Lecture - 34

Theory Of Mechanisms

Balancing of Mechanisms using Counterweights

We started force analysis and, you know,

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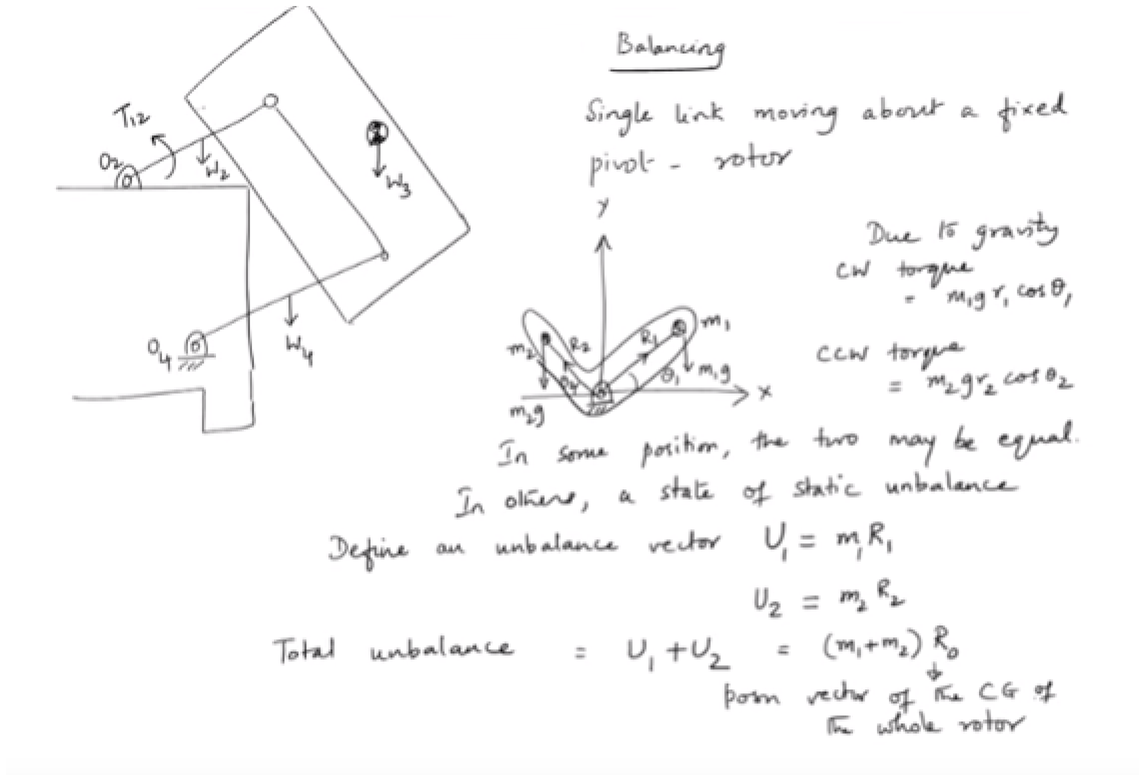

an example of, where you would use something like that. They say, you have, you have, say, something huge, like this, industrial oven. Okay? And you have the door of this, being operated by, a linkage. So you have something huge like this, obviously, the links also, will have significant mass. So in many cases, so something like this, the, it's not the high speeds, that you are worried about. but you know it could be more of, a quasi static analysis, but you would need to find out. So for instance, if it was in the halfway open position, what would be the torque, that I would require, in order to hold it open, there? Okay? So that may be something. So this is, sort of an example of where you would, do the force analysis. So this is a four bar linkage. So here even though, the individual links, may not have, accelerations, it's just the weight itself, can be a significant force, that needs to be, balanced and so the input torque, would miss, would in a sense, be trying to balance the, so in any particular position, of the linkage, you would be looking at, balancing this configuration. Okay? So you would do the force analysis, like we did last time, do the individual, flipper, setup, again you have nine equations, nine unknowns. And using that, you would find, say at different configurations, what will be the torque required? Now from, a safety issue, something like this. Okay? Say, say there is a motor, that's operating, the opening and closing and say it fails or there's a power cut, there’s no backup. Right?? This can be quite a dangerous thing. Right? The weight itself is significant, that. So ideally, you would want, this mechanism, to just, stay in that place. Said the power goes off, you want the mechanism, which is possible. So if I had a mechanism on a horizontal plane, I'm over to a certain position, it's going to stay there. Right? On its own, it's not going to move, to another. But in many cases, in real life, we are Operating, under the effect of gravity. You know? Many mechanisms will be vertical and will be under the influence, of gravity. So in a case like this, you want to be able to ensure safety. And also, you want to be able to, operate it, you know, even the motor. So you want to be able to optimize it, you don't want a really high torque motor, perhaps, to do this. Or, if you want to operate it manually. Then, that's when, we start looking at, what we call, balancing. Okay? So in balancing, essentially what we try to do is, you try to, fall in, as the mechanism moves. Right? You have these moving members, so the overall CG of the mechanism, keeps moving. Okay? Because of. If you can somehow, make that the CG of the mechanism, stay stationary. Okay? Then, so the potential energy remains constant. Then at every position, it's going to be in equilibrium. So to move it from one position to another, is going to take, I mean theoretically it will take zero effort, but practically it will take, only a little bit of effort, if it is exactly balanced, with, I should be able to move it from one position, to another, with zero torque, essentially. Because it's all, it's equilibrium, at that. So this would be a case where, in the new position it's equilibrium, so it’s at equal equilibrium. So at every position, it isn't static equilibrium. So balancing of linkages, has applications like this, where, you know. Say for, safety or to reduce the effort that you want to do. I showed you the example of the, standing wheelchair. Right? Your reducing the effort of, you have the weight of the person always acting, so you have to somehow, account for that, so that you can reduce the effort, required to operate the mechanism. So balancing essentially, you are trying to make the potential energy, constant or fairly constant. So that even if you have to apply a torque, it's sort of, way, you know, there's a small variation in the torque, that you, effort is more or less, uniform, when you are, operating the mechanism. Because otherwise, you're going to have huge variations, as this mechanism moves So balancing is also very important. In high speed mechanisms, you know, where we were, what you have done in K DOM, for instance, the balancing, that was more for balancing the inertial forces. So that would apply even if, say, I have a mechanism, that's on the horizontal plane. Okay? But I ran it at high speeds. I would still have to do, balancing, in order to balance, the inertial forces, in the mechanism. Here we are talking about, static, or merely static applications. But you are, balancing some constant forces, mainly gravity, which or it could be, a load, so some load has to be moved up and down. So you want that to, do it, do it with minimal effort. So mainly effects of gravity, can be, accounted for, by using balancing.

So this balancing can be done, using, counter weights or it can be done using springs. And we’ll, I just show you, a couple of examples, of both cases and you will have the opportunity to apply, that for your project. Okay? So this is basically three moving links. Let's just start with a single link. So if you want to balance a single link, a single link that is moving, about a fixed pivot, is generally called a. ‘Rotor’. So its motion will just be pure rotation, about the pivot. So let's just say, we have a link, that has, say, it's like a Bell crank. So it's. Say shaped like this and each arm, I can say, it has, you know, the CG is, located like this. So I have an m1 and an m2. Okay? Mass of this. And let's say, this is defined by the vector r1, this is defined by the vector r2, theta 1. Okay, so this. So I have, in this case, m1g acting like This, m2g acting in this manner. So if I have a link like this. Right? In the vertical plane, subject to the influence of gravity? Then, it's not, if, if I just leave it. Okay? Because of the two lengths, have different masses, it may, for some value of theta, you know, it may be, that it is balanced, it doesn't move from that position, but in most cases, it's not going to be the case. Okay? If I leave it, it's going to move, on its own. So that's when, we say we say, it's not Balanced. So you have a clockwise torque, which is equal to m1 G, r1 coz theta, cos theta 1 and I have a counter clockwise dot, on the sling equal to, m2g, r2, cos theta 2. Okay, this is again, this is, due to gravity. So at some position, these two may, be Equal, but, really we, at most positions they're not going to be equal. Right? In general it’ll, so that, we call this, a state of, so in, maybe equal. In others, we say, it is in a state of static unbalance, imbalance or unbalanced. So we define, an unbalanced vector, we define a vector, so capital letters, remember are vectors. U equal to m, u1, equal to m r 1, m 1 R 1 and u 2 equal to, m 2, R2. Okay? So the total unbalance, in the system, like this is, U1 plus U2 and if I define it in terms of the CG, this would be, m1 plus m2, into some R naught, where, this is the position vector of the CG of the system.

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For static balance, the total unbalance vector must be compensated by a balancing vector U_b such that

$$U_b + U_o = 0$$

$$\Rightarrow U_b + \sum m_i R_i = 0$$

$$\Rightarrow U_t = m_t R_t = 0 \quad \Rightarrow R_t = 0$$

$\Rightarrow$ The total CG after balancing lies at the pivot.

Balancing mass can be added or removed from the rotor

$$m_i R_i \omega^2 + m_b R_b \omega^2 = 0$$

So if I want it to be balanced, for static balance. Right? I want it to be, I need to compensate, the Unbalance, by another, balancing vector. So I say, the total unbalanced vector must be compensated by balancing vector Ub, such that, Ub plus U naught, should be equal to 0. That is, Ub plus sigma, mi, Ri equal to 0. So if I say, this is the total, so this whole thing, if I have, after balancing, I have all this mask, mass located, at some mt, Rt. Okay? That should be, this should be zero. Mass can't be zero, total mass can't be zero. Which means, this, has to be located, that means, the total CG, of the balance system, has to be located at the pivot. If the total CG, of the balance system, is located at the pivot, then there is no torque, due to gravity. Okay? And it doesn't matter, how you position the **rotor,** it's going to stay there. Because the total CG, of the system, does not change. Okay? So this implies Rt is equal to 0, implies the total CG, this is after balancing, so you add or subtract a mass, at a some distance, in such a way, that this becomes zero. Okay? The total CG after balancing, lies at the pivot. So no matter, how the rotor rotates, the CG will not move. So this is the basic principle of balancing. You, we did this, even when we did the balancing. So the balancing mass, can be added or removed from the rotor. So negative mass means, you're removing, mass from the rotor. Balancing mass, can be, usually if you are running the rotor at a constant angular velocity, you use the same thing. Because there again, you have mi, Ri, Omega square, the inertial forces, you know, that's the changing forces, that's what you want to balance. So you would, you would, similarly have this plus, you know, some balancing mass, RD, Omega square, is equal to zero.

You do the same thing with the rule. Here of course, we are looking at, gravity balancing, we are looking, we are not looking at the inertial forces. But this is what you would have done, in your Kaynom. Right? We did this. And then dynamic balancing is, when you are also looking at, another plane. You’re also looking at the. So this is, you're looking at this one plane, where you are adding this mass, but if you look at the bearings, so this will reduce the force, on the, the net shaking force, you'll be able to reduce to zero and you do this. But if they are separated, so axially if you have a length, then you have to do dynamic balancing, where you look at, that plane, and then you have to balance the moments, also in that way. We are going to look at planar, we've been looking at planar mechanisms and we are looking at static balancing of mechanisms. Okay? So that is. So, now, let's look at the same, I told you about this problem with this door. Right? So if I want to balance this Okay? I want to balance this mechanism, so that, no matter where it is, it will stay in that position. Okay? So in order to be able to do that, let's look at, because balancing a rotor, is fairly easy. Okay? You can just and counterweight, like this and balance that, in the case of.

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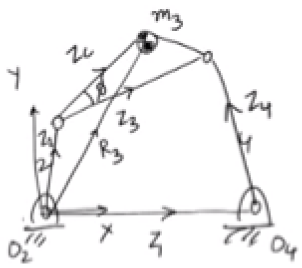

Static balancing of a 4-bar linkage.

$$Z_2 = r_2 e^{i\theta_2}$$
$$Z_1 = r_1 e^{i\theta_1}$$
$$Z_3 = r_3 e^{i\theta_3}$$
$$Z_4 = r_4 e^{i\theta_4}$$

Link 2 and link 4 can be balanced as rotors

Unbalance vector $U_3 = m_3 R_3$

$$R_3 = Z_2 + Z_c$$
$$\therefore U_3 = m_3 (Z_2 + Z_c) = m_3\left(Z_2 + r_c \frac{Z_3}{r_3} e^{i\phi} \right)$$

$$\underbrace{r_c e^{i(\theta_3+\phi)}}$$
$$e^{i\theta_3} = \frac{Z_3}{r_3}$$

Loop closure for the 4 bar

$$Z_2 + Z_3 = Z_1 + Z_4$$
$$\therefore Z_3 = Z_1 + Z_4 - Z_2$$

So if you look at the 4 bar, you have two links, which are rotors, link 2 and link 4. link 2 and link 4 are moving around fixed pivots. So I could separate those two and kind of balance them as rotors. So that the CG, of that, comes to this. But the tricky part, is this, the coupler, the coupler. Because it has, complex motion in the plane. How are you going to, as the mechanism move, moves, how are you going to keep, the CG, of that constant, in the plane? Okay? So that, that's what we are going to look at, now. So for the oven door, I'll give that to you, as a problem that you can, solve, the force analysis that you can do. Now let's look at, the, say for the same application, we want to do the, static balancing, of a 4 bar linkage. So let's say, this coupler, it's CG is located here, it has a mass m3, I have z2, equal to r2, e power i, theta 2, we call this Zc, this is Z3, usual notation and this, the location of the center of mass, of link 3 is specific, so this 5 would be constant, because this is one rigid body. Right? So so link 2 and Link 4, can be balanced, as rotors, so let's not worry about that, yeah, okay? They can be balanced as rotors. So you can bring their CG, easily to, that particular pivot. The motion of the coupler, is what we are interested in. Let’s say, this is located, R3 gives the location of the, center of mass of the coupler. So what I'm interested in balancing, is this unbalanced vector. U 3 equal to m 3, R3. Okay? So R3, I can write it as, Z2 plus Zc. So I can write U3 as m3 into Z2 plus Zc. I can write, the unit vector, along this. Okay? I can write this as, m3 into Z2 plus, rc into Z3 by r3. Okay? Into e power, i phi. I’ll I do that? Okay? This is, Z 3 by r3, is the unit vector along. So that's equal to e power, i theta 3. Okay? So I'm just, because this is rc, e power i of, theta 3, plus 5. Right? That I'm writing like this. Because e power, theta 3, I can write it as Z3 by r3. Okay? From the loop closure for the 4 bar, I have, here, Z2 plus Z3 equal to, Z 1 plus Z4. Therefore I can write, Z3 in terms of the other vectors as, Z 1 plus Z 4 minus Z2. Okay?

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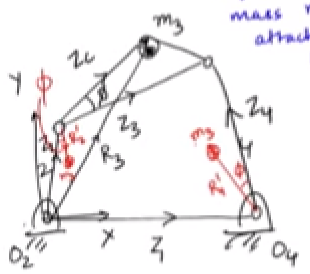
$$\therefore\ U_3 = m_3\left[Z_2 + \frac{r_c}{r_3}e^{i\phi}(-Z_2+Z_1+Z_4)\right]$$

$$= m_3\left[Z_2\left(1-\frac{r_c}{r_3}e^{i\phi}\right)\right] + m_3 Z_1\frac{r_c}{r_3}e^{i\phi} + m_3 Z_4\frac{r_c}{r_3}e^{i\phi} + \underbrace{m_3 Z_1 - m_3 Z_1}_{\text{add \& subtract}}$$

$$= \underbrace{m_3\left[Z_2\left(1-\frac{r_c}{r_3}e^{i\phi}\right)\right]}_{\substack{\text{equiv. to a}\\ \text{mass } m_3\\ \text{attached to}\\ \text{link 2}}} + \underbrace{m_3\left[Z_1 + Z_4\frac{r_c}{r_3}e^{i\phi}\right]}_{\substack{\text{equiv. to a mass}\\ m_3 \text{ attached}\\ \text{to link 4}}} + m_3 Z_1\underbrace{\left(\frac{r_c}{r_3}e^{i\phi}-1\right)}_{\substack{\text{unbalance due}\\ \text{to a non-moving}\\ \text{mass}}}$$



$$= m_3(Z_2 + R_2') + m_3(Z_1 + R_4') + \text{constant term}$$

I can substitute that back, into the unbalanced vector, as U3 equal to m3 into z2, plus rc by r3, e bar i5, into minus Z2 plus Z1 plus Z4. Okay? I've only substituted for r3 in terms of the other vectors. So I can now write this as, m3, Z 2 into, 1 minus rc by r3, e power, i Phi, plus m 3, Z 1, rc by r 3, e power, i Phi, plus m3, Z 4 rc by r 3, e power, i Phi. And then, let me add and subtract, m3, Z1, from this equation. Adding and subtracting. So this one now, if I group, I get, m3 into Z2, 1 minus rc by r32, e power i phi, plus m 3 into Z 1, plus, Z 4, rc by r 3, you will see why, I added and subtracted, e power, i phi. Okay? Plus m3, Z1 into rc by r3, e power i phi, minus 1. Now look at this. z2 is the vector here. Right? So this is like an unbalanced vector that is attached to, that is affected by the movement of z2. Okay? Because as z2 changes, this unbalanced vector will change. Look at this one. Z1 so my loop closure is, Okay? My loop closure is here. So if I look at this one, this is something, that varies with Z2. Because, everything else is constant here. Phi is constant rc, r3 are clear links, everything else is constant. So this is equivalent, to, it's like an unbalanced, vector, that is, attached to link 2. Okay? Power of mass, that is mass m 3, attached to link two, in some fashion, which moves with link 2, which changes with link 2 Okay? So this is equivalent, to mass m3, link 2. Look at this one, Z1 plus Z4, something. Okay? So Z4 is the varying, quantity here, everything else does not change, Z1 is your fixed link, does not change. So this is equivalent, to a mass m3, attached to link 4. Okay? Look at this one. This is a constant torque. Does not change as the, mechanism moves. So yes, it is an unbalanced vector, but it's not changing. Its, its constant. So it's just something, you have to live with, but you know, it it's not something that's going to, it's the unbalance, of a non moving mass, it's a non moving mass. So, so what it essentially means, is it will not affect, the CG of the system, because it is, it does not move, the CG of the system will not change, the CG will not be at, a pivot, like in the case of the other two. Okay? But it's also not going to change. So this will be something that will contribute to the CG, being someplace, other than at a pivot. Okay? So you have a finite, so this is the unbalanced, due to a non moving mass. So I can write, this the first term, I can write this as, m3 into, m3, into Z2 plus r2 dash, plus m3 into Z1, plus R4 dash, plus constant. So what is my r2 dash? It is, the vector Z2, okay, minus Z2, scaled by rc by r3 and then rotated by phi. Okay, so if I want to locate it, here? I can say, so I locate, this is r2 dash. Okay? And this would be at an angle, that angle would be, phi and this mass is m3, so this is attached to link 2. So and in this case, you have Z1 plus r4 dash, where, this is like this. This is the angle phi, this is r4 dash, this is m3. Okay? So, the unbalanced due to the

Coupler, can be reduced. So I can replace it by an equivalent mass m3, located on link 2, like this and on link 4 and then of course a constant term. Okay? Constant term, that's attached to link 1. So what this tells me is, now my link 2 and Link 4, are simply rotors. So in addition to their mass, which is located at their CG, their total mass, that I want to balance, is that one, plus due to this m3, located at r2 dash, on link 2 and similarly on link 4. Okay? If I do that, if I balance that, then I can say that, this, the varying component, is made 0. Okay? So there is no,

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∴ Unbalance vector due to mass of coupler can be replaced by unbalance vectors due to equiv. masses on link 2 & link 4

Link 2 & link 4 can then be balanced as rotors by including their own masses & the equiv. masses

The total unbalance vector of the linkage will not vary as the linkage moves

∴ The total CG of the linkage remains constant

so the, unbalanced vector, due to mass of coupler, can be replaced by unbalance vectors, due to equivalent masses on link 2 and link 4. Link 2 and Link 4 can then be balanced, as rotors, by including, their own mass, and the equivalent masses. Yes, gentleman. Take a look at that. I13 is changing, that is the problem. If it was a fixed point, yes, you could do that. And that's, that is what, you, you will end up with the same sort of result. So by doing this, the total unbalanced vector, that is after balancing, of the linkage will not move, when the linkage moves. Therefore the total implies that, the total CG of the linkage, remains constant. So you cannot make the unbalanced vector Zero, for the 4 bar. But its variation can be made 0. Okay? So it remains at a, constant CG. We will stop here. Tomorrow we will do, static balancing, using springs, instead of masses. So masses, this is one way of doing the balancing. The only problem is, you're increasing the mass of the entire system. You're adding mass to the system. So for instance, you know, if you had, you know if it is in a static place, fine. But if you, have to make the pull of, portable for instance. Right? You're going to carry, all the extra mass also, the balancing happens for the gravity. But other things, are going to be affected by the increased mass, it also increases cost. Because anytime you are using more material, you're going to increase the, cost of the system as well. So those are some of the drawbacks of doing, balancing with counter weights, like here. Springs offer another alternative, for doing, this kind of balancing; they come with their own challenges.