NPTEL

NPTEL ONLINE COURSE

Theory of Mechanisms

Force Analysis of Mechanisms-II

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So last class we looked at mechanical advantage where, so without doing a complete force analysis you can get it an idea of the relationship between the output torque and the input torque, and today we'll go into the details of doing a force analysis for a 4 bar in a systematic manner where you can determine the forces at each pin joint you probably want to determine, so for synthesis we usually deal with some kind of position requirements they may also be velocity or acceleration requirements.

So the force analysis with the, so if you want a certain set of velocities and accelerations, what would be the torque or forces you would have to apply to get the mechanism to run with those velocities and accelerations, so that is the purpose of the force analysis and from there on you will go on to do once you do a stress analysis and you know to determine strength, once you determine what materials you're going to use for your design, okay.

So we want to, the purpose of this is determine the forces and torques in the system that is the forces and torques that result from, and those that are required to drive the system to produce the desired kinematics, so this kind of an analysis where you know the kinematics and you're now trying to determine the forces in the system that's called inverse dynamics analysis. In engineering mechanics usually you apply a force on a block, and then you try to determine the acceleration in the system that kind of analysis is called a forward dynamics analysis, where you know the forces on the system and then you find the behavior of the system,

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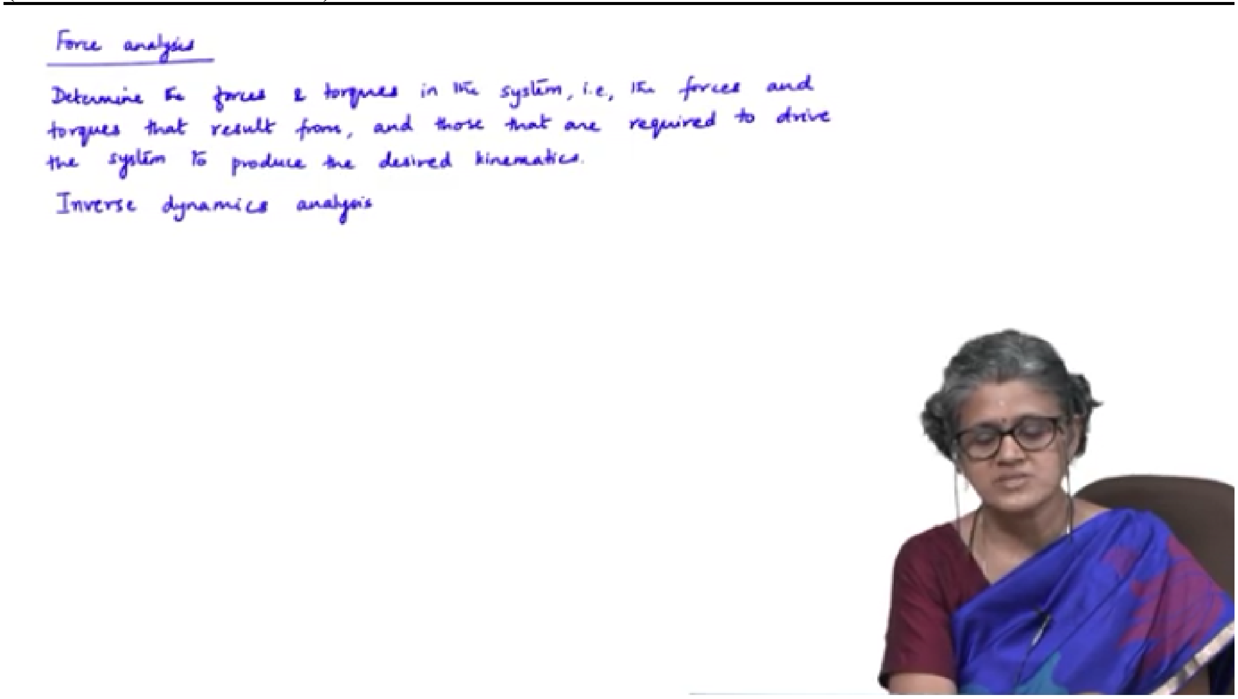

here from the kinematics you are trying to determine what should be the forces or torques applied to the system to produce the desired kinematics, okay.

So inverse dynamic analysis given the masses and acceleration you find F, forward dynamic analysis given F, given N and F to find A, so there are various methods to do dynamic analysis,

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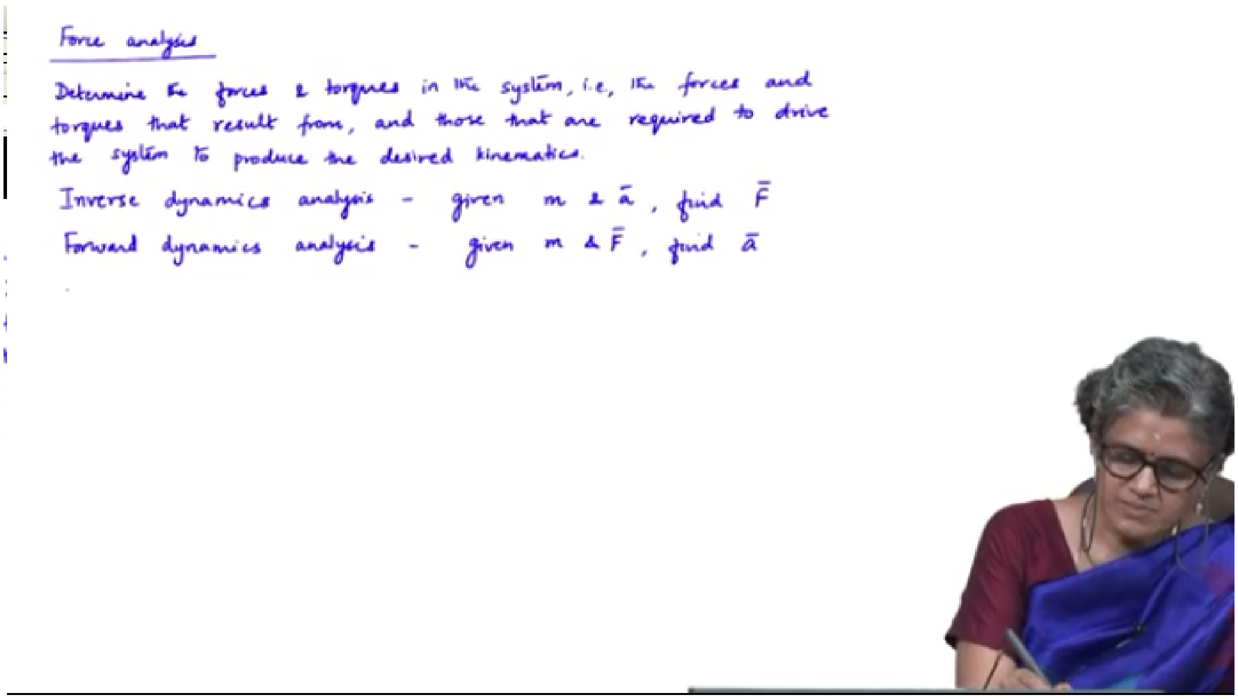

we will use the Newton-Euler approach there are analytical methods like energy methods etcetera, but we will use the Newton-Euler approach because this gives you the most information about the internal forces in the system, so for instance when we use in the previous case when we did T1 omega 1 = T2 omega 2, that's an energy method, because you are equating the input power to the output power, okay, so we didn't really care about what was happening within the system, you only cared about the input and the output.

So the Newton-Euler approach is used because it gives the maximum information about the internal forces in the system as well which is something you will need, when you are designing individual components you are going to need the forces on every component, okay, to see if they can withstand the forces that are applied on the system.

So essentially this applies the equations sigma F = M AG, where G is the, AG is the acceleration of this mass or center of gravity, sigma T = IG alpha where this is the moment of inertia about the center of mass, so for a 2D system, for a planar system like we have been dealing with, it reduces to the equations sigma FX = MAX, sigma FY = MAY, and sigma T = just IG alpha, okay, here I should say AGX, because the acceleration of points in the system vary,

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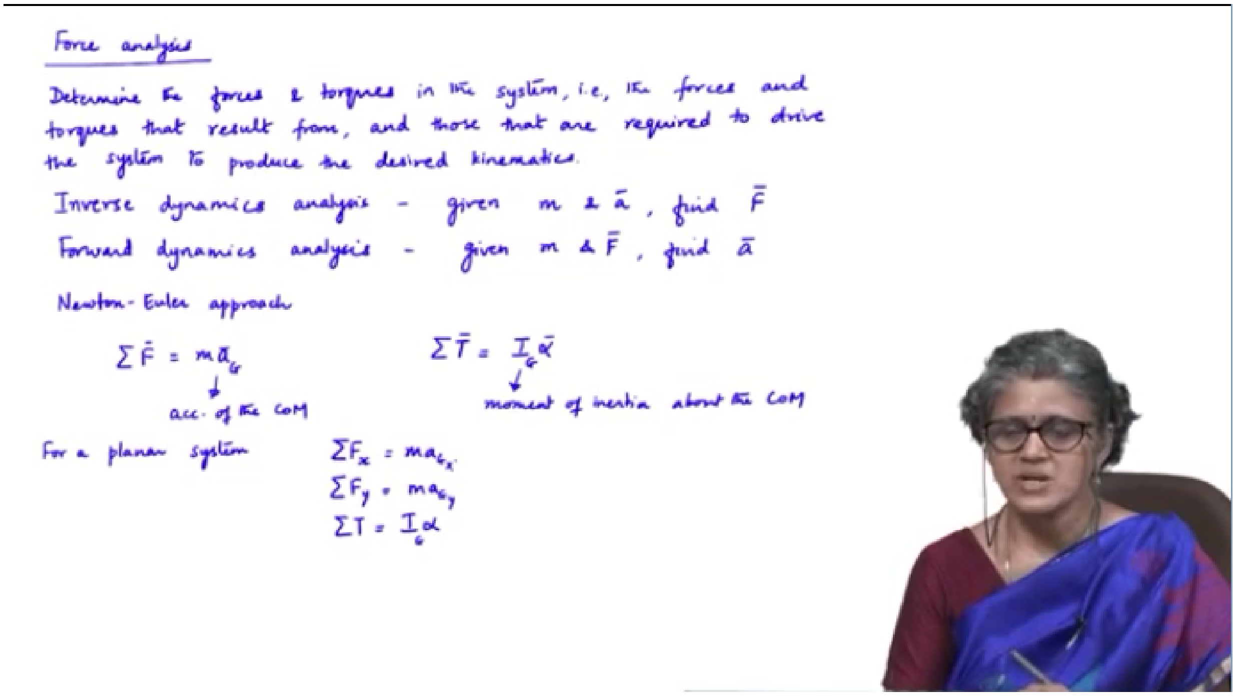

different points will have different acceleration, so we have to be very clear when we're applying Newton's laws we are applying it to the center of mass of the system, okay, so these will be the three equations that we will apply to each of the links.

And this is just again these are three scalar equations you only have, so this is basically the moments, some of the moments about the Z axis, okay for the planar system.

So we write these equations for every moving body in the system and then sort of assemble them together in a matrix to solve for the unknown forces, okay, so and typically we treat the weight of each link so sometimes you make the assumption that the links are mass less, if the accelerations are very high, okay, then you can't make the assumption because you could have significant inertial forces and those forces can be much higher than the weight of the system, so there you could actually neglect the weight of the link that you are using, but if the acceleration is low compared to the acceleration due to gravity, then you include the weight of the links as an external force, okay, again if the masses are very high you would have to include the weight as an external force on the system. So weight is treated as a external force acting at the CG of the member and it's going to be in a constant direction, right, so it doesn't matter how the link moves the weights always going to act downwards, okay.

So in order to do this kind of an inverse dynamic or kinetostatic analysis that's also called, you need to know the kinematics fully, so for all the points of interest especially the centers of mass of the various links you need to know the position, velocity and acceleration,
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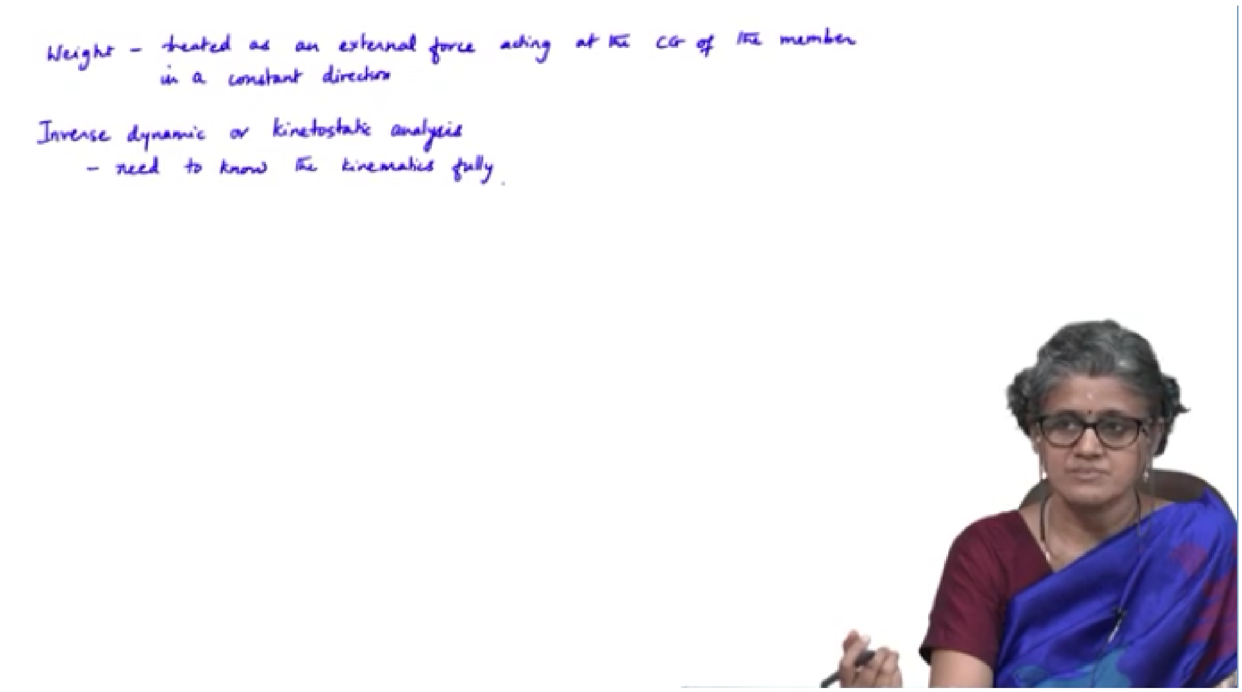

okay, so the analysis that we did earlier that should be completed for, you know earlier or points of interest where like say motion transfer points or other you know maybe a coupler point, when you do a force analysis you have to know where the CG is located and you have to know the position, velocity, acceleration for the CG of each of the links, okay, that will be your point of interest, you need to know the kinematics fully, and for all the positions for which you want to do a force analysis, okay, so for all the linkage positions of interest, and then you need to know the mass of each link, you need to know the location of the CG of each link, you need to know the moment of inertia about the CG of each link. You need to know all the external forces and torques applied to the, applied and their points of application.

And for the force analysis because, and for our linkage because you have things that are moving you need to follow a very systematic procedure so that you don't make errors, so we'll do that, follow a systematic method to define things and so that you'll minimize the chance of errors.

So the first thing that you do you set up a non-rotating local coordinate system on each link, for each moving member with origin, okay,
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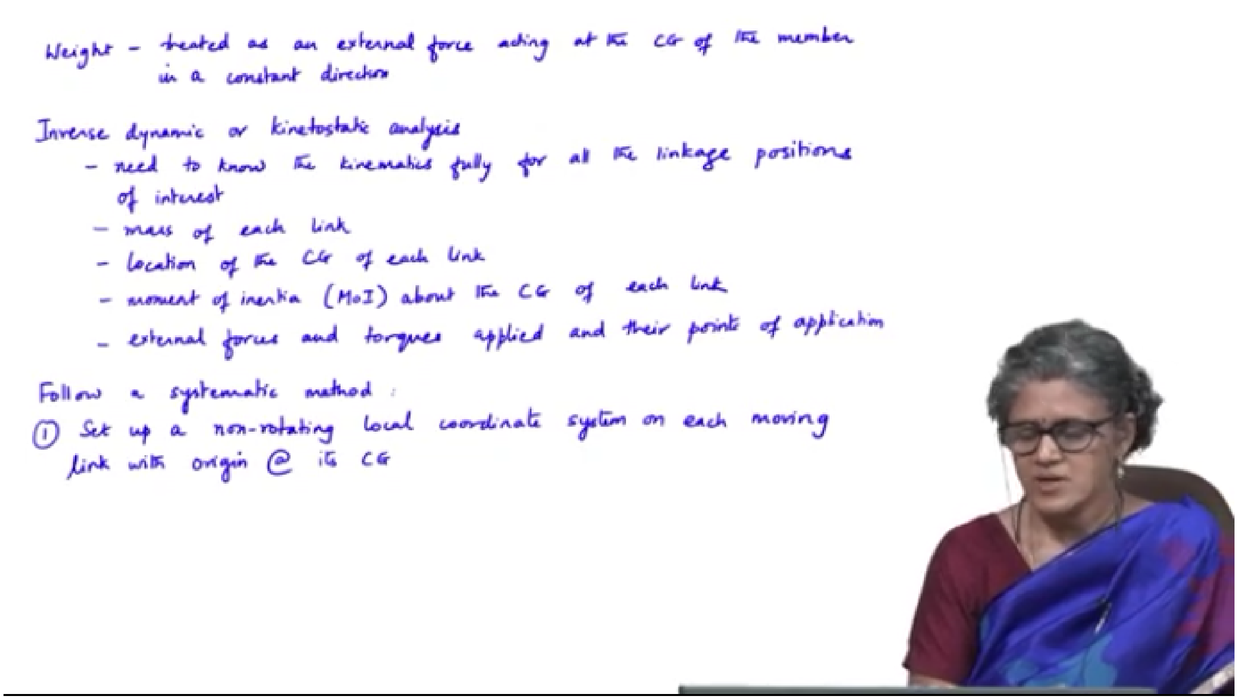

so then if I had the planar mechanism on this table, okay, and the plane was this, I don't need to consider the weights of the links, right, because I'm looking at a planar analysis and the weight is acting in a different, so if I had the same mechanism on a vertical plane then I'll probably need to consider the weights of the links, so it depends on the plane of consideration. So this will call this a local non-rotating coordinate system, okay.

So let's say in this local non-rotating coordinate system we'll locate the points of applications of all forces, these could be reactions or applied forces and they are all located in this local non-rotating coordinate system, we call it local because it's attached, the origin is at the CG of the link.

And this is the notation we'll follow, so F12 is the force of link 1 on link 2, so this will denote the vector if I say F12X, F12Y they are the scalar components of F12 in the X and Y directions, so we know that by Newton's third law F21 will be equal to –F12. T12 will be the torque delivered from the frame, by the frame to link 2, so this maybe through a motor that is connected at the joint between link 1 and 2, okay, so this is the torque required, remember a 4 bar for instance is a single degree of freedom, it needs one input, okay, so this would be the torque required to drive the links, so when we say, you know when we did all the analysis we said okay, this is been driven at some angle of velocity, some angle or acceleration,
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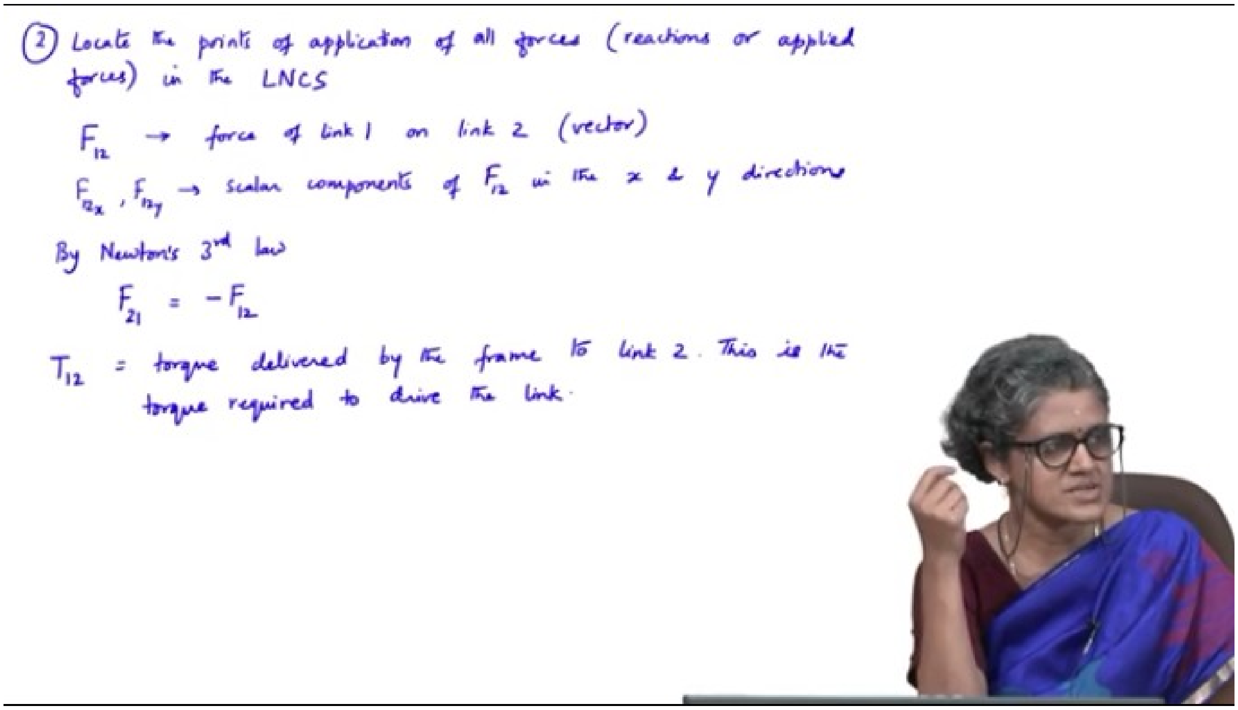

okay, so this would be the torque, you would deliver that input as a torque to the linkage, required to drive the link at the kinematically defined accelerations, so this would be one of the things you would be pretending out, because for your linkage what kind of a motor do you need to use, you know what should be the torque requirement for this.

And for a linkage as you know this requirement maybe may change as the link moves, as a linkage moves, in the case of gears it's a constant ratio, because the ratio of the output to the input torque remains constant in the case of gears, okay.

So let's look at first the case of applying this to the case of a single link in prior rotation so, let's say this is the CG, so I take, I establish my local non-rotating coordinate system at the CG of this link, okay,
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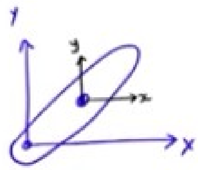

so I will have some force that link 1 applies to link 2, okay so I'm drawing the free body diagram of the link and what I want to determine is what is the torque that needs to be applied, now there may be some okay here I should actually let's say this is also, I may have some force that's being applied some known force, one of the known force is would be the weight of this link, okay, so that would be, suppose this is the link 2 I'll call this M2G that's a known force, there may be some other force that's being applied to this link, if it's an external force then I know it's magnitude and direction, okay, there may be some force that is being applied to this link, so this is my local non-rotating coordinate system the black one, and then I define the locations of all these forces, MG of course acts at the CG so I'll call this R12, sorry RP, okay, and this is R12,
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both RP and R12 are defined in this local non-rotating coordinate system, okay, and we will see why we do that. And this the blue one is the ground coordinate system fixed to the frame.

Now if I look at the pivot to which this link is attached, what will be the forces on those? F21, okay which will be, okay, I should write this as F21 which will be equal to –F12, okay, so it will be opposite, I'm taking F I'm calling this F21 and so it's actual direction will be opposite of this, yeah, I'm calling, okay, fine, this will be F21 this is –F12,

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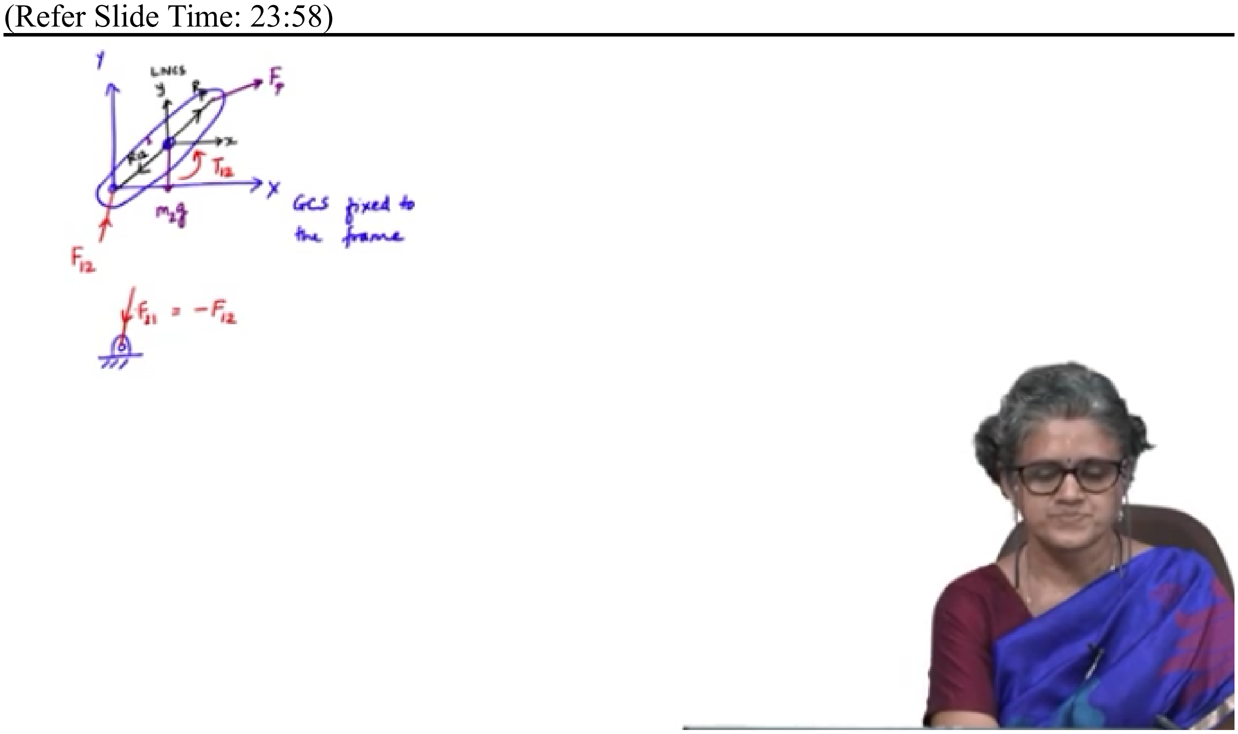

and then I'll have T21 which will be minus of T12 that will act on the frame, okay.

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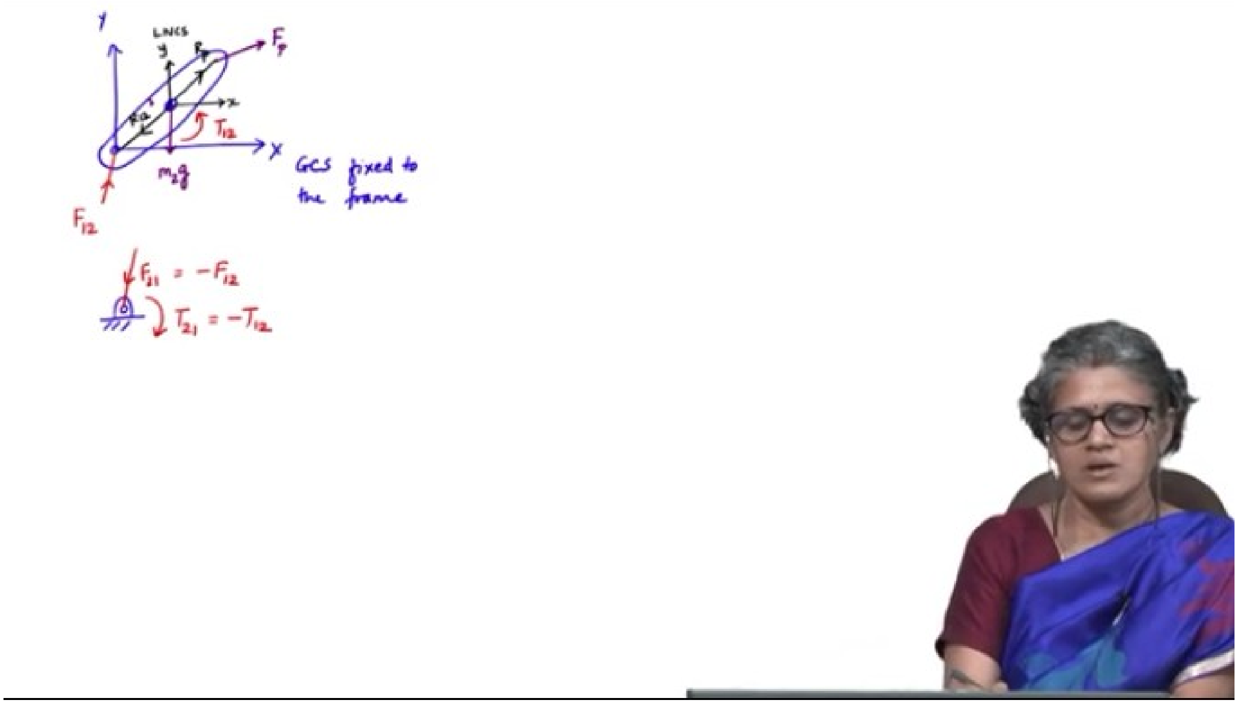

Now if I write in this case what are the unknowns in this, for this free body? FP is an applied force okay, so known applied external force, the unknowns in this will be M2G is known, what are the unknowns? There are two component, because I don't know the direction in which F12 acts, I have 2 unknowns there X component and the Y component and T12, so I have 3 scalar unknowns, okay, and our equations are I have, so my 3 equations F12X okay, or let me first write it in vector form so sigma F = 0 implies FP+F12 vector form, sorry MAG, this is equal to mass times M2AG, here I've neglected the acceleration, the weight otherwise this will be equal to without neglecting the weight of the link, I have FP+F12 – M2G = M2AG, so this gives me 2 equations, because I have FPX + F12X, M2G will not come into the picture there, AGY, so this gives these two equations.

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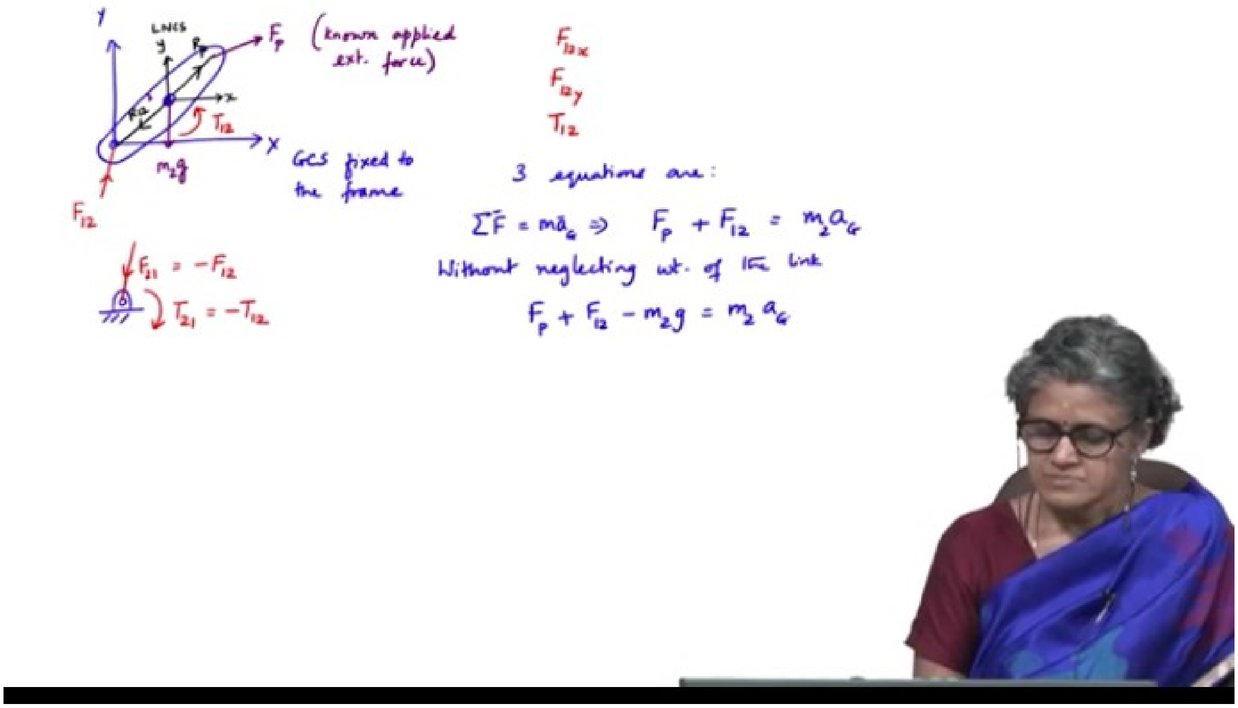

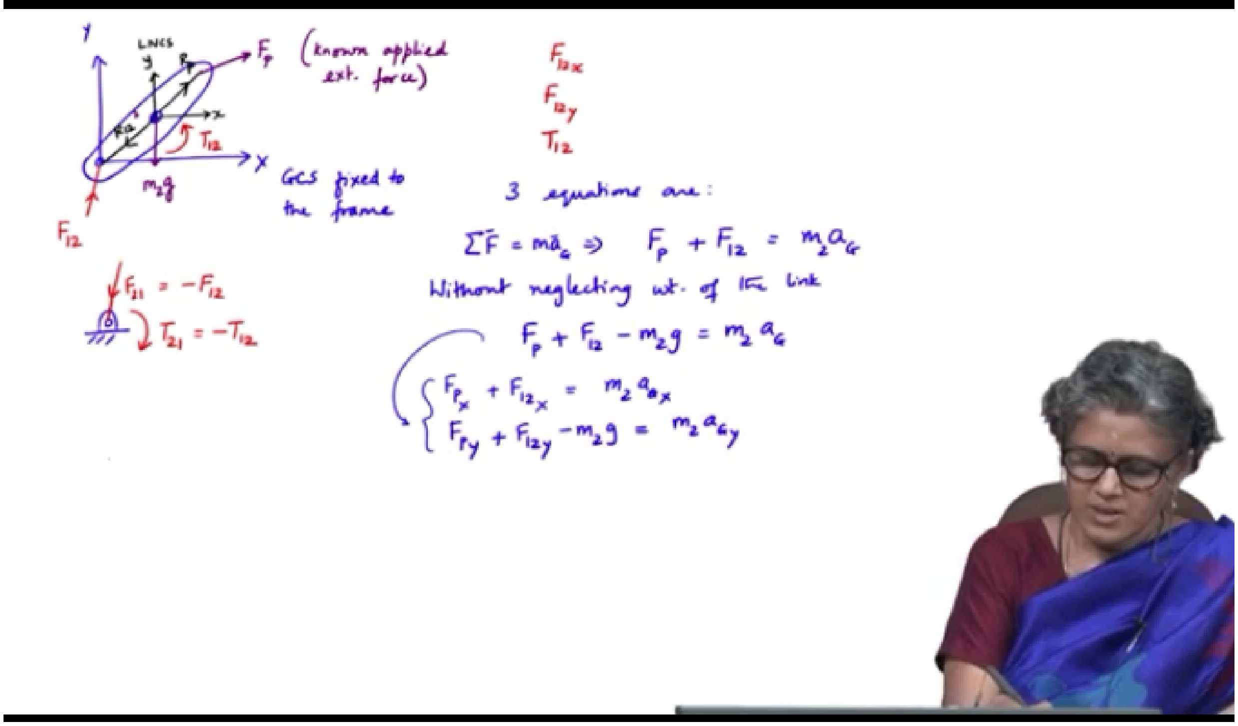

Then I have sigma T = IG alpha, so now I have assuming counter clockwise is positive, I have T12 okay + R12 x F12, okay, moment due to this force about +RP x FP = IG alpha.

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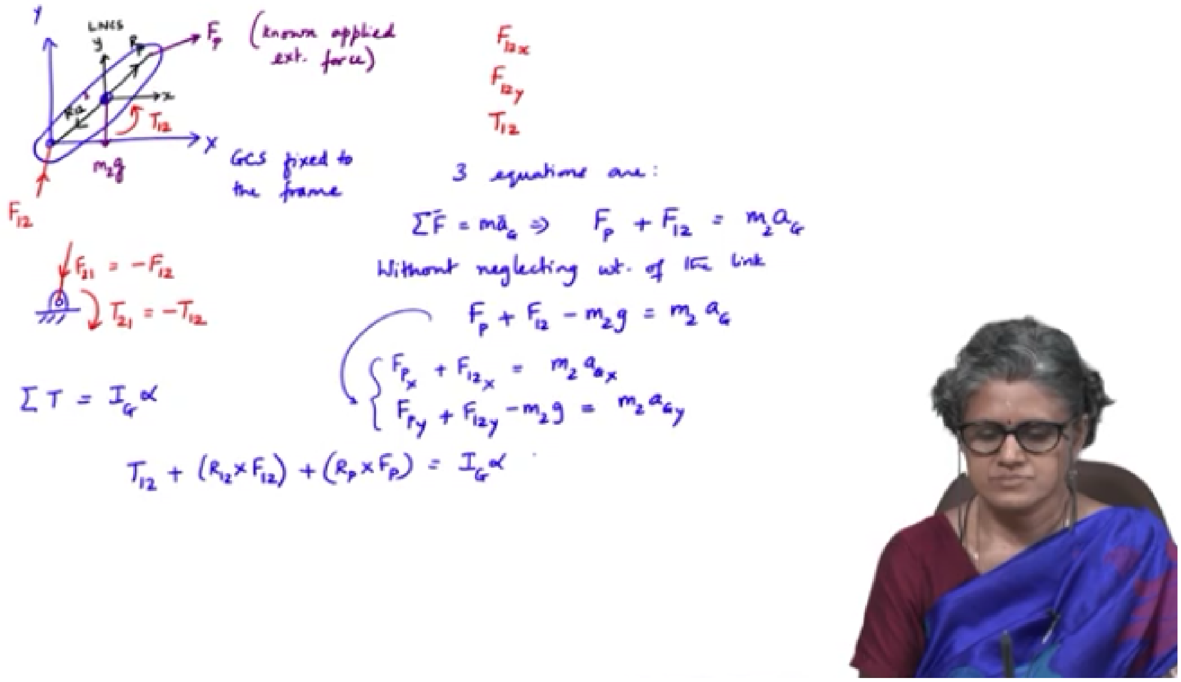

Question? T12 is, so I'm assuming a direction for T12, then if it turns out to be negative I know it's in the opposite direction, just as you do in engineering mechanics right, you assume for unknown force you assume a direction, then if it turns out to be negative you know it's the other way.

So this I can write it as there is only, this is only one equation for a planar case I can expand it as R12X F12Y – R12Y F12X + RPX FPY – RPY FPX = IG alpha. So now what are the, so this is, these are my 3 equations, the unknowns are F12X F12Y and T12,

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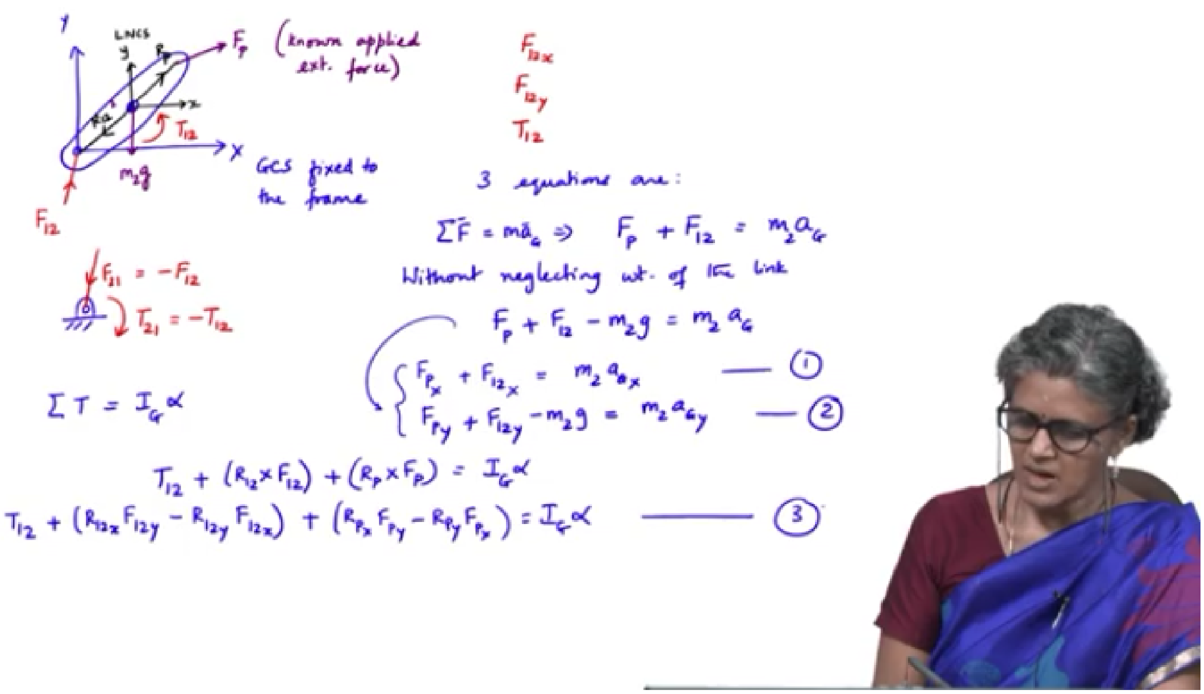

so I can put this in matrix form as 1 0 0, 0 1 0, -R12Y R12X 1, T12 and all the known quantities are M2AGX –FPX, M2AGY – FPY, AGY+ G and IG alpha – RPX FPY – RYFPX,
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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12y} & R_{12x} & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_x} - F_{P_x} \\ m_2(a_{G_y}+g) - F_{P_y} \\ I_G \alpha - (R_{P_x} F_{P_y} - R_{P_y} F_{P_x}) \end{bmatrix}$$

so I can then solve for the unknowns F12X, F12Y and T12, okay, so for a single link if I want to run it at a particular angular velocity and acceleration, then I can find out what is the torque I require in order to be able to do that, okay.

Now this then obviously be extended to the 4 bar linkage, okay so again you would just separate out, if you have a number of, okay, before that if you have a number of loads on the linkage you can just deal with them, you know you can either find the resultant of those and then deal with them or deal with each of them separately, it doesn’t matter, okay, for known loads you can have any number of loads on the system, it does not matter, it does not change anything about the form of these equations.

So this is different, so the local non-rotating coordinate system that we have taken is different from a coordinate system that’s attached to the link, this one is only located, the origin is located at the CG of the link, but it’s orientation with respect to the global coordinate system remains the same, because if you get in to rotating, so if you have because the links are all rotating, right, if you had a coordinate system that is rigidly attached to the link that will rotate with the link, and so it’s orientation will change, but that’s not what we are looking at here, the rotation, the inclination to the link is taken care of by the fact that you know this angle will be change, because this coordinate system remains at the same orientation with respect to the global coordinate system,
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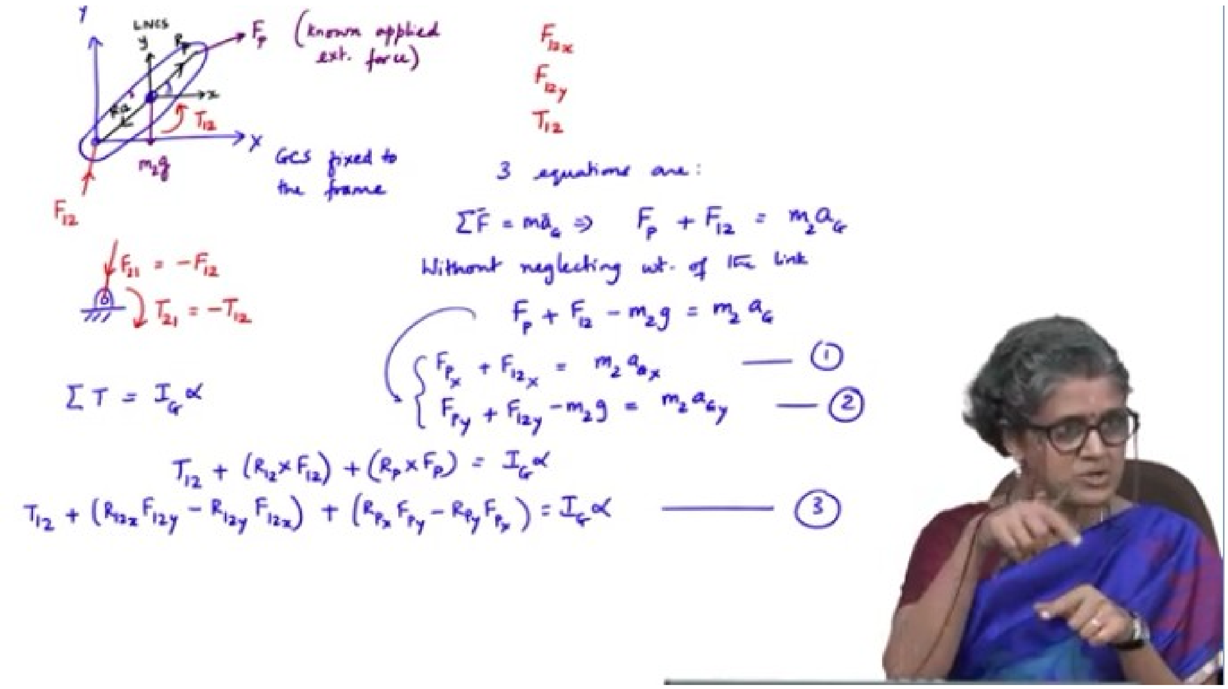

so those angles will come into play when you are defining these vectors RP and R12 and all that, okay that will come into play.

If you took a coordinate system that was rigidly attached to the link, then RP and R12 would not change at all, if they are defined with respect to that coordinate system, however then you have to take into account the fact that the coordinate system itself is rotating, okay, which is typically what you do in robotics, in robotics then you define the transformation, you say okay if I have a coordinate system that is rotating with respect to my ground coordinate system in a particular fashion, then I define the transformation between those two, and then vectors that are defined in that coordinate system automatically, it's the same thing we are doing, it's just that this is one way of doing it, okay, without using transformations, and at the same time you want to make sure that you're expressing everything in coordinate frames that are parallel to the ground frame, because with rotating frames then when you do accelerations you have to take into account when you express the accelerations into the inertial frames you have to take into account the transformation, okay.

So let's say we know have a 4 bar that could be the CG of that, okay,
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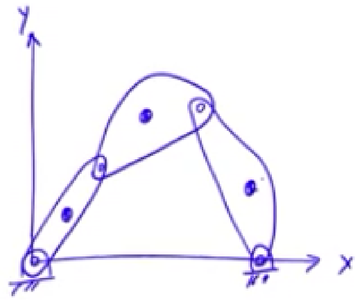

so the first thing you would have to do is for each of them you would, so when you do the free body diagram you take that, you would have establish, you would do the same thing that we did for the previous case, for a single link, right, the only thing you have to be careful about is to make sure that, that's where it helps to follow that convention to do it systematically, force of 2 on 1 you know expressing everything in a systematic fashion will help you to minimize mistakes, okay, but essentially you will have to do the same thing, you will draw the free body for each of these okay, and what will happen? You will have how many equations for each one? You will have 3 equations for each one, okay, and then so for these 3 moving links you will have a total of 9 equations, okay.
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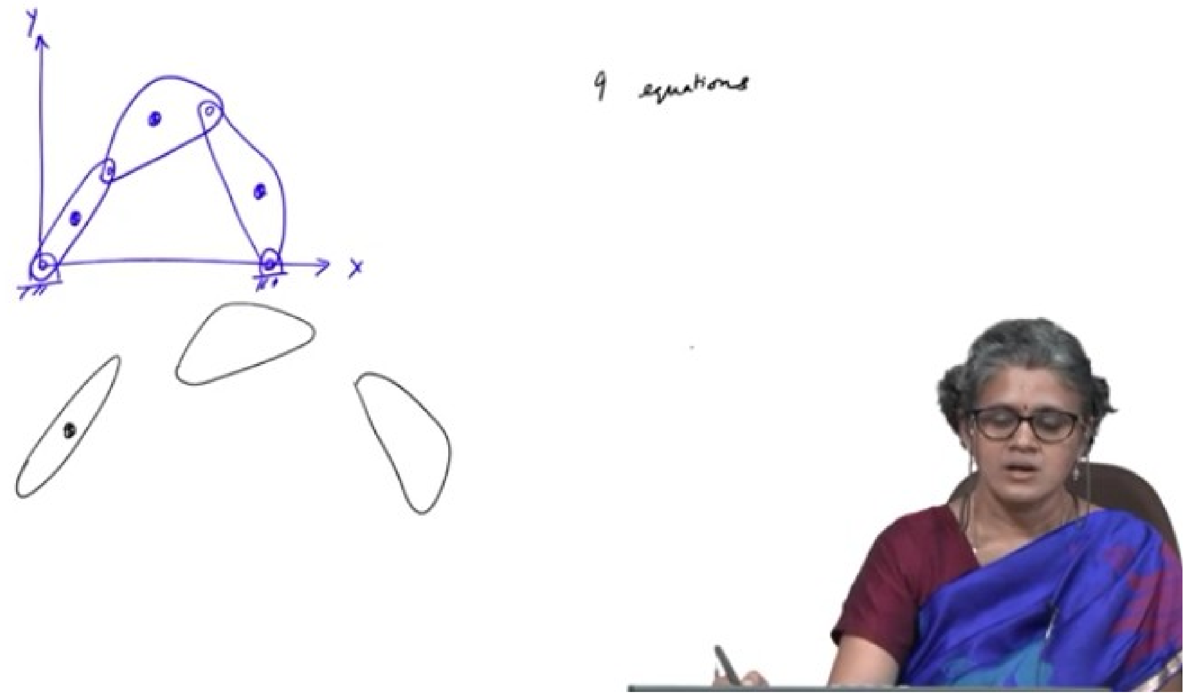

What will be the unknowns? You will have the joint forces which are unknowns, so how many of those do you have? How many joints do you have? 4 joints, is the reaction force at each joint, 2 components for each of them so that is 4 x 2, 8, so 8 unknowns are the reaction forces, what else is unknown? How I'm going to write this? So you will have a torque, it's not known, it's what you want to find out, it's the first thing you want to find out that will be T12,

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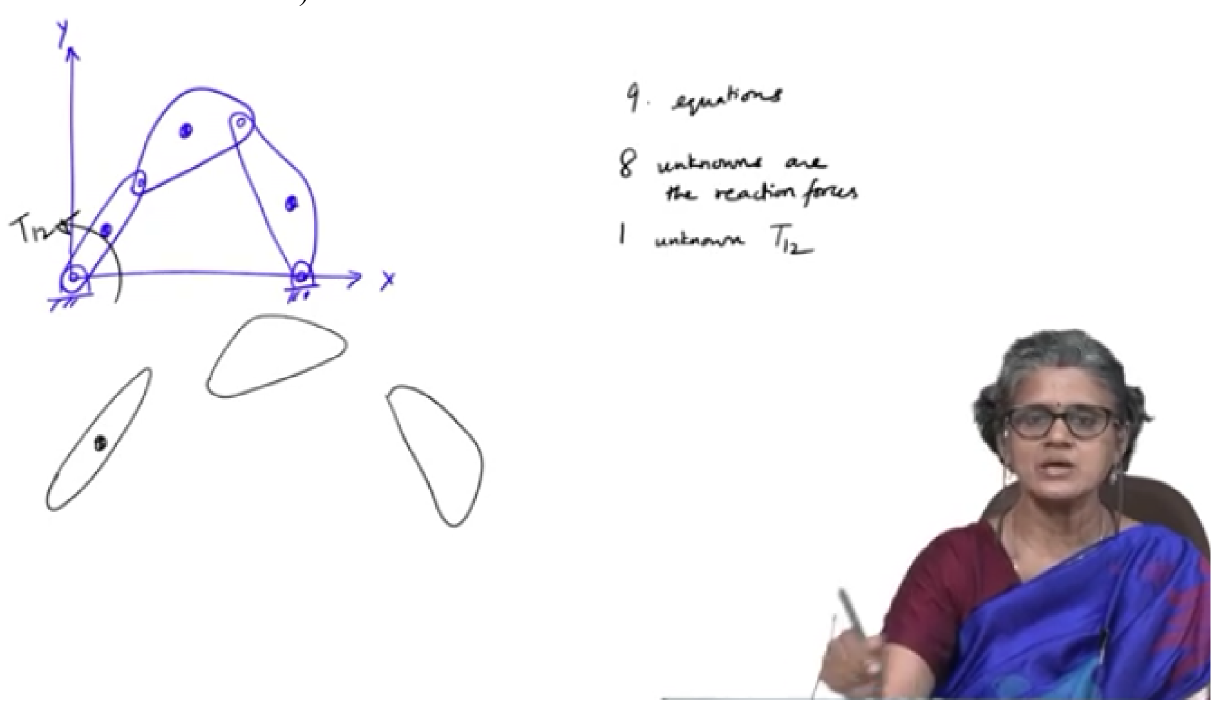

so this gives you 9 equations, 9 unknowns, you can set up the matrix and solve for this, okay.

Now you see that on the frame, so if I look at the frame you will have a reaction torque as well as forces F of 2 on 1 and also F of 4 on 1, right, so if I look at my frame I'll have some F21 possibly some F41 and T21, and this also because these two forces are acting at a distance, they will also cause a moment, so the frame will be subjected to, what are known as shaking forces and shaking moments,
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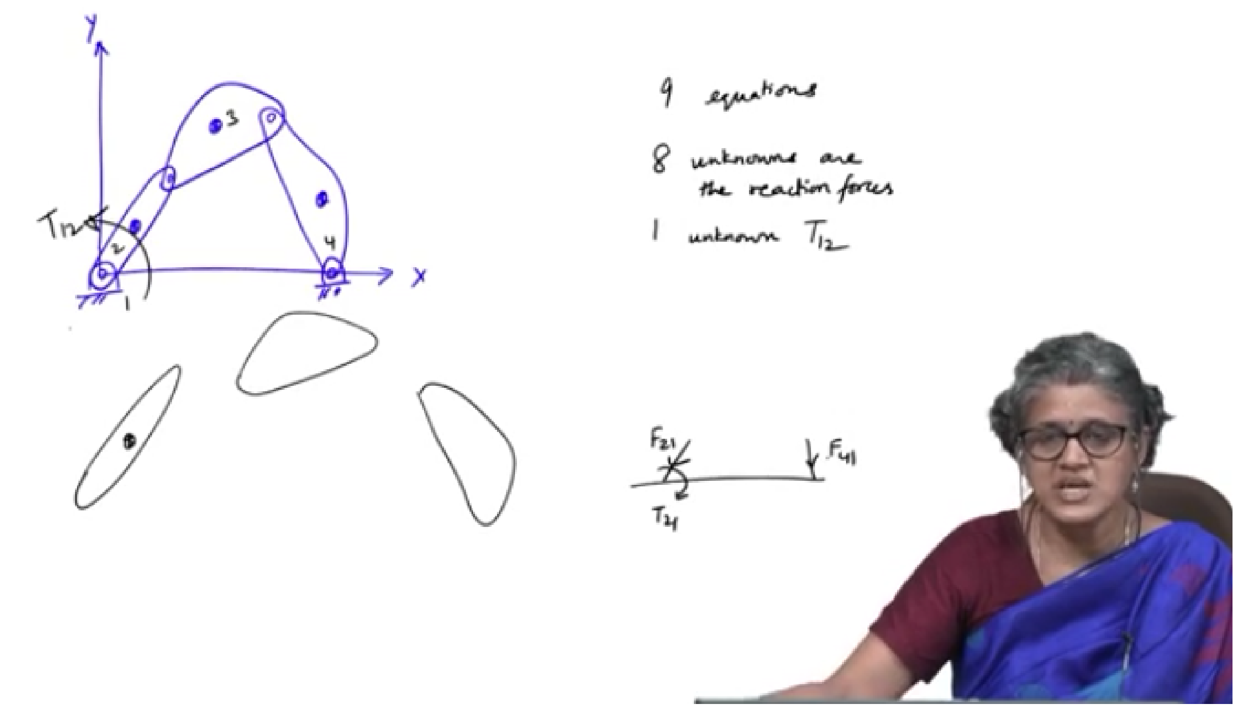

because all these are going to be transmitted to the frame through these, okay, and that is where balancing comes in, balancing is to help reduce this shaking forces and shaking moments, because if these are high then your systems going to be, there's going to be vibration, the ground is not able to take these, absorbed these shaking forces and shaking moments, then you're going to have vibrations induced in the system which can be detrimental to the system and therefore, but this force analysis will help you determine what are the shaking forces and moments that act on the frame as well, that are transmitted to the frame or the ground, okay.

So that, we use balancing in order to be able to reduce or eliminate this shaking forces and moments, you can't completely eliminate them but at least the problem with mechanisms is that because this shaking forces are not constant, right, see if it's a constant force it's usually not a problem, as long as the structure can withstand it, a constant force, if it's the force that keeps varying that's when vibrations will be induced in the system, and if that matches the natural frequency then you have further problems, right, so constant forces are not a problem, I mean you have heavy things sitting on something, sitting on the ground not a problem, okay, constant forces are never a problem, it's forces that are time varying that can cause a problems, and that's what you want to try to minimize the effects of this shaking forces and shaking moments, so this shaking force for this case will be F21+F41, okay.
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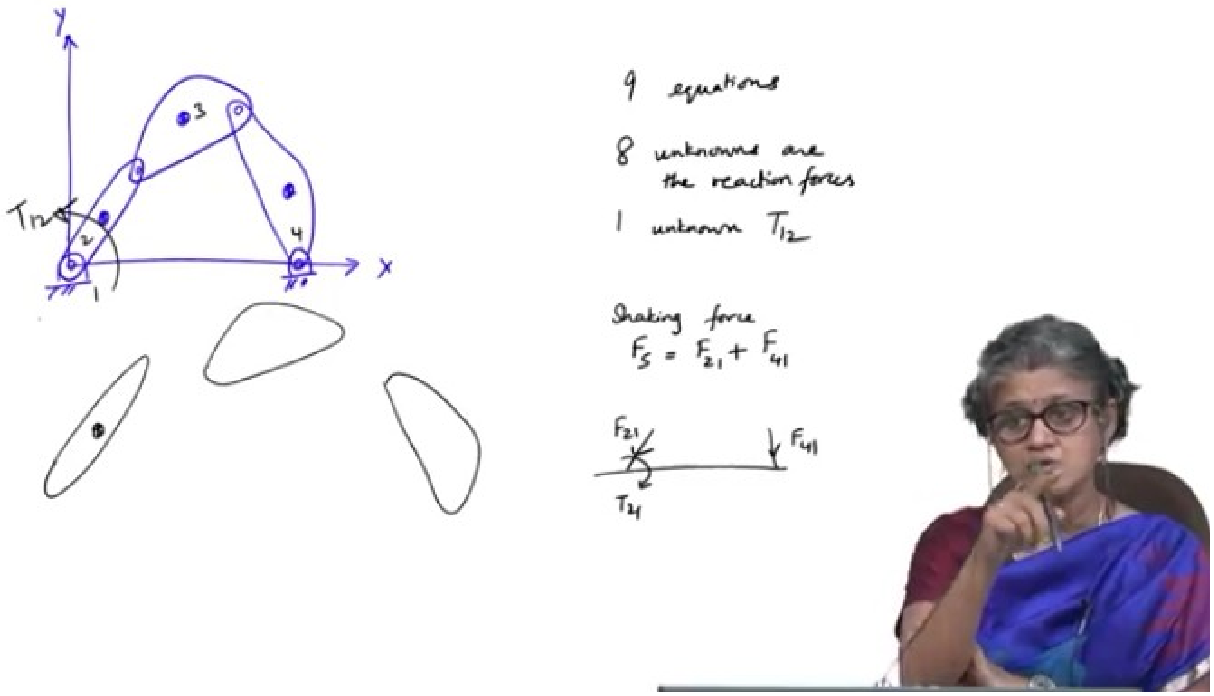

So one of the means of minimizing the effects of the shaking forces is to use balancing and we will talk about that in the next class, okay, we will see how to balance individual links then we'll see how to balance a 4 bar linkage as it moves.

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