Theory of Mechanisms

Lecture – 30

Acceleration Analysis: Analytical Method Contd.

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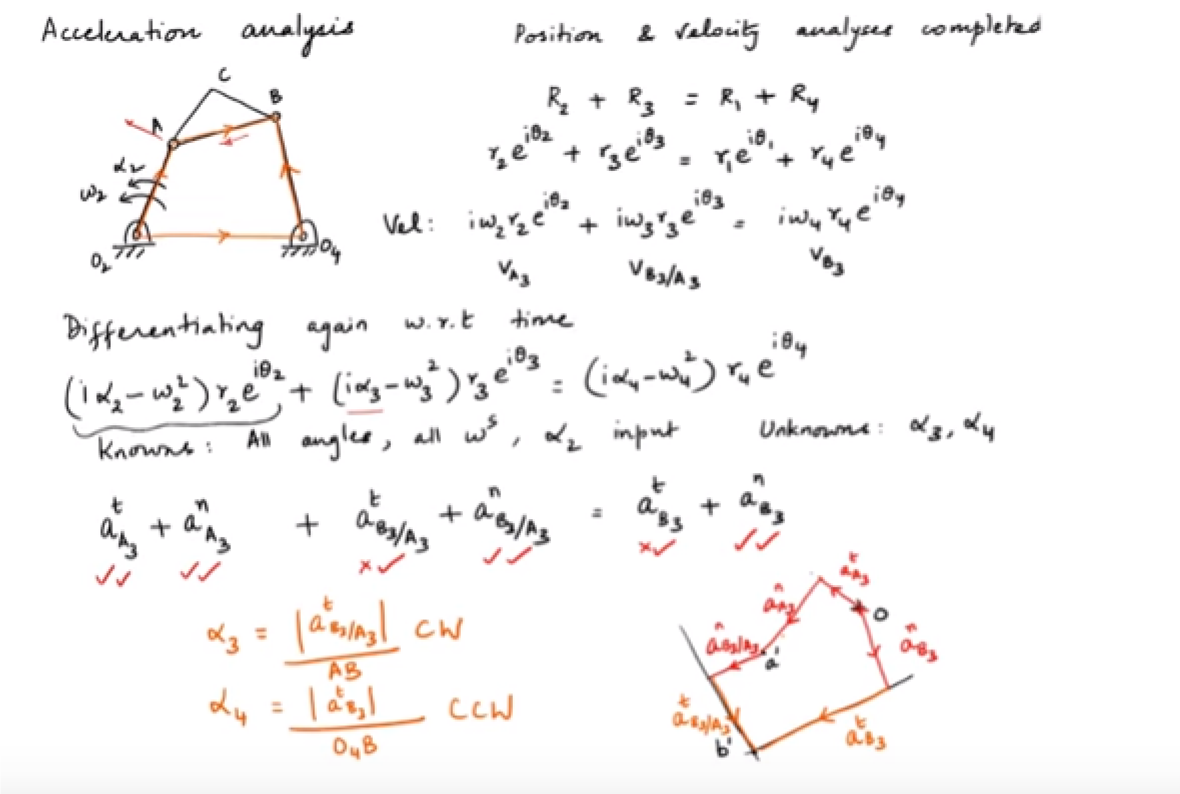

So we saw earlier, how the acceleration component or the acceleration difference, between two points on the same rigid body, has a normal component and a tangential component. the tangential component is dependent on the angular acceleration, whereas the normal component is only dependent on the angular velocity of the body. So once you complete your velocity analysis, you actually know all the normal components of accelerations of the various links. okay because it is the magnitude of the normal component is, Omega square R, right so if I have a then A. so let's so now let's apply this to the analysis of a four bar and we'll just, so we will assume the position analysis has been done and the velocity analysis is also been completed, okay. So we know all the angles we know all the angular velocities of the different bodies. Now let's say, I need an angular, I need an acceleration input, so that could be in the form of a constant angular velocity which would mean alpha 2 is 0 or I could have a finite, I could have some other alpha 2, a non zero alpha 2. Okay so we can again do our loop closure, will connect the loop closure and the acceleration diagram, but since we've already done this once for the velocity, I don't want to do it one after the other, we'll just do it simultaneously, okay. So I have R2 plus R3 equal to R1 plus R4, so R to e power i theta 2, plus R3 e power i theta 3, differentiating this for the to get the velocities I got I Omega 2, R2, e power i theta 2, plus i, i Omega 4, which basically related the velocities of points A and B on link 3, so I had velocity of A on 3, B relative to a equals the absolute velocity of B. We differentiate this equation again for acceleration and we saw, we get i alpha 2, minus Omega 2 square, R2 e power i, theta 2, plus i Alpha, 3 minus Omega 3 square, R3, e power I, theta 3, because if i differentiate each of these terms where alphas are basically D Omega by DT, equal to i alpha 4, minus Omega 4 square, R4 e power i theta 4, okay again 2 equations, one vector equation or two scalar equations, my unknowns are, known are, all angles link lengths of course because you're doing analysis so you have to know what linkage you're not analyzing, all angles you know alpha, sorry all omegas are known, because you've completed the velocity analysis and alpha 2, is input. So the only unknowns, in these equations are alpha 3 and alpha 4. so I can solve for those two unknowns, now let me solve this, so I can, I can write this as I can split this up as, a tangential of A on body 3, plus a normal A on body 3, ok Plus, this is a tangential, B relative to A, plus a normal, B relative to A, this is a tangential B on body 3, in normal. because these two, these two are absolute because they are with respect to the fixed pivot, O2 here, O4 here, that's what it means, when I don't specify the second point it implies that that is the with respect to a fixed pivot, okay velocity and acceleration zero.

So now I can plot this in an acceleration diagram. so I know so say I take let's say O, okay I have the known acceleration, so in this equation, okay let's first do this in this equation, this one is I know the magnitude, I know the direction, because alpha 2, is known and I know the direction it is perpendicular to O2a, the normal component again is completely known, because it depends only on Omega 2, here ,because this is equal to the acceleration of A on body 2, those two points are always coincident. In the case of B, with respect to a, the tangential component depends on Alpha 3, which is unknown. So I don't know the magnitude of the tangential component of acceleration of B, with respect to a, however, I do know the direction or at least the line of a, so this the magnitude is unknown but the direction is known, in this case again what is known here? you know the magnitude and the direction, because the normal component remember is completely known once you do the velocity analysis, so you know both, you know Omega 3 square, from your velocity analysis you also know the direction is directed from B to a. and here tangential component of this depends on Alpha 4, which is unknown however you know the line of action of that vector and in this case again it's completely known, because Omega 4, is known from the analysis, okay. So you see here again I have two unknowns, so I should be able to solve this vector equation, by only thing is in the velocity equation we had only three vectors to create the polygon, here we now have six, to create the polygon, the acceleration polygon, okay.

So let's start with what we know and what is absolute so if this is O, I know the tangential component of this which would be like this right, along that direction so this is a tangential, a three, known okay normal component would be along from a 2, O2, that direction that's also completely not okay so let's say that is this okay, so I know this point here completely not, this but normal component of b3, relative to a on body three, so I can add these vectors in any order right so this is known, so let me say that I can that would be this vector in this direction, right, from B to a, this would be the normal component of that, so I will say that this is the normal component of B3, a3, okay this one I know it lies so I only know the direction, so let me just do that in a different so it's perpendicular to this the tangential component is perpendicular to this but I don't know it's magnetically. then here I know these two are absolute accelerations, so I have, let's see a normal, B3, okay and the other component I know, the only thing I know is, it's perpendicular to this. Okay but once I do this now I get this point which will be, so this to this here this point is let's call D let's call it a dash, small a dash, for the acceleration, okay this point here a the whole acceleration of a was completely known it's this Plus this, so this point is a dash and I added the relative component there, so this point here will be B dash, the intersection of these two will give me B dash. So if I draw a vector from A to B dash that will give me the absolute acceleration of point B, okay. So now I know these two components. I know that what is this component here? and that is the direction because this Plus, this right the direction of this is what component is this? tangential component of B, with respect to a, here this is the tangential component of B, the absolute, so this is, so now can we find the, from here I basically solved this vector equation graphically. so now I can find alpha 3, is a tangential, the magnitude of that, by the distance AB, and what would its direction be? so it's like this it's perpendicular to AB, in this direction which means aB has been rotated the vector from A to B has been rotated clockwise, so this is clockwise okay what about alpha 4? Alpha 4 is, a tangent she'll B3, magnitude of that by O four B and what is the direction of that? So you have the direction perpendicular to OB like this, which means alpha is counter clockwise. This should be familiar to you. So this is how you solve the analytically you will just split it into real and imaginary parts, you'll have   two linear equations, in alpha 3 and alpha 4, and you would solve for those unknowns, graphically you would solve it in this manner.

Ok so although there are 6 terms in the acceleration equation, all the normal components are completely known from the velocity analysis ok. So you can solve this equation, but just as we saw for velocity image, right we saw that, I can find once I know alpha 3, I can basically find the acceleration of any point on this, ok so, so here the just as in the case of the velocity image you had, the velocity triangle, similar to the triangle in the linkage, but rotated by 90 degrees in that case and scaled by Omega 3, in this case, it's a different factor of scaling which you will find out and a different angle but it turns out that acceleration image also applies, so you'll find that, if I want to find the acceleration of Point C, all I need to do is, this reconstruct this triangle, ok in the acceleration diagram, ok and that will give me the acceleration of Point C. so the triangle in the acceleration diagram will be similar to the triangle in the position diagram. Ok so here how would I construct the accelerations? Alpha is it's a different angle, okay I'll leave that to you as an exercise to show how the acceleration image is obtained because if I go into that then I'll just stop, End up doing it anyway.

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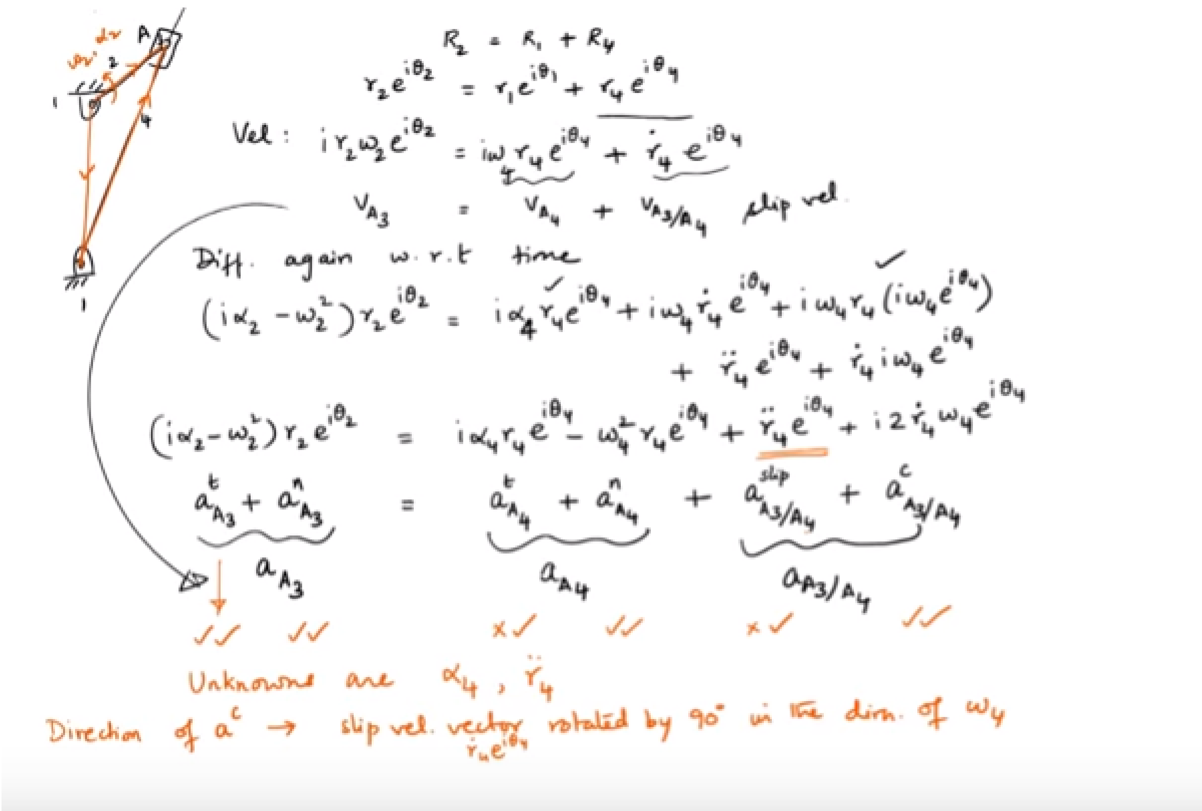

again slider-crank is fairly straightforward, you just don't have a normal component of acceleration because the slider is going to be moving linearly, so in the case of the slider block, you don't have a normal component, so in when you write that equation you'll only end up with the linear acceleration of the slider that'll be the there's no normal component there however I'll do the case where, for the inverted slider crank, okay. So let's say because that's where a new term comes in. So we saw earlier that in this case, we had okay let let's just do it with the loop closure, okay. So we have R2 equals, R1 plus R4, here I'm assuming my coordinate system like this, okay, so Thetas are all measured accordingly. so here velocity equation, when I look at that I have R2, i, r2, Omega 2, R1 is   constant and for this i had i r4, e power i theta 4 plus, r4 dot, e power i theta 4, when I differentiate this stuff because R4 is no longer constant in this case.

To close the loop you're going to have to go R4 has to change in length as you close the loop each time. so we saw that this is the velocity of a on body 3, which is equal to the velocity of a on body 4, plus the velocity of a on body 3, relative to a on body a point that is coincident at that particular instant on body 4 , ok yes ok so now we differentiate again to get the acceleration and we get i alpha 2, in this case minus Omega 2 square, R2, e power i, theta 2 equal to, here I get a similar form, now when I differentiate this now R4, is not constant, R4, is not a constant, so I will get two terms, when I differentiate this what are those two terms? i R4, dot, e power i, theta four, okay first I keep theta 4 constant, sorry here right, yeah thank you, I know something looked funny, thank you, so I have okay, so i alpha 4, R4, e power i theta 4, okay, plus i Omega 4, R4 dot, e power i, theta four, plus i Omega 4, R4, into i Omega 4, e power i theta 4, those are the three terms I get from differentiating this, yes? now from differentiating this term the second term I get, R4 double dot, r power i theta 4, so this was this slip velocity if you remember, okay this is the slips acceleration, plus r4 dot i Omega 4, e power, i theta 4, this is i alpha 2, is equal to, now if I group the terms, I will get i alpha 4, R4, e power i theta 4 and then I also get a minus Omega 4 square, right this one, R4, e power i theta 4, grouping this and this so this is taken care of this is taken care of then, I get R4 double dot, e power i theta 4 plus i 2, if I add these two R4 dot, Omega 4, i power i theta 4, Okay. so now let's look at these terms, this is straightforward, this is the last sorry, tangential component of the acceleration of a on three, plus the normal component of the acceleration of a on three and this is equal to tangential component a and 4, plus normal component of a on body 4. Now this is something different, these two terms, this is your slip acceleration of, a on body 4, okay R4 double dot, along this how is? How are the two points moving? so it's an acceleration of that a 4 relative to sorry a 3 relative to a four okay because I'm that's the slip acceleration and here this term is called the Coriolis acceleration of a this so the Coriolis acceleration appears then you have a rotating slider essentially okay right you have here you have Omega 4 and R4 dot, they're sliding with respect to us, with respect to a rotating path. okay so when you have both of those you will have this Coriolis component of acceleration, if you don't do this, I mean you can still write this equation straight away, by looking at this because this would be the acceleration of a on body three, this is the acceleration of a on body four and this is the acceleration of a on body three, relative to a on body 4.

So from this you know directly I, could I can look at this and write this okay but I have to remember what the components are, okay in this case it's not the same form for all three accelerations, when the two points were on the same rigid body I had tangential normal, tangential Normal, tangential normal. When you have a case like this where you have coincident points you have to take into account you have to remember to take into account not just the slip acceleration but also the Coriolis acceleration. Now if you see here, the Coriolis acceleration is again completely determined from the velocity analysis, because you know the slip velocity, you know the angular velocity, of this, okay. so that is completely known after you do the velocity analysis. so if I look at this equation again this term, if that is the input Omega I have Omega 2, alpha 2 say so this is your input so it's completely not Direction is perpendicular to the link o2a, this one again normal component completely known from the velocity analysis in the case of a on body four, again tangential component I don't know the magnitude because I don't know alpha 4,   Alpha 4 is not known, however I know the direction, it's perpendicular to the link. Normal component, completely known in this case the slip component, what do I know about it? if I look at this term here I know the direction it's along the link, but I do not know the magnitude of R4 double dot, so here R4 double dot, is unknown, but I know the direction for this term and for the Coriolis term, again it's completely known because of the velocity analysis, again six terms, two unknowns, the two unknowns are, alpha, what are the two unknowns? alpha 4, and the slip acceleration r4   double dot. Those are the two unknowns and you can solve for them either analytically or graphically. Any questions on this? You'll be comfortable solving this graphically? ok what is the direction of the Coriolis acceleration? it will depend on the direction of the slip velocity and omega 4, okay and it will be, so the magnitude will be like this and so it's basically this velocity vector see r4 dot e power i theta 4, is your slip velocity vector, so the slip velocity vector will be rotated 90 degrees in the direction of Omega 4, okay. So direction of Ac will be given by slip velocity vector, rotated by 90 degrees in the direction of Omega 4 that will be the direction of the Coriolis acceleration. so you have to be, so do plotting that in the graphical, will be it requires some attention, when you're doing the graphical analysis, okay and this will be along the vector, so it's sort of the same form, so this would actually be also perpendicular to this length, the Coriolis acceleration will also so these are two perpendicular components, one is along the link, one is perpendicular to the link, the direction will be determined by the direction of the slip velocity and the direction of omega. Those two will determine the direction of the Coriolis acceleration. Okay so this is for the cases that we have, again this you should be familiar with this is just a recap of what you already know. so again this velocity difference method cannot be used for certain types of mechanisms, just as we saw that the, it's obvious right if you can't use the velocity difference probably not going to be able to you use the acceleration different for the same kind of mechanism.

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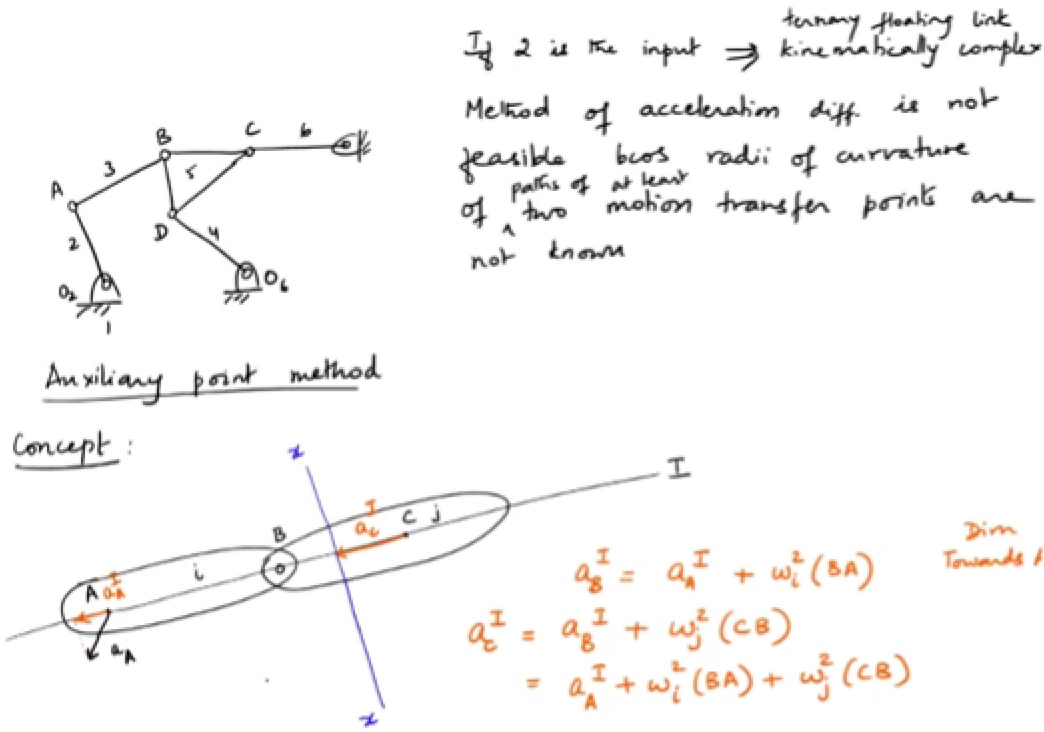

So let's look at a case where, thing to be half time what is known as a kinematically complex mechanism. So there are several methods, I am only going to do one of them because graphically for acceleration, is quite tedious, so most of these can be solved analytically, so I am just going to do one method, the graph are the auxiliary point method, for acceleration analysis. Okay, so here is a case where, if two is the input, it's a kinematically complex mechanism because I'll know the acceleration of a, but I'd know nothing about the path curvature of b, okay. If 6 or 4 is the input, not a problem, I can solve it okay that's just my four but I find the acceleration of B after solving the acceleration of this four bar loop and I can then and a is on a link that's connected to the ground so that is straightforward I can apply the acceleration difference method, but I can't do that if two is the input and using inversion and all that is a little bit more complex when you come to the acceleration analysis, okay so it's not you can't scale it directly like you do for the velocities because you saw that every point has two components of acceleration. So it's not as straightforward as the velocity analysis just inverting the mechanism. So in this case the method of acceleration difference or relative acceleration is not feasible because to apply that you need the radius of curvature of at least two points on thee of two of the motion transfer points.

Not know 2 is the input and you have a ternary floating link, therefore this mechanism is kinematically complex. So let's look at the auxiliary point method, I'll just explain the basic principle behind it and we'll do it next class so this method again similar to the auxiliary point method for velocities can be applied to most mechanisms, whether they are low or high degree, of complexity most mechanisms can be and the concept behind that see we saw that in the case of velocity the concept was that, along the auxiliary line, the component remains the same, in the case of accelerations it does not remain the same, but they can still be related so, I'll just introduce you to that and then say this is a point whose acceleration you know, okay?

You have body i and body J, you have some point C on body J, for which you are interested in you essentially want to find howdy some information about the acceleration of C. So if you look at you draw an auxiliary line, connecting a and the motion transfer point B, oka, so now if you look at this, this is the acceleration of a, along this line one, Auxiliary line one okay. What will be the acceleration of point B along this line; will it be the same as this? the velocity was easy to visualize okay velocity is going to be the same because the two points are on a rigid body and they can't move towards one another. The case of acceleration how is B moving with respect to a? o if I look at this I can write the acceleration of B along this, remember the acceleration difference between two points has a normal component and a tangential component. so I can write this as 1 plus BA okay so this is towards a, the direction is towards a, so I'm just writing it algebraically okay acceleration of B will be plus a component towards a. similarly the acceleration of C, will be acceleration of B, plus Omega J square, CB okay and then I can substitute for the acceleration of B, so this will be acceleration of a, CB. again all these are known from the velocity analysis, because once you know the velocities, so you can find, so I know one component, so let's say now from here I calculate ac along one, okay then necessarily, the head of the acceleration vector has to lie somewhere on this line.

Okay because they are the component two perpendicular components. so this plus. some other so the head of the acceleration vector of point C, will lie somewhere on this line X X, so if I know one more, if I know the acceleration of another component of the acceleration of C, okay using instead of just saying it's equal to you know again through some other point, if I can find another component of the acceleration of C, then the perpendicular to that wherever that intersects this line, is going to give me the location of the head of vector AC. So that is the auxiliary point method for acceleration. will solve a problem in the next class, but this is the basic concept behind the auxiliary point method, for accelerations slightly different from the velocity case but still quite elegant how they come up with this, okay