Optoelectronic Materials and Devices
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Electrical Properties of Materials
Lecture - 13
Equilibrium Carrier Statistics in Semiconductors: Qualitative Examination of Carrier Densities in Conduction and Valence Bands

Welcome to lecture number thirteen. So, today, we are going to… In fact, today and the next few days we are going to continue on semiconductor or equilibrium carrier statistics in semiconductors; that means determining n in conduction band – number of electrons per unit volume in conduction band; and, number of holes per unit volume in the valence band. Not only determining them; and, from their nature, their behavior, consequences, etcetera, is what we will be discussing in next few lectures. So, essentially, we will continue from the previous lecture.

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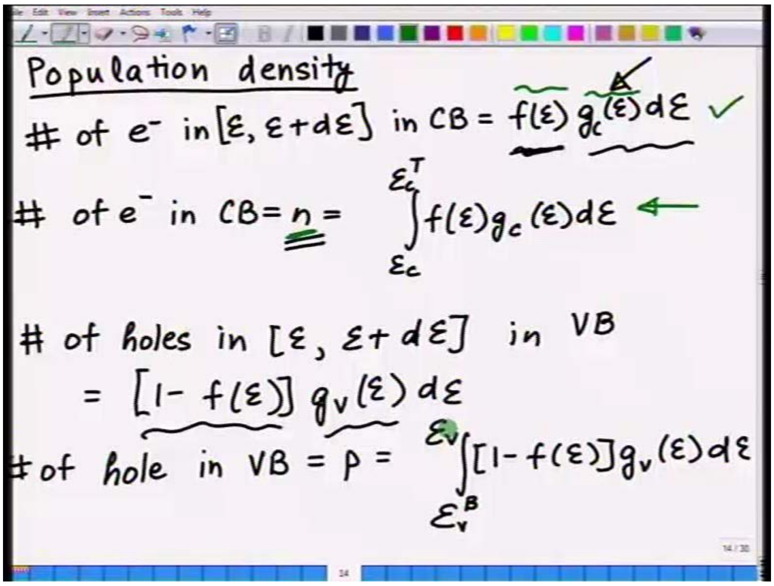

So, let us start with what we did in last lecture. We said that, what is the population density. We have written the population density of the electrons right here. We have written the electrons in conduction band in energy range E and E plus d E. We wrote down that as that which is Fermi distribution and the density of states, which is of the electrons state in conduction band. If so, then if we have, this is the number of electrons that are in this energy range. Then, all we have to do is integrate it over the entire energy range of the band E c to E c top and we will get the number of electrons that are in the conduction band. We had followed the same logic saying that, number of holes therefore are 1 minus f E, which is the probability of finding a hole at energy level E. And, g v E was the density of states for holes in the valence band. And again, if I integrate it over the whole energy range, which is on the bottom of the band to E v; then, band h – that means top of the band – valance band; then, we would get the number of holes in the valance band. So, this is where we ended last time. So, what I am going to do now is I had promised last time that, I will show you graphically what it means. So, that is what we are going to start doing in the beginning.

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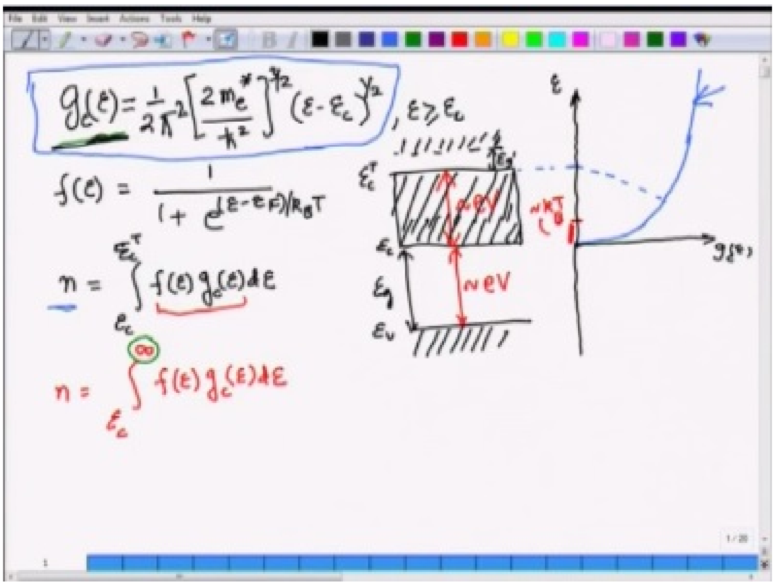

Now, I am going to write down the expressions, which we are going to plot. So, let me do that first. So, let us see what we have derived is a g c E is equal to… This is the density of states in the conduction band. We have derived it as 1 over 2 pi 2m e star divided by h bar square power 3 by 2 E minus E c to power half; and, this valid for E greater than or equal to E c. And, we have derived… And, this Fermi-Dirac function – the probability of finding electron at energy E was 1 plus e to power E minus E F is a quantity, which is a Fermi level. And then, we had said that, n is equal to… In that case, E c to E c top of f of E times g of c E dE. And, this is what we are going to derive.

Now, what I am going to do is I am going to do one more thing; I am going to write one more expression. And, the purpose of doing this graphically integration is to prove this point. I am going to write that, n will also be equal to – let me use another pen here – another color here – n would also be equal to E c to now instead of E c top, infinity f of E; everything else remains the same. So, now, what is going on? How can I change this E c top to infinity? So, there are several objections, which I had raised a minute ago also earlier also. And, I am going to show several objections to writing these equations 1 over here – use of g c as this and also this infinity, which I have just mark down. These are the two questions, which I am going to answer by doing this integration or there showing you some graphical constructs of these functions. So, let us start with it right now and point out to you why I am able to use this expression of g c and why this integral must go to infinity.

In fact, I will give you the reason right now. But, the proof will come as I start doing the plots. So, what I will do is let us plot just g c. Since I am interested in plotting g c; so, let us look at this function. Let us say they are right here is E c plotted little bit further here. Here is E c; here is E v; and, this is all valance band here; here all conduction band here. But, I am making a further statement that, there is a top to it; there is a top here. This conduction band ends somewhere, which is at E c top. Then, there may be another band gap may be; there may be another band gap of E g prime; and then, that is irrelevant, because all the only thing we are interested in is that, electrons are live up to here at 0 k up to this energy; then, there is a gap, which is the band gap E g. And, after that, there is a conduction band. And, this is the relevant band; this is the first empty band after the filled band right here. Then, that there are other empty bands also; there are other band gaps also in principle are irrelevant. This is because you will see that, all electrons live only here itself.

Now, if I want to plot this quantity g c, I want to plot this quantity g c; then, let us do that. So, here is… On this axis, I am going to plot g c E. And, on this axis, is of course the energy axis. So, if I plot this, then… Or, this whole axis is energy axis. So, you can see that, g c is defined only for E greater than E c; below that, of course it is 0. So, of course… Because this is density of strain conduction band; conduction band starts right from here at this point of time. Therefore, they cannot be… All these values – density of states below that is 0. This is empty. They have no density of states in the band gap any ways. So, we start plotting at from this point on. It goes as square root… In energy, it goes as a square root. So, this plot will look something like this. This I am plotting as this function; I am plotting this function here. This is what I have plotted. This is the quantity that I have just plotted.

But, the objection to it was that, if I look at the top of the… When you reach this energy, then obviously, density of states will become 0, because there is a band gap after that. So, I had mentioned that, there must be something like this. Actual density of state should be something like this; that means it goes as parabola given by this expression somewhere in the middle of the band. Then, it should become… Then, it should reduce and become 0 by the time it reaches top of the band. Yet in here when calculating n, I am going to use this blue curve – this blue curve, which corresponds to this expression right here. How can I do that when actual density of state goes something like this and then reduces like this. This is the first point I will have to answer. Graphically, I am stating to you that will be all right if I do this, if I use this expression in this integration. Yet I will get the correct value of n. That is one point I have to prove to you when I do graphical integration.

Second point I want to… And, the reason you will see is… And, the reason it will work is because these energies if you look at; if you look at, these energies are in order of electron volts. And, this band width of the band is also several electron volts. This is also few electron volts typically. And, what we will see is that, as we go above E c, while this function g c continues to increase as a square root of E; but, this product will go to 0 beyond some value of E c. As you increase E, this value, this integration, this product will very quickly go to 0 because this function f E is going to go to 0 as energy increases as you can see here. As energy increases, this function is going to reduce. After Fermi level, it is at 0 k; it goes to 0. And, even at finite temperatures after E f, this function drops off very quickly. Since this function drops off very quickly, this product will drop off, becomes very very small. And hence, as a consequence, whether g c was taken as this or this, it does not matter, because it is going to be multiplied by quantity, which is 0. So, whatever you take, that is the point I will make later one more time.

And, for the same reason, whether I integrate it from E c to E top or I integrated from E c to infinity, it matters none, because after E c up to only few kT of energy you will… Another point I will make is that, all I have to do is really few kT's is k B T. Up to few k B T is all I need to integrate this. This region is only… This product f E g c is going to live, will have some finite value only in few k B T at the most. And, hence… And, since this is electron volts, this is in electron volts; kT at room temperature is a 25 million electron volts. So, if you take 3 or 5 k B T, still it is 100 milli electron volts (( )) 100 milli electron volts of 0.1 electron volts; whereas, this whole length is in few electron volts. What that means is that, this integration lives only in a small portion of this entire... in conduction band. And hence, whether you integrate this from E c to E top, all the way you integrate from E c… If this expression from E c to infinity, it will matter none. These are the two points I am going to prove. These are the two points I am going to make.

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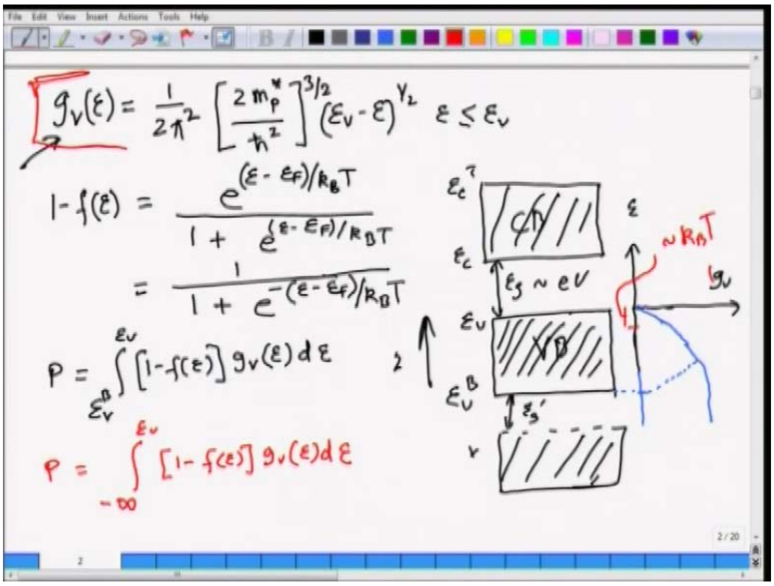

Simultaneously, let us do one more thing – let us write an expression for holes also. For that, we are going to use similarly, very similarly, g v… Expression for g v we have written down was 1 by 2pi square. I think I made an error in the first one; this should have been a square in here – 2pi square should have been there, which I missed out. So, 2 pi square 2 m p star by h bar square 3 by 2. Now, this time, E v minus E to power half; where, all energies are less than E v. So, this is valid for all energies less than E v. And similarly, we needed a probability function – probability of finding a hole, which was 1 minus probability of finding an electron that was e to power E minus E f divided by k B T divided by 1 plus e to power e minus E f by k B T; which we can also write by dividing numerator and denominator by this quantity. By this quantity, we could write this again one more time as 1 plus e to power minus E minus E f by k B T. So, this is the expression for 1 minus f E.

And then, we had said that, p was going to be equal to – integrate it from E v bottom to E v. And, this integration of 1 minus f of E g v E dE. And then, one statement at the end I will make is one more time I will make is (( )) and again I am going to write it in red pen is that, p is also equal to – I am going to replace it by minus infinity this time – E v 1 minus f of E g v E dE. This will lead to the same thing. And again, if I will do it the same exercise as here, as I have done here, I will continue. Now, this time, on the valance band side, hence write down… If I were to plot same way as I did earlier that, here is E c or conduction band; here is conduction band E c. And, this is E c top. And then, there is a band gap. This is E g. Remember again few electron volts. And, this is E v.

And now, we are going to say that, there is a E v bottom also. And, by same token, there may be another gap here and then another band here with another gap here – E g prime let us write it – E g prime. But, that does not matter; only point is that, electrons fill up all the way up to here. So, the only gap which is relevant is from where electrons jump from this point to this empty state. So, these are the only two relevant bands; we call this conduction band and we call this valance band. But, there may be other band gaps also below further underneath in lower energies. And, remember energy is increasing this way. If I do this time and do the same thing and I ask; let us plot on this axis, g v and energy of course increasing in this way, energy increasing this way and I take this expression and I plot this expression right here; then, this goes as a square root of course of energy and up to E v. For E equal to E v, the density of state is 0; right here it is 0. And then, as you go down in energy, this increases as square root of half. Exactly same argument as given earlier.

And now, we will argue again same way that, really, the density of states should have become 0 here; that is, there must be something – some part of the curve for g v should be like this. Also, the real g v should be going from here from – let us you something else – should have been going from here and then going like this. That is what should have been the g v. But, the g v we are going to use is this solid line. It is the solid line. In spite of the fact that the correct g v curve should have been the solid line up to here and then this dotted line like this, it takes it to 0 again at the bottom of the band. However, by same logic, we are going to say that… What we are going to say is that, again, this product 1 minus f E g v I will write here. And, I was saying that, only into few k b T – about this much. This is an order of k B T only up to this much energy, only up to this much energy – k B T. Only in this energy, we will see that, this product lies. And, it will happen though fact that, g v is being given by this curve right here. This equation is what I plotted as solid blue line right here.

But, nonetheless… And, I am going to use this in this integration. But, because 1 minus f of e will die out very quickly as we go down in energy; and hence, this product will die out. And, for same reason as we talked on previous slide, this quantity whether I use g v, this expression or I include the dotted line, does not matter; it does not matter. And, same token we can integrate this all the way from minus infinity to E v rather than E v bottom to E v; it does not matter, because this product is going to be 0 deep in the energy anyways. So, these are the two points I am going to prove to you by doing some graphical or showing you some graphically. This constructs as graphically also. So, let us do this.

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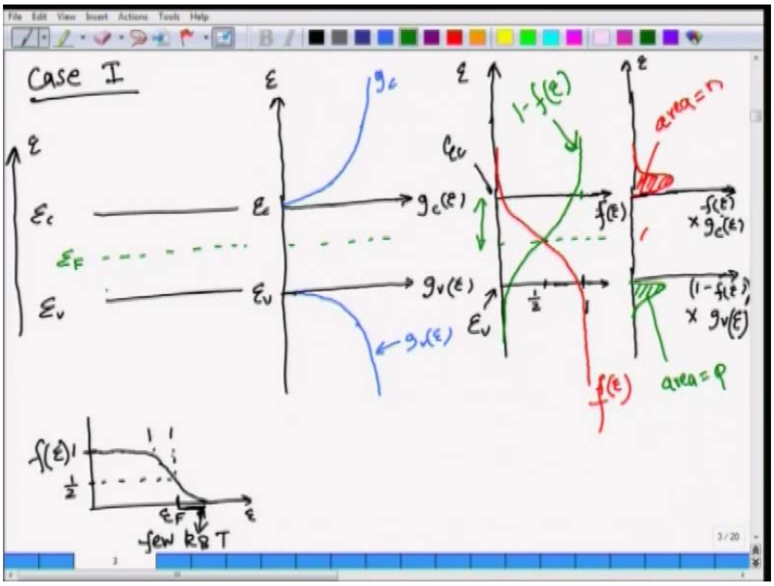

Let us go in third slide. And, I am going to make three cases. So, let us plot case 1. And, they are very similar to each other, but some points can be made. So, let us take this case 1 and let us do this; that here is therefore, E c and here is E v. And, energy of course increases in this axis. This is energy axis. Now, what I am going to do is I am going to plot what quantity? I am going to plot this quantity g c; which I have already done. I have plotted right here. And, I am going to plot this quantity g v, which I have already done here. But, I am going to put it on the slide also right here. So, on this axis, is energy and this is E; and, on this axis is time. I am going to plot g c of E. And, on this axis, I am going to plot g v of E. I am going to put number for g v of E here and g c of E there. What do they look like? I have already plotted on the previous one. I will reproduce this solid blue curve disregarding this dotted portion; and, I will justify that, why this solid blue curve was fine. And similarly, for g v, I am going to plot this solid blue curve here for the same reason on the third slide here. So, let us do that. So, this is the plot of g v, whose numbers can be read on this axis. And, this is plot of g c, whose numbers can be read on this axis. But, this is a plot of g c. So, this is the plot here.

And, now, third one I want to plot is… Let us draw this again. Four quantities I need to plot. So, I am going to keep this – all these things going here. So, again, this is the energy axis; this is the energy axis; this is the energy axis; this is the energy axis. And of course, these are the E c's (( )). So, these are the E c's. E c's level is here; E v is level is here; E c is here; E v is here. E v is here; E c is here. And, (( )) this is also visible. This is E v. And, this point is E c. And similarly, right here also. What I want to plot here? On this axis, I am going to plot now what function? I want to plot this function. And, on another plot, I am going to plot this product. So, this is my strategy. So, I want to plot this function f of E. So, let us do that. Let us plot f of E. Remember let me just do it – plot of f E right here for you. So, if I were to plot f of E; f of E remember was equal to… This was E f; if E f was here, then this is the energy axis; then, we had said that, this looks like something like this. This is 1 right here. This is the probability. And, if this is the dotted line… That at some finite temperature, it goes something like this; it goes something like this. And, this was few k T of energies in which this dies out; this curve dies out. This was only this or this was few k B T. This is only few k B T in which this thing got dying out. We have got few k B T only right here. So, this was dying out.

Now, I am going to plot this one, where energy's axis is going up and f of E is being plotted right here. So, this is one plotting f of E right here as f of E. So, now, how would it look like? Now, we have to know where the Fermi energy is, where E f is; it all depends on E f. That sets the three cases. In the first case, I am going to take the Fermi energy right here about middle of the band – band gap. I am going to take Fermi energy somewhere here. Somewhere here in the Fermi energy; somewhere here in the Fermi energy. So, what happens? If Fermi energy is here, let us take this axis as… let us take this value as 1. So, this value is 1. So, this value somewhere here would be half. So, somewhere here is the point, where this function becomes half. I am plotting on this axis, I am plotting f of E. So, right here is value 1. So, what happens now? You know that this function becomes half; this becomes half; its value becomes half at E f when E is equal to E f. So, when E is equal to E f, this function will be half. So, let us plot this; let us use this red to plot f of E – goes like this.

I have exaggerated this picture that I am plotting the same function that, for all values less than energy – less than E f, this function… For values E a… Energy is less than E f, this function tends to 1. So, for energies less than E f going this way, this function is tending to 1 and it falls off. And, I am showing you that… Remember in few k B T, I said that, this will fall off. And then, silicon – this band gap is about 1 electron volts; this band gap is 1 electron volts. And, k B T at room temperature like I said is 25 milli electron volts. So, I have exaggerated this picture that, I have shown that as if this curve is falling off fairly slowly, because this much will be 500 milli electron volts. So, within 500 milli electron volts, this curve – red curve should have died out to 0 value very quickly – nearly 0 value very quickly. But, I am exaggerating this picture and showing you some value just for schematic that, some value can be seen on this scale. So, this is what I have plotted as f of E. It is what I have just plotted. This is f of E that I have plotted.

Now, I want to plot also 1 minus f of E. That is simple. So, I want to plot this now. Next, one thing I want to plot is 1 minus f of E also for calculating the value of p, which is required. This 1 minus f of E is required for calculating the value of p. So, for this sake, I am going to plot 1 minus f of E also. And, that is easy to plot. So, since it was 1 right here; so, 1 right here. And, there will be mirror image of f of E. It will be simply 1 minus... Therefore, I will plot this as like this (( )) falling off. It is exactly symmetrical curve. Let me say if you add this… And, this is a plot of 1 minus f of E. This is plot of 1 of minus f of E. What I have plotted here is with green line is 1 minus f of E.

Now, if I want to plot, now, what I am going to do is I am going to plot… Next thing I am going to plot is… Look at this… So, f of E and 1 minus f of E is plotted here. g c is plotted to here. I am going to do. What I am going (( )) If you look at this, what do I need to integrate? I need to integrate this quantity. This is the quantity, which needs to be integrated – product of f of E and g c E in order to calculate n. So, what I am going to do is plot this product. Remember f of E was plotted by a red line and g c was plotted by blue line. So, I am going to take graphically, product of the red line and the blue line and then plot that. So, let us do that here, right here. So, I am going to take this g c and multiply it by f of E, which is a red curve. And, I am going to plot here… Therefore, f of E into g c E. That is what I am plotting on this axis right here. This is what I am plotting this axis on this axis. So, if I plot this… And, this is the product of course – blue line and the red line; let us continue to use the red pen for this. What happens that, for all values less than E c, which is right here; below E c if you notice, g c is 0. And, above E c, this f E function goes to 0 very quickly. So, what will happen to the product? Product will look something like this therefore; something like this. It will look like something like this.

And similarly, if I plot this product of f of… If I want to plot 1 minus f of E and g v; which will be used to calculate p; then, let us look how that will look like. So, you take 1 minus f E, which is a green line and multiply with this g v curve and plot that. That I am going to plot on this axis. On this axis, I am going to plot. Let me first plot it and then write this label. So, this… Multiply this blue curve with the green curve; notice that, for all values of energy greater than E v, this blue line is 0. And, for all values less than E v going in this direction, this (( )) function falls off very quickly. So, what will happen? This curve will also look something like this; it will look something very much like this. And, what have I plotted here? Let me level this also. This is… And, this is the scale I am plotting – 1 minus f of E multiplied by g v of E. That is what I am plotting on this axis. So, what do I have?

Now, you can see that, what is this area – this area and this area? This area is equal to n. Remember this integration. This integration leads you to n. And similarly, this particular integration leads you to p. And hence, I am going write that, this area – shaded area is equal to p by same logic. Now, what do you see is that, remember this function. Again I go back; look at this f E value. f E – this band gap is in electron volts for silicon for example and I have already mentioned that. So, this will be about 0.5 electron volts of 500 milli electron volts. This function therefore – this red function would have dropped down very quickly and these values will be very very small values. So, you can clearly see that, above E c, this function – this product cannot be having some finite value for anything more than few k B T. And, since this conduction band is also of several electron volts; so, I am very justified in using… I will be OK if I use this value of g c, because anyway, all that matters is integration from E c to only few k B T, where this product lies; after that, it is 0. Hence, whether integrated all the way to infinity or I integrated to conduction band top, it matters none. That is the first point I want to make.

And, second point is that, you are free to… You are welcome to use the g c curve as… Or, you are welcome to use g c curve as given by this expression right here. This is fine. Use of dotted line is not necessary. It may be possible for example, going back it to slide number 1; it may be experimentally possible to determine what E c top is. But, even if I do not know it, I need to integrate only up to infinity. So, I need no knowledge of this E c top as to where E c top is, because this product goes to 0. And likewise, same argument will apply for this also. Now, what is case 2? Case 2 now, I am going to draw rather quickly. What I am going to do is I am going to notice that, the way I have drawn is when Fermi energy in the middle, I have shown this area under red area and this green area to be approximately equal; n and p equal n; Fermi energy somewhere in the middle. You will see later when we do quantitatively. This case will correspond to something what is called as intrinsic semiconductor. Now, what I am going to do is I am going to take this Fermi energy very close to conduction band in second case. In third case, I am going to take this Fermi energy very close to valance band and see how these curves look like.

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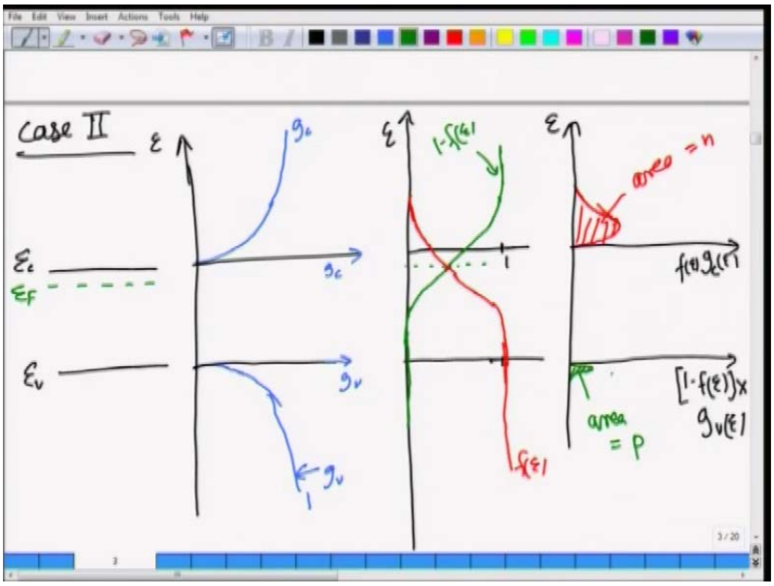

So, let us do that. This is case 2. And, I am do going to do very quickly now since you understand it. So, here is case 2. So, here is E c; here is E v. And, here is energy axis;

here is energy axis. And, I am going to plot this g c, which is going to look the same way, does not matter; it does not depend where the Fermi energy is. And, this is g v; this is g v. So, I am plotting this as g v and this is g c. From this axis, g c; on this axis, g v; I am plotting. And, it does not depend where the Fermi energy is. But, nonetheless, let me mark out where the Fermi energy is. Right here is the Fermi energy. In that case, if I plot again; where is this energy scale? And, I plot f of E and 1; here is 1; and, somewhere in the half. So, if I plot the f of E, now, remember Fermi energy is now here; Fermi is somewhere here. So, the value of f of E will be half right here. So, this f of E curve now, is going to shift little bit towards the conduction band. So, it will become something like this now. This is f of E. And, the 1 minus f of E will acquire a half value also right here and will be exactly mirror image of this. So, I am going to plot it something like this. (( )) 0 right here. So, this is 1 minus f of E when this is energy.

And now, if I plot same energy scale; on this, we are plotting now; this we are plotting g c and f E into g c E. And, on this, I am going to plot 1 minus f of E into g v E. Therefore, I am plotting… If I do that, then this time, since this Fermi (( )) this curve g c has remained the same. But, f of E values have increased, because this red curve has shifted up in the energy.

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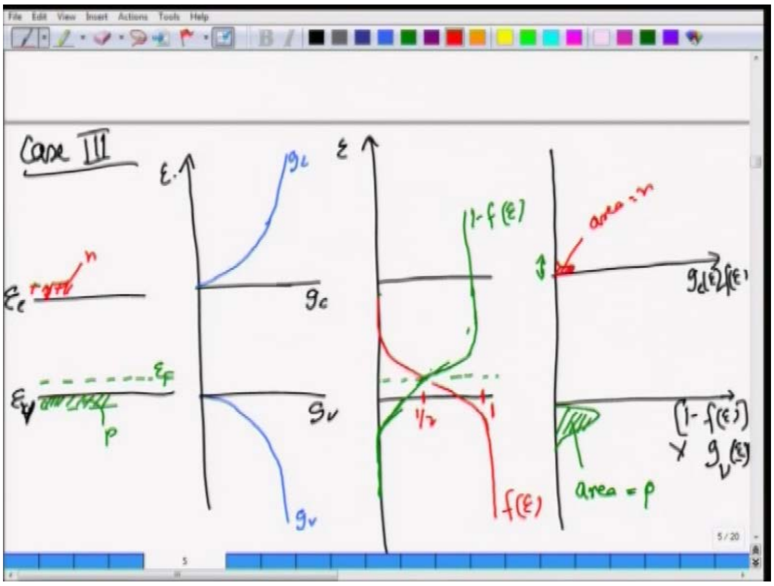

So, this curve this time is going to go little bit bigger than the previous case. So, I am going just exaggerating this picture and showing you this. And, this will be the area (( ))

again equal to n; and, this is the energy scale. And, by same token, 1 minus f E would have died down more. Nearly, 0 it would have become. And hence, this product would have become even smaller. This product would then in that case becomes smaller. And, this area would be equal to p. Notice what happened was that, at Fermi energy moves higher, this value of n is going to increase. As this Fermi energy goes up, and value of p will decrease.

Case 3 then will be… Here is now case 3 – last case. And, I quickly plot this also then; is here E c; E c; and, this is E v. And now, here is the energy scale. Same I am going to plot g c and g v under the blue curve right here like this. They do not change with the Fermi energy. So, this is g c and this is g v. And, this scale of course is energy scale. This is energy scale. And, the Fermi energy – we are going to this time keep it very close to the valence band edge. So, here is the Fermi energy. In that case, what happens? Let us plot the Fermi energy curve. Here is the energy. This is energy. Let us do one more time right here. Here is… I will be going to plot here on this.

We are going to plot on this g c E f of E. And, we are going to plot 1 minus f of E into g v E. g v E is going to plot on this axis. And now, Fermi energy of course is right here; not here, but below. Fermi energy is right here. Since Fermi energy is here; therefore, what will happen this time? This curve half… f E will become half here. So, it will become really 0 and something like this. (( )) f E if this is 1. This is 1 and this is half; this value is half. And, if I plot… This is f of E.

And, if I plot 1 minus f of E, it will be mirror image of course of this of course. Therefore, that will look something like… It should look like a mirror image something like this. It will look something like this. This will be 1 minus f of E. So, now, what happens? Because g c… Now, compared to case number 1, g v remains the same, but the green curve has move down. So, the 1 minus f E value has increased. So, this part will become little bloated this area will become little bloated. But, the red one… Because the f E has reduced now by shifting down; hence, will become very small. Something like this will become the… And, this area will be of course.

Again n will be equal to n. And, this area of course will be equal to p. So, what you notice? What you notice is that, few statements we can make. One… To summarize, few statements we can make that, the g c value we can take. As we have written the expression, the real value we need not take; real character we need not take simply because this integration has to be carried. This product of f E, g c and f E is only in a very small energy range – few k B T. And hence, we need not take. Only thing we need to do is, in that range, value of g c should be correct; which we have ensured. This should be the value.

Second thing is that, notice that, at value of n, electrons will not leave exactly at E c, because density of state is 0. But, electrons live only slightly above the conduction bandage; only in the small region here – right here they live; in the small region here electrons live; only in small region here do the… In small region here holes live; only on small and… Small region right here electrons live in this much energy. Very small energies electrons and holes live; rest of the part it does not matter; which also therefore, allowed us to carry out this integration to infinity for n. And, in that case, up to minus infinity for calculating p. So, with these arguments, we are going to calculate these red expressions. This one for p I am going to use this. And, quantitatively, I calculate p using this expression; and quantitatively, calculate n using this expression. So, let us move forward and calculate these quantities quantitatively. So, let us move on to that step. We are going to take that step now. So, when I substitute all these values… So, if I do that, then I am going to write again now n. So, let us start with n now. And then, same way we are going to do this for p.

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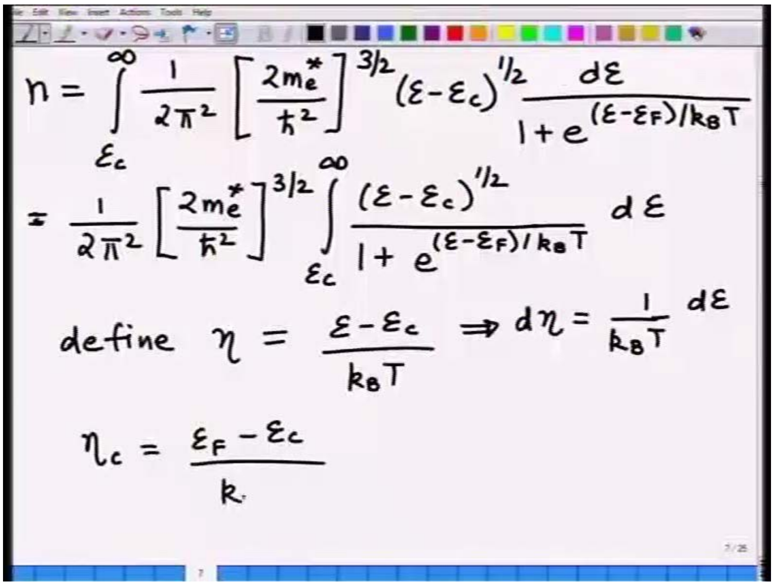

So, here I want to write n. Now, I am going to integrate only from E c to infinity; I am going to integrate up to infinity, not top of the conduction band; there is no need for it, because as you see, it is not so… Wrong choice of words; I will integrate from E c all the way to infinity; it makes no difference whether you integrated to top of the conduction band or infinity. That was the logic we had used. So, let me substitute now g c. So, what is g c? 1 by 2pi square 2 m e star by h bar square to power 3 by 2 E minus E c to power half. And, we are going to also use in the Fermi function, 1 plus e to power E minus E f by k B T dE. This is the integration we need to carry out. This is the integration we carry out and we are going to get our answer.

Let us write down. Let us simplify this. 1 by 2 pi square 2 m e star by h bar square power 3 by 2 E c to infinity E minus E c to power half divided by 1 plus e to power E minus E f divided by k B T dE. This is the quantity I want to integrate. There is no analytical solution for it. So, let us use some definitions first. Define… I am going to define the quantity called eta, which is equal to E minus E c by k B T. I am going to define this quantity. That implies d eta is equal to 1 by k b T dE; E c of course is a constant. That is the case. Then, we are going to define a quantity n c as a quantity, which is equal to E f minus E c by k B T.

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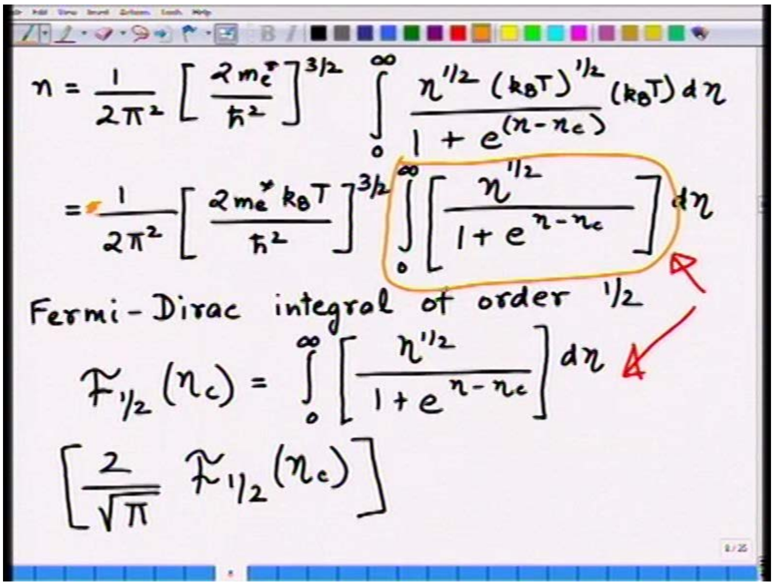

If so, now, I can write n as equal to 1 over 2pi square 2 m e star by h bar square to power 3 by 2. And, if I now want to integrate it; so, now, when E goes to E c, then eta goes to 0.

So, I am going to write this as 0. And, when E goes to infinity, then eta goes to infinity. So, this integration is up to infinity. So, now, what is this quantity right here? Is a quantity, which is equal to… In fact, I should write this as eta to power half E minus E c, is essentially equal to k B T to power half multiplied by area to power half. So, I should also write k B T to power half divided by 1 plus… Let us see what this quantity is; E minus E f by k B T. So, we should write this as E to power eta minus eta c. Then, we will see what happens. eta minus eta c is essentially equal to E minus E f. Therefore, I will write this as 1 by… And then, I am going to write this as dE. I am going to substitute as k B T d eta. So, I am going to write this as 1 by 2 pi square 2 m e star – I am going to include now k B T here – h bar squared to power 3 by 2. I will take k B T out; that means 0 to infinity eta to power half divided by 1 plus e to power eta minus eta c d eta. This integration becomes like this.

Now, let me introduce this Fermi-Dirac integral for order half as F of order half of parameter called eta c in this case. Eta c is just the parameter in there as being quantity equal to 0 to infinity eta to power half divided by 1 plus e to power eta minus eta c. So, this is a well-known integral, which is tabulated in many hand books. Or, you can calculate this as… You can calculate it numerically. So, point is that, this integration does not have an analytical solution. You cannot analytically write down this integral. But, we do have this as a… It is known as… It has a standard name called Fermi-Dirac integral of order half. So, you can use that. This Fermi-Dirac integral of order half is tabulated in many mathematical hand books; you can use that. But, now, they are at relevant computer age. You could just carry out this integration numerically as well. And, fairly simple, you could carry out this integration even in an excel sheet. So, now, it is simple. So, if that is the case, then I can write…

I am going to introduce not… This is often… This Fermi function is not tabulated; often it is common to tabulate instead a quantity called 2 by root pi this function and see which I have just written out. So, what I am going to do is I am going to separate out 2 over root pi. I will multiply this function by 2 over root pi and divide this quantity by multiply… I will multiply this quantity by 2 over root pi and I will also in front I will put inverse of that root pi by 2 that is. And, if I do that, then I can write this whole quantity n as… So, let us do that. Let us substitute all the values in there and carry on from there. So, I am going to rewrite the quantity n again.

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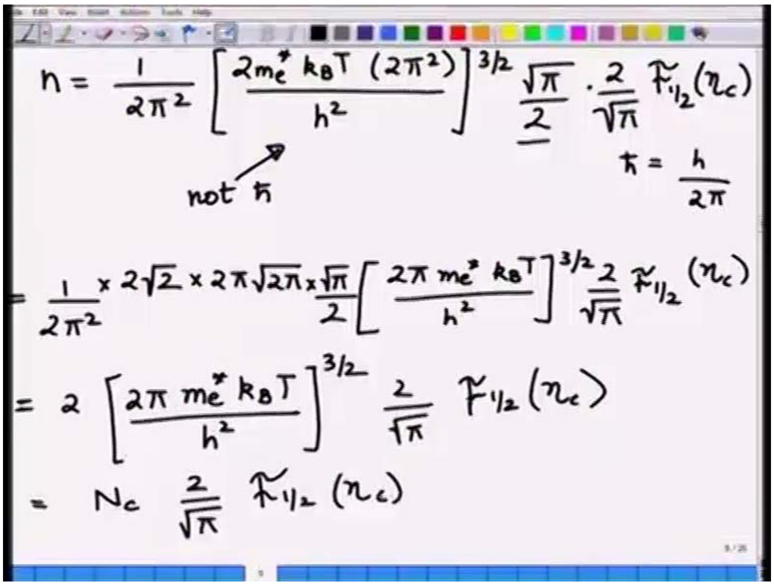

So, n now is equal to therefore, 1 over 2 pi square 2 m e star k B T. What I am going to also do is, I am going to write this as 2 pi square right here divided by h square now to power 3 by 2. This is one change I am making remember. h bar is equal to h by 2pi. So, this is not… This is h; not h bar any more. So, it is not h bar; it is h now. And, what I am going to do is I am again, what I am going to do is that, I will put now a factor root pi by 2; and then also, I will multiply… I am going to write this as 2 by root pi of order half now – Fermi-Dirac integral of order half. So, I am going to do a simplification of this coefficient and write the expression after that.

So, what we can do is in fact, you carry out the simplification. So, 1 over 2 pi square. What I am going to do is I am going to live in there the quantity 2 pi square; I am going to live 1 by 2 pi in there and I am going to take out this rest of the stuff out here; from here 2 to power 2 square root of 2, which is coming from here. And, I am going to take one 2pi outside. So, I am going to write 2 pi and the square root of 2 pi one more time here. I am going to write here. And therefore, I am going to write in bracket here as a quantity and I am going to take this also right here – square root of pi divided by 2; I am doing this all multiplication and I am going to write here now 2pi m e star k B T by h squared to power 3 by 2 2 by 2 root pi F half eta c.

So, if I carry out this, now, you can clearly see that, I have this 2 pi square; 1 pi is going to come from here. There is pi here. If you carry out this simplification on this, what you will find is, this quantity simply becomes then 2 into 2 pi. And, since I knew this answer, that is why I have just written it in specific form – k B T; otherwise, nothing is special. As all h now, remember; not h bar 2. That is a conventional way of writing 2 root pi. So, that is the value of… So, this is what n is equal to now. This is (( )) governing expression. Now, what I am going to do is I am going to define this quantity as… I am going to write this quantity as N c times 2 by root pi F or Fermi-Dirac function eta c.

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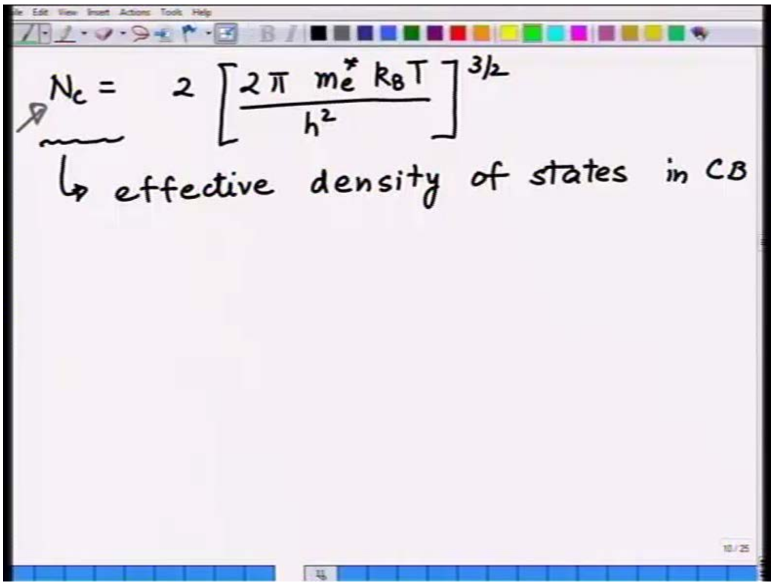

We have of course N c is equal to 2 into 2pi m e star k B T by h square to power 3 by 2. And, this quantity is called effective density of states in conduction band. Why this terminology? Will become clear in a minute; but, you can see from here… Not you can see, I would rather… I have simplified this expression in a symbol form; n c as the standard definition, whose coefficient… All this quantity is written out here; n is equal to N c 2 by root pi Fermi-Dirac function of order half. Having written this, let us now look at… Likewise, let us do it for holes in the valance band also. Let us carry out the similar exercise for holes as well. And, when we do for the same exercise for holes, let us go back. What I mean is now, remember if we are going to carry out this integration… I am going want to do this integration 1 minus f E multiplied by g v E. So, that I am going to substitute 1 minus f E like this and g v E like this. This is what we need to substitute in there. So, if you do the carry out this integration in there, let us see what happens for p. So, if you do calculate p…

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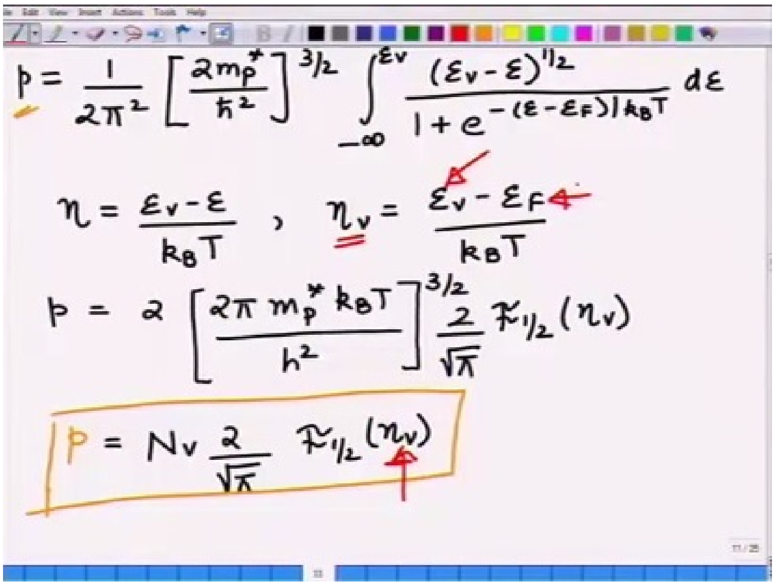

So, now, similarly, if you do this p, you will get equivalent expression and then we will handle both these things together. In that case, I am again… This time skips from steps – 1 over 2 pi square 2 m effective mass of holes by h bar square to power 3 by 2. And now, I am going to integrate from minus infinity to E v. And, what I am going to integrate now? 1 minus f E. So, 1 minus f E. I am going to prefer to write… Let us go back here. So, 1 minus f E; I can also write as…

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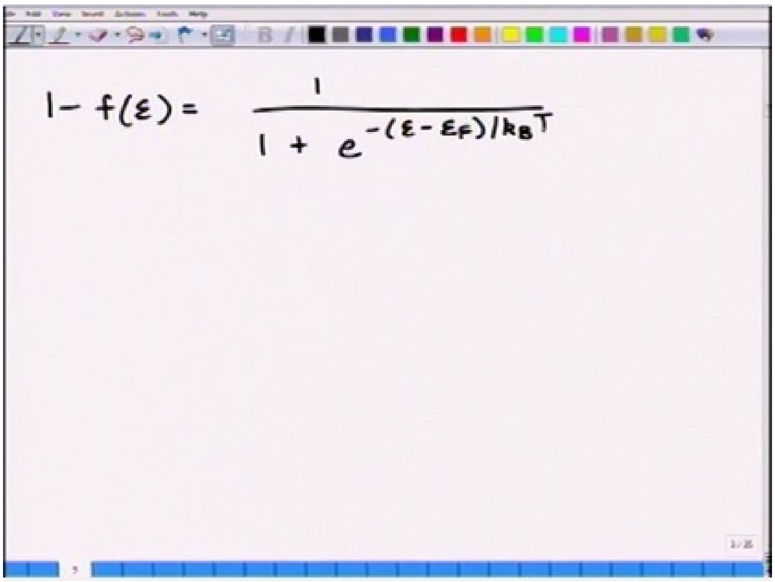

Instead I can write 1 minus f E – 1 minus f of E as also equal to – what I am going to do is divide it. And therefore, I am going to write this as 1 by 1 plus e to the power minus of E minus E f by k B T. I can also write it like this. Namely, therefore, basically divide the numerator and denominator by 1 by… Divide numerator and denominator by e to power E minus E f by k B T. So, I am going to substitute this for 1 minus f E. So, if that so, then I will write this as E v minus E to power half. You can see the form is very similar – 1 plus e to power minus E minus E f by k B T and dE. There are integration we need to carry out.

Now, this time, we are going to substitute eta as E v minus E by k B T. And, I am going to substitute n v as equal to E v minus E f by k B T. We substitute this. So, in that case, you can carry out the same exercise exactly same way, which we (( )) n. You find p as equal to 2 times 2 pi m p star by k B T by h square, not h bar square to power 3 by 2 F half eta v. So, exactly same way, I should in fact, at this 2 to power 2 and root pi over here also, exactly same way; which we are going to write as N v times 2 by root of pi F half – Fermi-Dirac function of order half eta v; exactly same way.

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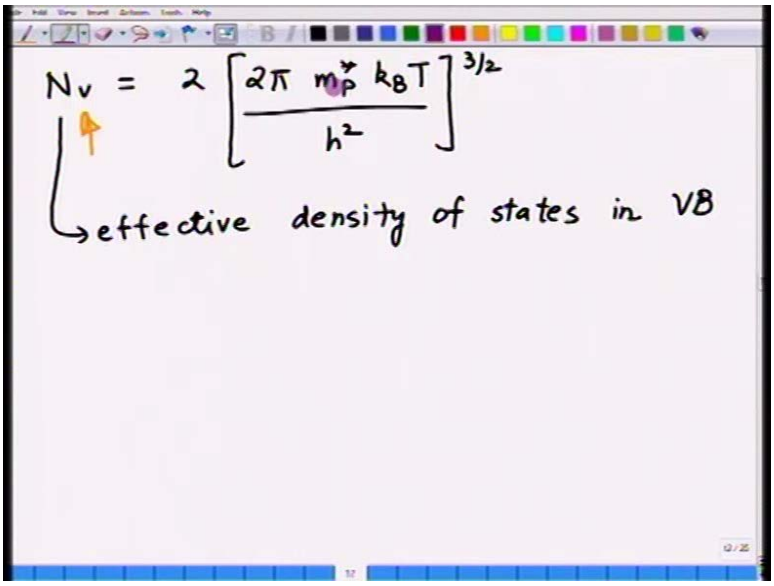

So, same form of equation, where N v of course is equal to 2 2 pi m p star – effective mass of holes – k B T divided by h square power 3 by 2; which this quantity is effective density of states in valance band. For holes that is obviously, in valence band. So, we have an expression for p. So, that is what essentially this is. So, I can calculate now n and p if I can calculate Fermi-Dirac integral, which is straightforward; that we should be able to calculate given that, where E f is… Notice that, in N v, what is the parameter? This property of material we know where that is. Therefore, only quantity, which we need to know is if you tell me what E f is – what the value of E f is; if you give me this is the value of E f, then I can calculate N v. And therefore, I can calculate this integral. And of course, N v is understood; thus, all fundamental parameters – Planck's constant and then of course effective mass of holes, which comes from the E-k diagram; recall that. And, temperature and Boltzmann constant, etcetera. So, I can immediately get effective density of states.

Same way for conduction band, I can calculate N c. And, if you give me E f, then we can calculate Fermi-Dirac integral of order half for N c – N subscript c. Therefore, I can calculate n and p. Only thing I need to know is where the Fermi energy is. You tell me where the Fermi energy E f is. We are in business; we can calculate what n and p are. But, we want to make some simplifications. So, this is effective density of states in valance band. So, let us move forward. So, as we continue, what we will do is now, try to get an estimate of approximation for this Fermi-Dirac integral. So, that is what we are going to do now. So, if you want to do… Actually, even before we do that, maybe let me give some numbers. Since we can calculate this quantity N c and N v, first let us look at N c and N v.

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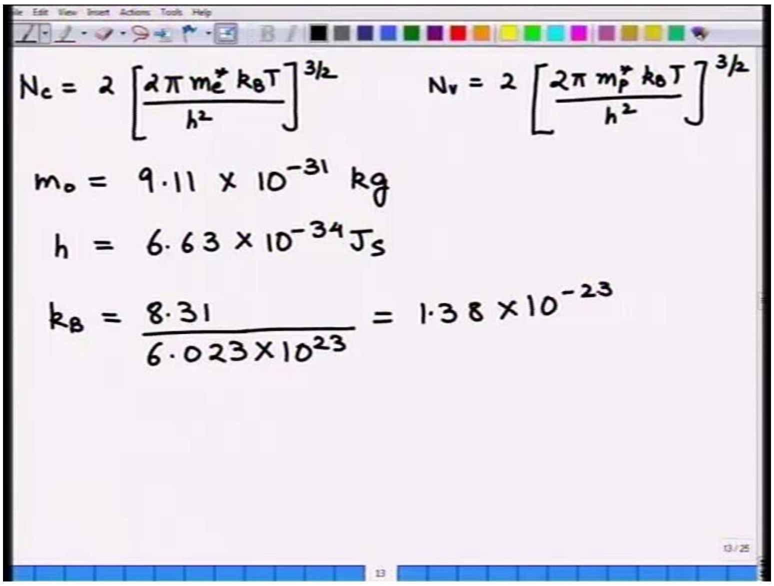

So, let us write N c. N c was equal to 2 times 2pi effective mass of electrons k B T divided by h square 3 by 2. N v equal to 2 – same expression, very similar expression – the effective mass of holes now, we have to use. So, these are the effective density of states. Now, let us just calculate these numbers; how much they are. So, what we can do is… First of all, what we are going to do is simplify it. So, you know that, rest mass of electron, which we are going to say as m 0 is 9.11 into 10 to power minus 31 kgs; h is equal to 6.63 into 10 to power minus 34 joules second. So, we can substitute in there a Boltzmann constant k B – of course is equal to 8.31 divided by 6.023 into 10 to power 23. So, you can carry out this calculation. Just check on this; where, I believe it is 1.38 into 10 to power minus 23, is what this quantity will be equal to k B therefore.

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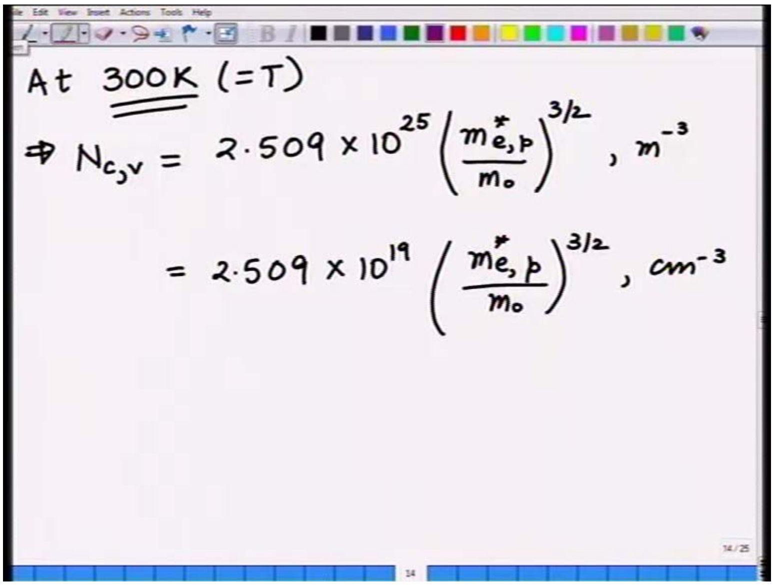

So, let us look at – at 300 k; what happens at 300 Kelvin, which is about room temperature; where, T is equal to – this is the value of T. So, T is equal to this quantity. So, what happens to N c and N v? Let us see what this quantity is. So, this first of all, we are going to write this as… Estimate this as… Substitute all the values except m e and m h and then what you get. So, I am going to write this as N approximate… or not approximate; I am going to simplify this expression as this quantity is equal 2.509 into 10 to power 2 2 6 – 10 to power 25 m either e or p star by m 0 to power 3 by 2. And, when you calculate this, you will get the answer in minus per meter cube. Or, therefore, I can write it as 2.509, which is more conventionally how we write it – 19 divided by 6. Therefore, m e p star by m 0 3 by 2. And now, it is in per centimeter cube. That is what density of states is. What I mean is that, if you substitute in here effective mass of electrons; in that case, you calculate N c. And, in case you substitute here effective mass of holes, then you calculate N v. That is what this comma separates. Pick one of the two. So, that is what the simplified expression is.

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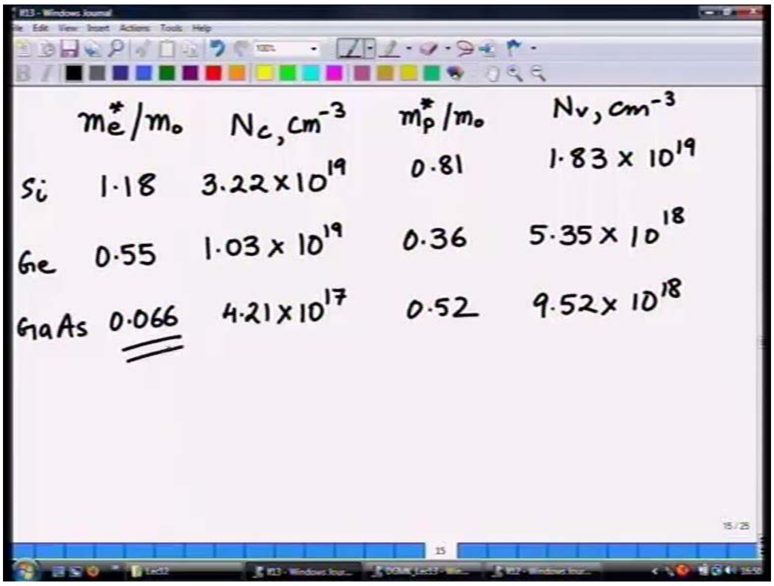

So, if I do that; if I is look at silicon for example; so, if I look at silicon; for silicon, the effective mass of electron is or divided by (( )) and with respect to the rest mass of electron… So, compared to rest mass of electron, what is the effective mass of electron in silicon? Then, you find that, this is equal to 1.18; that means electron is slightly heavier than the rest mass of the electron. And, if you calculate N c in terms of as I said in centimeters minus 3; then, this quantity comes out as 3.22 into 10 to power minus 19, is what this quantity is equal to. And, if I look at the whole effective mass for silicon, then there is 0.81 only; that means hole is lighter than… Hole behaves lighter in silicon than does the electron. And therefore, if I plot…

And, I calculate N v in units of per centimeter cube, then this number is 1.83 into 10 to power 19. What you see? The idea is the density of states – effective density of states is about 10 to power 19 per centimeter cube in silicon. Remember number of silicon atoms (( )) Number of number of silicon atoms are 10 to power 22 per centimeter cube. Yet, why we call this effective density of state is… Notice that, this number is not 10 to power 22 per centimeter cube, but it is just 10 to power 19. Similarly, if I… And, there is another reason we will see – germanium; if I write this for germanium, this is 0.55; lighter – electron is lighter; the curvature – depends on curvature remember; 1.03 into 10 to the power of 19; 0.36; 5.35 into 10 to power 18. This is in case of germanium.

Now, gallium arsenide – something really very interesting happens; the electron is really really light; really really light – 0.066. And, density of states is 4.21 into 10 to power 17 only; whereas, in relative terms, between electron and holes, hole is about same heavy as for example, in silicon and germanium. So, it is hole in case of gallium arsenide is about same (( )) equally heavy as in germanium silicon. But, electron is really really light. And, that has a consequence in electronic devices, because electron is… Electron devices therefore would be… Electron current can be very very large, because the mass is very small. So, mobility can be very very large. So, this has a very important consequence. Maybe I will point out eventually as we go along little bit later; not today, but few lectures down the line.

So, that gives you an idea of what the effective density of states are in these materials – 10 to power 19, 10 to power 17, 10 to power 18. And, here is an expression, which is given to you. You can substitute in there and you can try calculating yourself also to see if I had made any mistake. And, if I have, please correct it. Now, we are back to this. N c and N v I have estimated. N c, N v I have estimated; N c I have estimated likewise. Now, let us have an estimate for this Fermi-Dirac integral also of order half.

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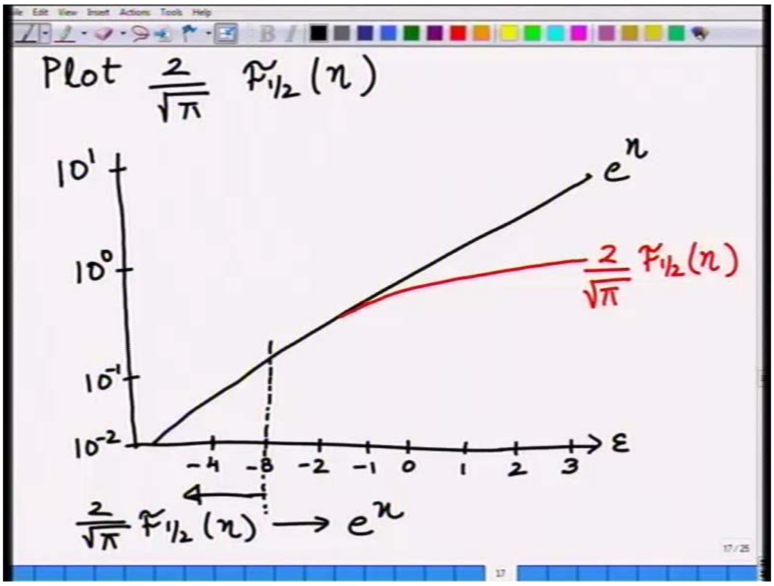

Let us try doing that one also. So, if I do that, (( )) I am going to do is I am going to plot 2 root pi Fermi-Dirac integral of order half. This is the quantity I am going to plot. So, let us do that. So, if I plot this like this, here is 0 and I am plotting eta; and, here is minus 1, here is minus 2, here is minus 3, here is minus 4, etcetera. And, this is 1, 2, 3, etcetera; and, 10 to power minus 2, here is 10 to power minus 1; here is on log scale, 10 to power 0; and, here is 10 to power 1. If I plot this, then at around somewhere here… So, if I plot this, this looks like – this curve looks like… I will just plot from here to somewhere here. Approximately, I am plotting this. So, somewhere here. This looks like this curve here. A straight line is here. Then, in that case, from (( )) 1 onwards, starts behaving like this. So, maybe I will plot it like this. This is a different pen. So, this is a plot of… This red line is a plot of 2 by root pi F half eta.

And, this black line is a plot of E to power eta. This two lines – black ones – a straight line. Of course, e to power eta when taken log will become a straight line. So, that is what this plot is equal to; which I have plotted here e to power eta that is. And then, (( )) What do we see? We see that, let us say up to here; let us be more conservative up to here. Till where all the eta values here in this direction. We can see that, in this range, we can find that, 2 root pi eta is approximately equal to E to power eta. So, for all values of eta less than minus 3, we are going to assume that, F – this function approximates to E to power eta. And, therefore, we will able to solve our problem within those limits. I will start from next lecture on this topic. And then, correspondingly, I will calculate n and p in the semiconductor.

Thank you .