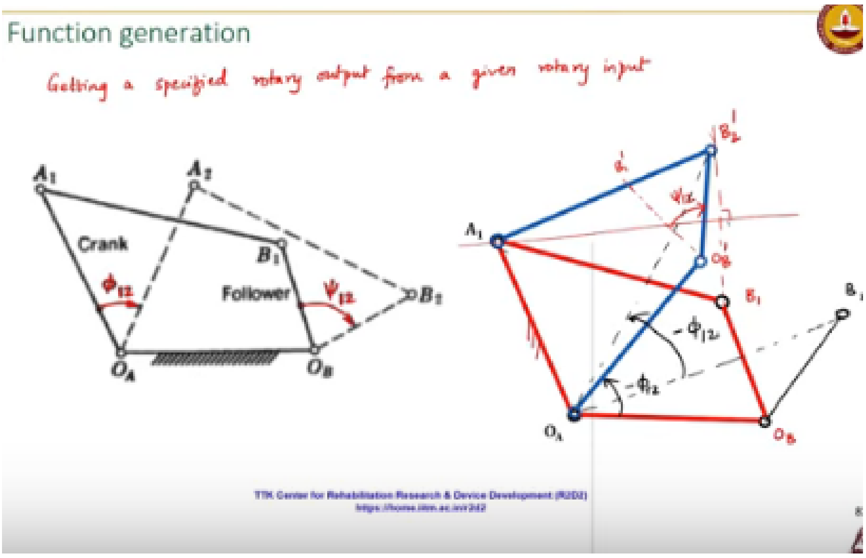
Theory of Mechanisms
Lecture - 9
Function Generation

So,

Refer slide time: (0:14)

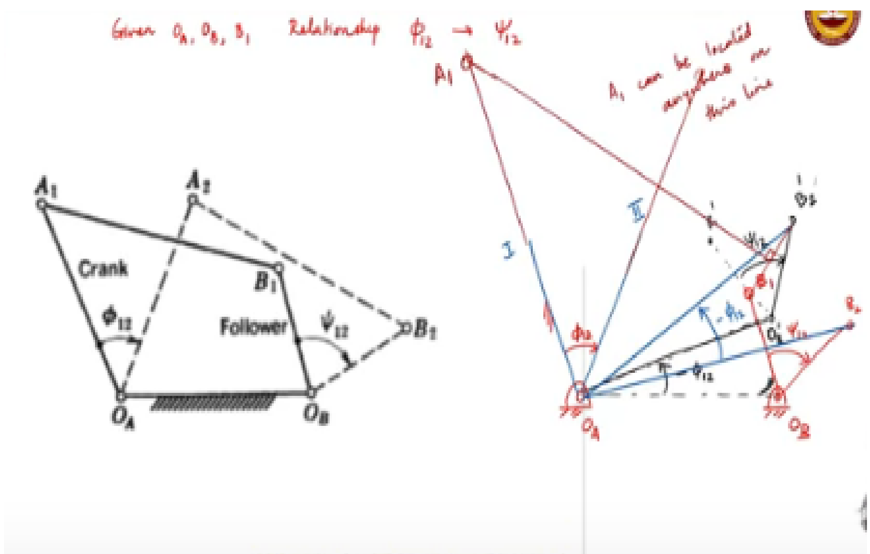


We started looking at, function Generation, synthesis for, function generation, where essentially we are looking at getting, a specified, rotary output, from a given rotary input. Okay? So if I have a crank in the follower like this, giving an input, of Phi 12, to the input link, gets me a specified angular output. Okay? So if I want. So we will first look at, what this looks like. So if I have a linkage, already designed, that does this. Okay? What is it? What is the relationship that I'm going to look for, that will help me in synthesizing it? Okay? So that is what I am looking at here. So I have OA and OB, this is my fixed link initially. And again, we look at inversion. Okay? I say instead of this being my fixed link. Okay? I look at the case where, this link is my fixed link. Okay? I change it; I changed my fixed link, to the input link, because my relative motions, among the links, have to be the same, the relationships among them have to be the same. So if I have this configuration, then regardless of how I, regardless of which link is fixed, this would be the relationship, among the four links. Okay? So we are using the principle of inversion. So if this is, a 1, B1, then I can look at this as. Okay, if now this is the fixed link, O a, a 1, is my fixed position. So that means, OA, OB, has to move by, minus Phi 1 2. Okay? Move in the other direction. And then if this is the original position, of B1, you know, say, I maintain, this move it, rotate the whole thing, O a, OB, B1 and then I have to, move this again by, s I 1 2, to achieve my final position. Okay? So if this linkage is designed for Phi 1 2, giving me s I 1 2, then this would be the relationship, between the two configurations, if I sit on the crank and observe the motion by inversion. So that means, this would be, the position of b2 dash. So b2 as observed, from the fixed crank. Okay? So I call that B2 dash.

And what this tells me is this link if I look at OB, b1. Okay? This is OB dash; OB has moved to OB dash, b1 has moved to b2 dash. Right? This okay. This should be, yeah this, this is B, in relation to, with just this rotation. So let's call this b1dash and then it moves to B2 dash, b2 dash should be its final position. Okay? So if I look at this, then, because of the fact that I'm observing it from the crank. Okay? That means, I have moved, so if I look at this link, OB, b1? OB, b1, has moved from OB, b1, to OB dash, B2 dash. So that is essentially motion generation with respect to, this is a fixed link. OB, b1 has moved to OB dash, B2 dash, which means, as observed from here, a1 would be the point, about which, B1 moves along an arc.

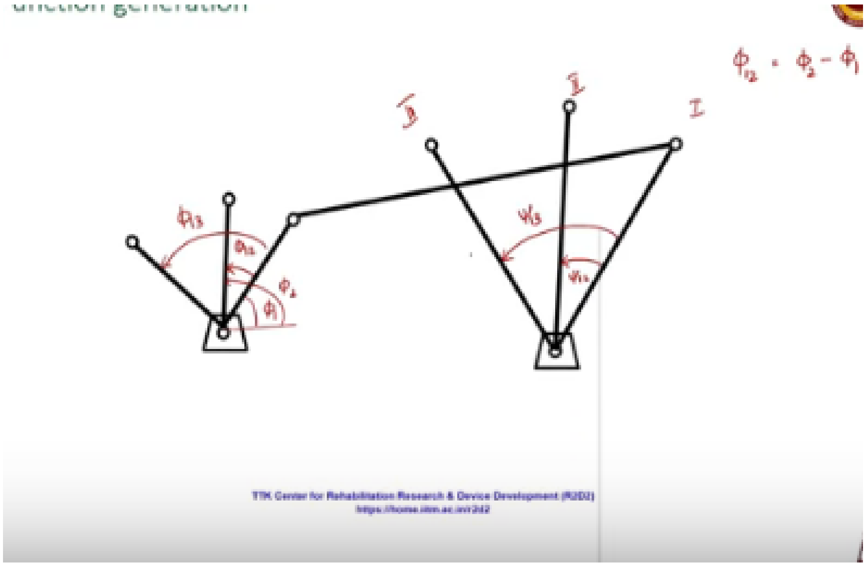
Which means, a1 essentially lies on, the perpendicular bisector of b1, b2 dash. So this would be the perpendicular bisector of b1, b2 dash. So this should be the relationship, between the various points, in the linkage, if this is satisfied. Okay? Now we are going to go back, okay, this is assuming, I have a linkage, which does it, I am looking at, what the geometry will be like, now I am going to Essentially use this to design the linkage. Okay? So suppose you have, I'm given this, I want to synthesize, a linkage that will do this. Okay? Say I'm given OA, OB, I am given the location of the fixed pivots and I have the link of, the link length for the input link specified, so a1 is specified. Now I want to synthesize the linkage, such that, it has this relationship. So I am given, OA,OB, A1, required relationship is, between Phi 12, Phi moving from 1 to 2, should give me, SHI 12. Okay? So now, if I look at this. Okay, let's do this. Suppose I instead of, I'll tell you why, but now. Let’s, let's just do this, okay, fine a 1, is, it'll be the opposite of what we did there, but that's okay. Okay, so let's say, I don't know where be one should be Located, but let's say this is, this angle would be. What would it be? SHI 12. Okay? Between two positions, this should be the angular relationship. So in this case, I know A1. So let me sit on position one. So I sit on the link about which I don't have enough information. Because my vantage point, okay the point from which I am going to observe, is going to be OB. Okay? Let's say. Okay, so I have if I'm sitting on position one, so let's say this is my fixed link now. Okay? So I have to move, so if I if you look back at this previous one, you can see that, essentially, if I do these two rotations, that's the same as, this is fine, so I have B 2 here. Okay? So if I look at OA, OB, B2, okay? If I rotate OA, OB, by minus Phi 1 2 and this by, SHI 1 2, that's the same as rotating this line OA B2 to by this angle, which is again, minus Phi 12.
Do you see that? This triangle it's, it's not a very good representation of it, but essentially, because this is a rigid body, now. Okay? I'm taking the second position, this is where the location should be, so rotating that by the angle, by which this whole thing should move, is, gives me B 2 dash, that's what I'm going to use here. So instead of doing this two-step, rotation, by minus Phi 1 2 and then SHI 1 2, I can instead, straightaway take, the location of the second position and rotate that by minus phi 5, 12.

Refer slide time: (10:28)



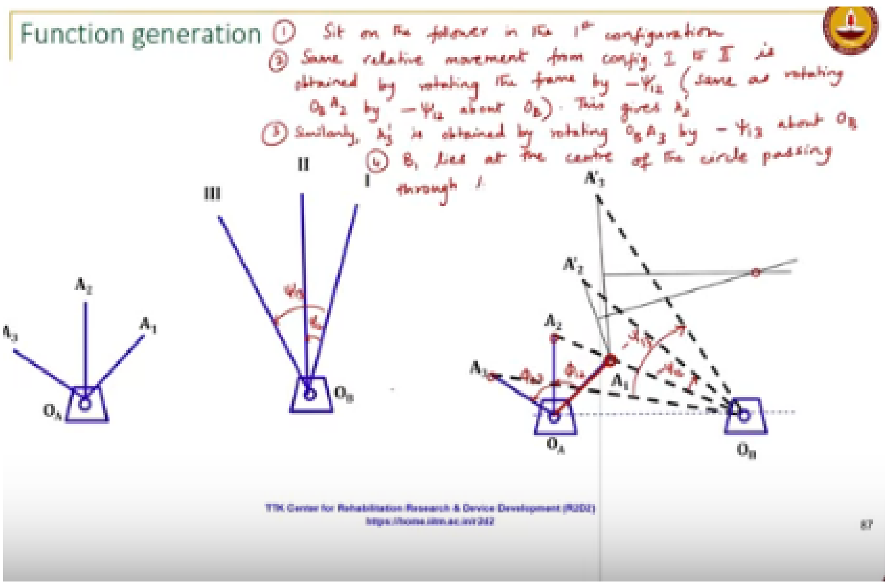
So in this case now, let me say that, okay, I have OA, I have OB. Okay? And I am given B1, so I'm given OA, OB and B1. Okay? And, relationship, Phi 1 2, gives me s I 12, in the output so this is, so I know this. Okay? It’s a two position synthesis, so let's just say, I have two positions of the input link, I know the angle should be, phi 12. Okay? This is also given, so let me just mark that in red. But I don't know, where exactly, so my synthesis is basically, to determine my crank length and my coupler length. Okay? Because I have an output link length, in the rocker length, I have the fixed link length; I want to find the input link length and the coupler link. If I find the pivot a, it gives me both of those. Okay? Because OA, A, will be my input link length, I know where B is, so AB will give me my coupler length. Okay? So now, I sit on instead of OA, OB, being my fixed link, I sit on link one instead, sorry, on position one and I say I'm going to rotate OA, OB and this to meet me at the new place. Okay? So let's just say, in the second position, OA, OB, this is closer to OA, OB, by this Angle, Phi 12. Right? In the second position, OA, A2, is rotated by Phi 12, towards OA, OB. So my first task here would be to, rotate OA, OB, by, bring it closer to this, by minus Phi 12. That is OA, OB. So this is OB dash, just do it in a, two step process. Okay? Then, in the second position, if this is, when I rotate, assume I don't change the inclination of OB, B1, first, I keep it as it was in the first position, so assume this is like welded right now, I rotate it, rotate the whole thing, as a rigid body, by minus Phi 1 2, so this is B 1dash. Now, but in the second position, I want OB, B1, OB, B2, to be actually rotated by s y 1 2, with respect to OB, B 1. So now this should be my, B 2 dash. This angle should be s I 12. Okay? This is the same, doing this two-step process, this, Plus this, is the same, as, if I get, if I take OA to B2. Okay? So if you look at this, OA, OB, B2, that forms a, rigid triangle, so whatever OA, OB, rotates by, in the second position, is what this would also rotate by. So that is the same as, rotating, so this angle will be, nothing but, minus Phi 1. You see that? So you can instead of doing it as a two-step process, you can directly take, B 2 to be B2 dash. Okay? So, the movement from b1 to b2 dash, b1 was in the first position, b2 dash is what you observed, from the inversion, that movement happened about a point a, on the input link. Okay? Such that, it moved in a circular path, so this, to this, so the pivot a, lies somewhere, on the perpendicular bisector, of that. So it could be anywhere on the perpendicular bisector, of this, because again my vantage point, is only OA, I am only, this movement happens about, this point OA. So if A can be located, A1 can be located anywhere on this line. One possibility is where it intersects one or you know? Or somewhere here, then this would give me, A1. If I do it to intersect, this line, then, this would give me A1, in this position itself. Okay? You can see, how. This should be my A1, then A2 would be on this, so when that moves by Phi 1 2, I will have B 1 move to B 2. Is that clear now? So it's a two-step rotation actually, the shortcut is to just rotate, that line. Okay? And depending on where you are sitting, that angle is what you have to take into account. You have to rotate in the opposite direction, of that angle in order, to get the new position. Okay?

Refer slide time: (18:32)



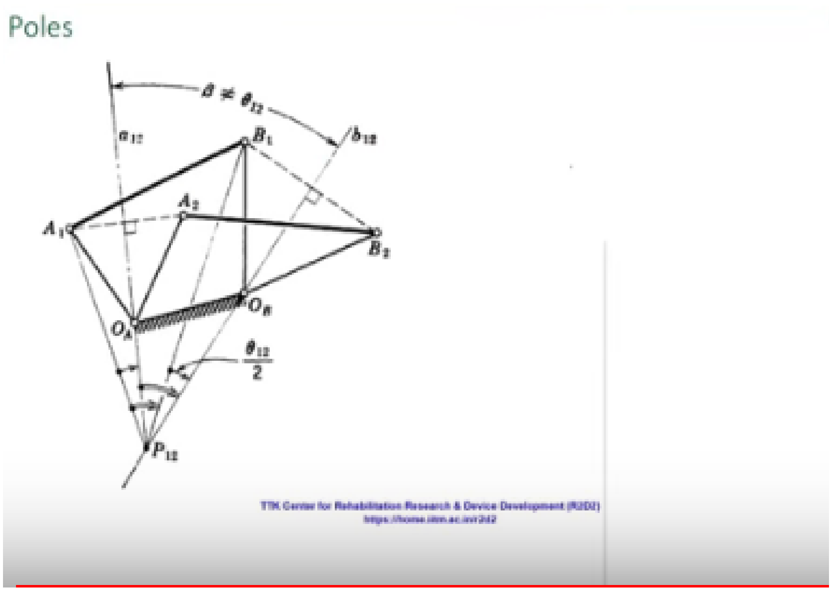
You can do the same thing for three positions. So you have, see what the angles, so I have say Phi 12, position 1 to 2, then I have phi13 and I have 1, 2, 3 here, 1. So this would be s I 1 2, this would be s I 1 3. So when I say, say if this is, some angle Phi 1 and this is Phi 2, Phi 1 2 is, essentially Phi 2 minus Phi 1. That’s the notation that is used. So this Is, s I 1 3. So let's go through the steps for that.

Refer slide time: (19:44)



This one I came prepared. So let's say, I'm given this, I have Phi 1 2, this would be Phi 2 3, I can look at it and I want s I 1 2, s I 1 3, or s I 23. Whichever two of the three, so this is what is given, given OA, OB, I am given, I am giving you a crank length, so I'm locating a 1, somewhere on that line. So first, I take OB, A2. What will I rotate it by now? Yeah. So I rotate to, A2 dash, by, minus s I 12. Okay? Then A1, A2 dash, so it's lies somewhere on that perpendicular bisector, but here I have one more condition, so I'm not ready to pick, that yet. Okay? Second step or third, OB, A3, that's in the third position. What will I rotate it by? Minus SHI 13. So this angle, minus s I 13. Okay? Now perpendicular bisector, from where to where? A12 , A3 dash, where they intersect, gives me, b1, yeah. That completes my linkage. Because I sat on position one and use the angles for position two and three, I draw it, this gives me B1. If I sat on position two and then I transformed, the location of a1 and a3, then what I would get? Is the linkage in the second configuration? So the B that I get there will be B2. Okay? So this is the, construction you will follow. Again it doesn't matter, if I give you B1, you will do the opposite of this. So sit on the link, where you have less Information, because you will need the location of the other link, after the rotation that is what this one. Okay this one is clear? This problem statement okay, I have OA, A1, rotating to, OA, A2 and this thing. And the output link, when this rotates by ph 12, this should rotate s I 12, similarly that. I have information about, the length of the crank. Okay. I'm given. So I know the location of A1, location of a2 and a3. Okay? So I sit, on essentially the first position. Okay? But with the vantage point OB. Okay? I sit at and I know that, OB, A2, should be my relative location, but with the whole thing, rotated by, so this is OB A2. Okay? What, the location of A2, would be in the second position and if I'm not moving from the first position, I essentially have to, rotate. So to move to the second position, I would have moved s I 12, but I am not moving SHI 12, so I had to rotate this, by minus SHI 12. I am staying in one inversion. Inversion means I am staying that is now my fixed link, I am sitting on the follower, on the first, at the first position. So I am rotating this, by SHI 12, which gives me the new location, for it. Okay? Relative positions all remain the same. For the third position, B lies somewhere on this perpendicular bisector. Because from the point B, the pivot point, that's my fixed pivot, now. Okay? That's my fixed pivot, from by inversion, the movement from a1 to a2 dash, occurs about a circular arc, that is centered at, b1. So my b1 lies somewhere on that perpendicular bisector. Same thing for the third position I join OB, a3 and rotate that, I'll write down some of this, if you want. OB, a3, rotates that, by the angle, minus s y 1/3, to get a 3 dash. Now again a 3 dash, also lies on the same Circle, that a 2 dash will lie. Okay? So, I can draw the perpendicular bisector, I could have also drawn the perpendicular bisector, of a2, a3 dash. It will still intersect at this point. Essentially a1, a2 dash, a3 dash, will lay on a circle, centered at, b1. Okay? Because by inversion, when this is my fixed link, the other links, OA and A1 will move on circular arcs, essential. Link 1 and here, you mean here, yeah. Because that would be too restrictive you won't be able to find a solution. Okay? So because this is a unique point now in the previous case I. I can still find a solution. But in the previous case, anything on the perpendicular bisector, would still work. So in the previous case, you have infinity of solutions, for the two position problem, in function generation. In the 3 position case, once you determine, a 1, once you fix a 1, you only have a unique solution, again as in the case with the 3 position motion generation, if I choose as different a1. Okay? If I choose, some other thing as my Length, I can get more solutions. Okay? So I could essentially have anything in the plane, because, if I am only concerned with angular displacements, three-position and function generation means, two angular displacements. 1 to 2 and 2 to 3. Okay? So I could actually have, more solutions, if I change where I pick A1. So I have an in, in the previous case, for the two position function generation, I have infinity, to the power three, infinity cube solutions. Because I can I have infinity square, for the location of, you know for the length, of one of the links, I can pick any point in the plane and any point on the Perpendicular bisector. So that is infinity of solution, Infinity square, into infinity. Here, once I choose A1, which is infinity square, choices, I only have one solution. Okay? So, I'll just sit on the follower, in the first configuration, same relative movement, from config, 1 to 2, is obtained by, rotating the frame, by minus s I 1 2, which is the same as, rotating, OB, A2, by minus s I 12, about OB. This gives A2 dash. Similarly, A3 dash is obtained by, rotating the frame, or OB, A3, by minus s I 13, about OB. B1 lies at the center of the circle, passing through, A1, A2 dash, A 3 dash. Okay?

Refer slide time: (31:38)



So for function Generation, there's a method using relative poles also, Instead of inversion. This is based on the properties of the pole that we saw in the one of the earlier classes. We will use that for function generation, in the next class.