# Lecture - 31

# Theory of mechanisms

## Acceleration Analysis: Auxiliary Point Method

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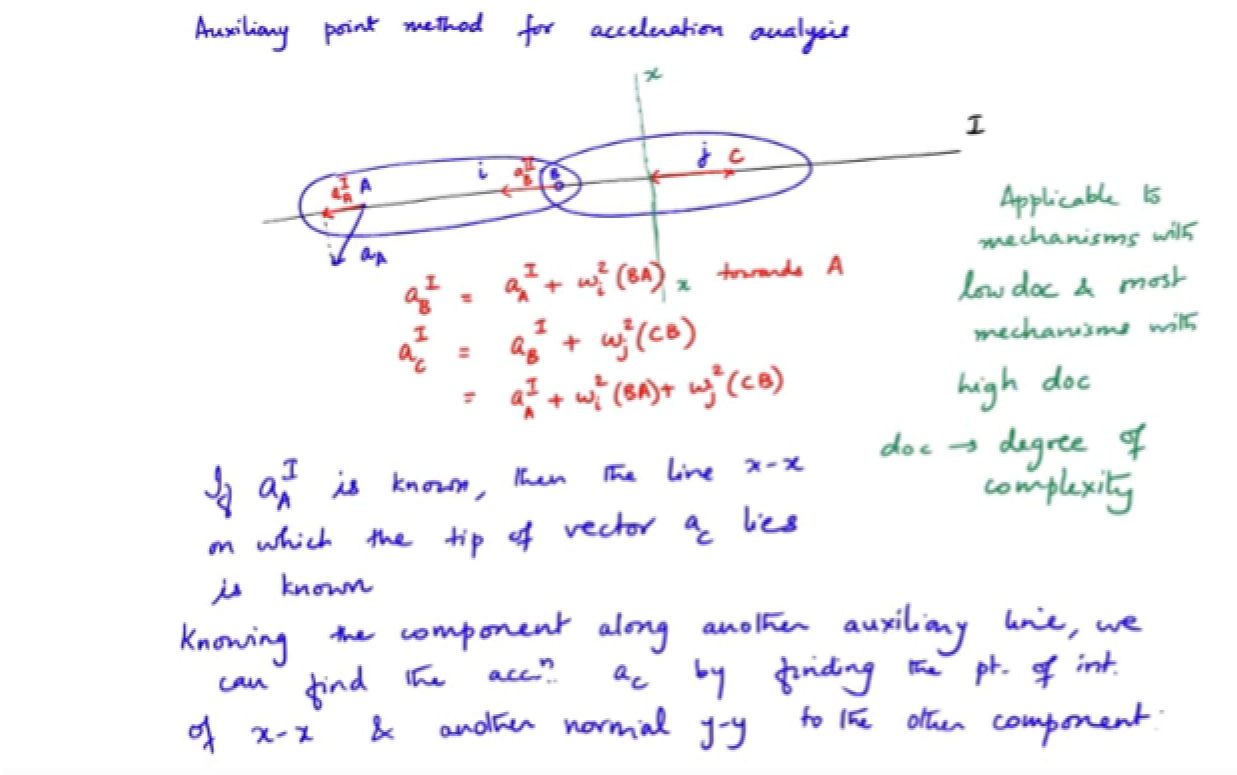


We started looking at, the auxilary point method, for acceleration analysis. So, if you have two bodies, that are connected, and if I know the acceleration of some point a. so, let's say this is I, this is J and I know the, acceleration of point a. okay? I can relate, the accelerations of point a and B along the line, that joins a and B in this manner. So, I can draw what's called an auxilary line, and then if this is, the component of the acceleration of a, along this then the component of B along this, B is the motion transfer point, is related to this one as this plus Omega I square BA. Okay? So, towards a. okay? So, I can say that, these two accelerations are related in this manner, in the case of velocities we saw that, they would be equal but ,here again because ,of the fact that a and B are points on a rigid body, we can only move along a circular path with respect to a ,which means the normal acceleration gets added by this ,there's an additional component for the acceleration along that line one, then if I take a point C on this auxilary line but, which is located on body J, I can say that the acceleration of C is this plus Omega J square C B. okay? And then I can substitute for the first one as, A,A I plus, Omega I square BA plus, Omega J square CB. So, I can relate points, along the auxilary line in this manner. Okay? So now, if I know, one component of this, acceleration I'm interested in finding the acceleration of C because, I know nothing about, body J so,if I know this is one component, then the other component, necessarily has to be sorry, the total vector ,the head of that vector necessarily has to be along this line, that is perpendicular to this component. Okay? So, that means acceleration of C is Somewhere, such that the arrowhead lies On, this line X X. so now, I have narrowed down mine Where, where the acceleration of C lies, using one component so, I'll need one more component .so, if I know the component along one other direction, and I draw the perpendicular to that, wherever these two normal's intersect, will give me the location for the arrow head for the acceleration of point C. okay ?so ,like that so, then I can determine the acceleration of point C, similarly if I can find one more point, that's also located on body J ,then I can find and if I can find the acceleration of that point, the acceleration of body J is completely determined. So, this method for acceleration analysis is applicable to mechanisms, of low and high degree of complexity, kinematic complexity. Because, all the Omegas, the normal component only depends on omega ,which is known completely known, once you've done the velocity analysis, most mechanisms that you encountered can be solved with this. Okay? I shouldn't say, DOC so, here once if a a I is known, then the line xx, on which knowing, a component along another auxilary line, we can find acceleration AC by finding and another normal YY To the other component.  Okay?

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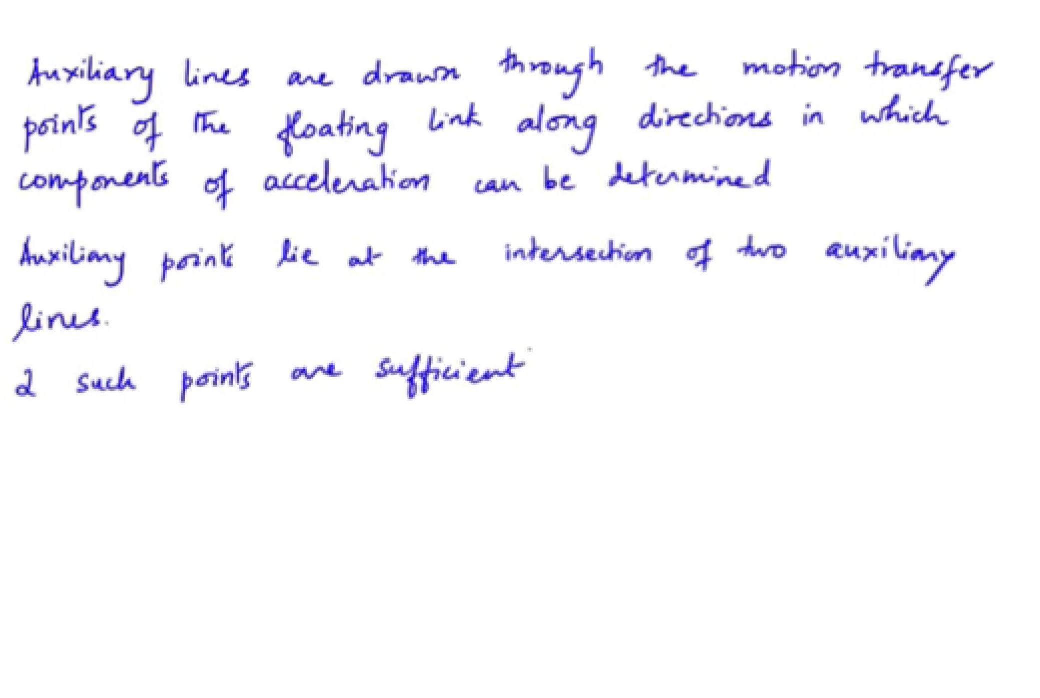


So, the auxilary lines are drawn through The and come back to it, through the motion transfer points, along directions, in which components of Acceleration**,** can be determined. So, they may be known either because, alpha is known you know either because, of the input that is given to the mechanism or they may be known because, their normal components which are determined after the velocity analysis .okay? so, those are the two cases where you would know the, and the auxilary points, which are considered as points on the floating link, of  two auxilary lines, and you only need two such points, are sufficient to completely determine the acceleration of the floating link, angular acceleration or acceleration of any other point, on the floating link. Okay? Let's do an example Yes, yes this is unique two points that's why, this is here, here you only know right? Now, I don't know the fully acceleration of this point C, yeah. that gives me the acceleration of point  C,that's not enough to give me the axle ,angular acceleration of the link, I need the acceleration of one more Point, to know the angular acceleration of the link. So, the question was is that sufficient to find the angular acceleration of the link, acceleration of one point, is not sufficient to find the angular acceleration of the link, that is why typically you need do auxiliary points ,not typically you always need two auxilary points, unless you know the angular acceleration by some other means. Okay? See rigid body three degrees of freedom, right? So, two the acceleration gives you two linear, for there you also need to know the angular acceleration. So, it's three quantities but, if here because, we're finding accelerations of points you need one more point to find the acceleration, angular acceleration of the body.

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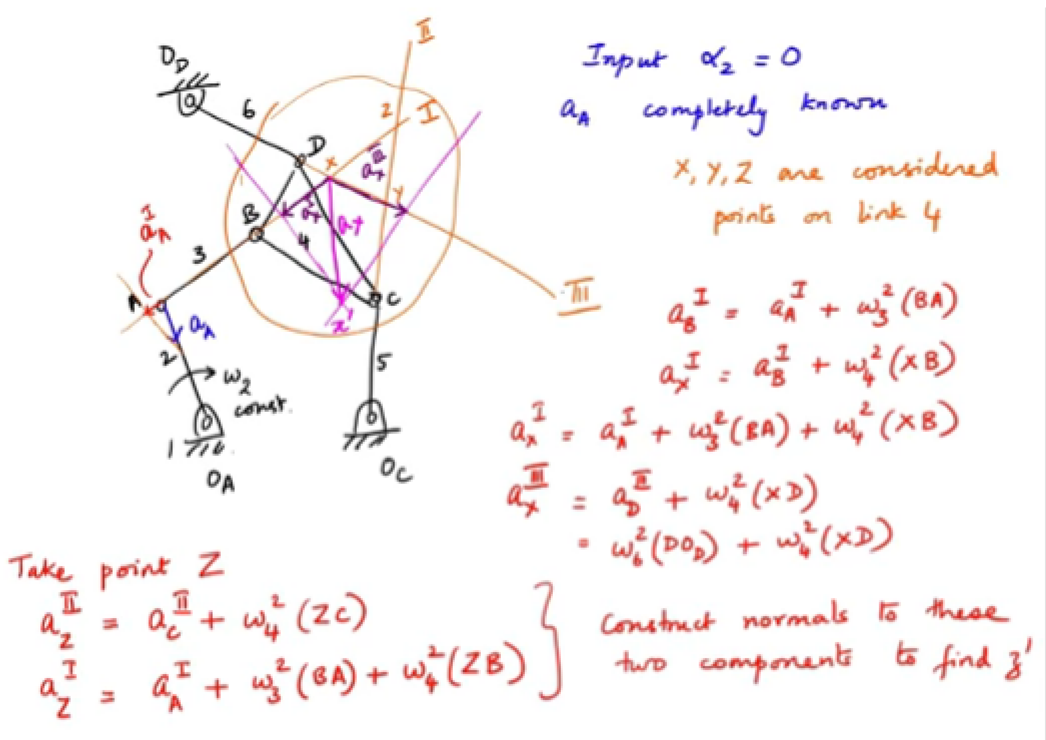


we if you are into AK so you have to know Omega before you get into acceleration analysis, you cannot venture into acceleration analysis, without doing velocity analysis, you cannot venture into velocity analysis ,without doing position analysis. Because, you need all the theaters. So, they have to necessarily follow that order, position then velocity, then acceleration ,this whole thing hinges on the fact that we know the omegas, right ?so, which, which you may have again found out using the auxilary point method but, there of course it was simpler because, the velocity is directly transmitted between two points in the body. So, let's do an example, how many degrees of freedom for this mechanism? Only one. Okay**?** but if this is my Sam given that my input is Omega equal to10 radians per second and it's constant, that's the input I'm given, let's assume the velocity analysis has been done. Okay? So, you know all the omegas let's say 1, 2 ,3, 4, 5, 6, for the velocity analysis also you probably have to do, you have to use the auxilary point method because, you can't use the velocity difference method and similarly for the acceleration analysis also because, i don't know how the path of this point, let me call it a,b,c,d, od ,oc okay .okay? I don't know the path curvature, for the point B so I cannot write, velocity of B equal to velocity of a plus B lost I can write it but I can't solve it so, I could write that still holds the relationship still holds. Okay? or acceleration assuming they finish the velocity analysis, this still holds however I don't know anything about the acceleration of B. because ,I don't know, the path curvature, I don't know, for B relative to a. okay? I don't know, what so, I don't in this one, I will have I don't know anything, about the magnitude or direction of the one, and about this one either. Okay? so, I can't solve this I have more than one unknown .so, I have to go to some other method, I can't use the acceleration difference method so, let's try the auxilary point method .okay? so ,what are the things that I know here, so, I know Omega 2 is constant therefore, my input, alpha 2 equal to 0, I am given 1 acceleration input, alpha 2 equal to 0. Now, so I know, the acceleration of a, completely .what is that going to be equal to? Yeah. So, it's just going to be, Omega 2 is given and it's given that it's constant. Okay? So, I know that the acceleration of a is just this, this is the full acceleration of a. okay? Now, B,C,D is my floating link. Okay? And I want to find I, actually want to find the accelerations of everything, else in the mechanism but, the problem is because BCD is a floating length ,this is a mechanism with , complexity. is it a high degree of complexity or low degree of complexity ,a sure, what's a low degree of complexity? I yeah. So, in this case, ok. Acceleration is not So, simple to do by inversion but, velocity at least I could do by inversion here, if I had six or five as my input, then I can easily find, it's a velocity analysis I can so, it as far as that is concerned it's a low degree of complexity ,kinematic complexity wise, it is a low degree of complexity because, if I had five or six as the input then this is not really a complex mechanism, there is a way to do that there are but, it's not directly, you cannot directly scale, again you know there's, there are methods called ,method of normal components there are methods, like that where you give it a zero ,angular acceleration. Okay? And then solve, for all the know the normal components, will not change because, the normal components depend only on the angular velocities.

So, they will not change, when the angular acceleration whether its present or not. So, there's a way you can solve you find the components, of the normal which will not change. and because, of that then you can go back to the actual angular acceleration, and solve for it, there is a method like that because, I don't want t go into too many graphical methods, I am NOT doing it because, that will also fail with some mechanisms, the auxilary point method, is the one that can be used with most of them .so ,that's the reason I am doing this one ,and not theirs, a that's called letting Goodman's indirect method, if you can if you want to look it up I can post some information on that. Okay? So, you can you can do that but, it's a lot more complex, in terms of you know, the graphical analysis, you have to do basically you have to do the analysis twice, once with zero angular acceleration ,and then use those normal components for your acceleration and then solve with the actual angular acceleration .but, yes. it is possible to do that. So, this is actually a mechanism of low degree of complexity, which can be solved by that method. Because, if I change the inputs to five or six then it is possible to do that, I think that is called the method of normal accelerations, there's also a Goodman's indirect method. Maybe, but they all the basic principle is the same, the normal components of acceleration are dependent, only on the angular velocities and do not change because, of the angular acceleration .okay? so ,here I know, the acceleration of a. okay ?so, I want to find the first let's say, acceleration of the ternary floating link BCD. Okay? Because, if I find that then I can basically find the other things also .okay? that's, that's the problem element there so ,how would I choose my so, the motion transfer points for this link are B,C,D, the motion transfer points do the ternary link are B,C and D. okay? Those are the motion transfer, that’s where it's connected to the other links. Now, I can choose mine so, what do I know, so, if I look at point B .okay? I know the acceleration of point a .okay? So, I could take, an auxilary line, along this, that could be one auxiliary line. Okay? What are the possibilities for other auxilary lines, O,C,C I know the normal component of acceleration of C. so, I don't even have to know, the full acceleration of the point C, of the motion transfer point, I know the normal component. Okay? I can use that so, since I know one component, I will take another auxiliary line, through that. so ,I take another auxilary line through that, and then I could take, similarly with O,D,D because ,it's again D is a point on a link that's rotating about a fixed pivot, I know that D has a normal component of acceleration, along O,D, d .so, I take my third auxilary line, route that. Okay?

So now, I have three auxilary points, I call this X, Y, Z, I only need two, I don't have ,I don't need all three. Okay? So, I only need two, the important thing to remember is x, y and z are considered points on, which link? Points on link four. Okay? So, you have to imagine that, link for encompasses all this. Okay? So, they lie on link four. so now, I'll apply the so ,I know a, a completely so, I know the component, I can find the component of a along, this so, this component here, is a a along one, that's known. Okay? So, I can use that to find A, b, one is nothing but, a a along one, plus Omega 3 square into BA.okay? omega-3 because ,B and a lie on link 3, a X motion is transferred to X ,which is on link four, that's equal to a B plus Omega four square X B, yes. So, I can write it in one shot, I can look at it as long as I make sure that I get the correct omegas, I could write this in one shot as, a X along one ,is a a along one, plus Omega 3 square b a plus Omega 4 square Xb. So, I know that Component, completely known because, my velocity analysis is complete. so, I know that, this component here is a X along one. Okay? Now, I need one more component for the acceleration of X.

so, I know that the acceleration vector of X, lies somewhere on this perpendicular, to this component. Okay? I need one more component, for the acceleration of X. so, look at another auxilary line, let's say, we take this one three .okay? so, this is a long I can write, acceleration of X, along this auxiliary line three, is what would be, what would it be acceleration of D along 3 plus, Omega 4 square XD, yes .what is it? What is this acceleration, this would be omega-6 square into d OD that would be acceleration of B along three, and towards OD you would have that acceleration, normal component of acceleration, plus Omega four square XD, YES. so now, I know one more component, which is say this, this is AX, the direction 3. So, now if I construct a normal to this, where it intersects this, I know my acceleration of X, is this. Okay? The acceleration of X can be split into, either these two perpendicular components or these two perpendicular components. So, I found the acceleration of one point on the floating link, take point Y o point Z ,let's take point Z. okay? So, this is for X. similarly now, I can write? I know, along two, the acceleration of Z along the line two, will be equal to acceleration of C, plus Omega 4 square, I don't need to consider Y because ,Y is just an intermediate point, I can two points have to be on the floating link. So, acceleration of C plus, Omega four square ZC. Okay? That gives me one component, acceleration of Z the other auxiliary line passing through Z is one, is acceleration of a, plus I have to consider B because, b is a point on common two because, omega-3 will be different. Okay? So, acceleration of a along I, plus Omega 3 square b,A plus Omega 4 square ZB .ok? So, I will get a component along this, component along that fully known, and then I construct normals to these two components. So, this second call for instance, I can call this X dash, X dash the point where the head of acceleration of X lies. So, I can construct normals to these two components to find, which point? Z dash, which will give me the acceleration of point Z. now, y and z are points on the same link, link 4.

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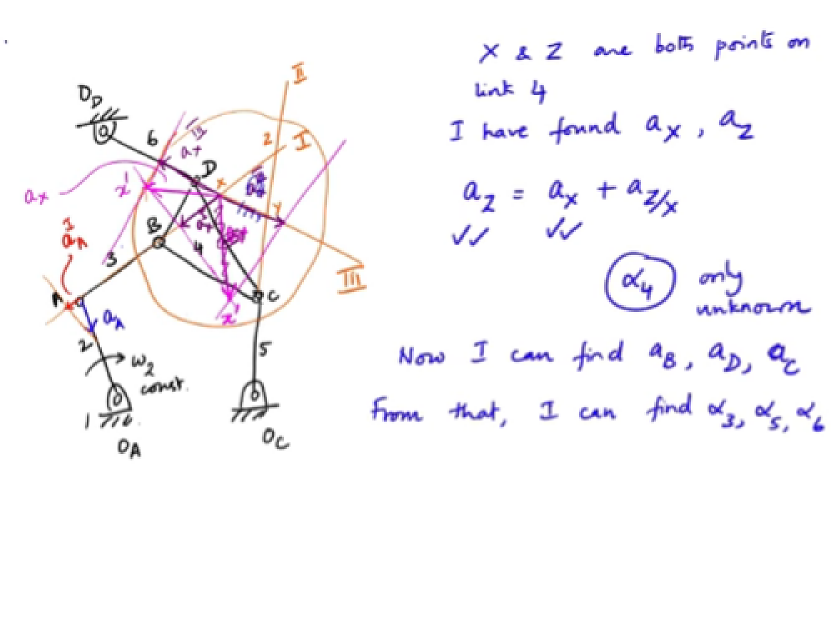


Sorry, X right? X and Z are both points on Link 4, I have found Oxidation X, Oxidation Z. Okay? So, if I write I can now, relate these two accelerations like this, completely known, completely known only unknown is alpha 4. So, if I know, I know the normal component of this, tangents from the, tangential component I can solve for alpha 4 which is the angular acceleration of that link, I can then easily find, the acceleration of B, acceleration of B or the acceleration of C .because, I can just use image, I can construct this figure, in the acceleration diagram such that, it is similar to the figure in the position diagram. Okay? So, I can get the absolute accelerations of or I can use the equation ,acceleration of X, is in acceleration of B, with relative to  with respect to X or to Z. and I can find because, I know alpha for now. so ,I can find the values of, I can find the vectors and once I do that ,then it's easy for me to determine, what is alpha 6? Alpha 3? Alpha 5? It’s start forward.  Once I know acceleration of B, acceleration of D, acceleration of C. okay? From that alpha 3, alpha 5, alpha 6. Okay? Any questions? A clear, what if you want to find, for multiple positions, this may not be the, the graphical method may not be the best way to do it. So, let's go back to, I made a mistake here which, the direction of, if you look at ax along three. Okay? ax along three, this should be towards OD. Okay? So, when I had drawn it the other way. So, I guess This, this should be ax along three, that should be the direction ,in which case again this is also wrong .so, I have one perpendicular like this ,the other normal is like this. So, this will be the point X dash, and this we'll be the acceleration of point X. okay? Just so ,ax along one is towards a, and ax along three is towards OD. Okay? I had it drawn in the opposite direction. Okay? So, this is the corrected, acceleration of X.

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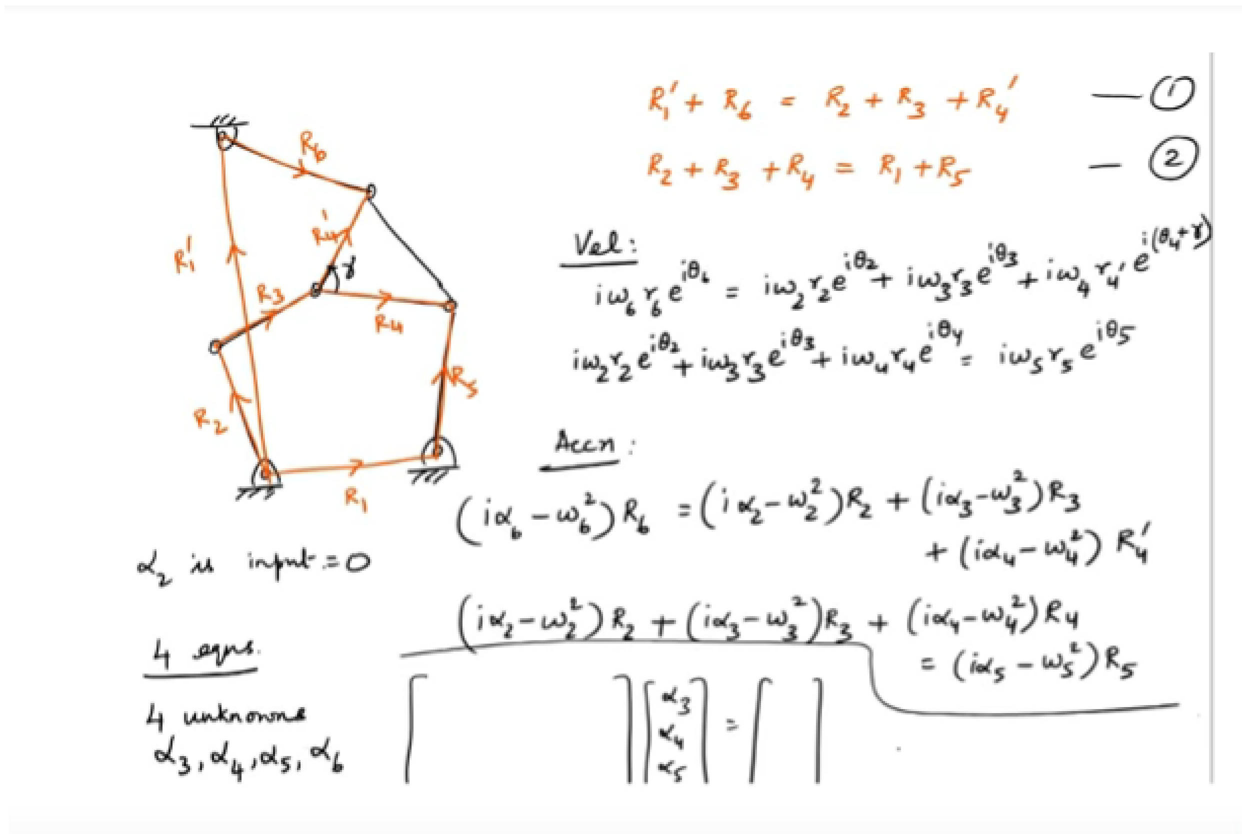


So let's see how we can, whether we can do these, sort of mechanisms analytically as well. Okay? So, my vectors So, I have one loop. so, let's call this R 1, R 2, R 3, R 4, R 5. So, one loop formed by r 1 through our faith, the other looked formed by, R 6. So, the other loop is r1 dash plus r6. So, let's, let's rewrite on the two loops, I have one loop r1 - plus r6 equals, r2 plus r3 plus r4 dash. Okay? the other loop is r2 plus, r3 plus, r4, equals r1, plus r5. Excuse me, okay? Two looks in this mechanism as it. So, let me directly write the velocity Equations. So, the I differentiate one I will get i r1 - will not be there, doesn't change. So, I Omega six, r6 e bar I ,theta 6, equals I Omega 2 ,R2, E bar I theta 2, I what will it be Omega 4 or 4 dash ,e bar I, theta 4 plus some constant gamma. Because, that angle does not change and for the second loop, I have I acceleration is, for each one of these I'll have, i alighted in vector form minus omega2 sorry, I Omega 6, alpha 6 minus Omega 6 square, R 6 I'm writing it back in vector form, R3, as I alpha 4 minus Omega 4 square R 4 dash.

And for the second equation, I have I alpha 2 minus, Omega 2 square, R 2 plus r5. Okay? So, how many equations do I have? For the oxidation, four equations, how many unknowns do I have? What are the unknowns? 4 unknowns, alpha 3 alpha, 4 Alpha 5, and alpha 6. alpha two is input, that's what I gave you know, in the previous same, same problem as earlier, alpha 2 is input equal to zero, in this case because Omega 2 is constant. So, I can set up the matrix and it's all linear equations, once my positional and analysis is done, velocity analysis is linear. So, is the acceleration analysis? So, I can just solve this for alpha 2, Alpha 3, alpha 4, I'm sorry. Alpha 3 to alpha 6 .ok? so, I could saw ,if I have to do it for several points ,this is probably the way to go so, that the analytical method also will work because, he basically just solving a set of simultaneous equations, even though the acceleration difference those methods don't work, this will work. So, will the auxilary point method?