

Theory of Mechanisms

Lecture 6

Driver dyad Quick-return synthesis

So last class

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2-position synthesis: driver dyad for $TR=1$ Transmission angle



Check Grashof

$O_A \rightarrow \infty$
 $O_B \rightarrow \infty$
 $C \rightarrow \infty^2$

$R = \frac{\text{time for forward stroke}}{\text{time for return stroke}}$
 $O_D \rightarrow \infty$
 $D \rightarrow \text{unique}$

Moving pivots - circle points
 O_A, O_B, O_C - fixed pivots
 Centre points

$O_D D_2 = O_D D_1 = \frac{1}{2} C_1 C_2$
 Crank: $O_D D$, Coupler: DC , Rocker: CO_A
 Fixed: O_D, O_A
 $O_D D C O_A$ should be Grashof-I
 $O_D D$ should be the shortest link of this 4-bar

Watt's six-bar

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We looked at the dead center synthesis using a crank-rocker, we first looked at for a specific rocker displacement and then saw that, we could use the same technique for a motion generation problem, if it is a two position synthesis problem, where the pole is located by easily okay, if we can use the pole for synthesis then you could use that as a rocker output, the new position motion problem, motion generation problem can be converted to essentially a rocker output problem and then you can design the driver diode, or the crank and the connecting rod in order to enable that to move between these two limit positions. So, that's what we looked at last time okay, in some cases, it may happen that, the pole may not be in a convenient position, so you may not be able to use the pole, to so this is say a 1, B 1, a 2, B 2, then it mesa, it may happen that, you're only option is to use a 4 bar, that you cannot use. So if I have, I can look at that and then for I can draw the perpendicular bisector of b1, b2 okay and I can locate O, a anywhere on this line and OB similarly anywhere on this perpendicular bisector, so that my linkage becomes O a, a 1 B, P 1, OB, ignore the fact that it passes through a 2 here for some reason ok, ok so this becomes my linkage, so I can move it, it'll move from A 1, B 1, to a 2, B 2, however I cannot, the way it is now its most likely a non - Grashof linkage, if you look at the dimensions, it's likely that, none of the links are going to be able to make a complete rotation and n to only that it's likely that these two are not limit positions, so O a, a 1 and o a, a 2 are not limit positions, so if I want to limit it to those two positions, now what do I do? I can just ignore the rest of the linkage and just look at O a, a 1. So if I look at O a, a 1 and O a, a 2 - okay, if I restrict the motion say that's my input link, if I restrict O a, a 1 that link to move between these two extreme positions, then essentially I am restricting a 1, B 1, moving to a 2, B 2 and back okay, so this basically reduces to the first problem that I solved, which is the rocker moving through this angle theta okay, so now I can pick any point on this link, or you know on this body, when I can

extend it but let's just say I pick a point here, I call it C, c_1 and the corresponding point would be C2 okay, then all I need to do is c_1 , c_2 , extend it pick my, what do I pick here? Some I can call it D, some fixed pivot okay, then half of c_1 , c_2 , will be, will give me D 1 and sorry, D 2 and D 1 ok. So oD , D_2 equal to oD , D_1 equal to half of C 1, C 2 and so this. So I'm adding these two lengths now okay, that's why we call it a driver diet, diet for two lengths, OD and CD ok, D_2 , C_2 , so OD , and CD , when I add it to this linkage. It's now limiting the motion of this 4 bar, so what, what kind of a linkage is this overall, so if I look at od ok, I have D, I have C, this should be, I should show with a, to show that this is one link okay, there are two links there, this I have Oa , aB , say OB , so this becomes my overall linkage, once I add the driver diet okay. Now which part should be the Grashof linkage? It should be, I should check the Grashof for OD , dc , oa , should be grasp of type 1 and ODd , should be the shortest length of this 4 bar. Okay, so now I could drive this with a continuous input, I could have a constant speed motor attached at OD , I could drive link ODd with that and have a_1 , b_1 , move between a_1 , b_1 and a_2 , B_2 okay, so I achieved motion generation with limits on the two positions, by attaching these additional two links, so what, what kind of a linkage is this one this, whole thing, do you recognize, how many links, it's a six bar, what configuration it's a what, it's a what configuration of the six bar, so this is a ,you have one four bar driving another, the output of OD , dc , oa , is the input for OA , AB , OB okay, so this is the, what's six bar solution to achieve this two position motion generation, with limit positions okay, so you've now synthesized a six bar, for you application started off with a four bar, then in order to be able to drive it with a continuous input, you converted it into the six bar, this is called attaching a driver diet okay, for a four bar, typically for a non-Grashof four bar, you can use a driver diet ,you can synthesize a driver diet, so that you can have a continuous same okay. So just with the one thing, that we learn to do we've now, extended it to multiple applications, any questions so far, so always ,after you synthesize, you check Grashof, in real life, you would also either build a model ,or simulate it over the entire range. Because, see when you do this to position synthesis, all that it guarantees is that, it can be assembled in those two positions ,it does not tell, you what's happening in between those positions, so if you encounter toggle positions for instance, I hope you remember ,what toggle positions are, when you can no longer drive, the link with the same input, you can't drive the mechanism it gets locked, in that position, if you encounter that then a_1, b_1 cannot move, to a_2 , b_2 and that is not something you can check, from the synthesis procedure alone, because this only guarantees, assembly of the linkage in the two positions ,so you would actually have to build a model, or use some kind of a simulation package, to check whether it is able to move from A_1 , B_1 to a_2 , B_2 okay, so that is something that, that check always has to be performed when you are designing a linkage, if it's a Grashof. You don't have to worry about that, because it will be able to drive through thee, the Grashof criterion ensures they rotate ability, so you don't have to check for toggle positions for that ,so this will be able to move from, but if this linkage has a problem, then you know if it was just this link ,it can move it between those two extreme positions, by use of this divert diet ,but whether it will be able to, actually whether this linkage will move from this position, to this position that has to be checked, the other thing that you might want to look at, when you are to look at the effectiveness of your design, is also the transmission angle okay, if you have forgotten what that is, please go back and look at it, so the transmission angle for smooth operation ,should typically be greater than 40 degrees or so okay, it's the angle between the coupler and the output link, which determines, so again those are design criteria that you would check after the synthesis is done. Okay ,to check whether it's a good solution ,later on you will see that, when we do analytical synthesis ,you'll be able to specify some of these as criterion, that you can then, so with graphical analysis you're somewhat limited ,because you solve something, then you analyze it, to see whether it's a good solution, if it turns out, then you have to go back and do the procedure again okay ,it's very quick, it's very

intuitive, because it's simple geometry, so it's very easy, to understand how graphical synthesis works and that's why it's very useful, because it's, it can give you like very quick solutions, but in some cases, if you find that you have to go back again and again to the drawing board to do this, then this may not be the best way to synthesize your linkage, that's when we will go to the analytical methods, which are harder to set up, so you know you would have to program all those equations, you know set, but once you do that it's very easy to cycle through multiple solutions and you can also specify criteria, you can check, so you can build in, the transmission angle calculation into your code, you can do all that, when you are doing an analytical solution, so here how many solutions are possible, what were our free choices. So, for a_1 , b_1 , so we said, this is the rigid body, you know a_1 , b_1 , had to be moved to a_2 , b_2 , we assume that a_1 , B , a_1 and b_1 have to be the moving pivots ok, moving pivots are also called circle points. Why is that? Because, they trace, they're attached to the link, which is pivoting about a fixed point, therefore they trace, the path that they trace is a circular path, so the moving pivots are A and B here, they're called circle points, $O A$, $O B$, also D , D is also a circle point, so a , b , d are the circle points here, $o a$, $O B$ and $O D$ are called the fixed pivots, or the center points, so you will see these this terminology, used interchangeably, so if I say Center point, it means fixed pivot, center point corresponding to point a , circle point a , is $O a$ okay, similarly, so circle points and Center points, are used interchangeably, with moving pivots and fixed pivots respectively, so if my, circle points are fixed, $o a_1$ and b_1 then okay, I don't have a choice, then but I do have a choice for $o a$, right, anywhere on the perpendicular bisector, so I will have an infinity of choices for that. I will have an infinity of choices for locating $O B$ okay, then how many choices do I have for point C , which has to be located on link $O a$, a . Why? It can, yeah I, I could have this shaped, any which way, right, so I can choose this point c , anywhere on this plane okay, so I have, so let's say, $O a$. I have an infinity of choices, $O B$ infinity of choices, point c I have infinity square, I don't necessarily have to take it on $o a$, a , I could take it anywhere on the body, $O a$ defined by o the pivots, $o a$, a okay. So that, then if it is, if the timing ratio is given, if timing ratio equal to 1, then $O D$ has to be on that line, joining c_1 and c_2 , timing ratio is, time for forward stroke, by time for return stroke okay here, here I was designing a driver diet for timing ratio, equal to one okay, so this $o D$ has to be on that line, so then my choice is for $O D$, I have an infinity of choice. I can choose it anywhere on that line, once I choose that, do I have a choice for D , no D is now unique, D becomes unique, because it has to be half of c_1 , c_2 . Okay, so you can see, I have there are a lot of decisions I'm making, lot of choices I am making, as I am doing this synthesis okay, lot of free choices that I have, which can give you multiple solutions okay, but you'll find that if I choose $O D$ very close to here, then I may not get a Grashof linkage, so some of these intuitively I choose okay, similarly here you know if my choice of o and $O b$, I could run into toggle positions, if I choose, in such a way that the configurations are different okay, so that is something, so unconsciously I am making some choices about suitable linkages, but it's possible, that you may not come up with the right, to come up with a suitable solution in the first trend, so for instance in the exam, if you come up with, if you do all this and then you come up with a linkage, that is non-Grashof, that's okay, because in real life, you will have more time to go back and fix It as long as your procedure is correct, I will not penalize you okay. But you have to make sure that you follow the procedure and show what you're doing okay, so here, if I look at for, for this linkage, I have $O D$, this is equal to, so my crank length is $O D$, d , my coupler is DC and the rocker is C , $O a$. It's not a 1, $O a$, because the linkage doesn't care, what points I'm taking beyond the pivot, as far as this linkage is concerned, the rocker is C , o , A and the fixed link is $O D$, $O a$ okay, so that's what you would use to compute the Grashof criterion okay, so you would do that and then check for, because as far as this linkage is concerned, a is simply a coupler point okay, say sorry a is simply a point another point on the rocker, so it's the kinematic ally the linkage is $O D$, d , c , $O a$.

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$Q_2 B_1 = r_3 + r_2$
 $Q_2 B_2 = r_3 - r_2$
 Solve for r_2, r_3

$O_2 A_1, B_1, O_B$
 Check Grashof

Quick-return
crank-rocker

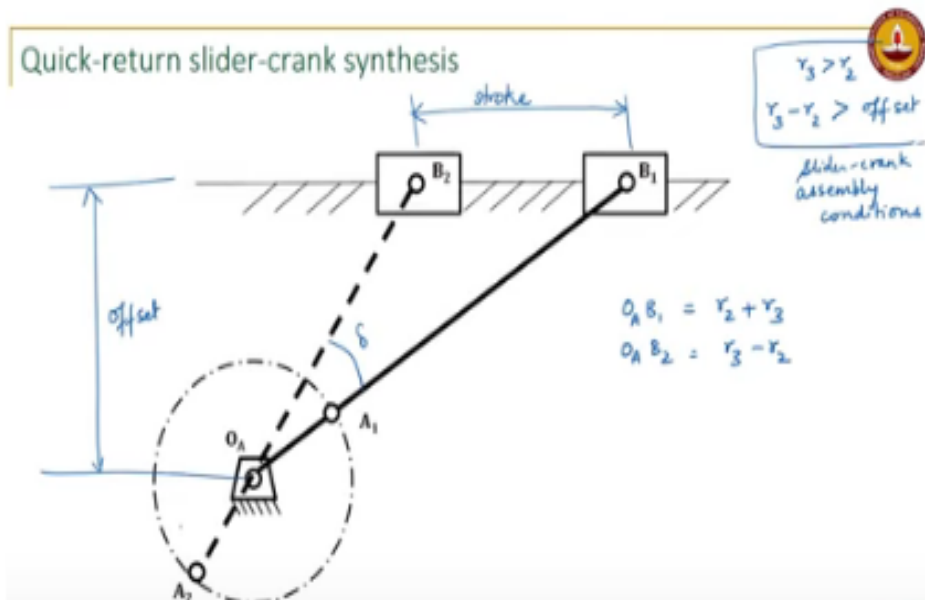
$TR = \frac{\alpha}{360 - \alpha}$
 Calc: $\delta = |180 - \alpha|$

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So this is for timing ratio of one okay, now in many cases, as I mentioned earlier, if these are the two extreme positions you can have so, in the case where the timing ratio is one, we always took, this pivot O a, on the line joining B 1 and B 2, so that should tell you that if O a, is not on that line, then your time for the forward stroke and time for the return stroke are going to be different, so here you can see the a1 to a2, traverses this angle okay, a 2 to a 1 traverses a small around it, so I am running it with a constant speed motor, the time it takes to go from a1 to a2 is going to be and therefore b1 to b2, is going to be longer than the time it takes to come back to from b2 to B 1 okay, so this is called a quick return mechanism, so a quick return crank-rocker takes a different amount of time for one stroke, than for the other stroke. So, let's say you are given a particular time ratio, so if I look at it here, I can call this angle alpha, this angle beta okay, so my time ratio, is essentially alpha by beta, or I can look at it as alpha by 360 minus alpha in degrees. Okay, I'll just explain this first and then I will explain the construction of the quick return mechanism okay, so now, I define a construction angle, Delta as mod of 180 minus alpha, or mod of 180 minus beta okay, so it's essentially, this ad that is a construction angle, that I am going to use for the construction of this quick return mechanism okay, so now if I look at this mechanism, that so I, I have my construction, so if know the time ratio, I can find the construction on it okay. Now I if I, if I look at this mechanism, these are the two extreme positions, these are the positions where the crank and the coupler are lined up right, the two positions one, one is folded on the other, the other one they are in line with each other okay, so here if I look at this, O a, b1 is what? Is r2 plus r3, so if I look at my four bar, or my crank-rocker and I designate this is one, two, three, four okay, two is generally my input link, here in

this case a crank, four is my output link and three is the coupler, so r_2 plus r_3 , is O_a, b_1 and O_a, b_2 equals R_3 minus R_2 , R_3 minus R_2 . Okay? So these are the, so if I solve these two, I should be able to get, so if let's look at the construction of this okay? I don't have, okay so I'll just use this, so I'm given that I want to move a rocker, through a certain angle θ . Okay? Using a crank-rocker and I want to achieve this, with a time ratio which is given to non-unity time ratio, so the first thing, I know the time ratio, so I can calculate the construction angle. So I'm given, so I pick some point, I know my angle, that I want to move this through, so I have this okay, and I'm given the time ratio, time ratio is α by 360 minus α , so I can calculate Δ , say 180 minus α okay, so now I have α , so once I construct my rocker, I pick some point in the plane, to locate my pivot O_b , fixed pivot O_b and then, I construct I pick some length for the rocker and construct the two positions, they may already be given to you as design variables or not, what you may have to choose them okay? So I can now, draw any line, through b_1 okay, then if I look here, this is a line through b_1 , if I draw a parallel line through B_2 , this angle should also be dealt okay, so I would use that for my construction. So if I take a parallel through here and then mark off my angle Δ here, then this angle is also Δ , so this gives me what? What is this point O_a , okay now I have O_a, b_1 , O_a, b_2 , so I know O_a, b_1 equals R_3 , plus R_2 , O_a, b_2 is R_3 minus R_2 . So from this construction, I solve for R_2 and R_3 okay, once I solve for that, then I can construct, the so I will know R_2 , so I can just, so this would be my a_1 , this would be my a_2 okay, so this completes my construction for a quick return, now again my mechanism is O_a, a_1, b_1, O_b , check Grashof okay, the other thing about this kind of a mechanism is, it doesn't matter what scale I choose initially okay, if we are talking about, yes sorry, yes it will be, this is Δ , this is, okay this will be a 2 , this will be a 1 , check huh. So I was saying, so I can scale this entire linkage, up or down without affecting any of it, I won't affect the time ratio, I will not affect the angular relationships okay, so I can design a crank-rocker, so I may be designing something, that's for an application where you know I have really large links, but for this for my graphical synthesis, I can choose a suitable scale, in the previous case for the motion generation, that's not possible, because when you are talking about motion generation, you are talking about a certain location and orientation of a rigid body in the plane. Okay? So that will change, look at that if I change the dimensions of the links okay, where I have unless, you know I just the driver part of it, I can expand or contract, but that part of it cannot be changed, so that part of the 4-bar cannot be changed, this has to stay the same, I cannot change the scale of that, because I am talking about the position and orientation of the rigid body, the absolute location and orientation in the plane, so for motion generation, I cannot change the scale of the linkage that I design and expected to perform the same function, for function generation, or if I'm just looking at you know rocker output, angular inputs and outputs, then by all means, I can change the scale of the linkage and still retain the input/output relationships okay, it's important to remember that, so this is your, this is how you design a quick return crank-rocker okay, again made a lot of free choices, for instance this line, that I drew through b_1 . I just took it at random, I made a free choice there, next we will look at how we, I can actually have a certain offset so in this case, what would do is, if I wanted a certain orientation of O_a and O_b , I would just design my linkage, then reorient it, provided I had flexibility in reorienting the rocker output, so if this gets fixed, then again I don't have that same flexibility okay, only if this is not fixed, then I can reorient the linkage still get the same travel, the change in the angle is okay, that move, that does not change when you scale, the linkage up or down Okay? So that's.

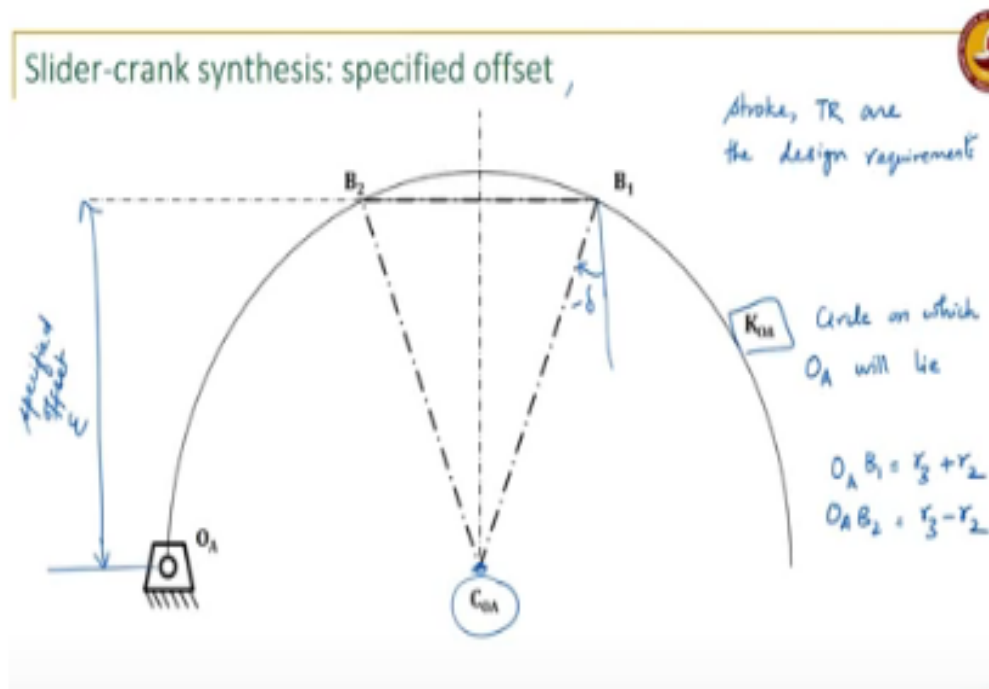
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I can do the same thing with a slider crank; the slider crank is just a special case, with the rocker pivot at infinity, so here B_1 and B_2 are points on a straight line. Okay? So if I have B_1 , B_2 in the previous case, there were points and an arc, pivoted about O_B , here they are on a straight line. So I could design a crank-rocker, sorry a slider crank, in the same way as the crank-rocker, again with B_1 and B_2 ok, here this would be my construction angle Δ okay, everything else remains the same. Again $O_a B_1$ equals r_2 plus r_3 , $O_a B_2$ equals r_3 minus r_2 okay, and we didn't look at, so the slider-crank also has, just like the Grashof criterion, you have some conditions, so that the slider-crank can be assembled in the extreme position, so if you have. What kind of a slider-crank is this? This is called in, this is called the offset of the slider crank, if this center was in the same line as your B_1 , B_2 okay, so that is similar to your timing ratio 1, so this moves 180 degrees, that moves 180 degrees, you are, that's called an inline slider crank, so when you have an offset slider crank, you automatically have a timing ratio that is not 1, you have a quick return mechanism. Okay? Now in the case of the slider crank, the offset is an important parameter and it can also determine, whether or not, you will have a functioning linkage, so there are some, just like the Grashof criterion, for the slider crank, you have conditions such as r_3 should be greater than r_2 and then r_3 minus r_2 should be greater than your offset, so these are conditions that apply to a slider crank mechanism. So that is something you should be, so in the case of quick return slider-crank, in many cases the offset will be specified okay, so in that case so here, we have chosen all these were free choices, the only thing I had was this stroke, so instead of the rocker angle there, for the slider crank, I'm given the stroke of the slider, I use that and the timing ratio, so the Δ based on the timing ratio to design this okay, so if I take a random line through B_1 , I'm going to get a some offset okay, if I fix one more thing in the slider crank and then so I get an offset, I get these two equations, which determine r_2 and r_3 , because once this O_a is fixed, this, these become fixed. Okay? So I can draw infinity of lines through B_1 , okay and that will give me infinity of solutions. Okay? Now if I fix one more thing in the slider crank, then my solution becomes unique, for a given stroke. Okay?

So I am given the stroke and given the timing ratio and I fix the offset r_2 or r_3 , then my solution, for the slider crank becomes a unique solution, so let's just look at the case of specified offset. Yes, due to, yeah, so the timing ratio is usually a design requirement, so you have to make sure that you design for that, right and then you check whether, your design needs these conditions, for it to be, for the slider-crank to be able to be assembled. Okay? To go like the limit, limits of assembling for the slider crack, so that's exactly what we are coming to next, so if you have so, if I specify the offset. Okay? To then, what are my options, so I know that maybe I don't want such a big offset? Okay. So instead of trial and error, I say okay look, I really need the offset to be no more than, you know or to be a certain quantity and then I want to design for that, so for that, what we are going to if you look at this angle Delta okay, so if I looked at b_1 , b_2 okay and I look at this angle Delta right, essentially I could have, I could create these sort of triangles right, with this angle Delta, what is that the locus of this point O_a , is going to be your circle, it's going to be a circle, where your called makes an angle Delta on the circumference, which means it has a central angle off to Delta, so if I draw the, if I construct the circle, for which this code makes an angle, to Delta at the center, then I get this infinity of choices, on the circumference of the circle, to get matter to find my offset.

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Okay? So that's essentially what this construction does. So how do I construct the circle? So I have b_1 and b_2 , draw the perpendicular bisector, now I know the angle Delta, so from the vertical I do this, so that Delta, I take it like the I construct this line, so where it intersects, gives me the center of the circle, on which O_a will lie, such that if I pick a point on that, I will always get that timing ratio that I want, because the timing ratio is a design parameter, it's a requirement. Okay? So once I find the center of the circle, obviously b_1 and b_2 , have to pass through that circle, so I know my radius, I construct the circle, so this is the circle on which O_a very light, that's my infinity of options for O_a , any point on that circle,

so now if I specify an offset, it's very easy, I know my line of, the line in which the slider moves, ok b_1 , b_2 , so from there, whatever my offset is, I mark it off on that circle. So this is my specific, specified offset say ϵ . Okay? So it could be here, so if my offset was specified here, then this will be my O_a , something else you know, anywhere on that circle, I take the perpendicular distance, from the line of action of the slider and that gives me the location of the O_a , huh so that's right. So you would take this as the major segment of this circle. Okay? Delta you are assuming that, O_a lies, you'll still have infinity of solutions. Even if I'm not considering the other part of the arc, so obviously if you, you know you know you're not going to pick away here, because then your central angle is different. Okay? So this gives you the unique solution, now for a specified offset I get a unique solution, because once I know the circle, once I specify the offset, I know exactly where point O_a lies. Okay? You could say it could be on this side, but still it's considered a unique solution and then O_a , b_1 , will be your R_3 , plus R_2 , O_a , B_2 will be R_3 minus R_2 . Okay? So this will give you the solution for a slider-crank, with a specified stroke and timing ratio, so specified offset, stroke and timing ratio also given, are the design parameters. That's what you're designing for. Okay? So we will stop here and continue in the next class, with the other cases, where we are specifying instead of say the offset, we'll specify R_3 , it's all based on, fairly simple geometry gives you an idea of the choices that you have.