Lecture – 29

Velocity and Acceleration Analysis: Analytical Method

Yesterday we looked at, you know the auxiliary point method and also, we did an example with that and then, looked at, how the, analytical method, using the loop closure equation, essentially, gives you, so what we have been doing with the graphical method, is not, anything different. It actually derives from the, loop closure equation, that you write for the mechanism and when you differentiate that, for the velocity.

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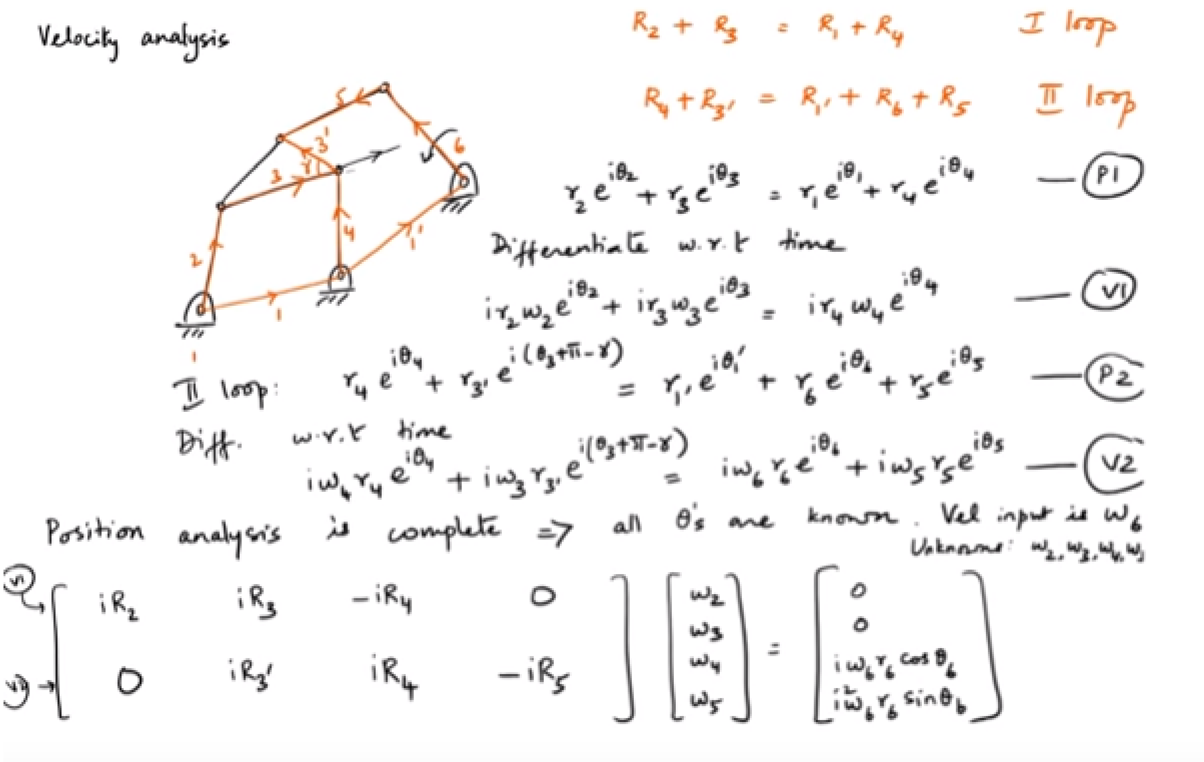

So for a mechanism, where we couldn't use, the or for a kinematically complex mechanism, let's just look at, applying the analytical methods or a mechanism with more than one loop, turnery, or higher-order, floating link, let's just look at, doing it analytically. Okay? So, velocity analysis. So this you know, the methods that we looked at, where, if this is the input, we found, we can't directly solve using the, velocity difference method. So you either use inversion, in this case, because inversion actually makes this a simple mechanism. Yesterday of course, we saw a mechanism where, that would also not help. But, so in this case inversion would help, to solve for the velocity analysis, of course, the auxiliary point method, is also, gives you a, direct way of solving it, without using inversion. So now we look at, the loop closure equation, to solve this, analytically. Okay? 1, 2, 3,4,5,6, again, ensure that, this, always check for a, check for mobility, when you see a, when you encounter a, new mechanism. This of course we've seen this, a few times now, but this is your, is a mechanism of mobility one. It's basically a, six bar Stephenson mechanism. So I have this, and call this 3 dash. Let's say this angle is gamma. So I have two loops, in this problem. So call this 1 dash. Okay? So one loop is, 1 2 3 4, the other loop is 1dash 4, 3 dash 5 & 6, I can even take 5, this way. No particular reason, but you can do that. Okay?

So if I write the two loop closure equations? I have, let me write the two, R 2 plus R 3, these are all vectors, equals R 1 plus R 4. Okay? This is the first loop. And the second loop is, r4 plus R 3 dash, equals, plus r6, plus r5, as my second equation. Okay? So assume the, position analysis is done. Like I said, you know, analytically position analysis is difficult. But what you can do is, so for a specific position, even finding the position, of all the points in the linkage, may be a challenge. Okay? because for this one, for instance, if I give you this angle, has the input. Okay? Knowing all the link lengths, getting this point, is still going to be a challenge. Okay? So typically what you would do is, you would say use, some other link as input and have a range of positions, you simulated for a range of positions that you are interested in. Okay? So for instance, if I, give 2 as the input and I have the position data, for the range that I am interested in, then I can just, pick whatever position I want, then to do the velocity analysis. Okay? I can write these as, R2, e power i, theta 2, R4 , e power i, theta 4, then, when I differentiate this, with respect to time, I get i, R 2 Omega 2, E power i, theta 2, plus I, R 3, Omega 3, E power I, theta 3, equals R 1 does not change, I get iR 4, Omega 4, e power I, theta 4. Okay? So this is one velocity equation. For the second one, I have R4, second loop, I have R 4, e power i, theta 4, plus R 3 dash, E power I. What is the angle for 3 dashes? I have, Yeah, so I can express it in terms of, the Angle for, so this is, angle of R 3, R 3 plus 180, minus gamma. Because this is all part of the same rigid body, so gamma is essentially a constant. Gamma plus or I could take gamma as, this angle, in which case, it would just be theta 3, plus gamma, instead of choosing gamma like this, I can. It’s just a constant angle, between 3 and 3 dash, because they belong to the, same rigid body, that's not going to change with time. This is equal to, R1 dash, e power I, theta 1 dash, plus r6, e power i theta 6, plus r5, e power I, theta 5. Okay?

This is, when I differentiate this, I don't want to call this 2. This is the position, first equation for the position, second equation for the position this, this the first equation for the velocity, when I differentiate this, I get i Omega 4, r4, e power i theta 4, plus I, R3 dash, will still be Omega 3. Right? Omega 3, R3 dash, e power I, theta 3, plus, phi minus gamma, is equal to R1 dash, does not change with time, I omega 6, r 6, theta 6, plus I, omega 5, r 5, e power i, theta 5. This would be my, second velocity equation. Okay? So these are both, vector equations. V1, all of them are vector equations, of, if I'm looking at velocity. So all my position analysis, is completed, position analysis is complete, implies all thetas, are known. Okay? Input, velocity input is omega 6 here. Okay? So I can now, if I if I look at v1 and v2, I can get two scalar equations, from V1 and two scalar equations from v2? And my unknowns, what are my unknowns? Omega 4 and Omega 5. Okay? So I have four equations, four unknowns, I can just set it up. These are my unknowns; this would be the known quantities. So if I take V 1, this is unknown. Right? So I have I, R 2, multiplied by Omega 2, I have I, R 3, multiplied by Omega 3, minus, I, R 4 and 0. Okay? From V 1, V 1 gives me this. From V 2, I have, and this side is, zero. So I should actually have 4. Right? 0, 0, then from this one, from V2, I have, iR4. Okay? Omega 4, for this position, this is iR3 dash. Okay? And Omega 5 dash, minus I, R5, when I bring this to this Side, there is no Omega 2 term and on this side, I will have, I, Omega 6R6, cos theta 6, I Omega 6, R6, i square, Omega 6, sine theta 6. Okay? Just, when I expand, each of these, you know, it gives you two. Right? So I would equate, so each of these, this is two equations, instead of splitting them up and writing in terms of cos and sine, I’ve just put it in vector form. Okay? But this is, two equations, this is two equations, 4r scalar unknowns, I can solve this for Omega 2, Omega 3, Omega 4 and Omega 5. Simultaneously solve for these two. Okay?

So analytical methods are always applicable, you can, do a simultaneous solution, for, it doesn't really matter, whether it's a complex, or mechanism, me, know, as long as you, because the velocity equations, are linear. Position is a different story. That's why you typically just simulate for range of positions and then. Okay? So this is how, you do the, velocity analysis, for multi loop or complex mechanisms, set up the equations and solve them. The thing is, if you want, you know, if you're interested in one particular position or it's the graphical is, a lot easier to do, yeah. And a lot more, you can get a feel for, ok, how is this mechanism going to move. Okay? If I give it this input, how is, my output going to be? You get a much better feel for it, than with all these equations. So this is. I will let you try the, problem that we did yesterday. With the two inputs. Same thing, you can set it up. The difference is, one of the link lengths will be changing, one of the vectors, the length will also be changing, as opposed to here, where it's all, rigid links, in the mechanism. Okay? We will move on to, acceleration analysis now. The ultimate objective, when you're doing an analysis, is to basically, like I said you know, what you're going to do with your project, is determine the actuation torque or force required, for the problem. Okay? And in this case, what you may be able to do, a quasi static analysis. So you may be able to get away with, not knowing, the accelerations. Okay? You only need the weights of the members. But you may be, but if you're operating things at high speeds, the accelerate, the inertial accelerations, could be much higher, than gravity or other accelerations, in which case, you cannot, neglect that. Those will be the major sources, for force in the mechanism. So the ultimate aim would be to calculate those inertial forces, in which case, you have to do the acceleration analysis, for the mechanism. And that can be done, only after you do the position, velocity and then, you do the acceleration analysis, because the ultimate objective, is your inertia force analysis.

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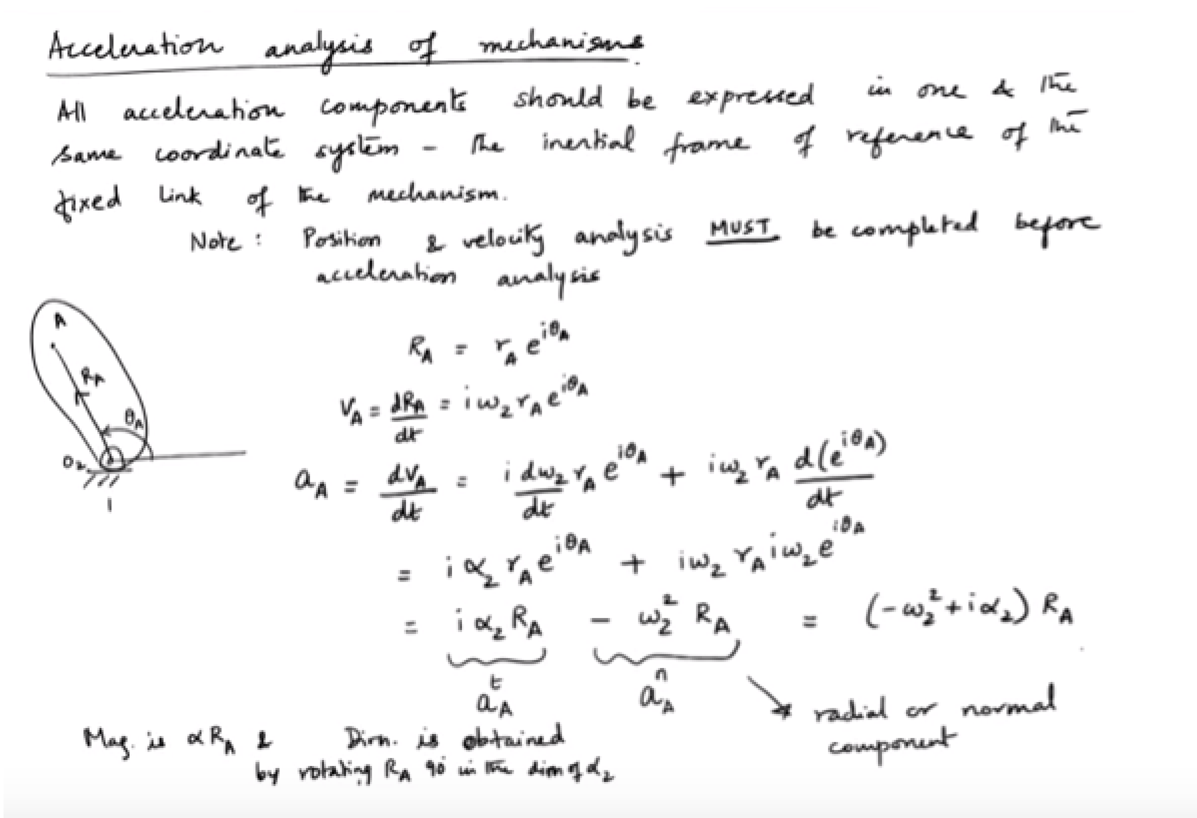

You need to determine, the forces, because you need to be able to see whether the, links that you have designed, can withstand, the stresses, that you subject them to. Yes, Jay.

So one of the important things when you're doing acceleration analysis, is to ensure, that all the acceleration components, are expressed in, the same coordinate system, which is an inertial coordinate system, which is a coordinate system fixed to the, usually the fixed link of the mechanism. So all, because Newton's law, applies, only in an inertial frame of reference, of the fixed link and the path of the point of interest, should be known, in this frame, in order to be able to find the acceleration. So we will look at, you know, similar to the velocity analysis, we will look at, accelerations of points, that are on the same rigid body. We will see what is the acceleration difference, between two points, that are on the same rigid body. We will also look at cases, where the two points, may be on different links. They may be and how we would then, express the acceleration of those points, depending on the relative motion, between the links. Okay? So let's first look at, the simplest case, of a single, of a link, rotating about a, fixed pivot. Okay? So note position and velocity analysis must be completed before. That is, at least for the position of Interest. Okay? Because there will be quantities, from the position and velocity analysis, you will need, for the acceleration. They need to be known, before you can solve for the unknowns in the, accelerations. Okay?

So let's take the case of, A point a, on this rigid body. Okay? That's defined by this vector, I theta i. So we saw that the velocity of A, would Be, Omega 2, I, Omega 2, RA, e power, i, theta A, because the velocity is dRA by dt. All the capitals are vectors. The acceleration of a, is defined as, the rate of, change of the velocity. So if I differentiate this. Okay? In this case RA is a constant. Right? Because O2 A, is part of this rigid body, RA is a constant. I get, I, d Omega 2, by dt, rA, e power i, theta A, plus, I also have this term here, so i Omega 2, r A, d of e power i theta, i by dt. So this gives me, i. alpha 2 , the, change in the magnitude of Omega 2, is called, 'The Angular Acceleration, Alpha 2'. rA, e power I, theta a, plus, this is I, Omega 2 rA, this is again Omega 2, i Omega 2, e power i, theta A. Okay? This, so this gives me, I, alpha 2, I can write this as the vector RA, minus, because I, into I, minus 1, Omega 2 square, R A, this. Okay? This is your familiar, tangential Acceleration, tangential component of the Acceleration, of a, and this is your, normal component, of the acceleration of a. So again tangential component is, a scaled version of RA, rotated 90 degrees, in the direction of, alpha 2. So for planar mechanisms, you can take alpha 2, as plus or minus, depending on whether it is, counter clock wise or clock wise and you have so the direction is rotated, Phi by 2, is, so magnitude, is alpha times, RA and direction is obtained, by rotating RA, 90 degrees, in the direction, of alpha 2. Similar to, what we got for the velocity, there in the case of velocity, you had the position vector, rotated 90 degrees, in the direction of Omega 2. Here it is in terms of alpha 2. And this is your radial or normal Component. And it is along RA, but directed towards the center of rotation. So it is opposite to the direction of RA. This you would have seen, several times, when in earlier classes. So this is the expression. So I can write this as, minus Omega 2 square, plus i alpha 2, RA. So the tangential acceleration, the i alpha 2, the alpha 2R component, is because of the change in the magnitude, of Omega 2. Okay? So if it's rotating, if something is rotating at a constant angular velocity, you do not have alpha 2. Alpha is going to be zero because the magnitude of Omega, is not changing. This arises, because of the, change in the direction of the velocity vector, change in the direction of the velocity vector.

So that's why, even if something is rotating, at a constant speed, constant angular speed, you still have, an acceleration because the direction of the velocity vector, keeps changing. Okay?
And that component is the radial or the normal component, centripetal component, also. This is due to the, change in the direction, of the linear velocity, of that, point.

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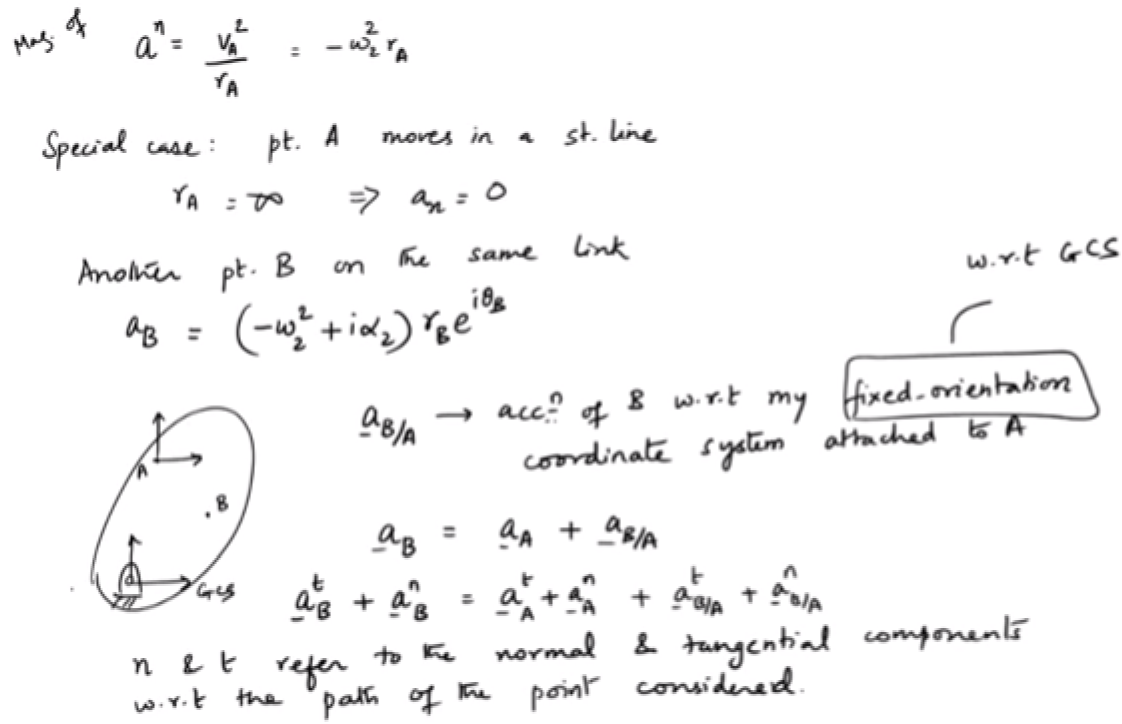

The other equivalent form, for the normal component is, VA square, the magnitude of that would be VA square, by rA. Okay? This is, is also equal to minus, omega 2 square, rA. So the special case when point a, moves in a straight line, what is rA? rA is infinite, which implies the normal acceleration is zero, from here. Okay? There's no rotation. Right? But from Here, you can have a particle, sorry, you can have a body, a point moving, along a straight line, with a particular velocity, and its normal acceleration, will be zero. So it's only component, is a tangential component, which makes sense. Right? Which, whatever direction, the particle is particle or the body is moving, is linear motion is an example of this, where the radius of curvature, is infinity, so similarly if I have another point B, on this link. Okay? Then, it's absolute acceleration, a B, equals, minus omega2 square, plus, i alpha 2, into rB e power I, theta B. Okay? So the direction of its tangential, even though, it's the same, this factor is the same, rB is changing, as well as, so, so there is an acceleration difference, between the two points, even if, the link is rotating, at a constant angular velocity. Okay? so even if alpha 2, is 0, Omega 2, so alpha 2 and Omega. Omega 2. are for the link. but specific points will have different accelerations because that will depend on, where they are located, on the link, so there is an acceleration difference, between B and a. And what that means is.

Suppose I am sitting at point A, what the acceleration difference means is, and there is a coordinate system, that is located at a, but, does not change its orientation, with respect to, the global coordinate system. So if this is the global coordinate system, attached to the fixed link, if I have, a coordinate system, that’s pinned at a, which does not change, its orientation, with respect to the inertial coordinate system and I observe the acceleration of B, from that coordinate system. Okay? So I am sitting at a. Okay? I'm a particle sitting at a, and I'm observing the acceleration of B, from a coordinate system, that remains parallel, even as the link is moving, this does not change its orientation, with respect to the ground coordinate system, if I observe the acceleration of B, that is the difference, acceleration difference, that I am talking about. So what is that acceleration of B? So the acceleration of B, with respect to a, is the acceleration of B, with respect to my fixed orientation, coordinate system, attached to A. It's a fixed orientation, with respect to, the ground coordinate system. It does, it remains parallel to the, ground coordinate system, it does not change its orientation, with respect to time. Okay? Typically you take them as parallel. Okay? So this would be, so I can write this as acceleration of B to A. So each term, has two possible components. So this would have a tangential component and a normal component. This would again have, a tangential, that we saw, each of these accelerations, has a tangential and normal component.

Similarly, there's really no difference, you know? It's like, my sitting at o2 and observing a, was now, I'm sitting at a and observing B. So this relative acceleration, will also have, a tangential component and a normal component. So in the case of two points, that are fixed, on a rigid body, the thing is, the tangential direction of this, is not the same as the tangential direction of something else. Okay? So you can't just club. I can't say I will equate all the tangential components. Okay? So they have different, directions. Okay? Tangential component of B, is along a different direction, so it's relative, and we talked about tangential and normal, it's with respect, to the path of that particular point, Tangential to the path of b, normal to the path of B. Similarly for a and for B, relative to a. Okay? So this, so n and t, refer to the normal and tangential components, with respect to the, path of the points considered. Okay? Because, a and B, are points on the same rigid body. The reason that, this acceleration difference also has, the same form, is because, if I sit at a, we can only move in a circular path, with respect to a. We cannot move, you, we saw that with the velocity. Right? The velocity component is necessarily, perpendicular to, a B and from that, you see that, the acceleration will again have a tangential component and a normal component. Because the motion of B, relative to a reference frame, that's fixed at a, is the same as or similar to, the motion of B, about o2 or the motion of a, about O2. So you will have the same two, tangential and normal components. Okay? So, let's look at, we will. Yes, yes, towards a, towards a, it's always towards the center of rotation. And the tangential acceleration will be in the direction of alpha 2, 90 degrees, so the, this vector a B. Okay? This rotated 90 degrees, in the direction of alpha 2 and scaled by alpha 2. Okay?

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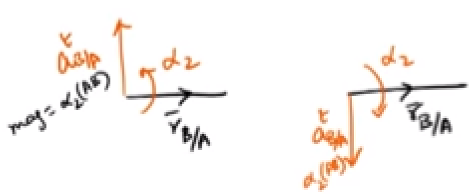

So if I have, if this is, R of B, relative to a. Okay? And my alpha 2 is, counter clock wise? Then this would be tangential component of, this thing. Rf r, and it will be scaled by, so this will be, magnitude will be, alpha 2r, here or I can just say AB. Here if this is the vector and my alpha 2, is clock wise, then this would be my acceleration, tangential component ,alpha 2, into AB. Same magnitude, different direction, depending on the direction of Alpha 2. Okay?