Lecture – 3

Theory of Mechanisms

Mobility of Mechanisms, Grubler's Criterion and Applications

The last class we derived the planar mobility criterion and then Gruebler's criterion is, nothing but

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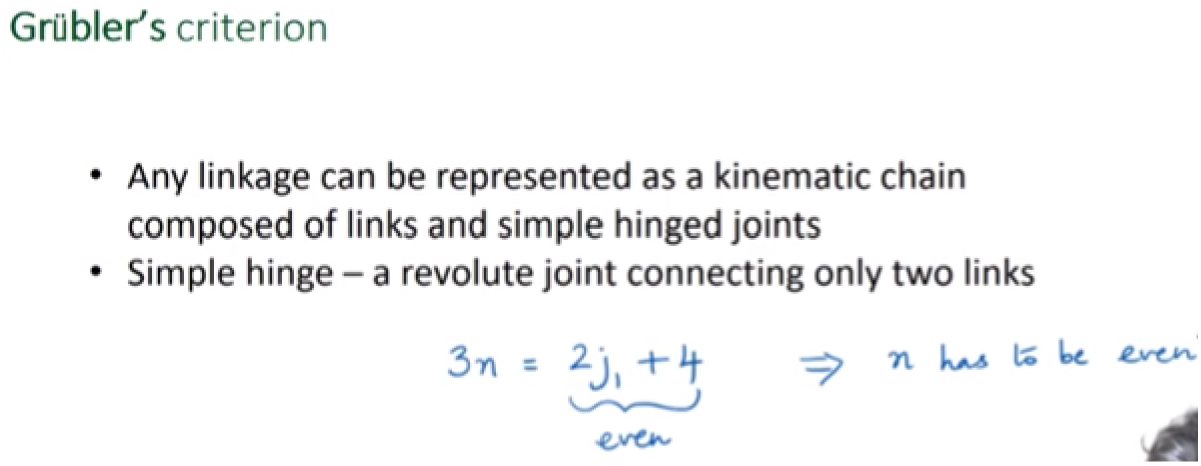

the mobility criterion for mechanisms with mobility equal to one and only single degree of freedom joints it's just a special case of the planar mobility criterion and the first thing that we saw from Gruebler's criterion is that, since this number is an even number, n has to be even. So if you want to create a mechanism with mobility equal to 1, you need an even number of Links. That's the first observation that we have from this ok.

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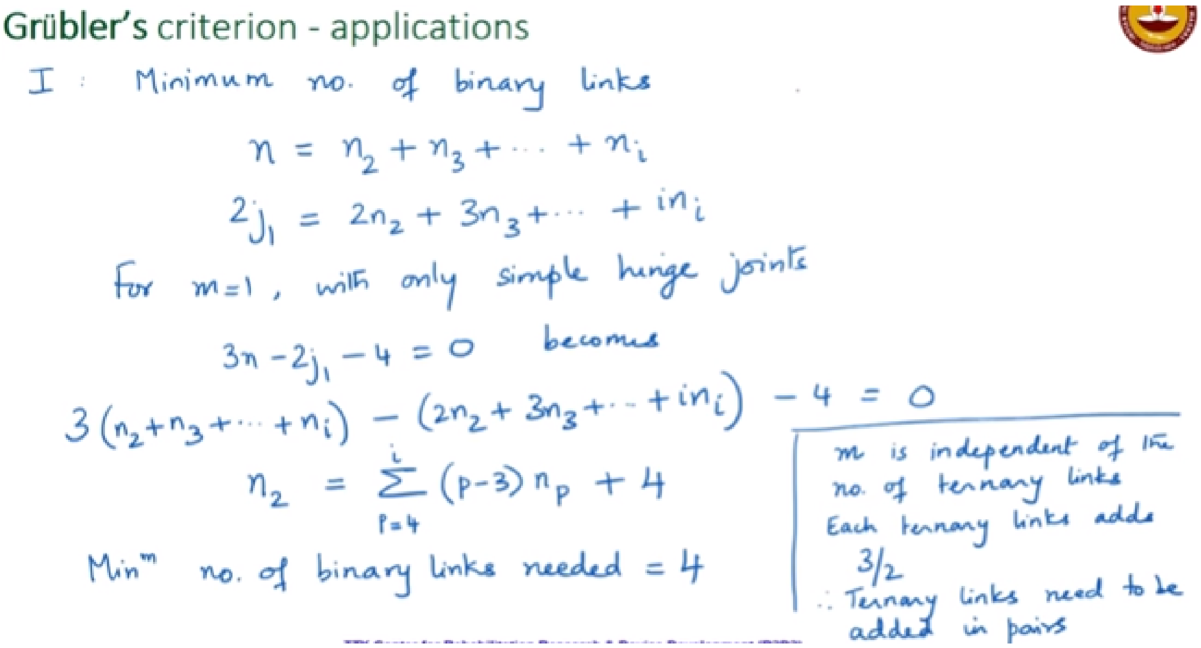

Now let's look at some applications, of Gruebler's criterion, because we looked At, so the first application is, what is the minimum number of binary links we need? And for this we go back to the form of the criterion so we say n, is equal to what did we have there? It's a closed chain so no singular links you have n2 plus n3 plus a link of order i, ok. Order i, meaning it has i nodes. Okay so we have that's your total number of links and then I can say that 2j1, equals, to 2n2 plus, 3n3 plus, so on up to ini. Okay remember that this applies only when you're dealing with simple hinge joints. So you have to be very careful if if you are trying to find the mobility of a mechanism, using this kind of an approach okay, so otherwise you could go wrong, because in many real-life mechanisms, you would have joints that have multiple links connected, you would have a hinge joint a compound hinge joint, okay so you would not have, where you have to count multiple joints. So don't use this unless you ensure that all the links have only simple hinge joints are connected by only simple hinge joints. okay so for m equal to 1, we can with only simple hinge joints I can write this as, 3n minus so, 3n minus 2j1 minus 4, equal to 0, Right, that is my Gruebler's criterion Becomes, 3 into n2, plus, n3 plus so on up to ni, minus 2n2 + 3n3 + ini, minus 4, equal to 0, ok.

So now what happens? I can write this as if I 3n2 minus 2n2, so I get n2 ok and if I take the other things to the other side, 3n3 & 3n3 all the ternary links are out, of this equation. So I get Sigma P equal, to 4 to i, P minus 3, nP plus 4, okay. So P equal to 4 to i meaning, links of order, 4 and above. So couple of things, I see from here from here what does this tell me about the minimum number of binary links I need to create a single degree of freedom mechanism, minimum number of links I need is 4. So minimum number of binary links needed, equals four, which is why your four bar is the simplest mechanism with mobility equal to one. Okay, so that is the first observation, the second observation is. The mobility equation in this form is independent of the number of ternary links, because the ternary links went out of the equation. So the mobility is independent subject to certain conditions, independent of the number of ternary mix. Why do I say subject to certain conditions? if I add one ternary link okay, how many joints does it add to the mechanism? Each ternary link adds how many J ones?

No, no in in the equation you get three by two, each one is shared with, say, if I add one link, I'm actually adding three by two, joins to the mechanism. So I cannot have a mechanism with fractional number of joints. so ternary links have to be added in pairs in this case, okay. Therefore ternary links need to be added in pairs. this is stills, you will still have to look at the entire mechanism you know, look at this thing to ensure that but the addition of ternary links in pairs alone, provided all the other things are met, will not affect the mobility of the mechanism, that's what this equation tells you, okay so if you if I add ternary links in pairs then, I still get an integer number of joints and therefore I should still be able to make a mechanism with mobility equal to one.

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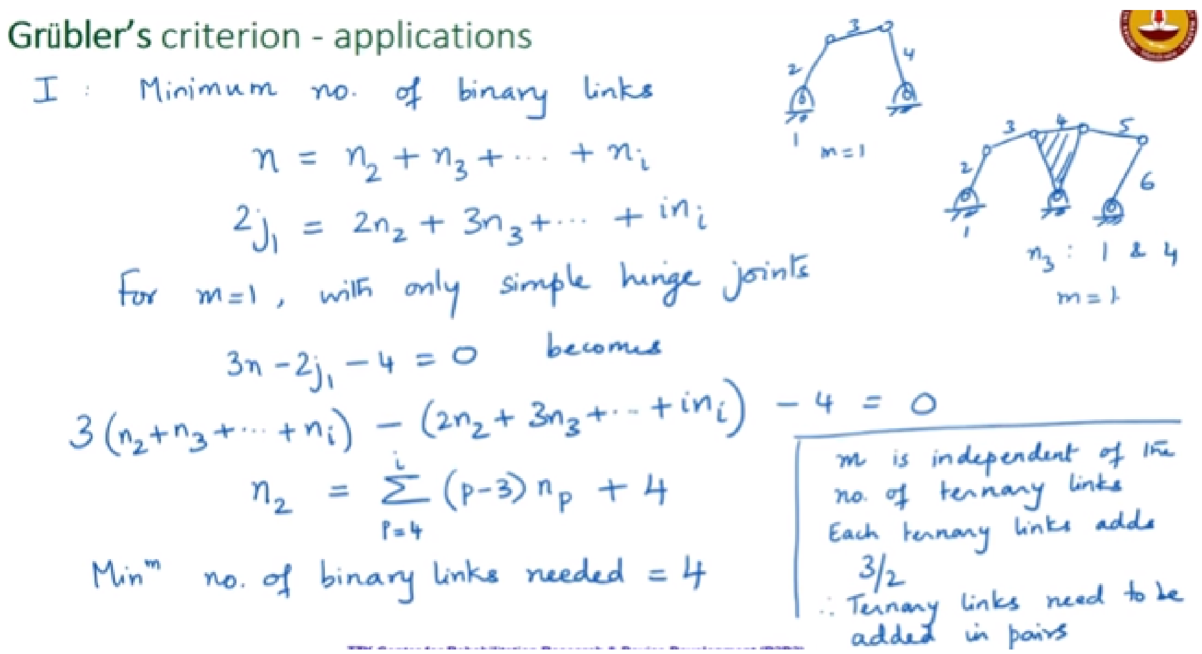

So if you look at a four bar, to a six bar, you will see that, I have a four bar, I have, so this is one, two, three, four, all simple hinge joints. Which are the two ternary links? so let me, one, two, three, four, five, six, ternary links are one and six, sorry, 1 & 4, 1 & 4, 1 & 4, are ternary links the mobility remains unchanged, mobility of this is also 1, mobility of this is also 1m okay. So this is an example where you add a pair of ternary links and the mobility of the mechanism is not affected.

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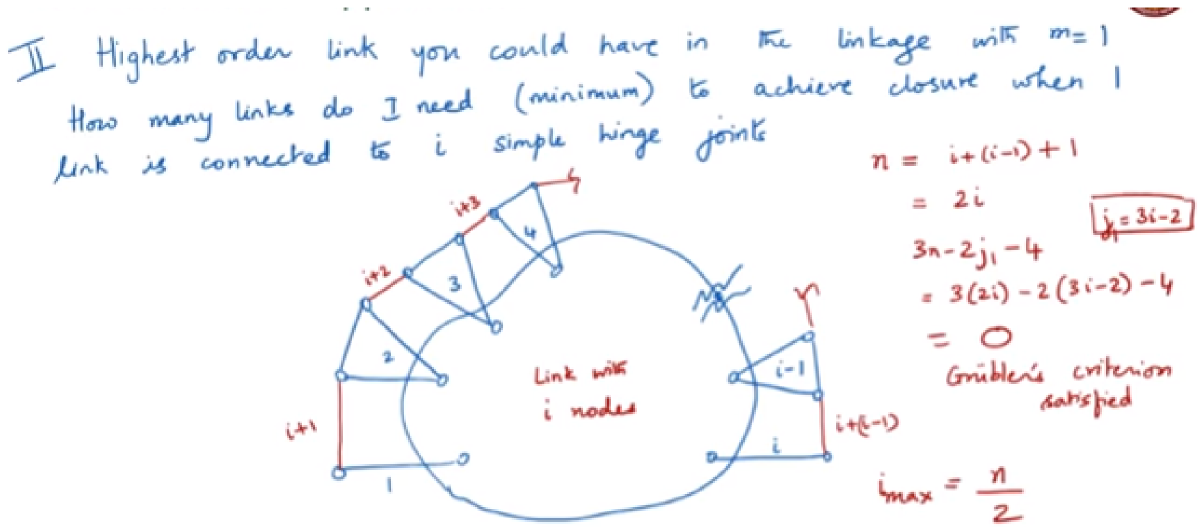

The next thing that we will look at is, okay so for a link of mobility one, sorry, for a linkage of mobility one, what is the highest order link you could have in the mechanism? M equal to one, so let's look at it from the, will approach this problem indirectly. So what we'll say is, okay I have a link, with inodes. okay let's say they discard this, okay, this thing will not let me cut it there, it is part of it, so I'll just go with this, okay, so let's say I have a link with inodes, so I want to find out, how many links do I need minimum number, okay to achieve closure, when one link is connected to i simple hinge joints, okay. How do I make a mechanism out of this? Basically I'm looking for a closed Kinematic chain, if I have one link, which has inodes, what is the minimum number of Link's I would need in order to achieve a closed chain? So let's start off with start connecting these two each of the inodes to links, okay. So start from, so I start with one, two, three, four, this is i minus one, i. so these are the links connected to the inodes, okay. now first link and i'th link, I can choose binary links, the reason I choose two, three, four, to be ternary links, so that I'll have simple hinge joints, when I try to close the kinematic chain, okay. Now let me say that okay, I connect, so Then this, this, this and so on, okay here I'll have something connected to that something coming from here, okay to form a closed chain. This is my link with I hinges or inodes yeah, so how many links? now count the number of links that I have, I have i links here, ok, so then this is i plus 1, this is i plus 2, it linked ok this link would be i plus 3, therefore this link will be i plus i minus 1, ok.

So what is my total number of Link's now? n equal to, i plus i minus 1, that's the total number of links plus 1, that gives me n equal to 2i. So if I have a link with inodes, I need a minimum of two i, to make a single degree of freedom mechanism, now I don't know yet that I have a single degree of freedom mechanism. so let's check Gruebler's criterion, okay. So 3n minus, 2J1 minus 4, should be equal to 0, so 3n is 3 into 2i, what is the number of joints that you have? Count the number of joints, what do you get there? 3 yeah, yeah, that would give you three i minus 2, 3i minus 2J1, equal to 3i minus 2, so I get minus 2 into, 3i minus 2 minus 4, so Gruebler's criterion is Satisfied. So I do get a single degree of freedom mechanism, with this 2i links, okay .so that tells me that if I have n links, iMAX can be what? f I have n links in the mechanism, iMAX can only be n by 2, the reversal, right. If I have, if I want to use a link with inodes, I need to have minimum 2n, 2i, number of links, if I have a mechanism with end links, imax can only be n by 2. Okay, so going back to my 6 bar, 6 links the maximum order of a link, in a 6 bar can be 3, so you can't have more a link of a higher order than a ternary link, in a 6 bar mechanism, okay.

Similarly in an 8 bar, if you want a single degree of freedom mechanism, a quaternary link is the maximum order of the link that you can, okay. So this is so this form of the Gruebler's criterion, gives you these additional insights, into when you when which we will use when we look at the next application which is basically, number synthesis.

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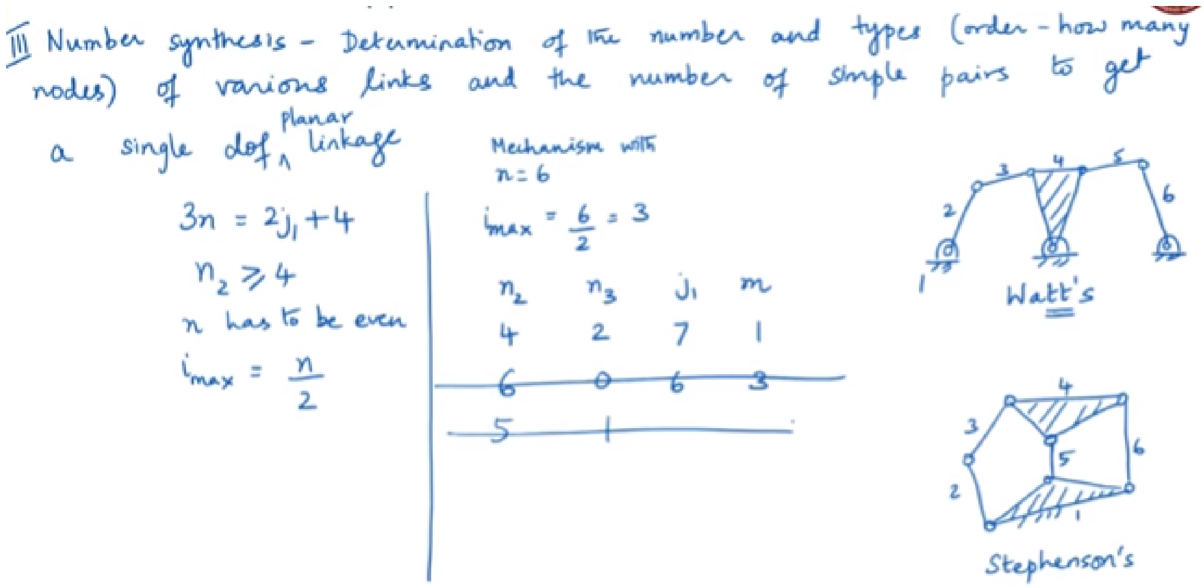

so number synthesis, is essentially finding the number of links and the order of the links, that you require in order to make a mechanism with a certain mobility. so this is just, you know okay. what kind of links do I choose? I have a box of links I want to create a mechanism with certain mobility, how do I choose my combinations of links in order to do that? so number synthesis is the determination and types, types meaning order, that is how many nodes, each link simple pairs, to get a single degree of freedom linkage, planar linkage, here we are looking at. so we already have, so we know 3n has to be 2J1 plus 4, okay we know n2 should be greater than or equal to 4, we know n has to be even, right and we know i max equal 2 n by 2, okay. So what combinations can we have if we want a design say a six bar? so four bar is pretty straight forward because I can only, I need to have a minimum number of four binary links, binary is the maximum I can have, in a four bar the number of nodes, so four bar is done, let's look at a six bar, okay. Let's see how, so the next one is, because n has to be even, n equal to 6 okay, mechanisms.

Now you may ask if single degree of freedom is what we want why not just use a four bar all the time, in most cases you do end up using a four bar, but in some in many cases you will also need to augment, you would need more links because you could get more complex motions, you could get, those of you who have seen before, sometimes if your 4 bar cannot do full rotations, you will need to add a couple more links and make it a six bar, in order to be able to run it with a motor, etcetera. so in many applications six bars, are quite widely used, you made the start of by designing the four bar and then make it a six bar, in order to just even be able to run it with a continuous input, okay. So with this we know when mechanism has i equal to six, sorry, n equal to six, i max is six by two, three, okay. So what are the options? it has to follow all these conditions, three n, it has to satisfy the number of joints, have to satisfy the mobility criterion, minimum four binary links, so what are the options so I can have, n2 and n3, let's say okay, no n4, is possible. So I have n2, n3, what do I get? my possibilities are I can have 4 and 2, that's the minimum number I need, I can have 2n3 so calculate j1, calculate mobility, the other option is again n has to be even, so n2 can be six, n three can be zero, those are my only two possibilities, yeah I think so what do you have for J? you get j1 equal to seven here, you get M equal to one here, you get j1 equal to, okay you can just solve simultaneously and then you you'll find that this is the only solution, basically, I think here you get j1 equal to six and mobility equal to three, so this will not be an option, okay. So your only option for a six bar is four binary links, 2 ternary links, okay and that's how you get the two configurations of the six bar, which you have seen before, one is the, do you remember? Anybody? So you have two ternary links, this is the Watts configuration. Watts configuration, the two ternary Links, are directly connected to one Another, that's one way to remember, direct connection between the two ternary links, in the Watts chain. In the Stephenson’s chain, the other chain is the, they are separated by a binary link.

The chain itself is called the Watts Chain, of the Stephenson’s chain, depending on which link you fix, you can, this could be fixed, or any of the others could be fixed, as we will see when we come to inversions. So Stephenson’s chain, the two ternary links are separated by a binary link. So you have again 1, 2, 3, 4, 5, 6, again j1 will be 7, in both of them mobility equal to 1. You can also think of the Watts as 2 four bars, one 4 bar chain, the output of one 4 bar provides the input to the other 4 bar. so 1 4 bar driving another is a watt strain, which is what you will commonly use when you do the driver diode synthesis. again here 5 is not, we could have tried 5 and 1, but again that's not possible, because J1 will be ternary links have to be in pairs, again so this is also not a possibility, because you will get a fractional j1, in this case.

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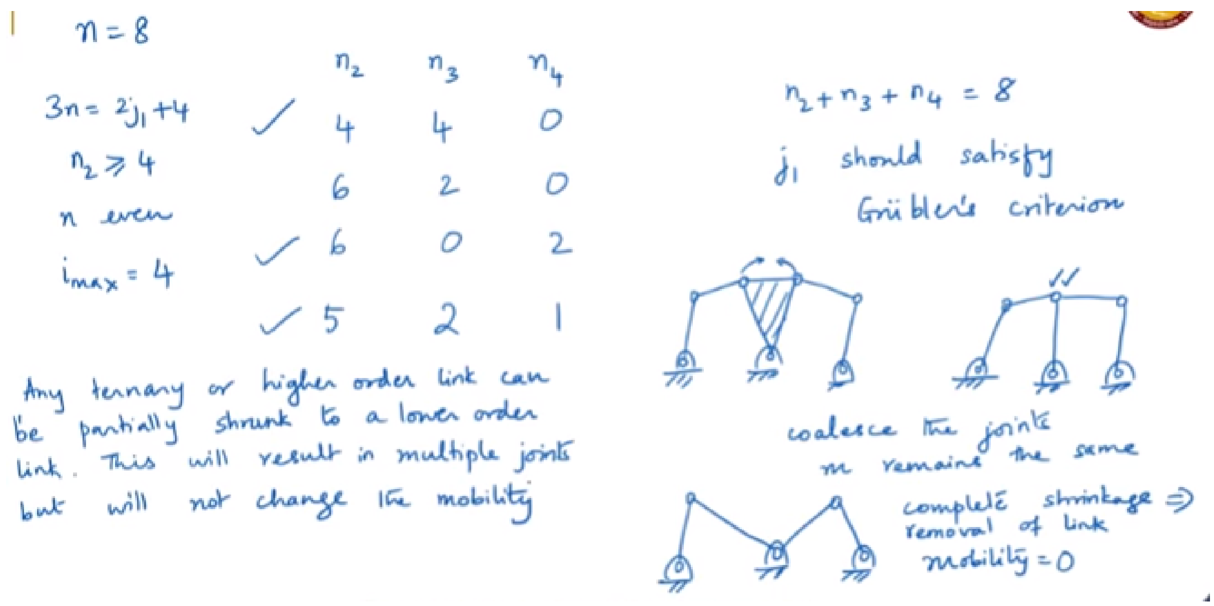

Now let's look at the next n equal to 8. Let’s do one more, see what options you get. so again imax is here 4, for n equal to 8, so I can have n2, n3 n4, 4, 2 and or 4,4,0 let's say 6,2,0 zero six zero two combinations which one which ones are possibilities, say basically solving n2 plus n3 plus n4 equal to 8, subject to these conditions and j1 should satisfy Gruebler’s criterion. Can have into, five, two and three, one and four, possibly, so tell me what are the possibilities? This one is possible, hmm 6, 0, 2, is possible, okay. So this is how you would do the number synthesis that would be the first step. Five to one is also possible, so this is an example of number synthesis. So the numbers and you know possibilities grow very rapidly once the number of links grows, okay. So the options become very many, in many cases, you don't really look at and in many cases also you're actually doing it as building blocks, you know, you take a four bar, then maybe you add two, more links,to drive it or you know add another parallelogram mechanism, to it mechanic and a bar etc.., so the mechanism is that you will encounter in real life can typically be broken down, into smaller units, you will see that even though overall if you count the number of Link's, it'll be an, a it's very rarely that you say okay I want to design an eight bar and you go about doing it this way okay. now once you get this, so this is for everything having simple hinge joints, okay, but you can sort of modify, you can shrink links, or you know make nodes, collapse and that will sometimes change the mobility or it will keep the mobility the same, so an example of something where, you collapse a joint, okay, so this is your Watts chain, right, I could collapse these two joints, bring them together, okay and do this, same thing, only thing when I'm calculating the mobility, I have to make sure that I calculate 2j ones here, so I collapse the joint it didn’t change the mobility of the mechanism, okay. So, so basically I coalesced the joints. Okay so, did I get the spelling right?

And M remains the same. Okay if I completely shrink a joint, okay then I mean if I take this entire link off and directly connect this, then what happens? It’s essentially removing the link so, what happens to my mobility here? Mobility becomes zero, so complete shrinkage, mobility becomes this link is shrunk, right. So from mobility you lose mobility, okay. So here mobility equal to zero, complete shrinkage leads to reduction in the mobility by one. So this is basically complete shrinkage implies removal of a link. So if any ternary or higher order link can be partially shrunk and to a lower order link, which will result in multiple joints, but will not change the mobility, okay, so that is the case here.

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Inversions and the Grashof criterion

- Inversions obtained by fixing a different link in the kinematic chain
- Relative motions between two links remain the same
  As many inversions as there are links – inversions may be distinct or non-distinct
- Grashof criterion for a 4R mechanism – gives the range of motion
- Shortest link s, longest link l, other two links p and q
  If $s+l < p+q$, then it is a Grashof mechanism and at least one link is capable of a complete rotation   Grashof – Type I
  If $s+l > p+q$, then non-Grashof or Grashof Type II
  $s+l = p+q$ – Grashof neutral.

Okay, so the next topic, so this kind of concludes the mobility equation and you know some applications of it and how you use it for number synthesis, okay. So now once we have a mechanism ,we look at the concept of inversion, you would have all seen this already, essentially when you have a closed kinematic chain, you if you fix a different link in the kinematic chain, the relative motion between the, links does not change, but if you look at the motion with respect to the fixed link, it will be different, okay but the relative motion among these links does not change, so an inversion is basically so if you have a four bar linkage, my possibilities are four inversions, because I could fix a different link in the kinematic chain each time, the inversions could be what are known as distinct inversions, or non distinct inversions, as many inversions as there are links. Inversions maybe, distinct or non Distinct, we will see what that is and for a four bar, or a 4R, okay 4R, four revolute joints, typically a four bar is referred to, as 4R mechanism, a slider-crank, as I mentioned earlier, would be a 3R,1p mechanism.

So the grashof criterion, is a geometric criterion, that gives the range of motion, for a four bar mechanism. Say if you have the four links, and you classify them as the, shortest link, longest link, and the other two links, you call them P and Q, then if s plus L, is less than P plus Q, then it is called a grashof mechanism, or some literature refers to it as grashof type 1, if it satisfies the grashof criterion it's grashof type 1, and then at least one link, so it tells you that at least one link is capable of a complete rotation, if s plus L, is greater than P plus Q, then it is called a non-grashof or grashof of type 2, a non-grashof is also called a grashof type 2 and the special case where s plus L, equal to P plus Q, is called a grashof of neutral mechanism, grashof neutral linkage. and for this you only need the link lengths, you don't even have to put the mechanism together the grashof criterion tells you, that with a certain set of links with the link lengths, you can predict whether, any of the links in the in the mechanism is capable of making a complete rotation and then depending on which inversion you are looking at, the nature of motion, of the links will change, ok the relative motion always remains the same so but if I fix a different link in a kinematic chain, I get a different

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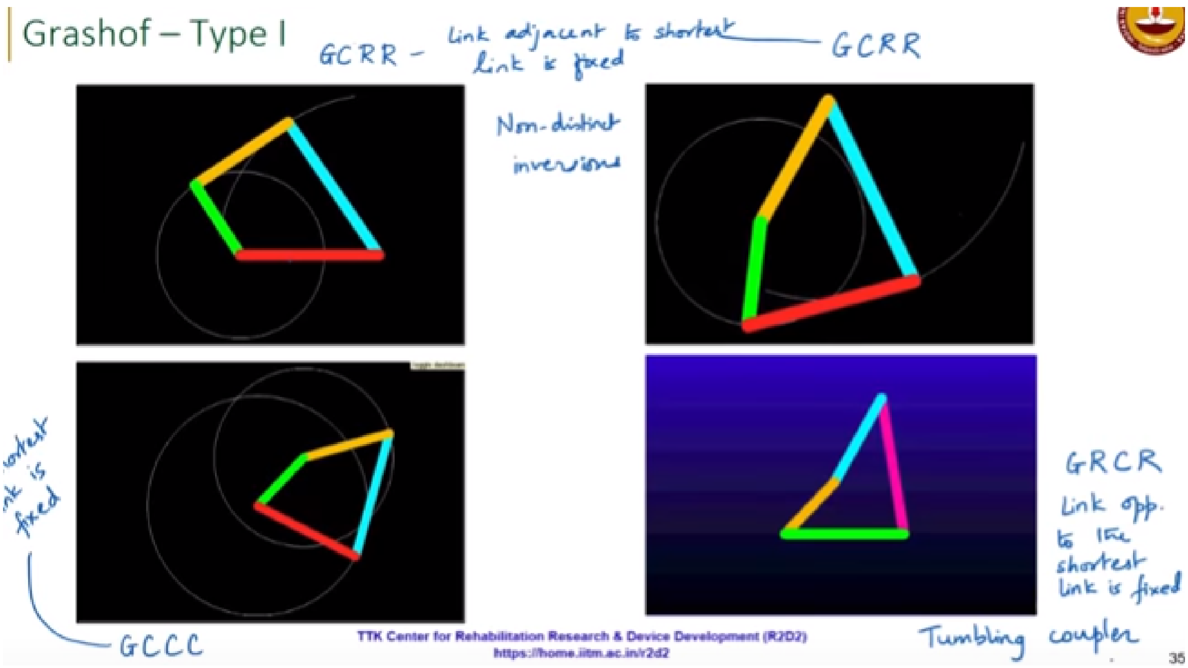

Behaviour with respect to the absolute frame, so in all these you can see that the relative configurations of the link is the same. Right? If the link next to this, so the green link here is the shortest link, okay the link lengths are chosen here, such that it's a grashof type one mechanism and if the link next to the shortest link, is fixed okay, then the shortest link is capable, of in all cases the shortest link is the one that is capable of the full rotation, here you can see that, the shortest link makes a complete rotation, the green link and the other two links, kind of go back and forth between two limits, okay. So you can see the blue rocker the limits of motion of that rocker, okay. So this is a this is called a crank-rocker, okay, or it's called a grashof crank-rocker, okay, which means this first link, so you have a fixed link, the link next to the fixed link is your crank, makes the complete rotation, the other two are rockers, in the case of this one.
So next to the shortest link I have the red link or the orange link, if I fix the orange link, now again the green link makes a complete rotation, the blue link rocks between limits, okay it's not able to undergo a complete rotation. so these two still give me crank rockers, either side of the shortest link if I fix, I get crank rocker, so they are called non distinct inversions, because the nature of the inversion is non distinct, although they had net you know if you look at the output motion, the rocker here moves, this is the motion of the rocker, here this is the motion of the rocker, but they are still called non distinct inversions, okay so this is again, grashof crank-rocker rocker or to distinguish you could say, grashof rocker rocker, crank, okay, just to follow them, but it's better to say this, because that way you know it's the link next to the, shortest link is fixed, so this is also a GCRR.

The third inversion where your blue link, sorry the shortest link itself is fixed. Okay in this case you will find that all three links, can make complete rotations, you can see even thee coupler is tumbling. so this is a case of grashof crank crank crank, this inversion where the shortest link is fixed and the fourth one, where you have the link opposite the shortest link is fixed, you will see that it's the coupler that's doing a tumbling, in the plane, okay it's not rotation about a fixed point but it is undergoing a complete revolution, so it's called a tumbling coupler. So this is you have, this is the link opposite the fixed link, so you have the grashof rocker crank rocker.

Okay so here, link both of these links adjacent to the fixed link, sorry adjacent to the shortest link is fixed, these are non distinct inversions, this one GCCC the shortest link is fixed and here the link, opposite to the shortest link is fixed. So this is the case with the tumbling coupler. do you have a question? Why is it the case that? so it doesn't say you you can connect the shortest and the longest together, grashof doesn't say how you have to connect the links, it only says, so you could have, yeah, it does not talk about so you could have many more options here if you have a certain kinematic chain, the same things apply, regardless of which link you put next to which link, in terms of the dimensions of the link, so just by knowing the dimensions alone the grashof criterion tells you how it's actually based if you look at the proof of it it's just based on the triangle inequality, you look at the extreme, you look at the positions you know the sum of two sides of a triangle, should be greater than, the sum of the third side, well the sum of a side of a triangle should be less than the sum of the other two sides, that's, that's basically what the proof of this is based on and there you will find out that the shortest link is the one that will make the complete revolution. Okay but other than that you can attach the links; you can attach that longest link, next to the shortest link, no problem with that. I picked certain dimensions in order to be able to show in some you may not see the tumbling coupler very clearly for instance.