

Lecture – 32

Theory of Mechanisms

**Force analysis of Mechanisms,
Mechanical Advantage**

So today we will start with,

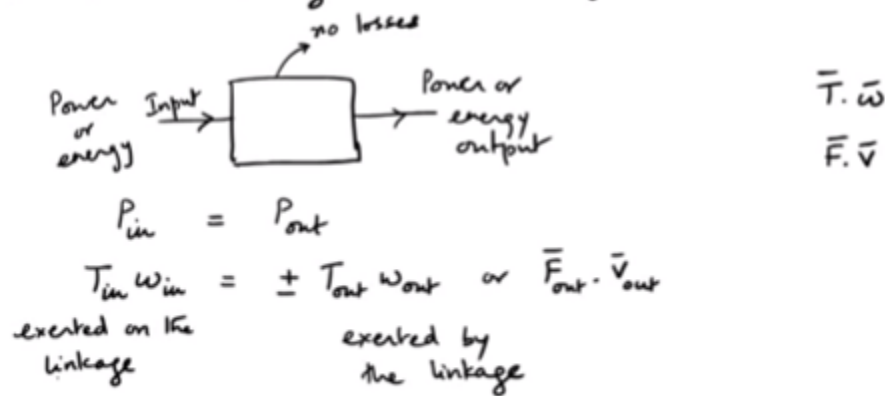
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Force analysis

Mechanical advantage:

Mechanisms used to transmit force or torque

Torque ratio is a function of the speed or angular velocity ratio between the input & output → functions of geometric parameters & will change as the linkage moves.



the Force Analysis of Mechanisms. Okay? And one of the first things we are typically interested in, so even before we go into, in many applications, we use linkages, to obtain a mechanical advantage. Meaning, for, so as human beings, our ability to apply forces, is limited and we may want to, multiply that, we may want to multiply that effort and being able to do that, is what is known as, using some device, to do that, is what is known as, 'Mechanical Advantage', as you all know, using levers, for instance, we do that. Now we are going to look at, the mechanical advantage of, mechanisms. So usually you're using a mechanism to, transmit, force or torque. You can also use, you know, other than Linkages, gear trains, are another example. Where you can, change the torque between the, input and the output. One difference between, a mechanism and a gear train, is that, a gear train will operate with a constant velocity ratio or torque ratio. Okay? The, you cannot, where as in the case of a mechanism, it varies, as the mechanism moves, because this is dependent on the geometry, of the linkage. Okay? So, so, the torque ratio is a Function, of the speed or angular velocity ratio, which can be determined from the, geometry, angular velocity ratio, between the input and the output. You remember, even well doing the velocity analysis, you know, many times we were only interested between, what is the output, you may not be interested in the intermediate links, so much, say, angular velocity, of the intermediate links. One reason is, because you are usually interested in the, torque ratio, between the input and the output. Okay? So and this is a function of, geometric parameters and varies, as the linkage moves. So typically we look at it as, in terms of work or power, you say, if this is your linkage, black box. Right? You have some power or energy input and you have some power or corresponding power or energy output, that you are interested

in. And if you assume, there are no losses, in real life, typically there will be losses, due to say friction, for instance or inertial effects. If you assume, that there are no losses, then you say that, the input power, is equal, to the output power, whatever you put in, you are able to get out. Okay?

So typically we assume, when you say no losses, we assume they are negligible, compared to the power that you are transmitting. So, in the case of planar mechanisms, you can write this as, P_{in} , can be written as, the input torque, times, the angular velocity. It's, it's typically, $T \dot{\omega}$. Right? The power is, $T \dot{\omega}$ or $F \cdot V$. The dot product, of these two vectors. In the planar case, both T and $\dot{\omega}$, will be along the, you know, yeah, you can, about the same axis. And so you can write this as, this is equal to plus or minus, $T_{out} \dot{\omega}_{out}$. Or you could also have, $F_{out} \cdot V_{out}$. So if it's a slider-crank, for example. Okay? So this is exerted, on the linkage, the output power is exerted, by the linkage. So you don't need the, plus, minus sign, if you if it's understood that this is on the linkage and that is by the linkage.

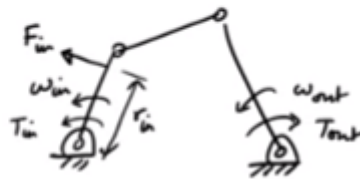
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$$\vec{F} \cdot \vec{v} = F_x v_x + F_y v_y \quad \text{where } \vec{v} \text{ is the velocity of the point at which } \vec{F} \text{ acts.}$$

$$T_{in} \omega_{in} = T_{out} \omega_{out}$$

$$\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}}$$

MA is defined as $\frac{F_{out}}{F_{in}}$



$$T_{in} = F_{in} r_{in}$$

$$MA = \frac{T_{out}/r_{out}}{T_{in}/r_{in}} = \left(\frac{r_{in}}{r_{out}}\right) \left(\frac{T_{out}}{T_{in}}\right) = \left(\frac{r_{in}}{r_{out}}\right) \left(\frac{\omega_{in}}{\omega_{out}}\right)$$

ratio of distances at which forces are applied

angular velocity ratio

So if you have, $F \cdot V$ here this is just $F_x v_x + F_y v_y$ where V is, the velocity, of the point, at which F acts, Okay? So if you have, ω_{in} , ω_{out} and you have, some input torque, let's say the outlook torque is this. Okay? Say there is a load, that's being lifted, load that's attached to this link, so it is, the resistance, is your output torque. Okay? So if you have something connected to this, which is lifting a load, this is the resistor. So then you have, T_{in} , ω_{in} , equal to, ω_{out} , depending on the directions, you would use, if both are in the same direction, it will be plus, otherwise minus. So T_{out} , by T_{in} , is ω_{in} , by, ω_{out} . And your mechanical advantage, is defined as, the standard definition is, F_{out} , by F_{in} . Okay? because usually the torque also, may be applied, as, a force, at a distance. So this could be r_{in} . Okay? So t_{in} , will be $F_{in} r_{in}$. Vectorial form, $R \times F$, but here assuming this is,

perpendicular to this, say f and r . The standard difference, sometimes in the case of linkages it may also be define as, T out by T in, if you're only, if you are interested in the torque itself. Okay? So where, say you're applying it through, a motor, then, rather than talking about a force applied at a certain distance, you talk about the torque itself, so. So MA , would be, T out, meaning, F out, sorry, T out by r out, by T in, by r in, which is, T out by T in, And this is, equal to, from this, you have, this is equal to this, by Ω in, by Ω out. So this is, the ratio of distances, at which the forces, forces are applied. And this here is my, angular velocity Ratio, which depends on the, geometry of the linkage. Yes? Yeah.

So you could define it as, F out by torque. So then, you would, yeah, the definition would be different. Because there are, you, you're resisting force, you're talking about a slider crank, crank rocker, oh, yeah, you can, so you can define it there as, T out by T in. Okay? The general definition of mechanical advantage is, F out by F in. Because, in that case you can just treat it as, you know, you have a constant, because R in and R out, if they don't change, you can say that, this is, you can define it, in terms of the input and output torques. So this is, it depends on the geometry, mainly the IC's, you can. From an instant center analysis, I can find the ratio of, ω in by ω out. Okay?

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What would be F_{out} & the MA of this linkage?
Neglect the weight of the link

$$MA = \frac{F_{out}}{F_{in}} = \left(\frac{\omega_{in}}{\omega_{out}} \right) \left(\frac{r_{in}}{r_{out}} \right)$$

$$= \left(\frac{\omega_2}{\omega_4} \right) \left(\frac{r_{in}}{r_{out}} \right)$$

$$= \left(\frac{I_{14} I_{24}}{I_{12} I_{24}} \right) \left(\frac{r_{in}}{r_{out}} \right)$$

$$F_{out} = F_{in} \left(\frac{I_{14} I_{24}}{I_{12} I_{24}} \right) \left(\frac{r_{in}}{r_{out}} \right) = 24 F_{in}$$

\downarrow \downarrow
 6 4

$$v_{I_{24}} = \omega_2 (I_{12} I_{24}) = \omega_4 (I_{14} I_{24})$$

So let's look at, let's say, I have a punch. Say this is a sheet of metal, into which I want to punch a hole. Okay? So this is the handle, so I would be applying, a force there, in order to punch a hole through this sheet. So I may have a table, that has, on which the sheet is placed and then it will punch the hole. Okay? Okay, so now, this would be, the input force I'm applying, applying at a distance, R in. Okay? This is essentially a four bar linkage. I have one, two, three, and four. And the output force will be what is being applied here and that is being applied at a distance, R out. So this is, F out, that's it. Okay? The punch is

going to apply that Force, on the sheet, that's F_{out} , acting at a distance R_{out} . Okay? So now what would be, F_{out} ? So given say the geometry of this linkage, what would be F_{out} and the mechanical advantage, of this linkage? Okay? Given some dimensions. Okay, let's say, we neglect the, weight of the links, because otherwise, there will be losses, due to the inertia, of the links themselves, so we are assuming, that there are, no kinds of losses, in this system. So I have, F_{out} by F_{in} , that's my mechanical advantage. That's equal to, I showed you, that, that is equal to, Ω_{in} by Ω_{out} , into R_{in} by R_{out} . Okay?

Now what is, Ω_{in} , by Ω_{out} ? How can I find that, any ideas? I can do a velocity analysis or I can just find the, instant centers. Right? I can find the instant centers, so here, 2 is my input, so I want in this case, and this is, Ω_2 by Ω_4 , R_{in} by R_{out} . Okay? So Ω_2 by Ω_4 , which means, I need to find the instant center, i_{24} , i_{12} , i_{14} and i_{24} , are the instant centers I need. So this is i_{12} , i_{14} and this is i_{23} , i_{34} . So this point here is i_{24} . So what is Ω_2 by Ω_4 ? They, so here I know that, the velocity of i_{24} , is the same, whether I consider it, as a point on body 2 or body 4, so I have velocity of i_{24} , equals, Ω_2 , into i_{12} , i_{24} , which is equal to, Ω_4 into, i_{14} , i_{24} Right? So Ω_2 by Ω_4 is, i_{14} , i_{24} by i_{12} , i_{24} into R_{in} by R_{out} . Okay? And in this case this is positive because i_{24} lies outside the line, joining i_{14} and i_{12} . Okay? So this is. So now, if I look here, my F_{out} is, F_{in} , times, this quantity, i_{14} , i_{24} , by i_{12} , i_{24} , R_{in} by R_{out} . You can see, this is going to be greater than 1. Okay? You can see this ratio is greater than 1, so is this ratio. Okay? So my F_{out} , is necessarily your, multiple, is multiplied. So you can take, so here if I just look at these distances, this is about, what? 6 times the other distance? To some. I could take that, you know, based on what I have here. And then, R_{in} by R_{out} , about four times. Okay? So, my input force is multiplied by 24 times. Okay? Typically, an operator may be able to Apply, like somewhere between, 5 to 10 kgs, up to 100 Newton. That can be multiplied 24 times. Okay?

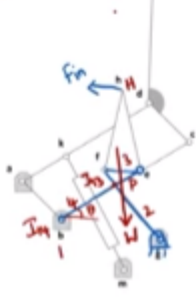
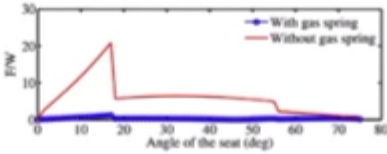
Using this linkage, if you were just trying to do, without the linkage, you wouldn't get, this kind of a magnification. So this and again, if I wanted, a greater mechanical advantage, I have control over, because this is all based on geometry. Right? All of these, can be, I can play with these parameters, of course, it's not going to be the same, at every point, in the, during the motion of the linkage. Okay? Because the instant Center changes. As, so here you see that, as this moves, i_{23} moves closer, to the line joining i_{12} , you, you get, this becomes smaller and smaller, and you get a high, mechanical advantage. Right? In some other position, where i_{23} , you know, where i_{24} may be here. Okay? You would, your mechanical advantage would not be high. Right? So as you are nearing the, toggle position, you find that, where these two links are, will be in line. And in that case, i_{24} and i_{12} will actually coincide. Right? Which makes it, which makes the mechanical an advantage, infinite, but obviously you cannot, the parts have to be able to withstand, the forces, so. But close to the toggle position, you can see that you can get a really high mechanical advantage. So most, punch presses or you know, anything that's where you want to generate, they'll operate close to the toggle position. Okay? So you look at, how you, and then of course, it's intuitive, that, R_{in} should be much larger than R_{out} . So if I increase that input lever. Okay? Then I can get, I can increase this, ratio, as well. Okay? So it can be a good design tool, when you are in the, because this is all dependent on geometry, so I can, look at, how I can modify, these parameters, so that, I get a good, even clamps, when you want to clamp something, you want a high force, to hold something in, this kind of a mechanism is used and it operates, near the toggle position.

If you look at brakes in wheel chairs, you will find that, it, it will have a four bar mechanism, It will be close to the toggle position, to operate close to the toggling position. Okay? Clamps in the workshop or punch presses, all operate, based on this. Okay?



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Standing Wheelchair

$$T_{in} = F_{in} (H I_{in})$$

$$T_{out} = W (P I_{out}) \cos \theta$$



Design Evolution

TTK Center for Rehabilitation Research & Device Development
<https://home.iitm.ac.in/r2d2>

So, I'll show you a real life example. So this is a wheel chair, this is a mechanism for a wheelchair, that operates, like a regular wheelchair, but it can, allow the user to, come up to the standing position. So many wheelchair users, they cannot stand on their own. Okay? because they don't have the required muscle strength, to stand up on their own. So they need something to, support the lower, especially at the knee. Okay? But lifting themselves, so for them, but it's standing is very essential, because if you are in the seated position all the time, I mean, even as able-bodied people, we are encouraged to get up and walk around, every hour or so. Right? They say, don't sit in one place for a long time. Because it's not good for your health, it affects the physiology, it effects. In the case of a person, who has, say, a spinal cord injury, and who can't stand and is sitting in the wheelchair all day long, they may come up with other secondary health problems, also. Okay? Their circulation gets affected, their bone density starts going down, because their legs are not weight-bearing, at all. So standing as therapy is considered very essential, for people, who have, to use a wheelchair all the time. But it's also very difficult, because you know; at least two people have to lift them up. You have to lift up the entire body Weight, in order to make, a person stand and then support it, because their legs can't.

So this is basically just a mechanism, that is integrated, into the wheel chair, so that, you know. It's like this. If you ask somebody to, exercise. Okay? You may do it, may not do it. You tell somebody, okay, know what? Just take the stairs, don't, you don't have to do anything special, but don't take the lift, just

start climbing stairs, just start parking far away, walk, you're more likely to integrate it, into your daily life, the idea is if it is part of the wheel chair then they will use the standing feature, more often. What is special about this design is, its completely mechanical, there is no motor, here. They are using only their upper arm power, to essentially, lift themselves up, to the standing position. And the other aspect that comes into is, so if you look at, the linkage. So this is the kinematic diagram, of the linkage, so here you see, if we use a parallelogram mechanism, first, for the standing, because you want the backrest always, remain straight, as the person, goes to the standing position. Okay? You can see as, he is moving up or Down, the inclination of the back remains the same. So the easiest way to do that is to use a parallelogram mechanism. But the mechanism that we are really interested in here, is this one. Okay? This is the 4-bar, just a 4 bar. Okay? So I, the other part is just a parallelogram, so let's just, you know, ignore, that for now. This is the mechanism that I'm using with the handle, to actuate. Okay? So this is this is just a couple of point, on this link EF, there, I have to make it, so that it is accessible to the person. So if I look at this, then the person is going to be applying, some force here, to actuate the mechanism.

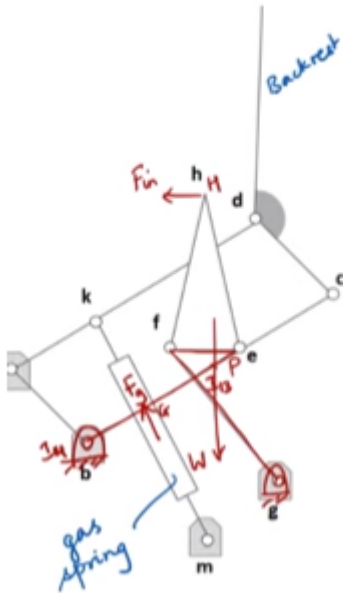
What you are trying to lift, is, what is the external load? The weight. Okay? Let us say, even we ignore, the weight of the links and all that. The weight of the person, is acting on that seat .Okay? So there has to be, the input torque has to overcome, the torque, due to the weight, about, this point. So this is, if this is say, if this is my 4 bar, this is 1, 2, 3, 4, then this point here, is, what is that point? This point here is, i_{13} , where those two links intersect. Okay? This point here is, i_{14} . Okay? Link three; I can consider it as, rotation about, this fixed favorite, i_{13} , the motion of length three. So the torque that's being applied, the input torque, is F_{in} , F_{in} , times the, distance, so, say, this point is H, the distance from H to i_{13} , that's the input torque, that I can apply, the weight, the output torque, that has to be overcome, is W times, let us say, this is $\cos \theta$. Okay? And let's say this is acting at a point P, on that link, p, i_{14} , $\cos \theta$, it's going to vary. But imagine, at θ equal to zero, to start, lifting. Okay? That's a pretty huge task, to lift this weight. This is the torque, you have to overcome. Okay? So you know, how long can you make this handle, so the longer the handle, obviously, you'll have, you know to, even get any mechanical advantage, you have to have, because, the amount of force, that you're able to apply with your hands, is going to be, a lot lower than the W . Okay? So this is actually not possible. It is only possible, because of, an additional link here.

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Mechanical Advantage

$$F_{in}(HIB) = W(PIL_u) \cos \theta$$

$$F_{in}(HIB) = W(PIL_u) \cos \theta - F_g(LIL_u)$$



So here, I'd have to apply a pretty high force. Okay? If you look at the dimensions, I can't make this too high, the H can't be, I can't try to, because economically then I won't be able to push, I'll be able to apply the maximum force at some optimum height. Okay? Beyond that if, if the handle is here, the force that I'm able to generate, because of, the way my body is structured, I'm not going to be able to apply the maximum force. So then, what do we do? So that is where, so, if you just had the mechanism, kinematically, yes, you can generate, you know, you can make it move. Okay? Kinematically I can simulate it.

But if I put a weight on it, Okay? I cannot apply enough force, to make it come up, to the standing position. Yes? No, no. So they're, in that the locking mechanism is by, ensuring that in the sitting position, it's at a toggle position. The standing position also, you come to a toggle position. And then you have a positive lock, but the toggle position, is what is locking the mechanism, so the linkage dimensions are chosen, such that, it toggles at the sitting position and toggles at the full standing position, so that you don't go beyond that. So that. Yes? Sorry? Do I have a simulation of the kinematic diagram? No. Hmm? What are the rigid points? These are, so these, are the fixed pivots. E? D, D is not a fixed pivot, D is the backrest, D is the back rest, of the. But this link, this is welded here. So it maintains the same orientation, its part of this, parallelogram linkage, ABCD. Okay? And because it's a parallelogram linkage, as this linkage moves, this orientation does not change and that is these backrest. So these, this just shows that, the backrest, is rigidly attached to this link but these are, part of the, frame of the, wheelchair. So what I was coming to was, with just, so this is my parallelogram linkage, to maintain the backrest, vertical and to that, I am adding the driver dyad, EF, FG. Because this is essentially two more links. So this is now a six bar that I have. Okay? To actuate, the driver dyad is to actuate the mechanism. Not necessarily, you know, full 360 degrees, but, that's my actuation for the parallelogram linkage. But, because the way it is pretty

high, you know, unless I have like a really long lever, I am not really going to get much of a mechanical advantage, so it's going to be a. That is where this spring comes in. So this is essentially a gas spring. So it's like a, you have a piston, moving inside a, cylinder and there is a force that it exerts at every point. So now what happens is, I have, I have this linkage, this 4 bar. Okay? This is i_{13} , i_{14} and say, this is W and this is F_N , so the person is pushing the lever, like that. Yes? Why are you saying it is not giving the input? My input is through EF , my, but H is a point on link EF . So driver dyad meaning, that is, you know, it's an additional two links, dyad means, two links, through which, you are actuated the mechanism. So here I am not actuated, through say link, FG , I am instead actuating through, link EF and H is a point on link, link EF . Okay?

So don't get caught up on the terminology aspect of it. It is still no additional links, through which, the original four bar, which is the parallelogram mechanism, is being actuated, activated, actuated. Okay? So it is, we can still call this, as driver dyad. So the, you have F in, in to, call this, point H , let's say, we call this point P , here where the, into H , i_{13} is equal to W into P , i_{13} , $\cos \theta$. Okay? In which case, F_N has to be very high, to do that and that may not be possible. And that's where, we use this gas spring, that now applies, an additional force, FG , on this linkage, which helps to, balance the weight. Okay? So I have FG , I'm taking this as perpendicular here, but that would act at some angle. Suppose, say, at that particular instant, it's perpendicular, then I have, F in H i_{13} will be equal to W i_{14} $\cos \theta$, minus, FG into, say this point is G , G into i_{14} . So you can get compact gas springs, which can apply, pretty high forces. So what that does is, it reduces the input torque that I have to, apply. This is sort of the idea, behind spring balancing also. So that we'll, no, that will also vary as the length changes. There will be a constant force, plus a force, due to, like a regular spring. Right? In a regular spring, you have F equal to KX . Here you will have a constant force, plus K times, the displacement. Okay? You could use a, the first design, that we used, used a compression spring or an extension, one of the two; we use an extension spring or con. The problem with that is, when the person goes up. Right? It just jumps. Because there's no damping, present in that. So a gas spring is better, because of its damp characteristics, so it's gradual, it doesn't just throw the person up, like that. Okay? So it, use of this, can reduce the, input torque that you require. So it's not just about the mechanical advantage, but also about reducing the input torque, required. And this is sort of what.

So we will, look at spring balancing, in one class, but balancing in general and then specifically, but this is a good example, where spring balancing, has helped you reduce the, input torque. Okay?