Lecture – 36

# Theory of mechanisms

Spatial Mechanisms



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So, this course has been largely about, planar mechanisms, where all the points on the mechanism, more want our parallel planes, and a majority of real-life mechanisms, fall into that category. So, you can actually load up the mechanisms that you will encounter, can be planar mechanisms, however there are, some special applications where you may need, a mechanism which has where the point paths are not in parallel planes, but you need them, you know, its general 3d space. Okay? So, there they have general 3d point Paths, and these kinds of mechanisms are called, spatial mechanisms. Where the motion. so ,you have you, you can't view the motion from just one directional. Okay? So, in any planar mechanism if you view the mechanism, from the direction that's perpendicular to the plane, in which the motion is happening you get the complete picture, of the motion. That's why most real life mechanisms, even though they are solid you know, they are you, you can move from a single plane like your folding chair. Okay? You can move the mechanism just from the side, you don't need the whole so, we say those, those 3d or spatial mechanisms are built by stacking planar mechanisms. That's not what we call her so, we would still deal with that as a planar mechanism, we wouldn't call that a special mechanism, a true spatial mechanism has point parts which are in three dimensions. So, take for instance, this is like an agitator mechanism in a washing machine .okay? You can see that there is no single plane, from which you can view this, view the motion of this mechanism or describe the motion of this mechanism. Okay? This, YouTube channel ,of this person Tank, has some very nice

animations, he's created animations of several mechanisms, you should go see, some of those, it's a, it's a very nice .so, I asked him, for permission to use some of these videos, and he agreed .

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Spatial Mechanisms



So, here is your wheel Attraction, in an aircraft you know, but it's not, just happening in a plane. So, you can see that this is, you have a revolute joint and you have this link with two and then you have a slider, the wheel attraction mechanism. Okay? And there's, a

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Spatial Mechanisms



Here's another example. This is a, mechanism for can you guess making dough, chapatti dough. So, they can also in big bakeries or you know, things like that they'll have mechanisms. That will do this. Let me just put it on repeat. So, it left certain you know, does the kneading of the dough. Okay? So, so there are, applications where you know, special mechanisms, are used.

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The special version of the fourth bar, let me show you this, this is our gondola 4 bar but ,in a special form .okay? the special version of the, 4 bar. So, you have the links are all in different colors, you can see the bearings, these are the, that's the fixed link, this is a special four bar.

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Examples: Bevel & worm gears Hooke's joint Universal jointe Hobson joint CV joint Human ann Parallel manipulators (eg. Stewart platform)

So, you have other examples can you think of any examples, you may have seen of special mechanisms, where the motion doesn't just happen in one plane, other examples can you, that you may have seen, not necessarily linkages but ,anything where you can't describe the motion in just one plane, bevel gears, yes, yes. So, the worm gears, bevel gears, they're all see bevel gear it's like two perpendicular axes, right? Yeah, so, you could yeah. You can't observe the motion, from the babble and worm gears Hookes joint, yes. I have a video of that. Okay? Hookes joint, where you have two, joints that are at different angles, and you're transmitting rotary motion, from one to the other that's a good example of a special mechanism. Right, right, right?

How do you spell that, Hobson joint? Okay? CV joints, yes. So, a lot of these transmission joints, you know where you have shafts, which may intersect, meet the access, may intersect, may not intersect you know, all kinds of combinations, when you want to transmit rotary motion, from one shaft to another, a lot of these joins are actually special mechanisms. Because, and you may also need it too even if they're expected, to be you know somewhat aligned you may use something like this, so, that you can take care of any misalignment in the shaft. okay ?you don't want things to bind up so, any constant velocity joint ,hope Hookes joint is not constant velocity but, there are other ,what you know in a general term we call is the universal joint, u joints, universal joints. They all fall under these, what else can you think of? Yeah. So, 3d we have already mentioned some gears, how about something close to you, can you think of anything that you use everyday bicycle, bicycle is planar more or less, by secondly most of it can be described using planar, I'm talking really close to you, human the arms. Right? So, as I am writing, my arm is making a closed loop mechanisms. Right?

And I can't describe the motion in the single plane. so ,as I'm writing I'm actually closing the loop, all robotic manipulators are also 3d many robotic manipulators are 3d mechanisms but, in many of them they may be open loop mechanisms but ,even if you look at like a closed loop, when I'm doing something with my arm. Okay? It forms a closed loop. Right? So, that's human arm is another example of a special mechanism. Okay? So, and then you may have heard of like stupid platforms, right? They're parallel manipulators, parallel manipulators. Okay? So, these are all examples of special mechanisms. Okay? cylindrical cam ,that's all again You, know you would have seen with a groove on it , and then it has you, you have a cylinder with a groove on , and that's moving of all, the followers moves in the group so, that's an example of a special mechanism .okay?

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So, just as you have in planar mechanisms, you can have revolute joints, in special mechanisms, revolute joint. so ,it's typically designated as, are in a special mechanism and its symbol would be so, in a special mechanism you would represent it like this .you have one link so, this shows that ,it can only rotate with respect to the other link and not translate. Okay? so your, so if this is your 3d representation, 2d representation we just showed planar, this is how we show the revolute joint. How many degrees of freedom do you have? Only one. The revolute joint still has only one degree of freedom .this is my prismatic joint, right? So, this is

prismatic, and here no rotation is allowed, yes. No so, you have one link, you have another link, when you connect them by a revolute joint is such that it allows only rotation of one link, with respect to the other, that's the only motion that's allowed, when you connect them where it doesn't matter what the direction of the accesses so, when I have a revolute joint, it basically restricts all other degrees of freedom between the two links, it only allows, this motion.

I can have you know something pointing in some other, it that will be the axis of rotation of the joint .but, the only motion that's allowed is rotation about that axis, yeah. Yeah. Yeah. so this is the general representation of a revolute joint. So, for a prismatic joint I have to address the rotation. So, if this is one link. Okay? One can move only, it cannot rotate with respect to the other. So, this is the general prismatic joint, in 2d we showed it like this, right so ,that is the 3d representation, again this is also a one degree of freedom joint. The only there's no rotation is not allowed between, the two links .okay? In the case of the cylindrical joint, you have two independent degrees of freedom. So, this would be the representation, this link you can see here, that I don't have these bars that denote the revolute joint. So, it can have sliding and rotation .so, it is a two degree of freedom join, this is a cylindrical. Okay? So, this is a two degree of freedom joint, and what would be the planar representation of this? It's not a planar joint, you do not encounter it in planar mechanisms because, and you have motions in two different directions. Okay? Sliding takes place and one, yet the rotation as observed in another place. So, because of that this will not, you will not encounter this in planar mechanisms, let me repeat the question again, do you consider pin joints as, yes .no if you have a pin joint, if you have a pin joint like this, if I connect two links with so, yes. In you know you would have a pin that goes through like this, connecting these two links like your door hinge. Right? You don't allow this motion, if it's a revolute joint I will not allow this motion, it cannot slide, one link cannot slide with respect to the other, what moves along the slot? So, if it is a, if it is a revolute joint, the only motion that is allowed is this, I cannot allow this motion, if I love this, and this, then it is a cylindrical joint. You are asking about a pin in a slot. Okay? So, when you look at a pin in a slot. so ,you have a cylinder, thank you, for so your question is , is that a cylindrical joint. it is not a cylindrical joint, because so, if I have a slot and I have a pin, this rotates, I can see their rotation in this plane, the translation is this way.

You are observing both the motions, there is no translation perpendicular to the plane, this is not a cylindrical joint, a cylinder is definition is ,where you have to this is, this is a planar cam joint, we call this a planar cam joint It the axis, yeah. The axis of rotation and the axis of translation coincide. Okay? So, it's translating along the same axis, about which it is rotating. So, the cylindric joint, the axis of and axis of translation coincide, right? the cylinder has only one that, that access so ,this translation happening about the axes ,as well as rotation about the axes, on the other hand this is the planar cam joint, pin and slot right? Here, the axis of rotation is perpendicular to the plane, of observation and the axis of sliding is also different, their two are different. But, thank you for clarifying the Question. Okay?

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And, what is the number of degrees of freedom of this joint? is one, we've seen because, the helical joint has DOF one, because, the translation and the rotation are related, by the pitch of the screw, so, actually the lead of the screw. Okay? So, you have a definite relationship between the two and therefore, it is a one degree of freedom joint but, again the axis of rotation and the axis of translation, are the same here like the cylindric joint but, in the cylindric joint, the two are independent, I mean the two motions are independent Here, the two motions are related degree of freedom is 1, and you have so, your representation would be and it's not a planar joint, not a planer joint. So, there is no plainer representation for us, this is your spherical, sometimes called referred to as a globular joint. So, the dynamic book refers to it as a Globular joint.

This is your ball and socket, ball in a socket joint. How many degrees of freedom? You have three degrees of freedom, and the general representation. so ,you have a ball and you have a socket ,common examples you know, the human body, your shoulder your hip, these are common examples of a spherical joint, or nearly, nearly spherical because, in the human body you don't have a perfect sphere going into a bucket, perfect capsule yes. I'll, I'll show you lots of, lots of applications for spherical joints. Okay? And this is the planar joint. Okay? So, in your mini drafter the scale, that's connected .okay? As it's moving you have the rotation as well as the translation of that. Okay? so ,that is an example, for a duster on a board, right? So, that would be any say if, if I was cleaning the board with the duster. Okay? My arm would form a mechanism, where the joint between the board and my hand is a planer joint .okay? because ,I can move assuming you know ,I take the duster as attached to my hand, then the join between those two links is a planar joint ,when I'm cleaning the board . or even here as I'm writing that's, that's a planar joint, I can move X Y and float it . Okay? So, this is a three degree of freedom joint, and its general representation is just a plane with another. Okay?

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So, typically planar mechanisms, are represented in terms of you know the joint so this would be like an R, S, S, R mechanism. So, this is essentially a four bar. Okay? you have this link so, this is link 1 which is fixed, then you have link to which is the crank, and then you have a link 3 you can see the so ,the joint between link 2 and one, is an R, is a revolute pair, they join between links 2 & 3 is a spherical joint. this is your here they call it globular but, that is a spherical joint, you have and then again the join between 3 and this is link 4, that is also a spherical joint and then 4 & 1, it's a revolute joint .so, this would be typically referred to as an RS, SR mechanism. It's a 4 bar, for as special 4 bars could be an RS, SR mechanism, I think I have a video of that, show that, show you that.

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you can see there, so you can see, this is the link that has, spherical joints on either side then you have so, this is their R,S,S,R 4 bar, special 4 bar, that. Okay? Now, we can do a similar mobility analysis, for spatial mechanisms, except that you have to take into consideration the fact that now , a rigid body in space, has how many degrees of freedom? Six. Okay? When we restricted it to the plane it only had three degrees of freedom. Now, you're talking about a general rigid body so, if you take n links and make a mechanism, you would fix one link so, this would be the total number of degrees of freedom, and then I start adding joints, I start adding constraints. so, if I add a joint, with a single degree of freedom, that is my like a revolute joint or a prismatic joint, what is it doing? How many constraints is it imposing? It's removing five degrees of freedom. Right? It's allowing only one that means it's imposing five constraints. So, for every j1, in the mechanism I'm losing five degrees of freedom so, that's what this means. similarly if I have j2, if it allows two, it's taking away four ,and so 4i2, would be the number of, and so on i3, 2i4, i5 you could have, you could have something, you could have like a ball-and-socket, that can slide in two different directions, in a slot which I allow sliding in two different directions. So, that would be a join that allows five degrees of freedom. Okay?

Like your pin in a slot, right? So, if you had a ball, which was in a slot that would I allow, sliding along two different directions, then you would have a j5. Okay? So, it's like a planer, it's a planer so, it will allow in two, what if that itself can slide? So, it's the ball is in a slot. Okay? So, the ball can slide within that slot but, that slot can slide enough in another direction in the plane .So that becomes a composite joint which has five degrees of freedom. Okay? so, that yes. So, we are coming to that we are coming to that .so, now if I look at this four bar. Okay? Let's do the mobility for this, R, S, S, R mechanism. Okay? So, my n is 4 how many J ones do I have? 2, the two revolute joints, how many I don't have any J 2, 0, J 3, 2. I have two Balan Sanat spherical joints, right? So now, if I do the mobility, if I apply the mobility equation, what do I get for the mobility of this mechanism? I get a mobility of two. But, it's quite clear that it only has you know that, that there is a definite relationship between, the input and the output .you need only one put, one input to make this, mechanism move in a predictable fashion, the input angle output angle can be related directly, which means there is an additional degree of freedom, what do we call that? Which is not affecting the input-output relationship; we call it an idle degree of freedom. So, what is this idle degree of freedom here? Idle degree of freedom is the rotation of coupler about its own axis. So, I can keep this mechanism in this position and I can just spin that coupler, right? I can spin this coupler, and it will not affect the input-output relationship. So, that makes it an idle degree of freedom, if with this input and output .okay? so, because it so the equation is not wrong, this mechanism does have two degrees of freedom, functionally we are only concerned about you know, this as input, this is output. So, functionally it has a sink, it's a mechanism of mobility one, and it has an idle degree of freedom which is the motion of the coupler about its own axis. Okay? So, if you had to draw the kinematic Diagram, of you would draw it here something like this. So, this could be moving in a, circle like this and this. So, you would have so, your R, S, S, R. depends on, what you define as cross and straight here, right? Everything is, it could have multiple branches here. Okay? The displacement analysis, the analysis special 4 bar is a lot more complicated, then but yeah.

If you just take so for instance you just take a planar four-bar. Okay? And you know, I put instead of making a revolute joint, I can always put a spherical joint there, right? So, I can have two branches of this. Okay? Why would I do something like this? Sometimes, and in fact they don't do this, even for some planar applications, to handle misalignment .so, they will put you know because, of manufacturing errors, there could be misalignment. So, for instance your steering mechanism will have this kind of an arrangement. Okay? So, they may use even in planar mechanisms, they introduce these additional idle degrees of freedom because, it can help to handle misalignment. But, there is a very specific relationship between input and output. Do you have a question? Ah. Okay? And let you think about that, really we'll talk, yeah. Why doesn't this, you know if I apply this for a planar case, it won't work because; they are applying the same constraint to multiple planar joints. So, they are not really independently taking off any so, you will actually get something that is over constrained, yeah, so, we so the question here is why this equation doesn't apply for a planar mechanism? if I take a planar mechanism planar four-bar and apply this, what will I get, I get, I get minus 2, I will get um an over constrained mechanism. So, that means there are three extra constraints, see one is I know, the mobility of that mechanism is one. But, I'm actually getting minus 2, which is 3 less, that's because, when I restrict it to the plane. Okay? I have already

said, that is my axis. So, the other three joints, the other three single degree, are not really I when you apply this equation, they are also taking away 1, 1 whereas they are not independent. Right? All of them it's the same constraint applied to all 4 joints, when you restrict it to the plane. So, you do not apply the criterion, the spatial mobility criterion, to a planar mechanism. Good question. Okay?

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So, this is your Sum more examples. So, a special case of the special mechanisms, is when all the axis, of rotation intersect at a point, and so you will see here, this is your hook joint, this is your hook joint, you can see that, it's transmitting from one shaft to another and you have this is a this is a 4r mechanism. Okay?

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This is another version, of it and we call these spherical mechanisms because, all the points move on concentric spheres. Okay? You can see that, the axis always intersect and the motionless spherical. So, these spherical mechanisms are, a special case of special mechanisms, yeah, yeah. So, but this demo shows that spherical nature nicely. Okay?

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Bennett Mechanism (4R)

And then, you can have other mechanisms so, this is like a twisting 4 bar to special mechanisms called the bonnet mechanism. So, I just wanted to show you that, I don't know if any applications.

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this is, this is the spatial version of a slider crank, this is your spatial slider crack. I want you to identify the joints, and calculate the mobility of this mechanism. So, tell me how you would designate this mechanism in terms of the joints this would be a? Okay, RS. Okay? somebody says, RS,RC look at what's happening there? Look at this what's happening there? if you look at this side, then you can see what's happening with that? Look at this end. So, he is right? it is an RS, RC. You can see, this thing keeps rotate, this rotates also rotates and slides. So, this is a cylindrical between, these two. So, now do the mobility for this, this is a j1, this is a j-3, this is a j1, this is a j2 .what do you get for the mobility? I share; the mobility of this mechanism is what? So this is a spatial slider crank Mobility 1.

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If we want it so, the question is? is it more efficient than the planar slider-crank, less efficient. Okay? less efficient you will have to then define efficiency. Okay? So, let's not go into that we can have the discussion later. Okay? Because, we need to define what is efficiency first, if you're going to talk about efficiency. Okay? So, here this is ,another slider crank, R,S,S, P and what would be the mobility of this one, the mobility of this one, it's similar to the R,S,S,R. right? Safe to do so, this is an R, S, S, P slider crank. Okay? And it's mobility is 2, why is the mobility 2 ? because ,it again there is predictable input-output motion but ,again it's the connecting rod the coupler can spin about its axis .so, you have one idle degree of freedom, coupler spinning about its axis ,which does not affect the input-output relationship. Okay? So, you'll find in many cases where you have two spherical joints in series, it's likely that you will have an idle degree of freedom. Okay? Unless for some reason that is desired but, that is required. Okay? That is part of the output that you're looking at.

okay ?then it isn't ideal degree of freedom, if you know this is ,attached to some kind of an indicator or something if , if that motion is of consequence then you would not call it an idle degree of freedom, essentially. Okay? You, you it might need some, then you would need an additional input to the mechanism essentially, is when you have mobility two, you need two inputs here, you can get away with one input because, the other one it does not matter. What the orientation of that coupler role is. Okay? Otherwise you would say that, that's an additional input you have to give the mechanism. Okay? So, that we will start looking at, the matrix method for analyzing, I'm just going to go over some basics of transformation matrices and how they would apply, to mechanisms ,robotics courses are much better equipped, to give you the full you know ,we obviously don't have the time to go over the entire analysis of spatial the mechanisms. But, there are I think several robotics courses that equal, they are dealt with better in robotic, robotics courses. Using but, I will show I'll, I'll talk to you about the basics of transformations. So, when you have just to give you a brief preview, I mean it's, when you have links that are oriented you know in, in space. Then it's easier to what you typically do is? You attach a coordinate frame rigidly to each link. Okay? And then you establish a relationship between, those coordinate frames. Okay? You have these things moving with respect to each other, the reason its convenient is that, if there is a frame that's rigidly attached to a link, then all the points on that frame, can be easily described with respect to that coordinate frame and it's not going to change as the linkage moves. so ,I know the relationship so, if I have a coordinate frame attached to my forearm, I know exactly where my wrist is, with respect to that or list or elbow or you know any ,any other point on my forearm, if I know the relationship of this coordinate frame, to say the ground coordinate frame. Okay? Then at any instant if I want the position of my wrist, as long as I know the relationship between, the two coordinate frames, I can find my position of the wrist grow globally. Okay? So, that is the ideas behind you know, in robotics, you attach coordinate frames and then establish the relationship between, those coordinate frames.

Because, then it becomes, easier to describe or the motion of all the points, that are of interest because, the points that are of interest on a specific link, will be completely known, with respect to the coordinate frame on that link, that's how you relate the various so, you if you have a manipulator, you have the gripper, right? if you know, the relationship of the gripper coordinate frame, to the ground coordinate frame ,then specific points on the gripper become, easy to describe. Okay? points then positions velocities acceleration there's a whole, we will not go into those, details in this course because, we've ,we are nearing the end of this course, and our purpose really was to look at planar mechanisms. So, this is basically just an introduction to special mechanisms and so, we will, I'll just introduce you to that concept in the next class, yes. Yeah. One second, in the slider-crank, yes, yeah. They're aligned here, right? When, when this, yeah. if you are saying, if this axis yeah , yeah, yeah .because, it's a singular point no, see because it's, it's, it's everything is in line it's like a toggle position, right? Everything is in line. So, what are you going to move? you're saying, the rep ,you're saying everything here is moved so, here you don't have this offset ,you don't have this link at an angle so ,you're saying I have a revolute joint, I have this revolute axis then I have this, I have this, and I have the slider moving. So, you are saying, he's saying, it's something like this, like that. Okay? It'll move on the so, it's, it's like a degenerate case, right? that you're ,I mean it still moves, moves in the sense it remains stationary, that that is the relationship between your input and output ,when you're moving this in a cone ,and that is along that the vertex of the cone remains, where it is? What would be the purpose of this mechanism? I don't know, if they use something like this, the mobility still comes out is 2, Yeah. Because, there is nothing, see the geometry is what is preventing the movement, that is so the output is for whatever angle of the input is 0, that is this specific relationship between the input and output, there is a specific relationship between, input and output, which means it has, it is not a structure it's not mobility equal to 0, where it can't move because, and you're moving this link. okay ?there is a specific relationship between, the input and output that's what you have here, and this rotation also exists, I can spin this and again that doesn't change anything. So, the mobility still remains the same. Okay? So, it's a special case of more you know, of zero output movement, like your dwell .right? See we don't say, just because the mechanism encounters our dwell, you don't say it's no longer a mechanism there, this would be like a dwell position. So, like he said if there's some way you can change, it and you don't want it to move then this, would be some way of doing that.

So, it would be a way to create a dwell in the mechanism for instance, yes, yes, yeah, yeah. so, because you know, you may be, this may be, in some way this may be operating something else, as this slider-crank is moving, you may want to use this degree of freedom, to operate something else, in which case the orientation of that probably matters, it's no longer an idle degree of freedom because, you may have to actually specify. So, this could be connected to something else, right? It could be connected to another linkage, and the rotation of this may actually be the output of something else, it's possible, in which case you're going to have to control that rotation. So, anytime you have a degree of freedom it's an opportunity to control the mechanism, when you say it's an idle degree of freedom, what it means is okay? for this input and this output, I don't care how, that thing is moving but ,if that matters, then you have to control, that you have to give an input that, which will influence some other output, as far as this relationship is concerned it's I may have some other use, yeah, which one, yeah. This one you're talking about this one or the next one, the 4-bar, yeah. So, it depends, right? It depends on how the revenue job so, so that's what here, here that is the case I could have these, two revolute axes parallel to each other. Okay? In which case, I can visualize it as a play not with just additional. Now, which one are you talking about? Okay? This is a four bar, this is the projection .okay? Let's go to that, if you don't have a projection like that because, there are access of this different. You want to make this into a planar

mechanism. You already know, how to make a planar slider-crank what, what are you trying to do. Okay? And if you I can, I can do that no I can replace any revolute with a spherical.

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See, I can have this I can put a ball like this. Now, how would I create a planar? Okay? So, attached to this slider, a ball attached to this slider which moves No, no, no, no, he's talking about, he is still talking about a planar thing see here, when this moves .okay ?when I rotate this crank ,this is going to move on a cone. Okay? So, that distance they out this block will not move so, his question was okay? for doesn't move is it still does the mobility equation still apply for this case, why is it not showing mobility equal to zero, again special geometry.

Right? it is not a structure, where nothing moves, if you have mobility equal to zero, nothing moves in that, like you have a truss ,three links pinned together nothing moves here you are moving the input link you have no motion of the output link it is a special case of movement where you are getting advent mobility is still one for this in this case mobility is still two because you have that other independent motion about the coupler axis yeah so that that's exactly what you did here no I mean in the special case special geometry always you know you can you can always play around with the geometry to come up with something special like you have your four bar right typically your coupler has complex motion you have a parallelogram linkage your coupler has only translation can always play with geometry that's always possible so yeah but this is a very special I mean what are you accomplishing with this mechanism right I mean it's a three link mechanism

true but you know and that is because yeah of the special geometry, so you could still have a three link mechanism in 2d, when you have a pin in a slot if you look at a pin in a slot so if you have a higher pair for instance you can have can't cam and follower that's a three link mechanism it's a planar mechanism so it's not that you don't have see when we talk about the mobility Gruebler's criterion and whatever we say specifically that it is simple hinge joints we very varied if I marry clearly define what are the conditions in which the Gruebler's criterion then you need minimum four links if I want higher pairs a cam follower is a cam follower in the fixed link it is a three link mechanism fixed link the fixed pivots for the cam and the follower to rotate about.