both the links. Now let's look at, I, two, four. Okay? So now, I have I, 1, 3, I've determined I, 1 ,3, the other one that I want is, I ,2, 4, so from here, I can see, it's I, 1,2 , I,1, 4, it lies on the line, I,1,2 I, 1,4. Okay? And I, 2, 3, I, 3, 4, so it's this, yeah, this will be I, 2, 4, is the velocity of i-2, 4, 0. No it only has the same velocity, whether you consider it, as a point on link to, so imagine that, link to be extended to include, I, two, four. Okay? And you can also imagine that, link 4, is extended, to include I, 2, 4. Okay? What this tells me is ,what is the motion of link to ,the absolute motion of link to, is pure rotation about I, 1, 2, the absolute motion of link 4 is, rotation about I, 1,4, fixed pivot, so that means ,what would the velocity of this point B, I, 2, 4, if I consider it, so ,from because it is the instant Center, velocity of I ,2, 4, will be the same, whether, I consider it as a point, on link 2, or link four. Right, what is the velocity of this point, if I consider it as a point on link two, let's say this is Omega 2, velocity of I, 2, 4, equals Omega 2, into the distance I, 1, 2, I, 2, 4. Okay? And this is the same as Omega 4, into I, 1 ,4, I, 2, 4, because, link 4 is rotating about I, 1, 4, therefore I can directly get Omega 4, as I,1,2, I,2, 4 ,by I,1, 4, I, 2, 4, into Omega 2.

So I don't have to go through that, with the other method, I would have to calculate Omega 3 and then. So here, in the 4 bar, you don't see the use of it so much, but if you when we look at the next, the 6 bar mechanism, that we saw earlier if you want to find Omega 6 here, you can find that, by finding, the instant Center that relates 2 and 6, I, 2, 6. Okay? So let's do that. Let me do another example quickly.

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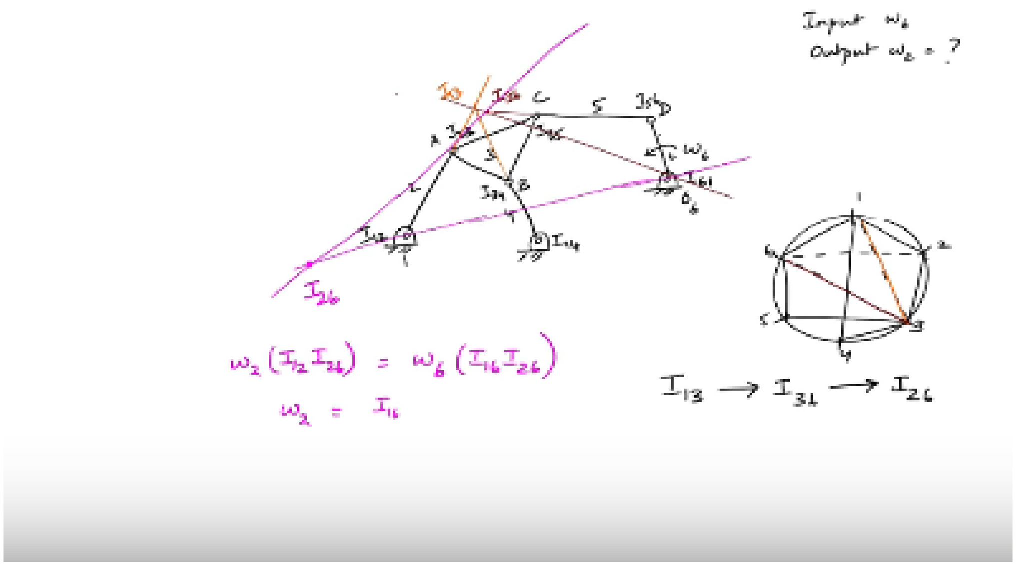

So, so let's take. Okay? This is similar to the mechanism that we saw earlier and let's say, I want to find. Okay? Here's an interesting situation here, suppose I give you Omega six, as my input. Okay? I can't even solve this, by the velocity difference method. Okay? I'm telling you, I can't solve this by the method that I used earlier because, you see one, two three, four, five and six. Okay? So a, b, c if i look at this loop. Okay? This particular loop, the coupler is not attached to any link that is part of the frame, link five, so if i try to find, say i know the velocity of point d, because that's my input, that's based on my input Omega six, now to move to the other links, i need to find another point on link five, for which i know something about its motion.

But in this case, C is not part of a link, for which I know the, curvature of its path, in the previous case, C was part of a link which was pivoted to the ground, so I knew the direction of its, I knew the curvature of its part and hence, I knew the direction of its velocity vector, in this case I don't know that. Okay? So I cannot even solve this, by the velocity difference method, by the relative velocity method. Okay? We'll talk about another technique to solve this tomorrow, but now let's just do, let's see if we can do it by instant centers, so I want to find input is omega six, output Omega 2, equal to what? So I have, my circle diagram, with now six marks on it, three, four, five, six, I ,1, 2, I, 2 ,3, these are all the known instance centers ,3, 4, I,3,5, I, 5, 6, I, 6, 1, I, 1, 4. So all the known ones, you mark on your circle diagram, 1, 2 ,2, 3, 3 ,4, 3, 5, 5, 6 and six, one, also one, four, these are all know these are all permanent incident centers that I get from here. Right, now if I want to find, this is what is missing, I need to relate, if I want to relate two and six, I need to find I, two, and six. What is the Path? So if I want I to six, it have to be a common base for two triangles. Okay? So, maybe i need, i to five, or i, three, six. Okay? let's see ,if suppose i find I, 3, 6, then I can find I,2,6 right because 1, 2, 1 6, 2, 3, 3, 6, good now to find 3, 6. What do I know? What do I need? Here if I look at this, if I find, so I know three, five, five, six if I find I, 1, 3, then 1, 3 ,1, 6, 3 ,5, 5, 6 that will give me I ,3, 6 ,so I have to find I, 1, 3 the order in which, I go is I find I, 1 ,3 ,then, I can find I, 3 ,6, then I can find the 5 to 6.

Do you see that? I, one, three is straightforward, because this is just a four bar here. So I have this is I, 1, 3. Okay? Now. Okay? Once I know, I, 1, 3, I, 1, 3, I, 1, 6. Okay? That is I ,one, three, I once, I, one, six and then I, 3, 5, I,5 ,6, by 3, 5, I,5, 6. Okay? So, somewhere here, is I, 3, 6, so now, I have found, I, 3, 6. Once I know, I, 3, 6, I ,2, 3, I, 3, 6, so I want to find I, 2 , 6 now I ,2 ,3 and I ,3, 6, I, 2, 3, I, 3, 6 and I, one, two, I, one, six ,notice - let me just extend this, I ,one, two, so this gives me my I, two, six, so if I find, I ,two, six. I directly get Omega 2, because Omega 2 into I,1, 2, I,2 6 ,is equal to Omega six ,into I,1, 6, I, 2, 6, because I,O, 2, 6, is a point that's common to both ,link 2 and Link 6, therefore Omega 2 is I,1, 6, I, 2, 6, by I,1, 2, I,2, 6, Omega 6.

So, in some cases where it's not possible to solve by the velocity difference method, instant centers may work, but with instant centers, suppose the instant Center is located out of the plane, in which you're, I mean, even when I say out of your working area, on the plane, then you're in trouble, if you don't have a big enough sheet, then you know for a particular position, you may not be able to do the velocity analysis, again it's. Okay? If you want to just do it for specific, graphical methods always are. Okay? If you want to just do it, for a few positions. Okay? If you want to do it for multiple, for the range of motion, you'll have to go to an analytical method and we look at a couple more interesting method okay.