Optoelectronic Materials and Devices
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Electrical Properties of Materials
Lecture - 14
Equilibrium Carrier Statistics in Semiconductors: Quantitative Examination of Carrier Densities in Intrinsic Semiconductor

Welcome to lecture number fourteen.

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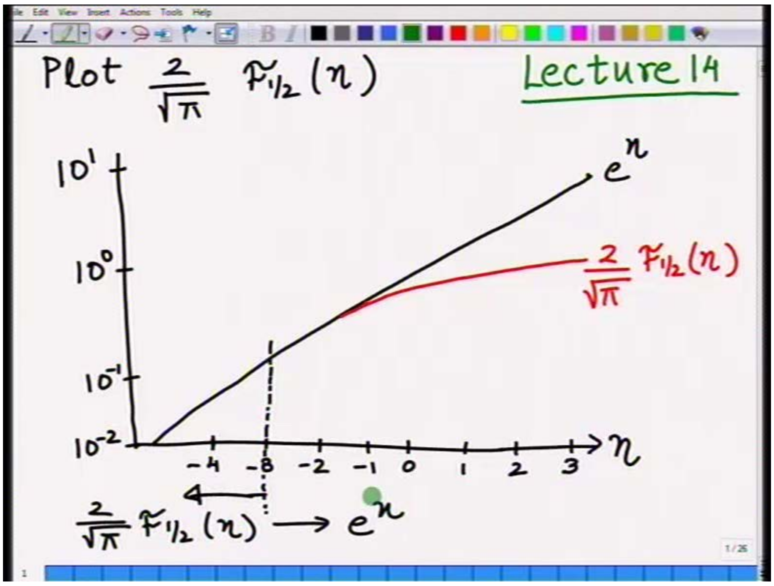

So, we will continue from where we left last lecture. So, let us recall what we did. Remember we calculated that n – the number of electrons per unit volume in conduction band is equal to N c times 2 by root pi – this Fermi-Dirac function of order half eta c; where, eta c was equal to E F minus E c by k B T. And similarly, we have derived number of holes per unit volume; that means holes density was equal to effective density of states of valance band for holes or electrons for that matter – Fermi-Dirac function of order half eta v; where, eta v was equal to E v minus E F by k B T. That is what we have derived. And then, what we started doing was that, we started approximating this Fermi-Dirac function – integral rather not function; rather this particular integral we had started evaluating as to what these two integrals were may be equal to we started approximating it.

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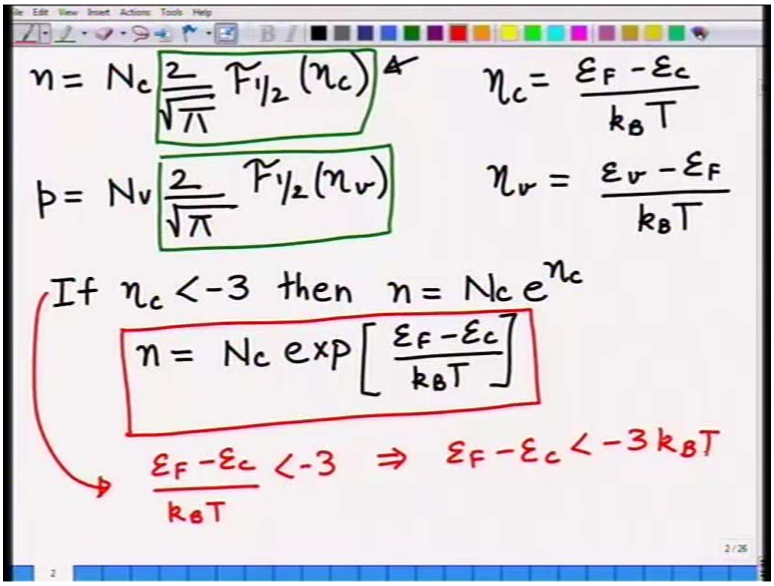

$$n = N_c \boxed{\frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c)} \qquad \eta_c = \frac{\mathcal{E}_F - \mathcal{E}_c}{k_B T}$$

$$p = N_v \boxed{\frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_v)} \qquad \eta_v = \frac{\mathcal{E}_v - \mathcal{E}_F}{k_B T}$$

If $\eta_c < -3$ then $n = N_c e^{\eta_c}$

$$\boxed{n = N_c \exp\left[\frac{\mathcal{E}_F - \mathcal{E}_c}{k_B T}\right]}$$

$$\frac{\mathcal{E}_F - \mathcal{E}_c}{k_B T} < -3 \Rightarrow \mathcal{E}_F - \mathcal{E}_c < -3 k_B T$$

So, in that context, what I had done was that, I had plotted this graph for this function as a function of eta. And, here is eta and then this is some values here and this red line – this red curve was this particular function, which appears in these equations also.

Then, what we also did was I simultaneously plotted e to power eta right here. And, I was saying that, at least for eta less than minus 3 as this value right here – on the left of this value, we could approximate this function as equal to e to power eta as this indicates. In fact, you could pick this approximation at minus 2 also, maybe minus 1 also; depends on how good approximation you make. So, I am taking… If am stating that, if it is less than minus 3, then this particular function – this integral – Fermi-Dirac integral could be approximated as e to power eta minus n. If so, then now, we continue from here. Then, in that case, how do you approximate n? So, what are we saying? That means if eta c is less than minus 3, then I can write n to be equal to N c times e to power… I can replace it by replace it by eta c. So, I can replace e to power eta c; I can replace this function, which is right here – this integral which is right here; I can replace it by e to power eta c. Hence, this is we know what eta c is. I can therefore write n being equal to N c times e to power E F minus E c divided by k B T. So, here is n. Here is the value of n that we can derive as N c times e to power E F minus E c by k B T under the assumption that, N c is less than minus 3. We can further look at eta c is less than minus 3. So, we can further look at this. This is when… What is the condition? Condition is, that means, E F minus E c by k

B T should be less than minus 3; which means E f minus E c – energy c should be less than minus 3 k B T.

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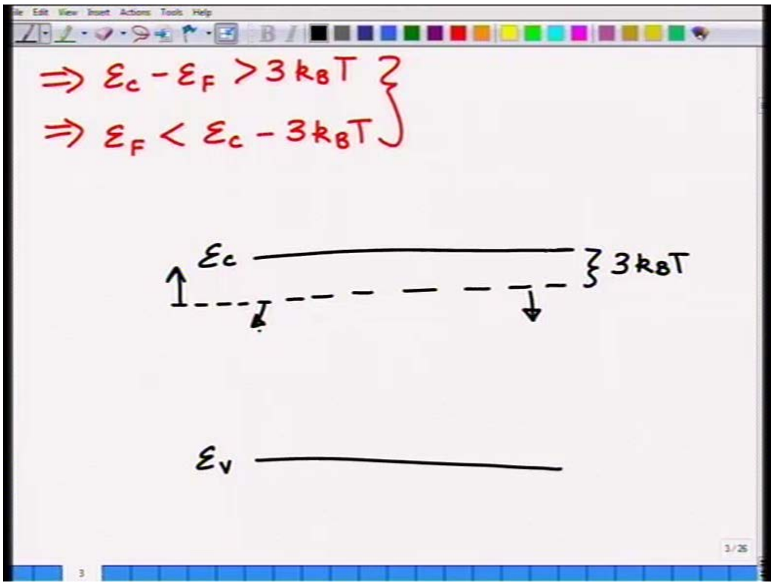

$$\Rightarrow \mathcal{E}_c - \mathcal{E}_F > 3k_BT$$

$$\Rightarrow \mathcal{E}_F < \mathcal{E}_c - 3k_BT$$

Which means I can write this as E c minus E F as being greater than 3 k B T. And, that means E F – Fermi energy should be less than E c minus 3 k B T. So, that is what our requirement is. If so, then we are OK. So, E c (( )) You can look at this or this; either way. What that means is that, if I plot this band diagram as follows; if I have something like this, this is E c and this is E v as long as Fermi energy lies 3 k B T below E c level; that means as long as Fermi energy lies, if this is 3 k B T; then, as long as Fermi energy lies – E F lies below this, then this approximation can be used; this particular approximation can be used. And, n will be then given by… n will be given by this. If not; that means if Fermi energy lies above this, then of course, in order to calculate n, we must go back and calculate it through here; which means in that case, we must determine what this Fermi-Dirac integral of order half is; what will be the value given an E F of course; that is, if we know where the E F is; if we know the value E F, then we can determine n in general by using this equation always.

But, if E F is 3 k B T energy below this value here – E c level; that is, if it is below that, then in that case, we can use this approximation as here – n equal to N c e to power n c – e to power E F minus E c by k B T. But, why 3 k B T? Same old question. If you are happy with 2 k B T, go ahead use 2 k B T; that is approximation. So, that is the judgment you have to call based on what kind of approximation you want to make – what level of approximation you want to make, how much accuracy you want. In that sense, normally, 3 k B T is common and it works fine. Let us now do the same thing for… Let us evaluate also p – what the value of p will be – number of holes in valence band. For that, we will repeat the procedure exactly same thing; whatever is in this box here, we will replace it by e to power eta v now.

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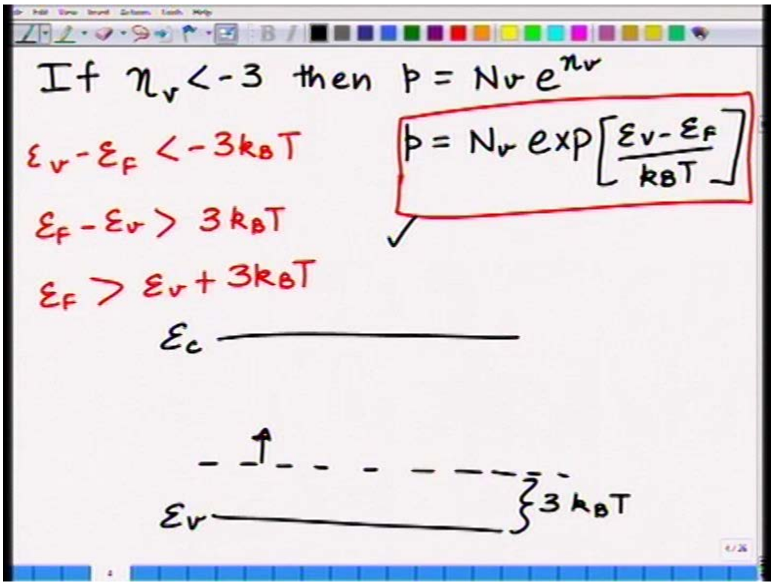

If $\eta_v < -3$ then $p = N_v e^{\eta_v}$

$\varepsilon_v - \varepsilon_F < -3k_BT$

$\varepsilon_F - \varepsilon_v > 3k_BT$

$\varepsilon_F > \varepsilon_v + 3k_BT$

$$\boxed{p = N_v \exp\left[\frac{\varepsilon_v - \varepsilon_F}{k_BT}\right]}$$

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In that case, let us write down p. Therefore, p… So, we will write again – if eta v is less than minus 3, then I can write p as being equal to N v e to power eta v; which means I can write p is equal to N v e to power… And, you recall what eta v was; eta v was E v minus E F by k B T. So, that was E v minus E F by k B T. So, that is the formula for calculating p in case this is true; which of course means that E v minus E F should be less than minus 3 k B T. This approximation also means therefore that, E F minus E v should be greater than 3 k B T; which means E F should be greater than E v plus 3 k B T. What does that mean? So, E F should be greater than E v plus 3 by k B T; that means we do the same thing here to show you here that, if this is E v and this is E C; then, what should happen is that, if this is 3 k B T – as long as Fermi energy is more than 3 k B T; that means on this side, Fermi energy is higher than this line I have shown you – dotted line I have shown you; that in that case, we are OK to use this formula. We are OK to use this formula in case Fermi energy is greater than 3 k B T from the valance band edge; it is higher than that.

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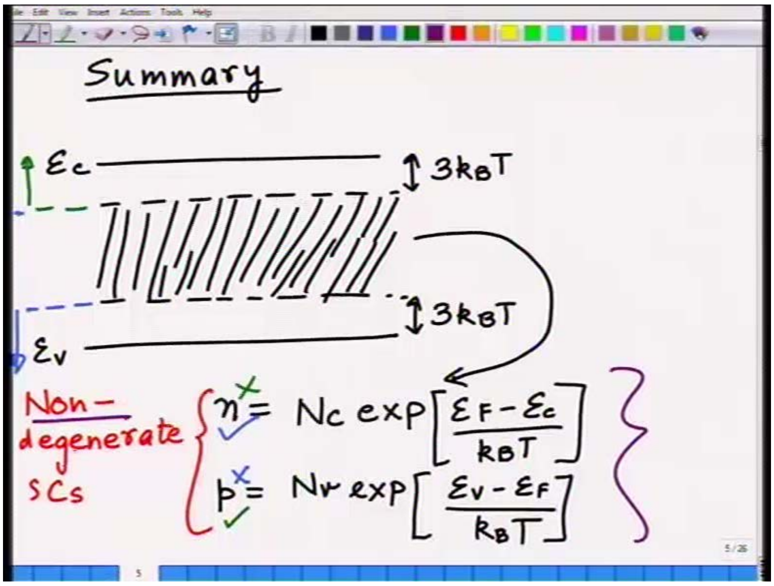

Summary

Non-degenerate SCs

$$n = N_c \exp\left[\frac{\mathcal{E}_F - \mathcal{E}_c}{k_BT}\right]$$

$$p = N_v \exp\left[\frac{\mathcal{E}_v - \mathcal{E}_F}{k_BT}\right]$$

If so, then this formula when we used; which means that, overall, if we summarize… So, if I write the summary of this whole thing, then I will write it like this. This is 3 k B T and this is also 3 k B T. In that case, what happens, I can write in this region. When Fermi energy lies in this region, then I can write in this region… If Fermi energy is in this region, then I can write n as equal to N c e to power E F minus E c by k B T. And, I can write p as equal to N v e to power E v minus E F by k B T. I can write it like this – Boltzmann constant times temperature. So, these two formulas are valid in the region if you are 3 k B T away from the band edge. Such semiconductors are called non-degenerate semiconductors. Such semiconductors are called… When Fermi energy lies 3 k B T away from the band edge, then such semiconductors are called non-degenerate semiconductors.

Notice if the Fermi energy goes higher than this point; move from this side; then, what happens? In that case, I can continue to use this particular formula. Let us use another pen. See Fermi energy is higher than this; I can continue to use this formula; I can calculate p according to this formula. But, this particular formula will not be valid, because now, I must use the Fermi-Dirac integral if Fermi energy goes on that side. Then, in that case, while because of E F is still greater than E v plus 3 k B T; so, this formula for p is OK, but not for n; it is not OK for n. And, I have to use then Fermi-Dirac integral in order to calculate n.

Similarly, if the Fermi energy is below this; if Fermi energy is below this point – this energy right here, which I have shown you as dotted line; in that case, the formula for n is valid. Then, it is OK, because Fermi energy is below this dashed line. As long as Fermi energy is below this dashed line, this formula for n is valid. And hence, this is OK. But, then this will not be valid any more. In that case, p has to be calculated using Fermi-Dirac integral. So, I hope you understand what these non-degenerate semiconductors mean. It means that the Fermi energy lies 3 k B T… And, 3 is obviously, arbitrary here; you could make it 2, etcetera. But, thus… Now, these are the formula we will be using often. And, you will see in most of our cases, (( )) are of concern to us, this condition will indeed be satisfied; that I will show you in few minutes from now. So, now, what else? In this region… Let us continue on these non-degenerate semiconductors.

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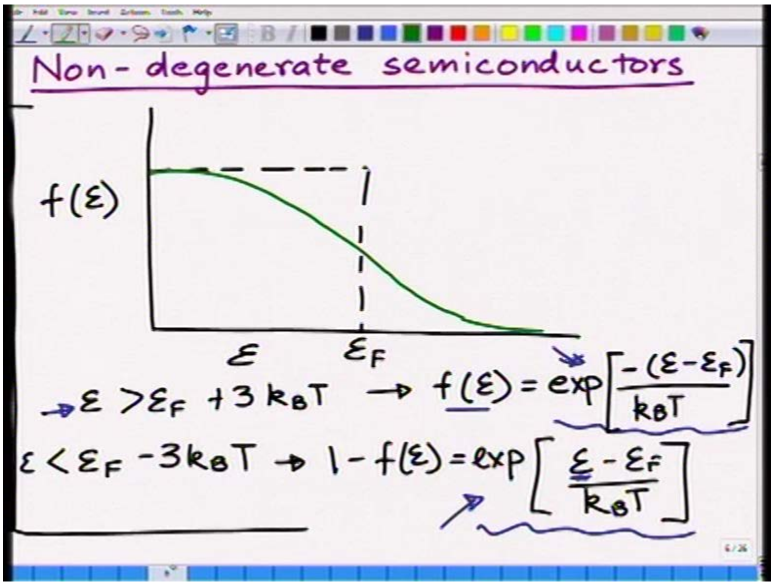

Non-degenerate semiconductors

$\rightarrow \varepsilon > \varepsilon_F + 3k_BT \rightarrow f(\varepsilon) = \exp\left[\frac{-(\varepsilon-\varepsilon_F)}{k_BT}\right]$

$\varepsilon < \varepsilon_F - 3k_BT \rightarrow 1 - f(\varepsilon) = \exp\left[\frac{\varepsilon-\varepsilon_F}{k_BT}\right]$

Now, you should recall what I had taught you about Fermi-Dirac function – that Fermi function, probability of finding electron at energy level. Recall that. We had said that, when we are plotting this curve like this – Fermi function f of E verses energy, I had plotted something like this. That if this is for E F, then I had shown to you that, at any temperature like this, this curve behaves like this; and, I approximated for E greater than E f plus 3 k B T. We had approximated F of E as being equal to e to power minus E minus E F by k B T. And, we had approximated for E less than E F minus 3 k B T; we had approximated 1 minus f of E as being equal to e to power E minus E F by k B T. This is the approximation I had shown you earlier. At some point of time, few lectures ago, couple of lectures ago, I had shown you this approximation. Notice that two are same thing; that means for a case of electron… In case of electron, when you are trying to calculate this n in conduction band; if Fermi energy is below this level, then all energies we are talking about – this E that we are talking about… If Fermi energy is below this level, then all energies we are talking about here E are indeed greater than 3 k B T. And, therefore, what happens is that I can use in fact, for f E, I can use a classical distribution – a Maxwell-Boltzmann like distribution. So, I can use this for conduction band, a Maxwell-Boltzmann distribution.

Similarly, for all energies greater than E v – greater than this dotted line, I am showing you here above E v. For that, you notice that, all energies here in the valance band, where the holes are; that means this energy right here – this energy right here – this energy will indeed be… This energy E indeed will be 3 k B T below the Fermi energy. Fermi energy lies somewhere above that – this line; in that case, this E F minus 3 k B T – this condition will be satisfied. So, 1 minus f E can indeed be approximated again by a Maxwell Boltzmann like distribution right here – a classical distribution that is, instead of Fermi-Dirac distribution. The point being that, non-degenerate semiconductor is… So, if you substitute in fact, for 1 minus f E by this or f E by this, you directly… And, you carry out the integration; you will get directly these expressions; which we have done later by approximating either that integral – Fermi-Dirac integral of order half to e to power eta. We have done later.

But, you can directly substitute… Under these conditions, you could substitute this instead of f E… For f E, you can substitute this; for 1 minus f E, you can substitute this expression; and, you will get back these equations exactly. The point therefore being means that, an electron behaves – behavior of electrons is somewhat classical when the Fermi energy lies in this range. And, that is the… So, you can either think of this as a non-degenerate semiconductor or a classical semiconductor when the Fermi energy lies in this shaded region; otherwise, of course, you have to use the Fermi-Dirac distribution for f E – the full distribution. And, therefore… Which results in Fermi-Dirac integral of order half. And, that has to be evaluated in that case.

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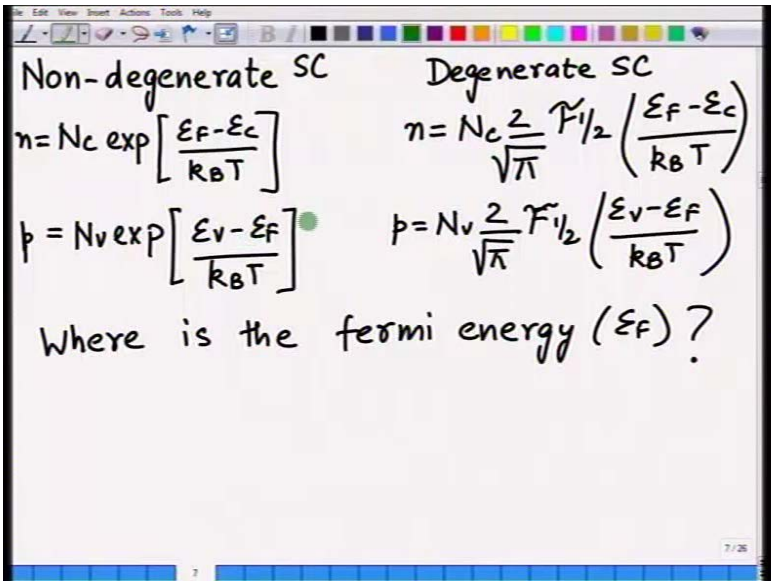

Non-degenerate SC

$$n = N_c \exp\left[\frac{\varepsilon_F - \varepsilon_c}{k_B T}\right]$$

$$p = N_v \exp\left[\frac{\varepsilon_v - \varepsilon_F}{k_B T}\right]$$

Degenerate SC

$$n = N_c \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}\left(\frac{\varepsilon_F - \varepsilon_c}{k_B T}\right)$$

$$p = N_v \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}\left(\frac{\varepsilon_v - \varepsilon_F}{k_B T}\right)$$

Where is the fermi energy (ε_F)?

I hope that gives you nice good picture of what it is for non-degenerate case. For non-degenerate semiconductor, which I will… Semiconductor – I will abbreviate like this versus degenerate semiconductor. Exactly opposite of that; that means Fermi energy does not lie in shaded portion. In that case, we call it degenerate semiconductor. So, the formulas are n is equal to N c times e to power E f minus E c divided by k B T; and, p as N v e to power E v minus E f by k B T. That is our formulas for this; otherwise, here n is equal to N c times 2 by root pi Fermi-Dirac integral of order half – and, maybe if you wish, I can write this as a function of now, E f minus E c by k B T, because function of that eta c. So, I substituted eta c in there. And, p as equal to N v 2 by root pi Fermi-Dirac integral of order half for E v minus E F by k B T. So, these are the formulas. We will be clearly… For most of our cases, we assume that, Fermi energy (( )) semiconductor is non degenerate and we will be using these formulas. Then, next question – where is the Fermi energy? Where is this quantity? Where is Fermi energy? Unless I know Fermi energy, remember…

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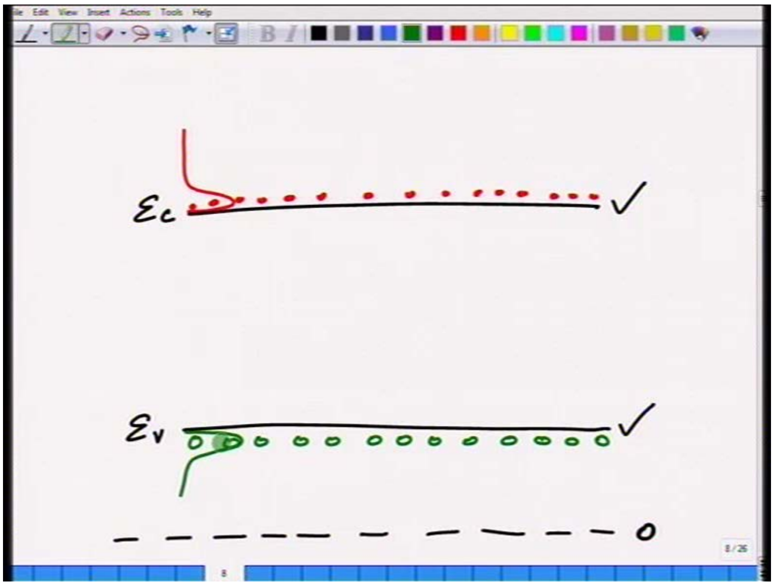

So, let us just do one more thing here. Now, E v and E c for a semiconductor are known. They are very well-known. You may argue that, how can you be so precise about E v and E c? Of course, they are always determined in reference to some common energy. So, some energy; that means, if you choose your reference to be of energy to up to to be 0 here; then in reference to this value, then we know where E v is, we know where E c is. Those are the two numbers we know, because of its material property. And of course, notice this reference of energy is not important, because always you have difference of two energies involved in here – E v minus E F. So, if I determine E f, I have to determine or I have to state E f – what E f is; then, I have to state that E f with respect to the same reference. As long as reference is same and since we are dealing with difference of energies here, therefore, where you choose the reference of energy does not matter.

To that extent, depending on where the reference you have chosen, with respect to that reference, we always know what E c and E v are since these are material properties. Since we know these, therefore, only question in order for us to determine what is the value of n in conduction band, how many electrons that are present in this conduction band and how many holes are present in this valance band; how many means… Whenever I say how many, I always imply per unit volume. So, everything is about densities. So, it is per unit volume; how many holes in valence band per unit volume; how many electrons in conduction band. If you want to know, we need to state where the

Fermi energy is. So, let us do that. Let us try to first determine where the Fermi energy is. So, let me define some…

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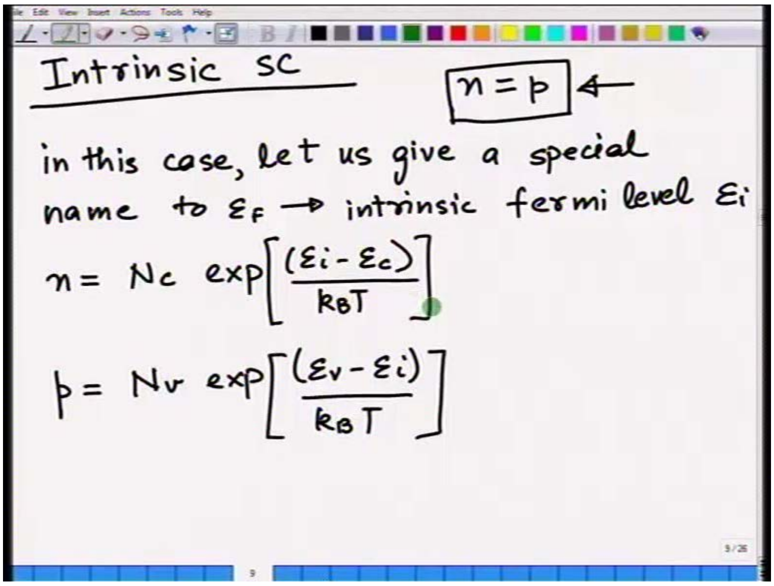

Intrinsic SC

$\boxed{n = p} \leftarrow$

in this case, let us give a special name to $\varepsilon_F \rightarrow$ intrinsic fermi level ε_i

$$n = N_c \exp\left[\frac{(\varepsilon_i - \varepsilon_c)}{k_B T}\right]$$

$$p = N_v \exp\left[\frac{(\varepsilon_v - \varepsilon_i)}{k_B T}\right]$$

We will first talk of intrinsic semiconductor. So, we will first talk of intrinsic semiconductor. What is intrinsic semiconductor? This is a terminology you have heard right from your school days; that semiconductor being intrinsic and extrinsic. Remember when it is doped; which we will cover immediately after this. So, what is intrinsic semiconductor? Often it is taught as if it is… I said a semiconductor, which is chemically pure is an intrinsic semiconductor. To some extent, such a definition may be OK; it may be OK. However, it is not always correct. What I mean by that is there are examples such as let us say there are the defect semiconductors, let us say, zinc oxide or for example, you take gallium arsenide; they may be chemically pure, but still they have some defects; they have defects in there. The point defects for example; as an example; and, these defects could be charged. And, these charged defects – they could behave in a way as if the semiconductor was doped. And hence, it is not always true to say that, a chemically pure semiconductor is intrinsic semiconductor.

More precise definition of an intrinsic semiconductor therefore is that, n is equal to p. n is equal to p. This is basically the definition; that number of electrons in the conduction band is the same as number of holes in the valence band. Notice where I am drawing by the way. Remember… Recall that we have something like this. The density of state goes something like this; and, density of holes goes something like this. You will recall that, this is what you had plotted earlier. And, since this maxima occurred very close to the band edge; therefore, we always show all these holes lying right near the band edge – right below the band edge. And similarly, for electrons, we show all these electrons located just above the conduction band. And, that is the point I had already made to you yesterday. So, given this condition… And, we say that, if n is equal to p, what we mean is area under this red and area under this green curve should be equal. So, where is that Fermi energy? You will recall such a situation occurred when Fermi energy was somewhere in the middle. I had shown in the first picture when I showed the Fermi energy was smack in the middle… or, not smack in the middle, somewhere in the middle. Then, I had shown that, these two curves are approximately equal; approximately I say.

Now, I will be little bit more precise. So, let us find out. Let us say under this condition; in this case, let us give a special name to E F. We will call it intrinsic level – intrinsic Fermi level. And, we give a name symbol E i. So, what the question? Our question was what is E F? So, we are now trying to determine what E F is. For intrinsic semiconductor we state the definition is n is equal to p. We will use this to determine where E F is. And, under this condition, for intrinsic semiconductor, we are giving a special name to the semiconductor and we are calling it E i. In that case of course, I will write n as equal to… I will use those formulas right up from here, from here; write these formulas; we will use these formulas right here. n is being equal to N c times e to power E i minus E c by k B T. And, we will use p as equal to N v e to power E v minus E i by k B T. In writing these equations, I have assumed this semiconductor is non-degenerate. Eventually, I will prove it to you that, indeed this will be true, my assumption is not wrong.

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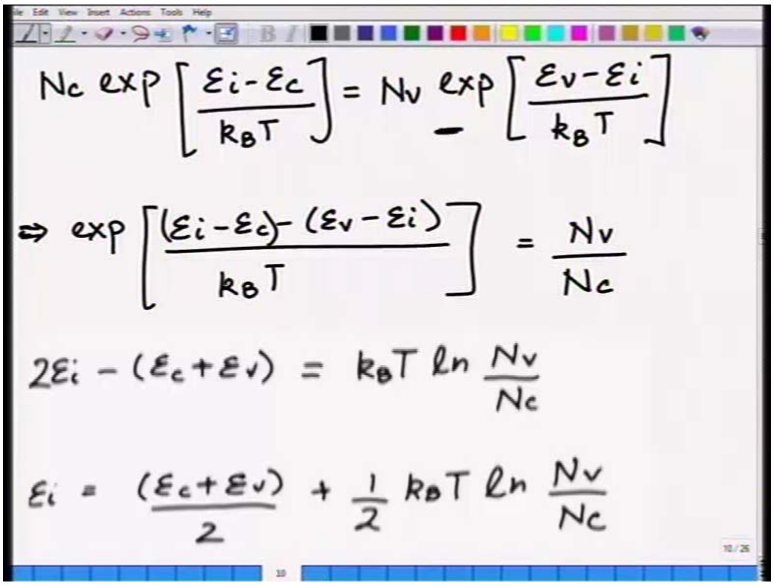

$$N_c \exp\left[\frac{\varepsilon_i-\varepsilon_c}{k_BT}\right] = N_v \exp\left[\frac{\varepsilon_v-\varepsilon_i}{k_BT}\right]$$

$$\Rightarrow \exp\left[\frac{(\varepsilon_i-\varepsilon_c)-(\varepsilon_v-\varepsilon_i)}{k_BT}\right] = \frac{N_v}{N_c}$$

$$2\varepsilon_i - (\varepsilon_c+\varepsilon_v) = k_BT \ln\frac{N_v}{N_c}$$

$$\varepsilon_i = \frac{(\varepsilon_c+\varepsilon_v)}{2} + \frac{1}{2}k_BT \ln\frac{N_v}{N_c}$$

So, now… If so, then let us try to determine E i by equating an n and p; that means if N c e to power E i minus E c by k B T should be equal to N v e to power E v minus E i by k B T; which implies… So, we do it like this. We take this on the left-hand side – exponential on the left-hand side. So, I will write this as e to power E i minus E c minus E v minus E i – this whole thing divided by k B T. I am taking this on left side and writing it N v by N c. And, now, let us take log of it on both sides. So, what do we have? Of course, this is simplified here. So, we can a get E i. So, I am going to get 2 times of E i minus E c plus E v is equal to k B T log of N v minus N c; which means E i is equal to E c plus E v by 2 plus half k B T log N v by N c. So, what does that tell you? If… Let us repeat this one more time where E i is.

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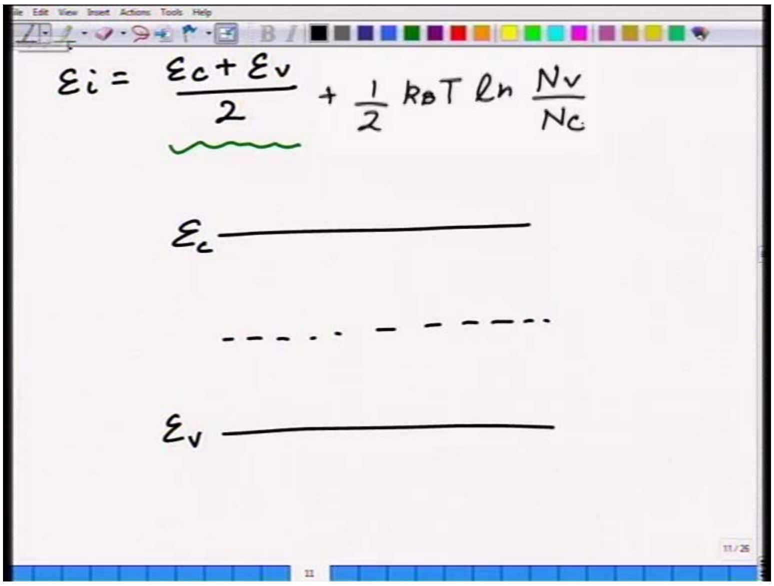

$$\mathcal{E}_i = \frac{\mathcal{E}_c + \mathcal{E}_v}{2} + \frac{1}{2} k_B T \ln \frac{N_v}{N_c}$$

Therefore, we are going to write E i as same thing; I am just repeating it; E c plus E v by 2 plus half k B T log N v by N c. If this is E v, where does the Fermi energy lie for an intrinsic semiconductor – that chemically pure semiconductor? Or, in principle, what should be read as… What we should be calling as n equal to p; where, n is equal to p. Where is the Fermi energy in that case? In that case, we find that, Fermi energy is right in the middle. Where is this point? This point is right here. From where in the… Exactly in the middle; exactly in the middle is E c plus E v divided by 2; exactly in the middle.

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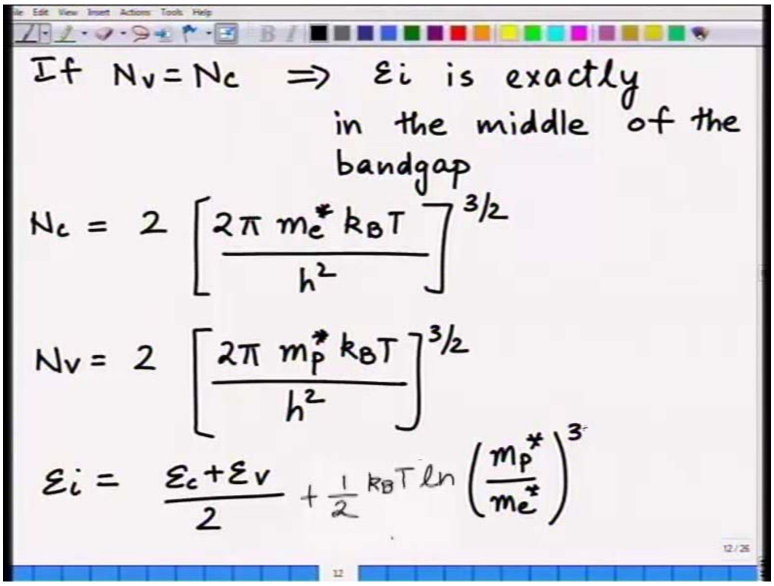

If $N_v = N_c \Rightarrow \mathcal{E}_i$ is exactly in the middle of the bandgap

$$N_c = 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2}$$

$$N_v = 2\left[\frac{2\pi m_p^* k_B T}{h^2}\right]^{3/2}$$

$$\mathcal{E}_i = \frac{\mathcal{E}_c + \mathcal{E}_v}{2} + \frac{1}{2} k_B T \ln \left(\frac{m_p^*}{m_e^*}\right)^3$$

Now, Fermi energy is not exactly in the middle, it is exactly in the middle in case if N v is equal to N c; in that case, E i is in exactly middle of the band gap – right in the middle; precisely, exactly in the middle is the Fermi energy. This is E i in that case; the dotted line is exactly the… In that case, this E i level, which I am showing you here is exactly in the middle of the band gap. But, recall what N c and N v were. So, N c you will recall was equal to… N c was equal to 2 times 2 pi m e star k B T by h square, not h bar square; keep the remind. And, N v was equal to 2 times 2 pi m p star – effective mass of hole – k B T by h square to power 3 by 2; that means if effective mass of electron was equal to effective mass of hole, then N c and N v will be equal and this would be precisely in the middle; which means I can write this E i one more time; I can write this E i to be equal to E c plus E v by 2 plus half k B T log of what? N v; that means m p star – hole effective mass by electron effective mass to power 3 by 2.

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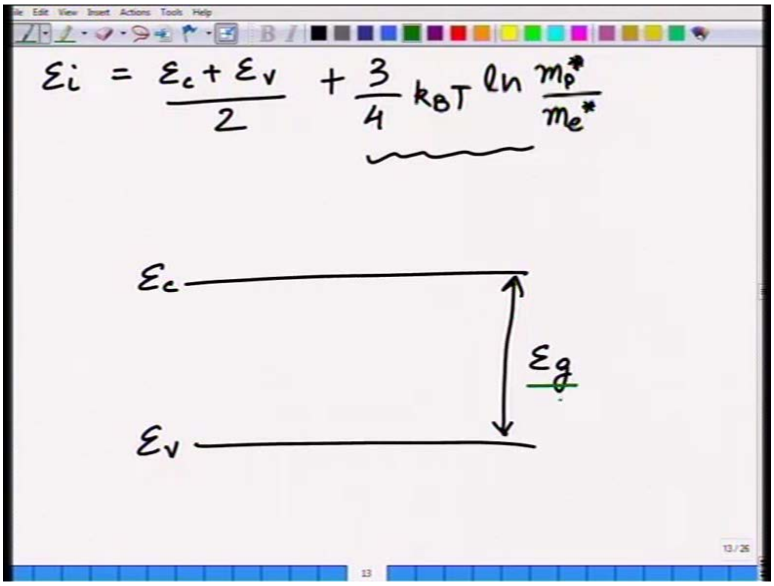

$$\mathcal{E}_i = \frac{\mathcal{E}_c + \mathcal{E}_v}{2} + \underbrace{\frac{3}{4} k_B T}\ln\frac{m_p^*}{m_e^*}$$

Or, I can write this as E c plus E v by 2 plus 3 by 4 k B T log effective mass of hole divided by effective mass of electron. So, I can write it in this form, where E i is… So, E i is exactly in the middle if effective mass of electron and holes are equal; if not so, in that case, if they are not equal; in that case, it is shifted from center by amount this. Indeed this is a small amount, because for most cases, you have seen I have given you the effective mass in silicon or germanium, except in gallium arsenide, where this may be slightly more. But, in silicon or germanium, indeed effective mass or of effective electron holes are approximately equal; some is 1; one is… other one is 0.5. So, even this… Then, this number is a small number. So, the shift from the center is only a small shift. This indicates the shift from the center; whatever the center is, shift from that. So, that is what anyway… So, that is what an intrinsic semiconductor is. So, this is intrinsic semiconductor.

That is where… That is what by definition, intrinsic semiconductor means n is equal to p. In that case, Fermi energy lies in somewhere in the middle slightly shifted according to the effective mass of electrons and holes. But, nonetheless, you can see you justified to use the formula for non-degenerated semiconductor, because these band gaps are an order of what we are talking about, except if we are talking about something, which is a infrared detector, where the band gap has to be very very small. Or, other than that, we are talking about these band gaps to be an order of 1 electron volt or plus; more than that; in case of silicon, it is 1 electron volt; in case of germanium, it is 0.66 electron volts. So, in 3 k B T is 29 milli… k B T at room temperature is 29 milli electron volts. So, 3 k B T; let us say 100… Let us say about 100 milli electron volt per or 0.1 electron volts. So, as long as we are about 0.1 electron volts away from the band edges, we are to use non-degenerate semiconductor statistics at room temperature. And clearly, since E i lies in the middle and band gaps are above an order of 1 electron volt; hence, we were justified in using those formulas for non-degenerate semiconductors. This is all about intrinsic semiconductors.

Now… If so, now… So, now, let us do this. So, what is n – we have still not determined. We have determined n equal to p. Now, once we know E i, we should be able to determine what n is equal to. So, what is n equal to? Let us find out. In fact, that let us do in a minute afterwards. In fact, let us do this. Let us make general comments for it.

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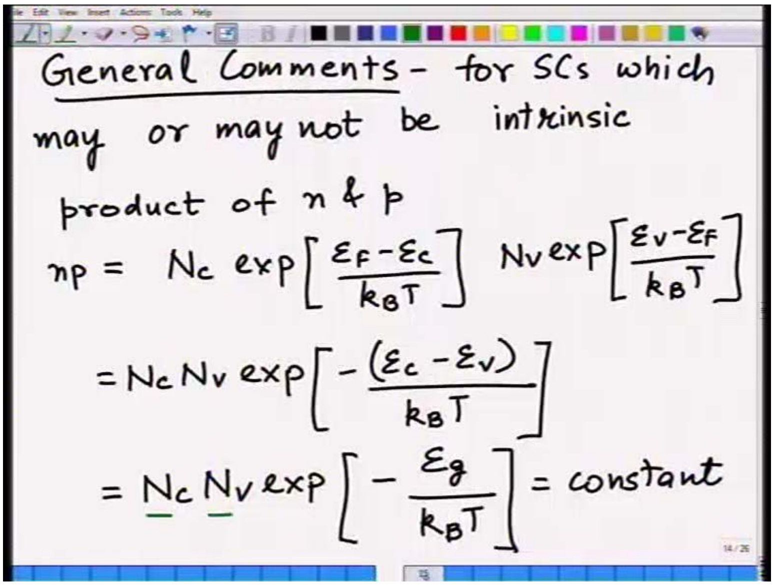

General Comments – for SCs which may or may not be intrinsic

product of n & p

$$np = N_c \exp\left[\frac{\mathcal{E}_F - \mathcal{E}_c}{k_B T}\right] N_v \exp\left[\frac{\mathcal{E}_v - \mathcal{E}_F}{k_B T}\right]$$

$$= N_c N_v \exp\left[-\frac{(\mathcal{E}_c - \mathcal{E}_v)}{k_B T}\right]$$

$$= \underline{N_c}\,\underline{N_v} \exp\left[-\frac{\mathcal{E}_g}{k_B T}\right] = \text{constant}$$

We will just determine n i; just hold on. First, general comments for semiconductors, which may or may not be intrinsic; they may be intrinsic, they may not be intrinsic. First comment I would like to make is this product of n and p. Remember I am not assuming that it is necessarily intrinsic now. So, it may be intrinsic or may not be intrinsic. So, n… – product n and p is what I wanted to know. So, n p. n p is equal to N c e to power E F minus E C by k B T multiplied by N v e to power E v minus E F by k B T. I am assuming non-degenerate semiconductor. I am continuing to assume that. We will always assume this in this class from now on, unless specified otherwise. Thus, let us look at this – N c N v e to power – now, E F will cancel out and what I am going to get is – minus E c minus E v by k B T. So, what do we have? We have this as N c N v e to power minus E g by k B T. Notice this; this is a constant.

Notice that, this is a constant. Why? N c is a material property; N v is a material property. Let us take a look. N c was defined right here. What is there? This temperature, Boltzmann constant, this Planck's constant – all these are constants. Once you say a material silicon, then effective mass of electron gets fixed. Similarly, effective mass of hole gets fixed; that means N c and N v get immediately specified the moment you say you name the material. The moment you say silicon or gallium arsenide, these N c values of N c and N v are specified; and, we have calculated those. So, that means… Hence, also, the moment you specify the material, band gap becomes known; the band gap is also… It is independent of E F remember. This product has become independent of E F.

E F was the only quantity we did not know. The moment you specify the material, E g is known, N c is known, N v is known. And therefore, it is a fixed number. Whatever the Fermi energy, wherever there is a Fermi energy, product of n p is a constant; product of n p is a constant. That is a very important result and this is a general comment; that means semiconductor may be intrinsic or semiconductor may not be intrinsic; does not matter for all semiconductors, as long as they are non-degenerate semiconductors. This must be true that, this product n p must be constant; which means as long as I know this constant, I need to determine only one of them; I can determine n. Then, immediately, since product n p is constant, therefore, I can immediately determine what this other p is. If you know p, then we can immediately determine what n is.

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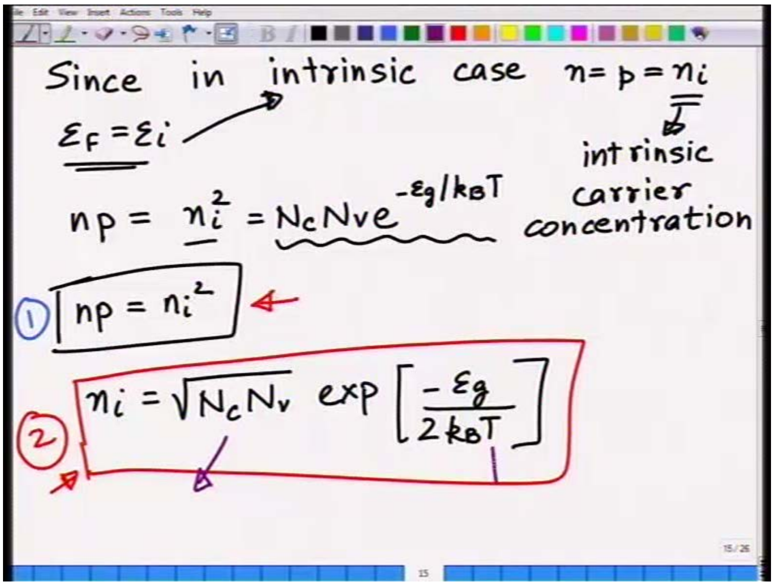

Since in intrinsic case $n = p = n_i$

$\mathcal{E}_F = \mathcal{E}_i$

intrinsic carrier concentration

$np = n_i^2 = N_c N_v e^{-\mathcal{E}_g/k_BT}$

① $\boxed{np = n_i^2}$

② $\boxed{n_i = \sqrt{N_c N_v}\, \exp\left[\frac{-\mathcal{E}_g}{2k_BT}\right]}$

Since in intrinsic case, n is equal to p, is equal to n i; we are giving it a name now. Just like we had given a… For E F, we had called it E I; we had given it a name as E i. Similarly, we are calling as intrinsic carrier concentrations. Number of electrons or number of holes per unit volume – that is for a semiconductor, which is intrinsic. If it is an intrinsic semiconductor, then in that case, n is equal to p is equal to n i; that means product n p is equal to n i square; that means since n is equal to p and they both are values are called n i, that must be true. Therefore, N c N v e to power minus E g by k B T. What does that mean? That means… Since this… Remember this is a material property. The moment we say silicon, this number is fixed. And, since this is fixed; that means this quantity is also fixed quantity; that means say fixed quantity once we specify the material. If it is a silicon, n i is fixed immediately. And, this is always true. In intrinsic case, n p is equal to n i square. And, since a product n p is always constant, that means, in all semiconductors, whether semiconductor is intrinsic or not intrinsic, product n p should always be n i square. This product n p is n i square whether or not semiconductor is intrinsic or not. In case of intrinsic, it is indeed n i square; that is obvious. But, since product n p is constant, it is the same whether it is intrinsic or not. Therefore… Or, rather since it is constant, and in special case, intrinsic semiconductor is n i square; that means in all cases, it should be equal n i square.

Now, with these two results, therefore, I have; one is that now, n i… One result I have written here itself. This is the result number 1; that product n i – general result is that n p – product n p is equal to n i square. And, I can now determine what is intrinsic electron concentration is also. I can now write this as square root of N c N v e to power minus E g by 2 k B T. So, I can determine as I determine n i. So, these are the two results, which I have; I can determine n i and I can… I have already written this as a general comment and I can written determine n i. You recall from here; there is T to power 3 by 2 dependence on this. Therefore, this t by (( )) N v and N c. So, when I am writing this as follows, so, this would have a dependence of… This would have a dependence of T to power 3 by 2 dependence, will show up here also; and, the temperature dependence will show up here as well.

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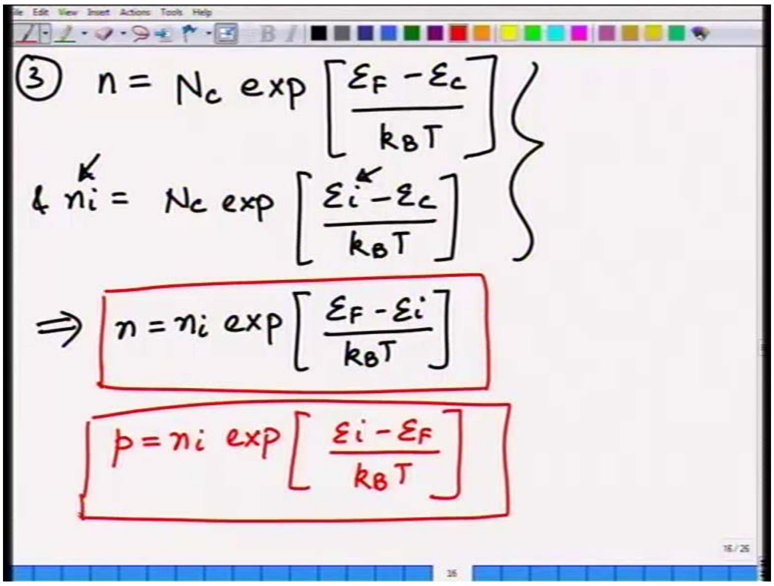

③ $n = N_c \exp\left[\frac{\mathcal{E}_F - \mathcal{E}_c}{k_B T}\right]$

& $n_i = N_c \exp\left[\frac{\mathcal{E}_i - \mathcal{E}_c}{k_B T}\right]$

$\Rightarrow \boxed{n = n_i \exp\left[\frac{\mathcal{E}_F - \mathcal{E}_i}{k_B T}\right]}$

$\boxed{p = n_i \exp\left[\frac{\mathcal{E}_i - \mathcal{E}_F}{k_B T}\right]}$

Now, one comment… Third comment… Third statement I want to make is, in general, since I now know n i, I can use this… n i is a material property (( )) specify silicon I know what n i value is. I can use this as a reference to calculate n in all other cases. Since n is equal to N c e to power E F minus E C by k B T. And, n i is equal to N c e to power E i minus E c by k B T. Remember when Fermi energy is E i, then the electron concentration is n – n i. These two equations imply that, n… Use these two equations; and, you can write therefore, n i as e to power E – I can write this as… Use this; so, I can write n equal to n i times E F minus E i by k B T in general. And similarly… So, this is what I will… So, n can be determined as follows. In general, n can be reference to n… Instead of N c, I can write it in reference n i and I can write this as n i times E F minus E i by k T. Similarly, I can write p as equal to n i times e to power E i minus E F by k B T. You can derive this one also just by same method. So, n and p can be written in general. In general, this is not for (( )) intrinsic semiconductor. When E F is equal to E i, then of course, n will come out as n i; otherwise, given what E F is, then in reference to n i, you can determine what n is. So, these are some other formulas, which we could remember now. So, these few things which I am going to point out; these are general comments. They apply to all semiconductors whether they are intrinsic or not.

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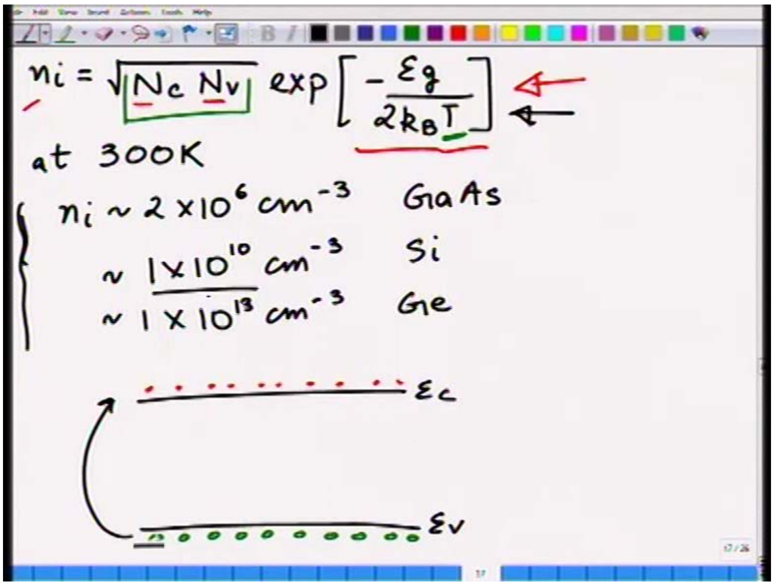

$$n_i = \sqrt{N_c N_v}\, \exp\left[-\frac{\mathcal{E}_g}{2k_B T}\right]$$

at 300K

$n_i \sim 2\times10^{6}\ \mathrm{cm}^{-3}$ GaAs

$\sim 1\times10^{10}\ \mathrm{cm}^{-3}$ Si

$\sim 1\times10^{13}\ \mathrm{cm}^{-3}$ Ge

Now, let us calculate n i. What is the value of n i? n i value is N c times N v e to power minus E g by 2 k B T. So, you can… You know the band gaps; you have been given the band gaps for different materials; you know what N c and N v is. This is also given to you. So, at room temperature, at 300 k, let me give you some numbers – n i is equal to about 2 into 10 to power 6 per centimeter cube number of holes and number of electrons; number of electrons in conduction band and valence band for gallium arsenide, which has a band gap of about 1.5. And, this is about 1 into 10 to power 10 for silicon; and, this is about 1 into 10 to power 13 per centimeter cube for germanium. What do you see. So, these are the values for n i; that means in the semiconductors, these many electrons per unit volume of course will be present in the valance band and these many holes will be present in electrons in the conduction band; and, these many holes in the valence band would be present. Notice that, as band gap decreases, this number increases; germanium is a band gap of 0.66 – about 1, about 1.5. So, band gap is increasing and this number is decreasing; which is fairly obvious as you can see here. (( )) E g minus E g in there; higher this number, smaller this n i I would get.

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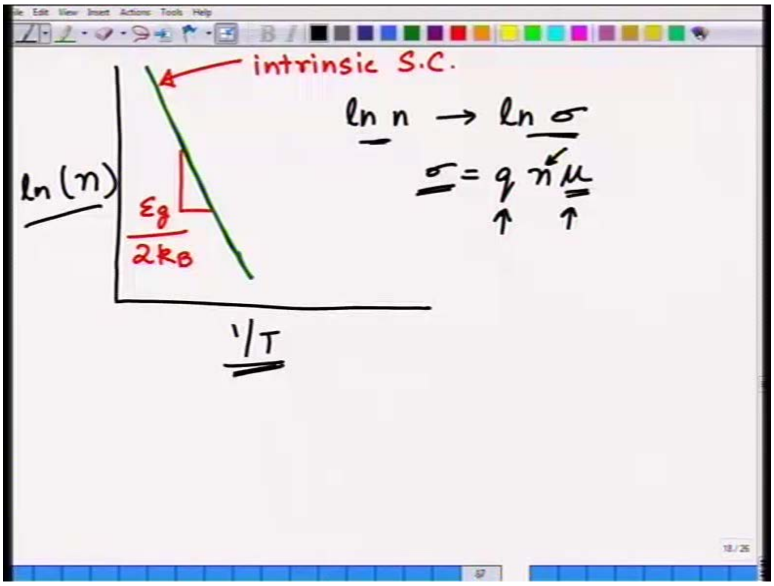

So, if I plot this like this as a function of temperature actually; if I plot this as 1 by T and I plot n i; or, if you wish log n i; then, what I will get is something like this – a curve, which looks something like this. This is the plot of n i. I leave it at not n i, but I will leave it at log of n – the carrier concentration n – number of electrons in conduction band. Then, in that case, this is for intrinsic semiconductor; then, this equation will be valid. This equation will be valid for intrinsic semiconductor and I am going to get a straight line if I take log of n i. If I take log of n i – natural log, then it will have a slope of E g by 2 k B. The slope will be E g by 2 k B. So, this could be a method of determining a band gap; that means if you measure… For example, if you measure this carrier concentration, of course you can notice that, why is it straight line. This is straight line is because you see there is temperature dependence here; there is a temperature dependence right here also. But, this is much much weaker temperature dependence in N c and N v; has a much weaker. And, that is what I will… That is why I was pointing out to you this temperature dependence. Since this temperature dependence is much much weaker, then the exponential temperature dependence. Therefore, this curve looks more or less a straight line.

If instead of n, if you were to plot; instead of log n, you will plot log of conductivity. And, remember conductivity would be – you go back to our idea of conductivity. We had said the conductivity would be equal to number of electrons charge and mobility – mobility of the carriers. So, if instead of log of n, if you plotted log of sigma; then q obviously does not have temperature dependence; temperature dependence of n is already stated here – right here. Then, if mu has certain (( )) temperature dependence also, interns out that temperature dependence of mu is – as temperature increases, this quantity decreases – mu decreases, which I will talk about eventually.

And, if temperature increases, N c and N v increases; this quantity mu decreases. In fact, sensitivity to conductivity therefore, has a function of 1 by T becomes even less. And, the straight line becomes even more straighter. Therefore, if you were to plot conductivity; and, conductivity is relatively easy to measure. So, if instead of log n, if I plotted log of conductivity, then notice this could be a method and you measure it as a function of temperature; then, this could be a method to determine what the band gap is. E g is because the slope will be divided by 2 k B will give you the band gap. So, this is what I wanted to tell you all about intrinsic semiconductor and also the general comments. So, remembering this, we will start using now… From the next lecture, we will start with extrinsic semiconductor; that means semiconductors which are doped. But, before I do that, maybe I make one more comment before I close this.

Now, let us look at this n i value actually.

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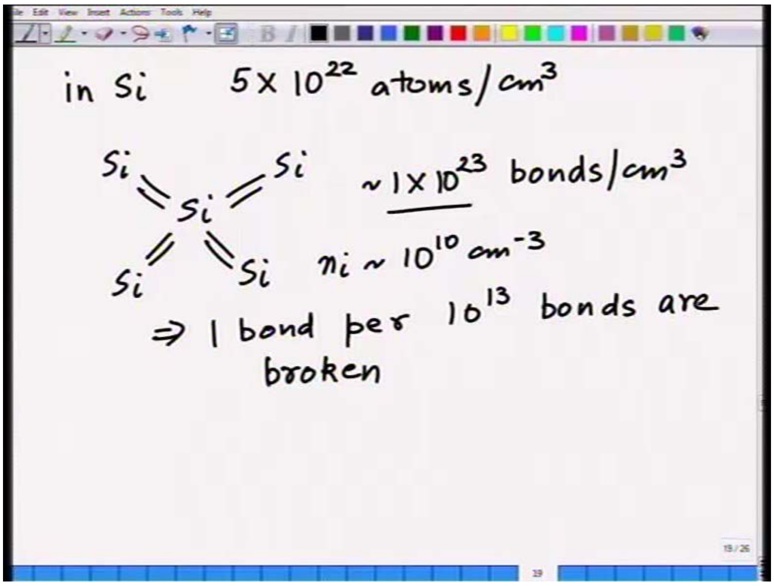

in Si 5×10^{22} atoms/cm^3

$\sim 1 \times 10^{23}$ bonds/cm^3

$n_i \sim 10^{10}$ cm^{-3}

$\Rightarrow$ 1 bond per 10^{13} bonds are broken

Let us look at these numbers also; these numbers, which are n i, which you determined. Remember how many bonds are there in (( )) silicon; in silicon or for that matter in other cases also, we have about 5 into 10 to power 22 atoms per centimeter cube. Each silicon has four bonds. Each silicon has four bonds with other silicon. Remember that crystal structure for silicon was shown, where tetrahedral bonding. So, multiply that by let us say 4. So, that means about 1 into 10 to power 23 bonds per centimeter cube – that number bonds. So, if I have these many bonds; and, in intrinsic conditions at room temperature, if I have 10 to power 10 in here as intrinsic carrier concentration; and, if I think that, each bond breaking leads to one electron going up here and creating a hole here. If that was the process is that because the bond breaks, an electron goes to the conduction band leaving behind a hole; then, I am having about 10 to power 10 bonds in silicon at room temperature, which are broken; which means we can… which can say we can expect so, about…

Since n i is about 10 to power 10 per centimeter cube, that implies 1 in bond per 10 to power 13 are broken. So, that is the order as to how many bonds are broken in silicon at room temperature. Of course, this number could not have been very large. If it were very large and all the huge majority of the bonds are broken, then obviously, this would not remain a semiconductor at all; it would not remain a material, not semiconductor; I should say it would not remain a material with all the bonds broken. So, with this, let me close this topic today of intrinsic semiconductors and the general comments which apply to all semiconductors intrinsic or not. And, from next lecture, we will start with extrinsic semiconductors namely, the doping of semiconductors.

Thank you.