Theory of Mechanisms

Lecture 4

Inversions, Inversions, Grashof Criterion, Kinematic equivalence

So last class, we looked at some of the applications of Grubler's criteria and number synthesis.

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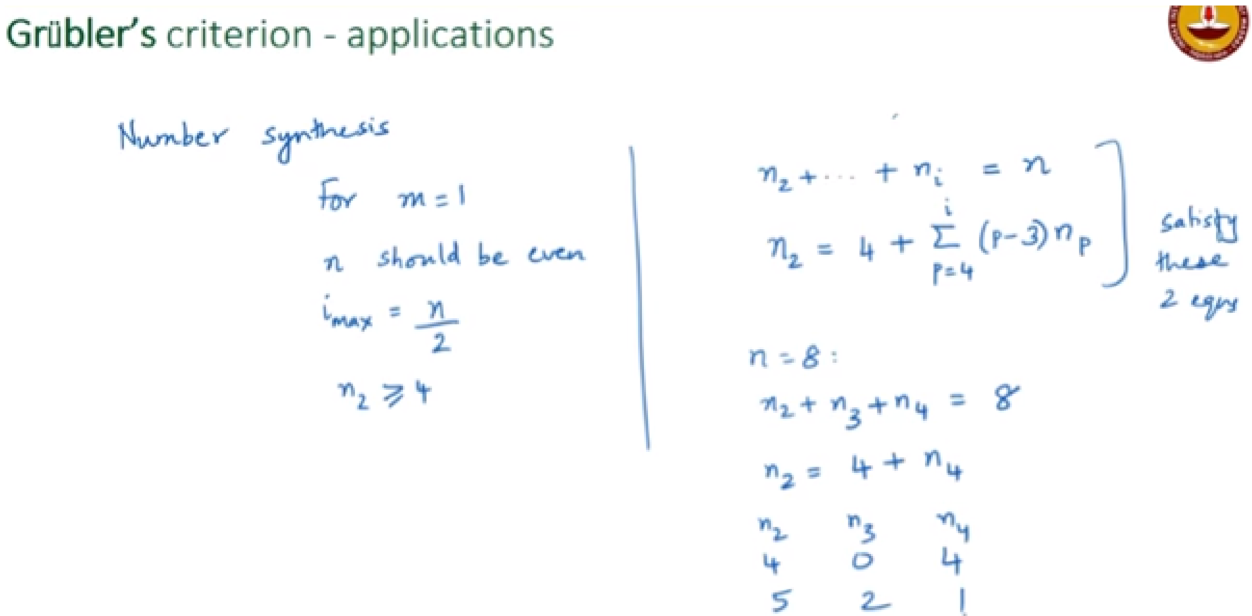

Basically, determining the numbers and types of lengths you would need, for say a single degree of freedom linkage, would mean that you would have to for mobility equal to one, you have n should be even, I max which is the maximum order link, would be n by two, the order of the maximum number of load nodes in a link can be n by two and then you have, you have to satisfy minimum of binary links greater than or equal to four and then you have to basically satisfy the equations and two plus whatever and I equal to the total number of links and from the mobility criterion we saw that you have n 2 ,equal to 4 plus, Sigma P equal to 4 to I, P minus 4n p. Because this we obtain from the mobility criterion, so if they satisfy these two equations, then they give you possible solutions for the number synthesis okay, so satisfied pay minus 3 n p. Okay, so far say n equal to 8, you would have n 2 plus okay ,you only have I equal to 4, so n2 plus n3 plus n4, will be equal to 8, and then you'll have n2 equal to 4 plus n4 okay, so as long as these are satisfied you have your possibilities could be n 2, n 3 ,n 4. So, you could have 4 & 4, or you could have 5, 2 ,1 etcetera, say you couldn't, because these all satisfy these equation simultaneously ok, so that is your number synthesis, then we moved on to looking at inversions.

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Inversions and the Grashof criterion

- Inversions obtained by fixing a different link in the kinematic chain
- Relative motions between two links remain the same
- Grashof criterion for a 4R mechanism – gives the range of motion
- Shortest link s, longest link l, other two links p and q
  If $s+l < p+q$, then it is a Grashof mechanism and at least one link is capable of a complete rotation

And the Grashof's criterion, so if s plus L, the shortest plus, the longest link, is less than P, plus Q.

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Grashof – Type I

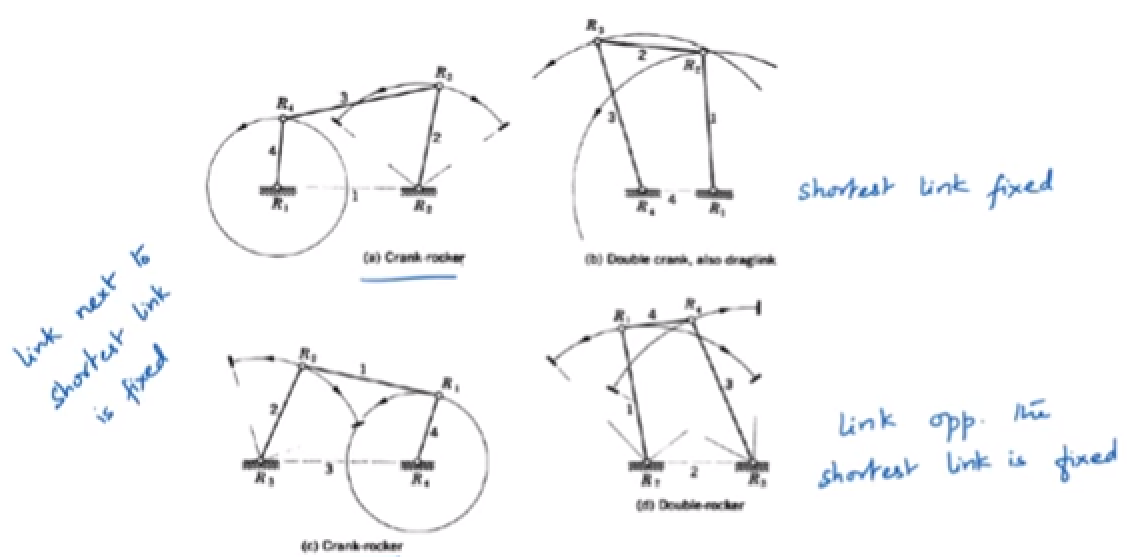

FIGURE 2-29 Inversions of the four-bar linkage.

Then you have, the Grashof or the Grashof type 1 linkage, where if the link ,next to the shortest link is fixed, then you get a crank-rocker ,so if 4 is the shortest link in this one you have one or three fixed, here one is fixed, you get a crank-rocker, if the shortest link itself is fixed, you get the double-crank or also called the drag link mechanism, so these two are called non distinct inversions, link next to shortest link. And in this case, they link opposite the shortest link is fixed, so you're only concerned with the shortest link and its location with respect to the fixed link, to determine the kind of motion that you have in the case of the Grashof double-rocker, the coupler which is the shortest link makes the complete revolution, ,so it functions like a crank.

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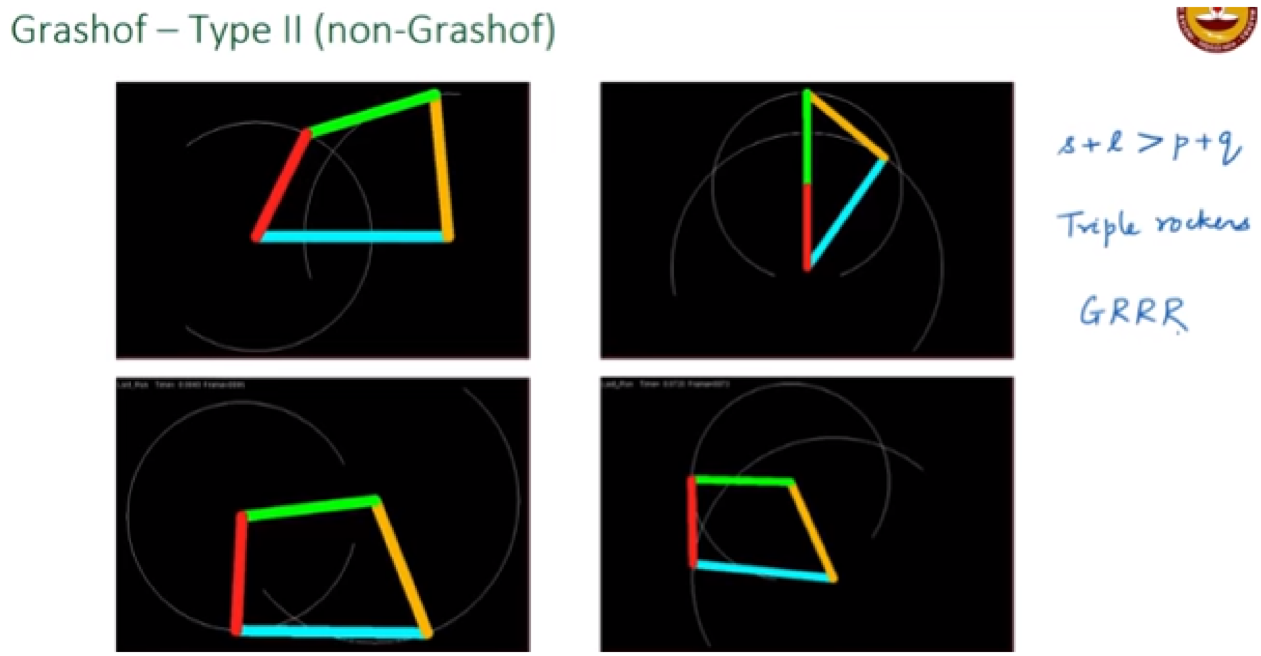

And I showed you these animations, I will not do it again, okay, so in the Grashof type - which is s + L is greater than P plus Q, all four inversions function as double rockers, or actually correctly speaking we call them triple rockers, because all three links can only rock between limits, there is no link that can make a complete revolution. So they are typically denoted as R, R, R okay, you have no C in the notation, so that is the Grashof triple rocker, example there's nothing wrong with being a Grashof triple rocker, it depends on the application, there are plenty of applications where you don't need a link to make a complete revolution okay, so it's perfectly it really depends on the application, as to whether you need a Grashof or a non-Grashof linkage will do okay.

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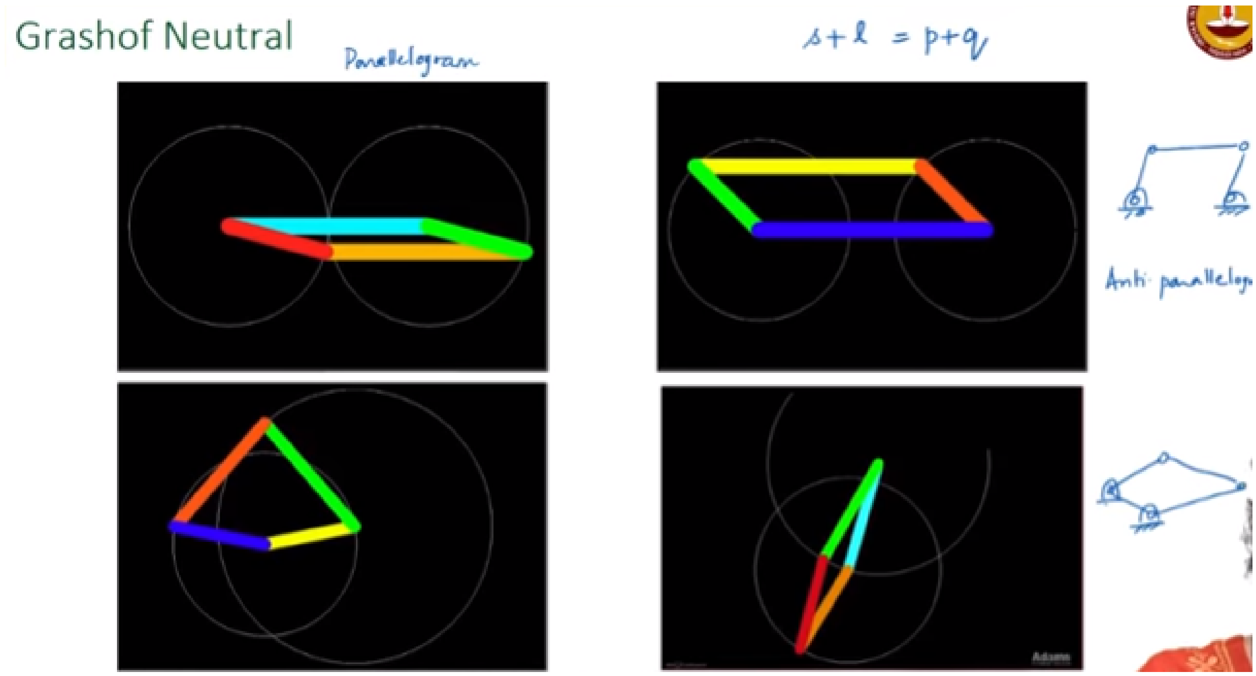

Now the special case, is when s plus L, equal to P plus Q and some of the a couple of special linkages are called the parallelogram or kite form of these linkages, so obviously if you have a parallelogram then s plus l, equals P plus Q and you'll find many ,many applications of the parallelogram form of linkage, so here you see that the input and the output motions are the same in the parallelogram form and you will see that the coupler is essentially just translating in the plane okay, so the orientation of the coupler does not change at all, so you will find many applications where this is very useful and it's also very useful when you want to basically reproduce the input motion at another point ,so the parallelogram mechanism is very useful for that one of the problems with the parallelogram mechanism which you don't see in this particular simulation but you will see in the ,so this form here the second one, so this is the parallelogram form and this one where it's crossed like this, this is called the anti parallelogram form, so it's the same links, in the anti-parallelogram form, so you can see here, when the links line up like that okay, that's called a change point and there is some unpredictability associated with that change point, so you started off with an anti parallelogram mechanism okay, you can see here it's switched to a parallelogram mechanism, so your output becomes unpredictable, in this case we didn't see this because the inertia of the links if the inertia of the links are able to carry it through, then you don't have a problem.

So here I think the yellow link the mass of it is enough to sort of carry it through and not have it, switch to the anti-parallelogram form, but otherwise that's real possibilities ,so when it goes through a change point ,the motion becomes unpredictable and so parallelogram mechanisms can be troublesome, if you are using them over the entire range of motion, so one way to deal with that there are a couple of ways of dealing with that, one is you put another parallelogram mechanism that's out of phase, with this so that when one is at the change point the other one is not so it drives it in the direction that you want okay, the other thing is don't run it over the entire range, if you ,you may not need to run every mechanism over the entire range of motion, so if you need the parallelogram action only for a certain you know, in say a rocker fashion, you can you're perfectly justified in using that between those limits, you don't have to necessarily run it through the entire, you will still get the duplication of the input motion in the output motion, it all depends on the application that you need. The other form is the kite form, where you have the two short links it's like this, so it could be like this or the other way, so that's called the kite form it looks like a kite and here you have the input motion, the speed of the input the speed of the output motion will be half that of the input motion, you can see that here okay. So these are the Grashof neutral mechanisms.

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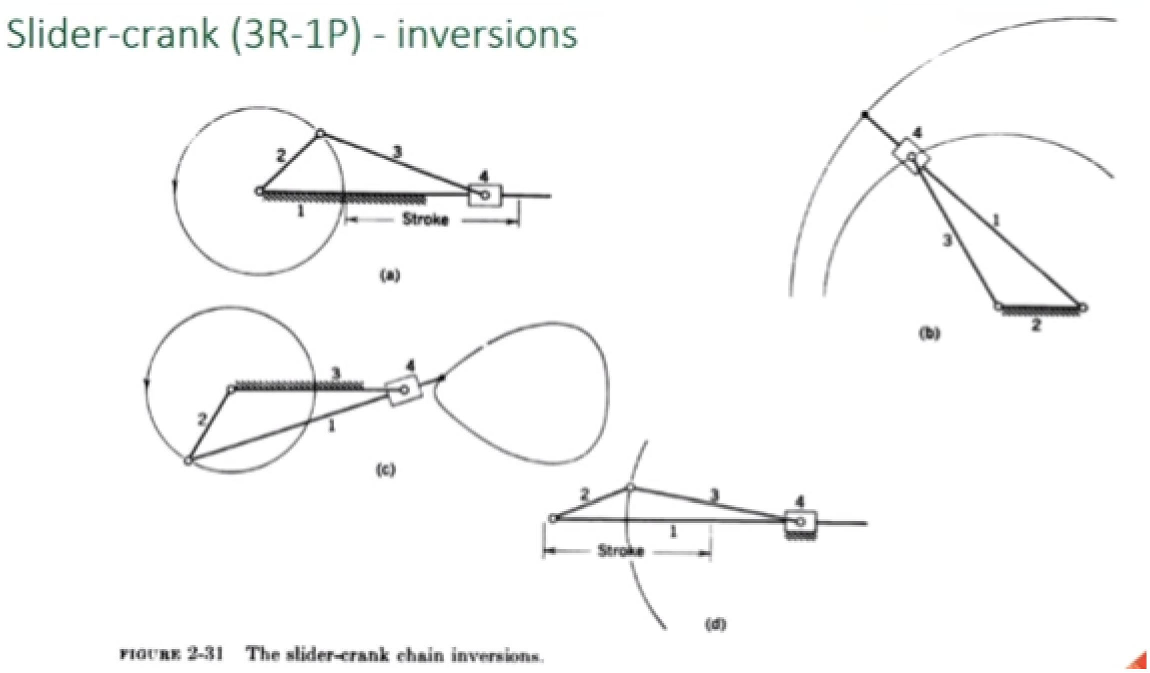

Similarly for a slider crank, you could again have four inversions. Where you have the side adjacent to the and you can see, I will show you the possible inversions.

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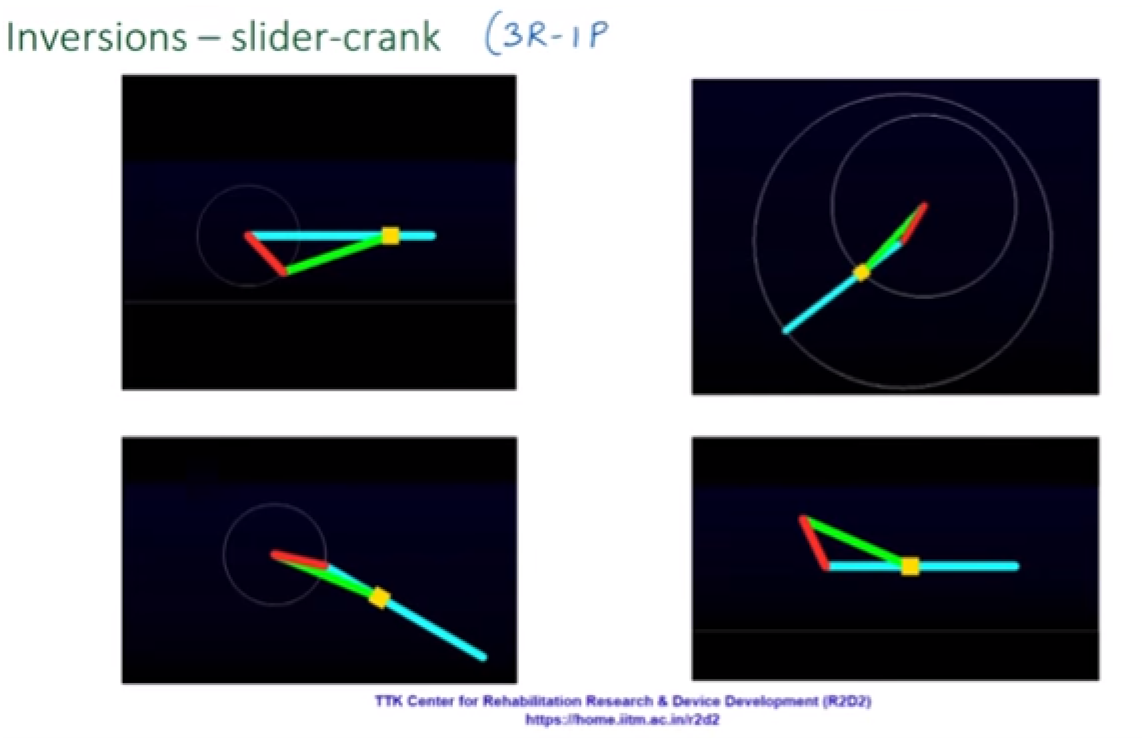

So here, you can see that all four of them are essentially the same, the same slider crank, but in each of these a different link will be fixed. So let's start with basically the path is fixed and the slider block is moving back and forth on that but, so the slider block has translatory motion in this case, then here you see that the slider block has more complex motion okay, so here the shortest link is fixed ,then in this case this link is fixed ,the link to which the slider block pivots, so the slider block can only give it about that point, okay and the other link moves with respect to that ,fourth case the slider block itself is fixed, which is similar to you know kind of the path being fixed, so that, so these are the four inversions of the slider-crank ,or the 3r-1p mechanism.

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Scotch yoke (2R-2P) - inversions

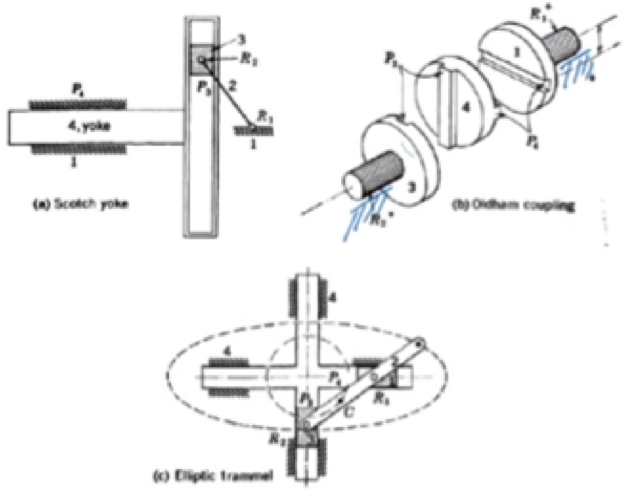
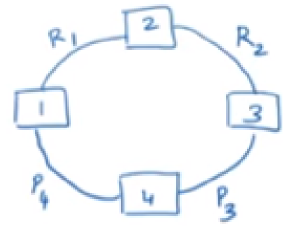


FIGURE 2-30 Inversions of the Scotch yoke.



So a scotch yoke is a mechanism, which has two revolute and two prismatic joints, so if I have link one, two, three and four, they join between one and two. Sorry ,this should be our one, our one this is a between two and three is a prismatic joint, no we're okay this is two and three so that is R2 , that's the second revolute between three and four is your prismatic joint and between one and four, so this is the kinematic chain, you have two prismatic and two revolute joints and then depending on which link I fix in the kinematic chain, I can get so here link one is fixed and I get a scotch yoke okay, so if I fix link to okay. Then I get what's known Allison Oldham coupling okay, so you see between one and two is a revolute, so that a link to is not shown which is fixed, the bearings are, so that's your link two, between two and three is also, between two and three is a revolute joint and between one and two, so basically for misaligned shafts you can use a coupling like, this which is essentially a kinematic inversion of the scotch yoke, so you have three and four, you can see that this link you have translational motion and then orthogonal translation motion between these two legs, one and four. Okay? So this is not a planar mechanism this is not a planar mechanism, but it's an example of the two-r, two p mechanism, so it's typically used for coupling misaligned to take up misalignments and shafts, so that the rotation of three is transmitted, to the rotation of one or vice versa okay, so in this case ,you have in this inversion you have linked three, know which ,which link is fixed here, link two, three ,four, four is fixed link 4 is fixed here in this inversion, link 4 is fixed so between 4 & 3 is a prismatic joint okay and then between 2 & 3 is a revolute ,between 1 & 2 is again a revolute joint, so this is called an elliptic trammel and you can see that this point traces an ellipse, it's called an elliptic trammel. If you fix 3, it will be similar to the one fixed, so it will just be a non distinct inversion of that okay ,so this is how it's a relative motion, sub the links remain unchanged, but overall when you look at it with respect to the fixed link, what you get is or your fixed coordinate system you get different motions in the plane, different output motions.

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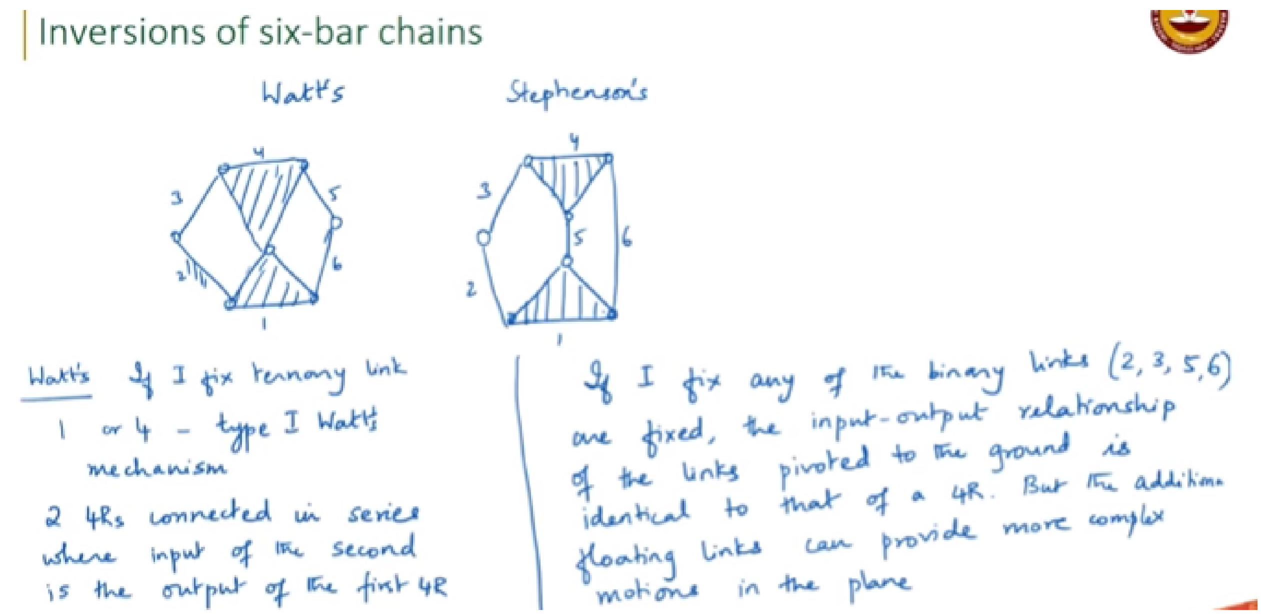

Let’s also look at the 6 bar chains that we have seen, the 2 chains, so what are the two chains, the watts and the Stevenson. So if I look at the chain, remember the watts has the two ternary links directly connected to each other, that's your watts chain, then this Stevenson chain, you have the two ternary links, separated by a binary link. Okay, so again you have so this is the Stevenson chain. In the case of the Watts mechanism or the Watts chain, if I fix ternary link 1 or 4? Okay? Say I fixed one, what do I get? I get one for bar chain, driving another 4 bar chain. Okay? So if I take this two as my input, 4 will be the output for the first 4 bar chain, which will be the input to this, 4 bar chain, 1 4 5 6. Okay? So that is and whether it's 1 or 4, basically they are non distinct inversions, in terms of the form of the inversion and that's called a type 1, Watt mechanism Watt's mechanism. So, these are two 4R’s, connected in series, where input of the second, is the output of the first, 4R. If I fix any of the binary links, so this is. Okay? The other four binary links, if you look at the form, it's pretty symmetrical. So if I fix any of the binary links, what do I get, say a fixed link 2? If I look at this closed chain, right? It's essentially a 4 bar. 1,4,3, but, with additional, coupler links. Okay? So I have four, five, six, three, coupler links, in the mechanism. So I can get more complex motions, in the plane. But essentially if I just look at input and output, it's essentially just a four bar. Okay? So unless I am interested in that complex output motion, of the coupler, if I can use a four bar for this application, I should use a four bar. Okay? Because, it doesn't really give me anything else, if I am just looking at three and one, as my output and input links. Okay? And it doesn't matter which of the binary links I fixed, I get a similar configuration. So if any of the binary links, namely, two, three, five, six, are fixed, the input and output relationship, if I just look at the links that are pivoted, to the ground, is identical to that of a four bar, but it has three floating links.

But the three floating links, can provide more complex motions, in the plane. I shouldn't say the three, because four will do what a four-bar does, five and six, but the two additional floating links. Floating links can provide more complex motions in the plane. Okay? So those are, so if you look at distinct inversions with the Watt’s chain, you are basically getting, two different types, two distinct inversions. One is if you fix the ternary link, one is if you fix one of the binary links. Actual motions of course, will be quite different, depending on which link, you fix, because that will be dependent on the dimensions of the links.

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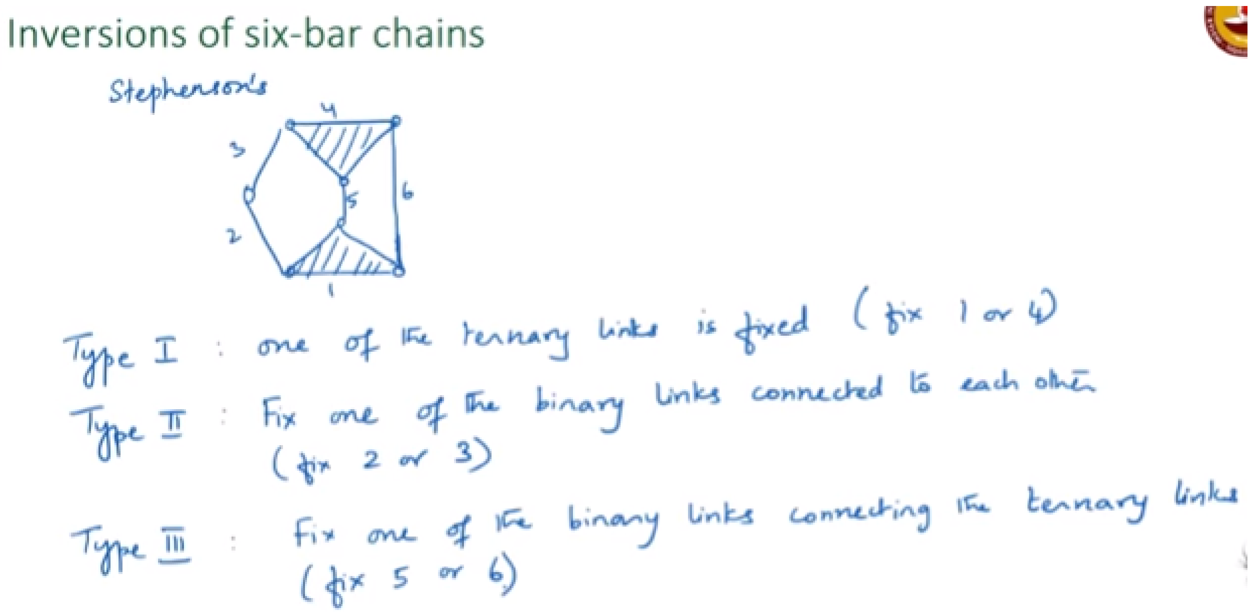

Now let's look at the Stevenson chain. I could fix one of the ternary links, let's see if we get something different. Okay? So, inversions of the Stevenson's Chain. You actually have three different, kinds of inversions you could get. So let's say, type one. One of the ternary links is fixed. Okay? Type two, one of the binary links that are connected to the, that are connected to, each other. Fix one of the binary links, connected to each other. Which will be, which one's? So say, called one, two, three, four, five, six. This would be fixing two or three. This is fixed one or four. And then the type 3 inversion, is fix one of the, binary links, connected to the ternary, connecting the ternary links, sorry. That is fix, five or six. If you fix six for instance, you are essentially getting a, four bar, similar to the Watt type 2, do is what you will get, with this. I'll get some simulations for these and show you. Okay? For this, because these motions, are more Complex than in the case of the, Watt Chain. So these are in inversions, the concept of inversions, we will use quite extensively, when we come to synthesis. Because inversions in, in a kinematic Chain, the relative motion, between the links is the same. So we will use that concept repeatedly, when we do synthesis of linkages. So it's a very important concept, for you to remember.

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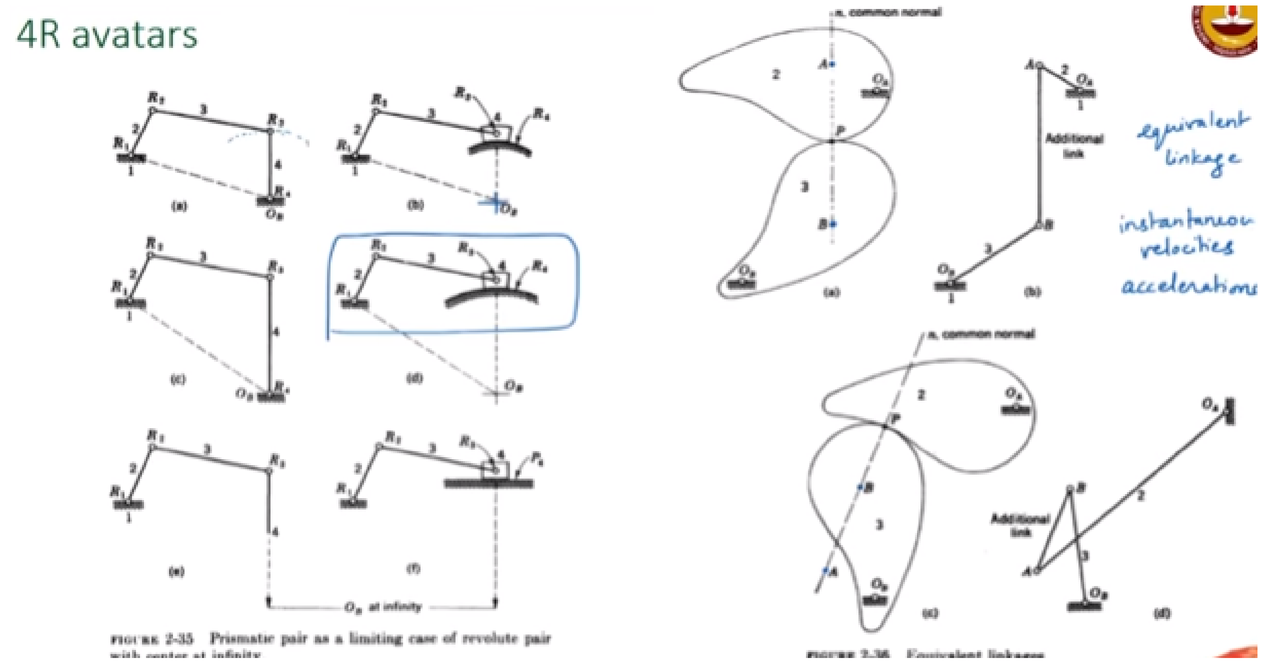

Okay, so in many cases, if you remember we said, we could, when we did the mobility or the type synthesis, we made this assumption that any linkage, can be represented as a pin jointed linkage. Right? You can make, it can be represented as links, connected by simple hinge joints, something. So here is how the slider crank, the reason we focus a lot on the four bar, is, it turns out that ,many linkages can be represented as four bars, at least instantaneously. Okay? So in many cases, you can make a kinematic representation, of a link, say, with higher pairs or with other kinds of joints, as a four bar, for analysis, and therefore. So say if you, for instance, if you take this four bar, essentially if you look at the motion of this point, right? This link 4 is pivoted to the 4, so this traces an arc. Okay? So this may be represented, as a block, with that curvature, moving on the curved surface. But this is not really considered a prismatic pair. Okay? Because essentially, because of this, because of this, radius of curvature, it has a finite radius of curvature, so this is actually equivalent, to this 4bar. Okay?

Sometimes practically you may use something like this, because of space constraints, for example. So if you look at, so say I increase the length of link 4. Okay? Then the arc that I get, will be flatter. Okay. Now if you keep increasing it, to Infinity, then what I get is basically, a prismatic pair. Okay? But even in some cases, say I need a really long radius of curvature, really large radius of curvature, for the path of this point, at the end of link 4, the pivoting point, I might use, form like this. Because it is, it gives me some advantages from a practical consideration. But when you look at the kinematics of it, you are essentially looking at, what's on the left hand side. Okay? So it makes analysis simpler, when you are able to identify, the 4-bar in some, in something that, appears more complex, it makes the analysis, a lot easier. Okay? And essentially the slider-crank or the prismatic pair, it's just a limiting case, of the 4-bar. That's why we always talk about, the four bars and the slider crank, in the same breath, you know? We talk about them as together, because essentially one is a limiting case of the other, when the rocker pivot is at infinity. This is even more useful, when this kind of what we call kinematic equivalence or where the two linkages are equivalent, for the purposes of, say, calculating velocities and accelerations. This kinematic equivalence is very useful, especially, when you use mechanisms that have higher pairs in them. So you take for

example, this cam, driving another follower, that is, that has this, complex shape. Okay? So in this case if you look at the centre of curvature, of this, so you have cam 2 and cam 3. Cam 2 is driving cam 3, say.
So if you look at the centre of, centers of curvature, at the point of contact, say, a is the centre of curvature, so P is the point of contact, and then B is the, centre of curvature of the other link, at that particular instant. Okay? Now the property of the centre of curvature, is such that, say if you draw the, the common normal at the point of contact, will obviously pass through the centers of curvature. The centre of curvature, do not change for three infinitesimal positions. Okay? That's a property of the centre of Curvature. And so using that, we can say, that the velocities and accelerations, of this mechanism, will be equivalent. So if I replace it, with a link, connecting the two, along the common, normal, connecting the two centers of curvature, then I get this 4bar. So at this particular instant, the relationship between two and three is the same as, the relationship between two and three, in this four bar. Important point to remember is that, it is at that particular instant. Because, one of the things that you can do with the cam follower system, is you can constantly change, the radius of curvature, between the two connected bodies. Right? So at a different instant, you'll have a different link a B, connecting the two. Okay?
So you can use that to analyze, so analyzing the 4-bar, is a lot easier, than analyzing the velocities and accelerations, for a linkage with these higher pairs, for a mechanism with higher pairs. So this is a way to represent, a linkage with higher pairs, with an equivalent four bar, that is valid at that particular instant and then use that for analysis. The other side of the picture is, what makes the cam follower system, so versatile, is that, it's essentially a series of changing four bars. So you can look at a cam for, so, you have much more flexibility in with a four bar, once you have a set of links, what motion you get, is what you get. Right?
With a cam follower system where, you, the input and the output members are directly in contact, you can see that, it's essentially a series of changing four bars and therefore you get a lot of flexibility in, designing the input-output motions. But cams of course, have other problems, especially with respect to where, maintaining contact between the links, etc. So in some, so linkages typically are, preferred, but of course, again it depends on the application. That's the caveat, everywhere it always depends, on the application. Here is another cam follower system where you can see that, in this case, the center of curvature, for the link 2, is here and for link three is here, so you get a crossed four bar linkage that represents the system, at that particular instant. Okay? So you can use this for velocities and accelerations, not for higher derivatives. This kind of an equivalent linkage can be used to compute, instantaneous velocities and accelerations.

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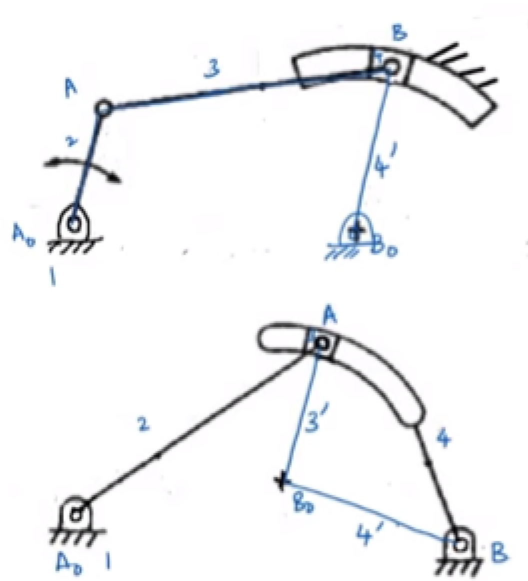
Kinematic equivalence



Here are some more examples, similar to what I just showed you. Essentially here, if this is a not a, if this is B, then this will be B naught. So my four bar will be, a not a, a B, D not. So this will be my equivalent 4 bar. Similarly here, if this is, this is, this will B not, this is the center of curvature of that. So this will be my, so if I call this, one, two, three, four here, this is 4 dash. 1, 2, 3, 4 dash, is my equivalent, 4 bar. Here I have, 1, 2, say the block is 3 and this is 4, now this is my 3 dash, this is my 4 dash. The equivalent 4 bar would be, 1, 2, 3 dash, 4 dash. Okay? So the input output relationship will be the same, between two and four dash and here between, again two and four dash, at that particular instant.

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Kinematic equivalence

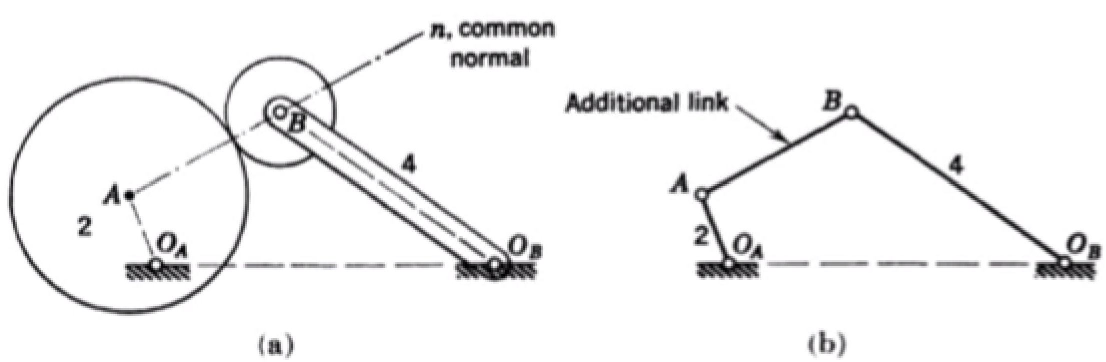

FIGURE 2-37 Cam with constant curvature profile and its invariant equivalent linkage.

In a few cases, so here is, a cam-follower system, but it uses a circular cam. Okay? Which is just pivoted, with an offset. Right? So you have in this case, because the centres of curvature do not change, throughout the motion. This can be represented by, an invariant, equivalent, linkage. So it's the same four bar, that I can use to analyze, the entire the motion, over the entire range. Okay? So the instantaneous four bar, is the same as the four bar, at every instant. Okay? Which, makes life a lot simpler analyzing this, is a lot easier than, analyzing this. Okay? So this is the use of, equivalent linkages.

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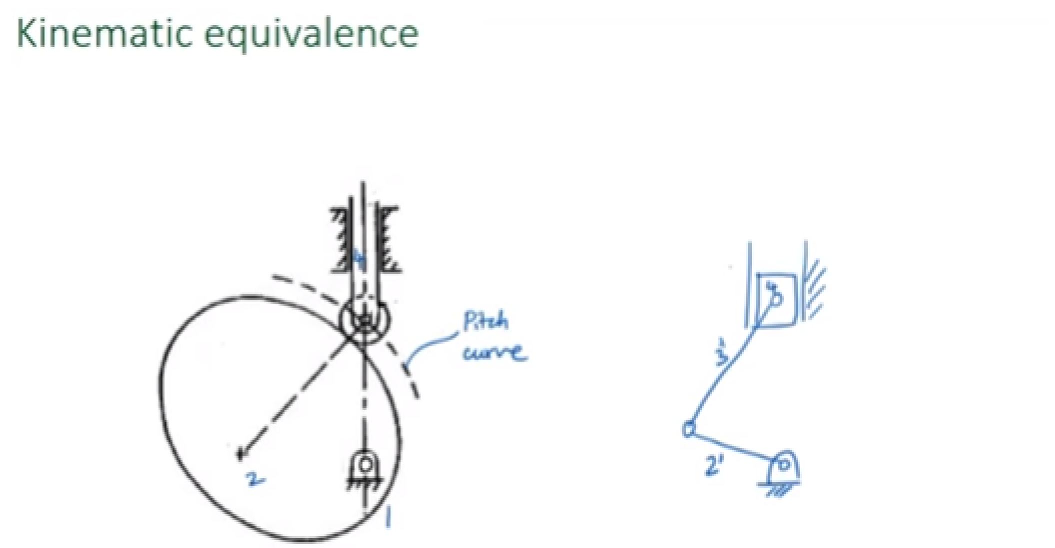

One more. So here I have a cam with the roller for follower. What is this curve called? Pitch curve, yes. So you use the pitch curve, the, so I have. So what would I, what is the nature of the motion I have? This is the center of curvature, of the pitch curve and the motion that I have is, what would I do? How does the follower move? Sorry, this is not fixed. I have one, two, and let's just call this four What is the nature of motion of 4? 4 is essentially a block. It's moving with respect to, one. Okay? And the motion of this, would be equivalent to, this would be 2. So this would be the kinematically, equivalent system, with just lower pairs. We saw earlier, for the pinion slot. Okay? You separate the higher pair motion into, a lower pair, or two lower pairs, basically. So I'll give you some exercises. Yes. It is only instantaneous. Because the pitch curve keeps changing, so the center of curvature, will keep changing. So this link 2 dash will keep changing, so will 3 dash. Okay? That was only in this special case.

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Kinematic equivalence

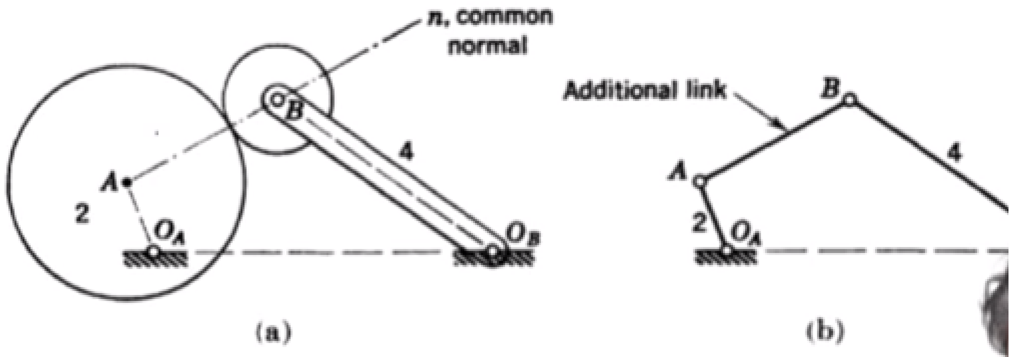

FIGURE 2-37 Cam with constant curvature profile and its invariant eq linkage.

In this special case, because the radius of curvature of the scam does not change. A is always the center, it's a circular cam. So here you can represent, it's a very special case, where you can represent, by a four bar, that does not change.

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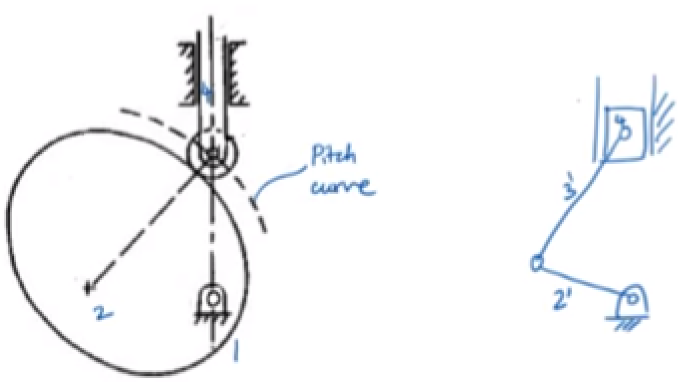
Kinematic equivalence



In most cases, it is only applicable at a particular instant, next instant everything changes. Okay? So with this kind of thing, it may be difficult to do, you know, unless you have the equation of the, cam surface, you know the, rate, it's a little tricky to do the analysis. But if you are interested, say, in only certain positions, you are interested in the, velocities and accelerations, then for those positions, you can represent them, by an equivalent linkage, and then do the analysis.

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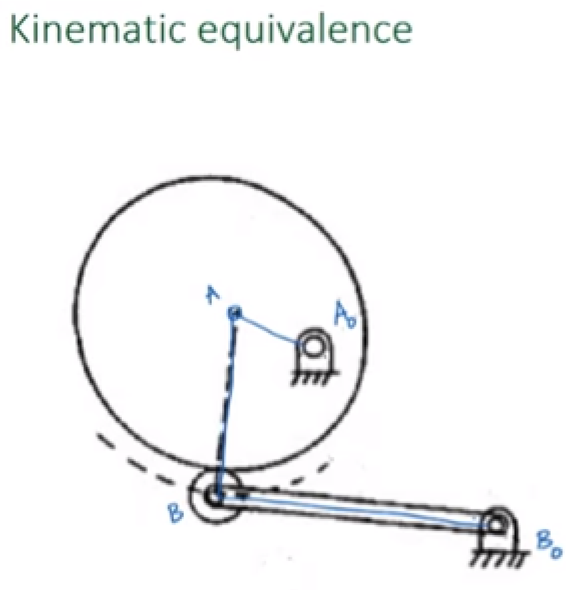

This one, you have a, this is B, b not. So my equivalent linkage, is this. Okay? Lot, easier to analyze. So this is the concept of, kinematic equivalence.

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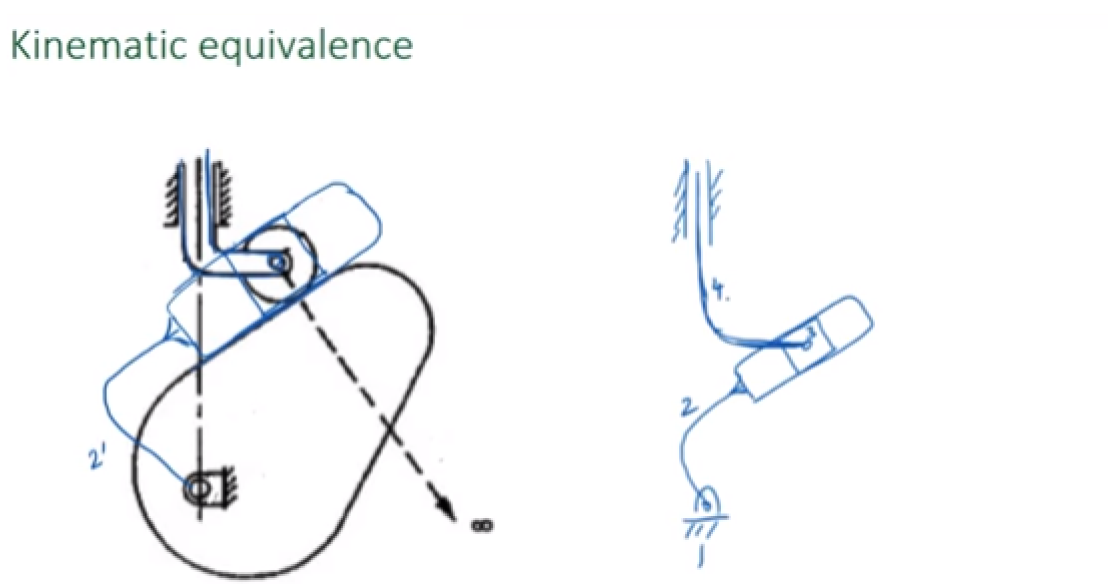

One more, you have time. Okay, so this thing is moving along the, this portion here, in camps you know, you can have, sometimes you can have portions that are straight lines. Right? So, the radius of curvature, is at, is infinite, the center of curvature is that infinity. In this case, you can have, you can represent this by, so this motion is up and down, so I can have a link, with a slot. Okay? And instead of a roller, I would represent it, sorry, by a block. And then attach this link to it. Okay? So this would be my 2 dash, so this is welded here. I draw this separately; maybe something like, because the follower is moving, in line with this, so it's essentially, like this, this link is pivoted. Okay?