

Optoelectronic Materials and Devices
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Module - 01
Lecture - 16
Equilibrium Carrier Statistics in Semiconductors: Complete Ionization of Do pant Levels

Welcome to lecture number 16, we will continue from where we left, so let us see where we left.

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$$\frac{N_{D^+}}{N_{D^0}} = \exp\left[\frac{E_D - E_F}{k_B T}\right] \quad N_D = N_{D^0} + N_{D^+}$$

$$\frac{N_{D^+}}{N_D - N_{D^+}} = \exp\left[\frac{E_D - E_F}{k_B T}\right]$$

$$\frac{N_D - N_{D^+}}{N_{D^+}} = \exp\left[\frac{E_F - E_D}{k_B T}\right]$$

$$\frac{N_D}{N_{D^+}} = 1 + \left(\right) \Rightarrow \frac{N_{D^+}}{N_D} = \frac{1}{1 + \exp\left[\frac{E_F - E_D}{k_B T}\right]}$$

So, this is where we were in the last lecture, what we have done was that we have derived this expression for ionization of dopant. So, if you have a dopant and I am considering now a monovalent dopant, namely that which has only one ionization; that means, it can go from 0 charge state to plus 1 charge state, a donor rather I should do pant I should say donor here, the d stands for Donor.

So, we have a donor which is monovalent meaning, that it can go from charge state to from 0 to plus 1 like phosphorous in silicon, where with it pie electron phosphorous is 0 when it gives away one electron, then it becomes phosphorous plus; that means, d plus. So, if, so then if I add, so much quantity of 10 to power 16 per centimeter cube, of phosphorous and silicon. Then question we asked is out of these 10 to power 16 atoms

that we have added of phosphorous in silicon, how many of them are in how many of what fraction of this 10 to power 16 goes into plus 1 charge state.

Of course, the remaining is in then there for 0 charge state. So, that how much what fraction goes into plus 1 charge state, is given by this expression in red and similarly, if were we have a acceptor, monovalent acceptor meaning there by something which is a 0 charge state normally, when it take and then it has it can also go in minus 1 charge state by accepting a electron.

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The image shows a whiteboard with the following equations:

$$\frac{N_{A^-}}{N_{A^0}} = \exp\left[\frac{\epsilon_F - \epsilon_A}{k_B T}\right] \quad \underline{N_A = N_{A^-} + N_{A^0}}$$

$$\frac{N_{A^-}}{N_A} = \frac{1}{1 + \exp\left[\frac{\epsilon_A - \epsilon_F}{k_B T}\right]}$$

The first equation is written in red. The second equation is written in green. There are arrows pointing to the terms in the second equation.

So, we have acceptor like boron for example, if I have added 10 to power 16 boron meaning this number here N_A , if I have added 10 to power 16 number of boron atoms then the question we ask is, how many of them would go into minus 1 charge state. And that for all what that fraction will be that fraction, will is given by this expression in which is written out in green alright. If, so then remember now, all these quantities are depending on where the Fermi energy is remember ϵ_A level is known, ϵ_D level is known.

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Handwritten derivation on a whiteboard showing the relationship between donor concentration and ionization fraction:

$$\frac{N_{D^+}}{N_D} = \exp\left[\frac{\epsilon_D - \epsilon_F}{k_B T}\right] \quad N_D = N_{D^0} + N_{D^+}$$

$$\frac{N_{D^+}}{N_D - N_{D^+}} = \exp\left[\frac{\epsilon_D - \epsilon_F}{k_B T}\right]$$

$$\frac{N_D - N_{D^+}}{N_{D^+}} = \exp\left[\frac{\epsilon_F - \epsilon_D}{k_B T}\right]$$

$$\frac{N_D}{N_{D^+}} = 1 + \left(\right) \Rightarrow \frac{N_{D^+}}{N_D} = \frac{1}{1 + \exp\left[\frac{\epsilon_F - \epsilon_D}{k_B T}\right]}$$

So, only thing n N_A is known this is what you have added, we have added and N_D is known this is what we have added. So, we can determine this N_D plus and N_A minus, if we know where the Fermi energy is, where this Fermi energy is that is a question we will answer little bit later. But, this is expression which tells us how much of it will ionize, at this point of time may be let us move on now, to our to our lecture now.

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Handwritten notes for Lecture 16, including energy level diagrams and equations:

Lecture 16

$$\frac{N_{D^+}}{N_D} = \frac{1}{1 + \exp\left[\frac{\epsilon_F - \epsilon_D}{k_B T}\right]}$$

$$\frac{N_{A^-}}{N_A} = \frac{1}{1 + \exp\left[\frac{\epsilon_A - \epsilon_F}{k_B T}\right]}$$

ref $\epsilon_V = 0 \leftarrow$

$\epsilon_A = 10 \text{ meV}$

$\epsilon_D = \epsilon_g - 30 \text{ meV}$

The diagram shows energy levels ϵ_C , ϵ_D , ϵ_A , and ϵ_V with a band gap ϵ_g . A 30 meV gap is shown between ϵ_C and ϵ_D , and a 10 meV gap is shown between ϵ_A and ϵ_V .

first question let me just try to write this down again one more time for you this n N_D plus over N_D we are deriving as, 1 by 1 plus e to power e F minus e D by k t k B T . And

similarly we have deriving n_A^- / n_A has been equal to $1 / (1 + e^{-E_A / (k_B T)})$. One thing which I did not mention in last lecture and something, which may confuse you sometime about what these energies are. So, first let me point that out to you, that if you have this as a E_c level this as a E_A level somewhere is the 0.

Let us say here is the 0 energy. So, this reference to this 0 energy is this E_A and E_c level, so sorry E_A E_c and E_v level valance band energy level. And now, if I add the do pants, then as I have already mentioned that do pant, donor is the this is the E_D level and this is then there for E_A level, as I have written there is no confusion, everything is measured with respect to this reference energy.

So, E_A is this level which we are talking about; that means, it is measured from this origin here, from this reference here for a reference here. However, often if you look at a text book or if you look at a hand book, for this data, if you look at phosphorous and ask for where is the do pant level, then often it is said at 30 mille electron volts. For example, what they are then coating is that this energy difference, in that case the energy difference being coated is not really the E_D level, with respect to some 0 because, at 0 is not specified.

If if a book gives you or a text book or a hand book gives you a number, if 30 mille electron volt for E_D for example, then; obviously, it is assuming some reference. And you must know what what reference energy it is using, when they say some number like 30 mille electron for a do do pant then it is always with respect to the relevant band edge. In case of donor since since, it exchanges it is electron in conduction band, then this reference energy really is, with respect to the conduction band energy.

That means, 30 mille electron volts is this energy, then in that case this energy which I have just marked out here. If it and it similarly for boron, in silicon if a energy is given as 10 or 20 or 30 mille electron volts, as E_A level acceptor energy level, then what is being coated is, this energy difference with it is with respect to the band edge. So, there for when you when you work on this, then you a need to keep track of it then when you substitute this E_D and E_A in here, you cans substitute this as 30 mille electron volts.

You have to substitute it, with respect to a common reference common reference which is somewhere somewhere we do not know where that is is, choice it does not matter

because, you always dealing with difference of energies. So, whatever is the reference that will cancel out, the only point is whatever is the reference, you choose for E_D here same reference you must choose for E_A also and then whatever the Fermi energy you will deal with, must have also the same energy level, as long as you doing that then you are.

So, for example, if this was 10 mille electron volts, mille electron volts and this was let us say 30 mille electron volts then what then I will first choose a reference. Suppose, I choose a reference as E_V equal to 0, suppose I choose my reference energy to be right here itself, E_V to be 0 in that case I will choose E_A then I will say is equal to 10 mille electron volts, is what I will choose and somebody ask me what is E_D , then in that case E_D will be, if this is whole thing E_g , if this whole thing is band gap E_g .

For example, in silicon 1.12, then I will write it as, I will write E_D as E_g minus 30 mille electron volts, of course E_g also has to be taken in appropriate units or 30 has to be converted into electron volts also, you have to keep sure make sure the units are same, but you get the idea. That E_D in that case is not 30 mille electron volts, but band gap minus 30 mille electron volts.

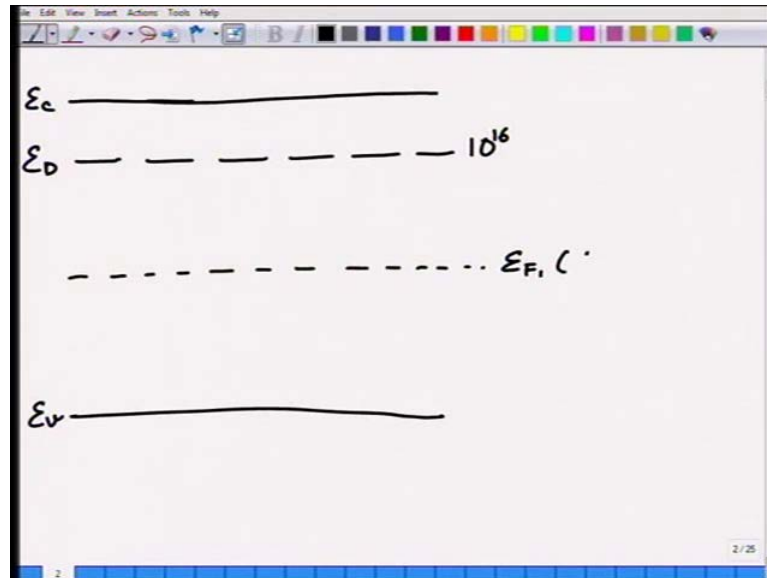
And when you then, use Fermi energy or you calculate Fermi energy, then that of course will also be calculated with respect to, whatever comes out, number comes out that always has to then in that case measured from, whatever you reference was which is E_V equal to 0. So, these are point we need to keep keep in mind because, you will find often the numbers coated, in hand books which are with respect to the band edges respective band edges and you must, if you are going to use them for calculations, you must correct the you must make sure that you are using at least a common a common reference.

It does not matter what reference you choose choose, you are welcome to choose anything as your energy reference, as long as it remains the same reference for in throughout the calculations for every quantity, energy quantity all the energy quantities. So, thus one point we need to keep in mind now, second thing is let us look at using these expressions, let us look at ionization and a qualitative idea of where the Fermi energy would be.

So, let us begin with this begin with this and and we last lecture we are finished at at the point where we talking about, complete ionization and what does that mean, let us try to

understand that for a minute. So, one thing I would like to point, out let us construct a picture again.

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That here is my e c conduction bandage, here is my valance bandage and let us add some donors to it for example, let us just or add only donors no acceptors. So, if I add donors to it, like this here is the donor I have added added in it that is a 10 to power 16 of these or whatever that number is schematically speaking I have added here 1, 2, 3, 4, 5, 6, of them here alright. So, if, so then question I ask is can you determine qualitatively, let us look at how the ionization will proceed. Suppose, the Fermi energy was here, let us call it $\epsilon_{F,1}$, suppose Fermi energy was here, this is suppose...

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Lecture 16

$$\frac{N_{D^+}}{N_D} = \frac{1}{1 + \exp\left[\frac{E_F - E_D}{k_B T}\right]}$$

$$\frac{N_{A^-}}{N_A} = \frac{1}{1 + \exp\left[\frac{E_A - E_F}{k_B T}\right]}$$

ref $E_V = 0 \leftarrow$
 $E_A = 10 \text{ meV}$
 $E_D = E_g - 30 \text{ meV}$

Then let us look at this expression what happens. Once ask, what fraction of donors are the phosphorous atoms in silicon that I have added, how many of them are ionized what fraction is ionized. Now, notice $e F$ in this case $e F$ minus $e D$ is what appears.

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E_c —————

E_D $\oplus \uparrow \oplus \oplus \oplus \oplus \oplus \oplus \cdot 10^6$ $(E_{F2}) \quad \frac{N_{D^+}}{N_D} = \frac{1}{2}$

----- E_F (Suppose)

$\hookrightarrow \Rightarrow \frac{N_{D^+}}{N_D} \approx 1$

E_v —————

Now, if you look at $e F$ 1, so there for if you look at $e F$ minus $e D$, in this picture that I have show I am showing here, this picture I am showing here, you see that this number would there will there will there for will be very large negative, will be very, very negative number. Because, $e F$ is much, much smaller than $e D$, so it will be a large

negative number what is that mean. That means, that what; that means, is that this exponential quantity will be almost 0 this quantity will be almost 0.

And hence, what will we have this n ratio of N_D implies, this implies N_D plus by N_D is almost nearly equal to 1 that is what this implies. That means all donors will be ionizing, that you can clearly see that if I if i have; that means, I will be left with all only plus t plus or donor plus atoms here, at this level I will show p plus the electron, the fifth electron which was with it is gone it is ionized. Why it this happen, you must also understand this Fermi energy, represents average energy approximately not exactly precise the, but ave it represents approximately average energy of the entire electron system.

If, so you can see the since Fermi energy is much, much smaller than e_D there for, why should electrons continue to sit at e_D level, there we can lower the energy by going to e_F level. So, will we since e_F they, so also rough roughly speaking something like that, that there for they will rather live at e_F level, if that represents a average energy. So, there for, it is logical to think that all phosphorous atoms would be ionized and, as long as e_D minus e_F is substantial.

Let us say, more than three k t more than three k t in that case, what do will have is that all the phosphorous atoms will be completely ionized. Now, suppose Fermi energy is right here, this is if su if you suppose Fermi energy is moved right here, I am writing as e_{F2} let it little bit more cleanly here. So, it is Fermi energy is moved here, just where the e_D level is, if we move at up to e_D level, then what happens in that case let us look at this expression again, let look at this ex expression then we get e_F is equal to e_D ; that means, N_D plus by N_D this fraction is exactly half, this fraction is half.

So, if Fermi energy le leave riches the donor level then what; that means, is half of the phosphorous atoms, in silicon would have given their electron, away where as half of them would be half of them would have give given its electrons, half of them will be not ionized. That means, half of these would be carrying the electrons, with it e_D level was right there, let us go back again to one more thing e_F level just see it in this context, since even e_f was very low then where does the fifth electron go.

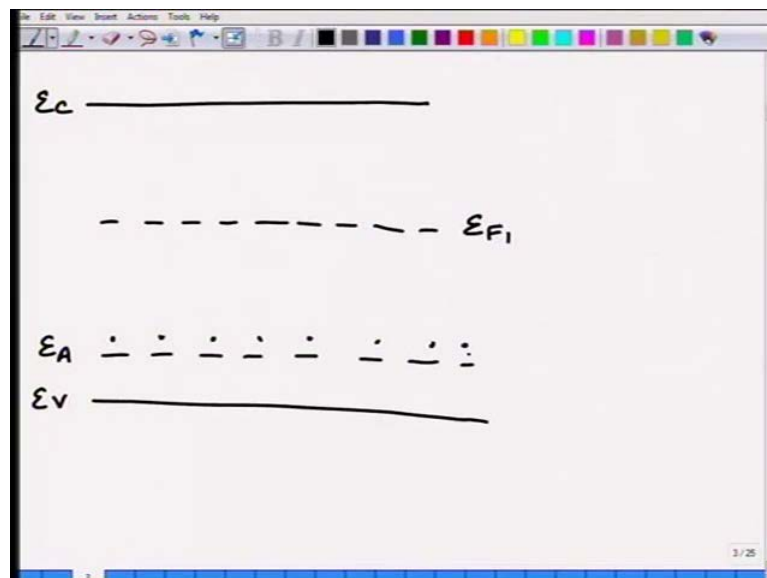
We think that this fifth electron off's goes to the conduction band, but then you can ask that, why what do you mean by that. Well remember, this average energy of electron

system this e_F is e_F which suppose to represents, if suppose to represents all the free electrons all the free all the electrons. So, electrons in the valance band and also the electrons, in the an and the electrons, in the conduction band, it suppose to represent that. So, if there is electrons in conducts, so some though we say some electrons have gone in there, this average of all these energies is e_F .

So, it is that electrons, in case of e_F is le lower than e_D , that they go into conduction band because, then they are there effect is being taking care of in the, average energy calculation. So, you can think like this, now similarly of course, if Fermi energy continues to move upwards, if a Fermi energy was still higher, than in that case this ionization would be, what will what will happen then, in that case e_f will be much, much greater than e_d when it, so happens.

In that case, this number can become very large and then what will have is $N_D N_D$ plus by N_D would then start going towards 0. In that case no atom will be of phosphorous in silicon, no donor atom would be ionized; that means, all of them neutral their fifth electron will be tied up to, the do pant itself. So, thus the picture you can look at how by moving Fermi energy the ionization occurs.

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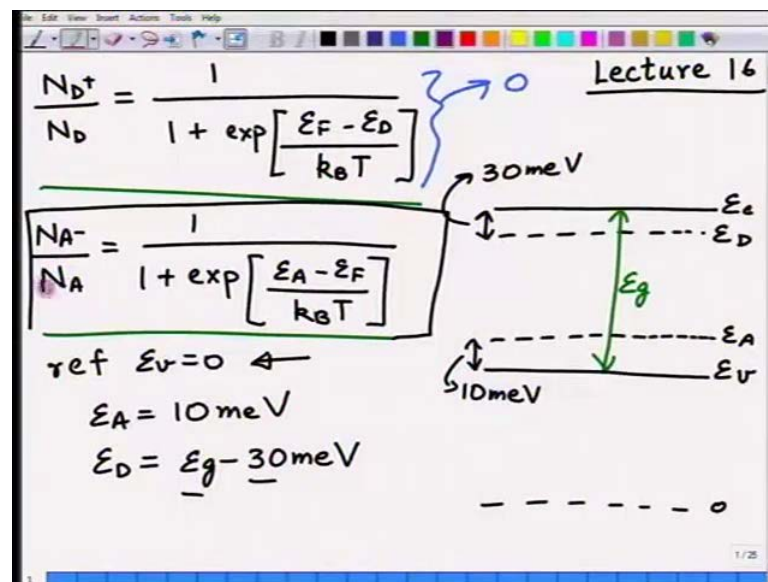


Exactly, same argument applies for val for acceptors also what I mean is here acceptor levels, this is e_c , this is e_v and this is e_A if, so then you can clearly see, if Fermi energy was high up here. Then what will happen e_F , if energy was higher up here, then what

will happen without looking at the without looking at the mathematical expression qualitatively you can see that, if Fermi energy was higher, that is the average energy of the electrons and e A level is lower.

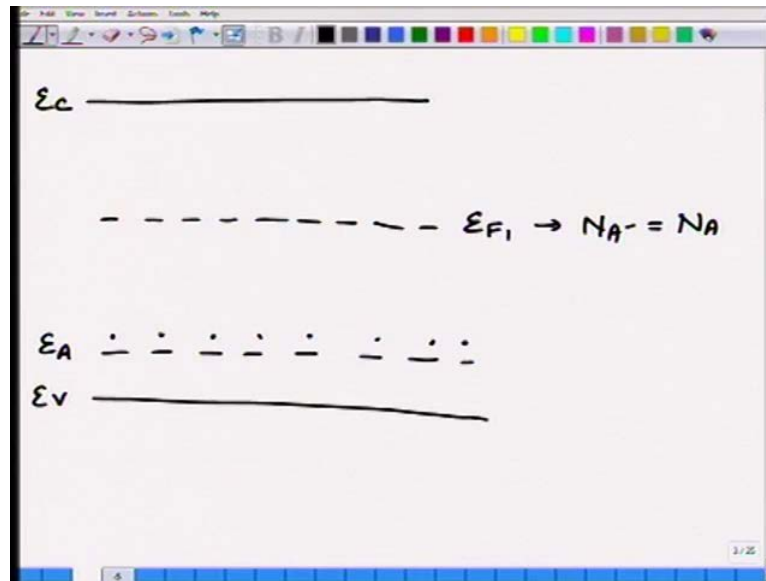
Than might as well these electrons, why would they are not occupied, they would then occupy lower energy state would be that they occupy the acceptor level. So, in that case if electrons are going to be found if Fermi energy is here, then we should be able to find electrons over at on e A sight. So, there for; that means, acceptor are ionized, so these acceptors are boron; that means, they are not they are in boron minus state, they have accepted the electron.

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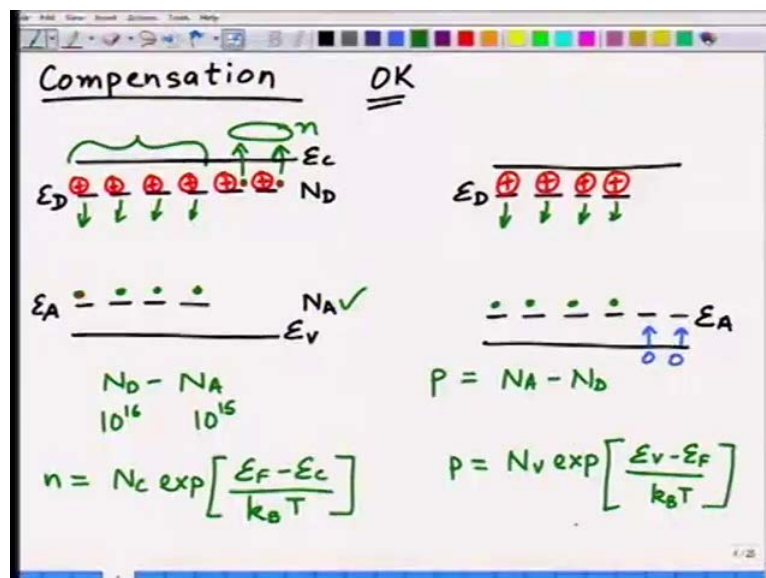
And that you can now see from this mathematical expression also, if e F is much, much greater than e A, then this difference is a large negative number and hence this is approximately 0 and then N A minus by N A.

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In that case N_A^- is equal to N_A ; that means, complete ionization, all the boron atoms are completely ionized. You can proceed this same argument, when ϵ_F Fermi energy is exactly at ϵ_A , then half the boron atom should be ionized and half would be neutral. And similarly, if you go further down in ϵ_F further down in Fermi energy, below ϵ_A level, in that case then no boron atom will be ionized, they all will be neutral in that case.

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So, that is the big picture which you can look at, second aspect before I move on, I will talk about compensation. Now, what I am going to do is, I am going to add two types of two types of both a two types of dopants, in a semiconductor n type, p type back to the same picture and now, let me add this time both acceptors and donors simultaneously. And I am showing fewer of, fewer of acceptor levels and more of donor levels similarly, I could draw something like this, more of acceptors and fewer of donors.

So, what would happen in this case let us in this case what will happen, suppose number of donors I have added is N_D that is the number of donors I have added, per centimeter cube and N_A is number of acceptors that I have added per centimeter cube. So, I ask the question, that let us say temperature is 0 K for temperature is let say 0 K what is the picture at 0 K. Now, notice what I am going to do is here is the phosphorous atom with its electron fifth electron here is phosphorous atom with its fifth electron.

Now, what happens to rest. So, let me draw the ion for even at 0 K the fifth electron, you see at least four of those in this schematic picture could have gone down here, they can system can lower its energy, but sending the electron four electrons down here, which were which I could have put here, here, here and here. So, even at 0 K the ground state would be that I would have, two atoms of phosphorus in this schematic, in this schematic two items of if use the different colors or hairs electron my electron here, electrons are right here.

So, here is the four electron this electrons. So, four of them have ionized, four of these electrons has gone down to boron side that way system can lower its energy, total energy of electron system can be lower, even at 0 K. So, if I now raise the temperature how many electrons can go to the conduction band, only these two only these two can go to conduction band, when I raise the temperature. If, so there for what do you expect, the electron number of electrons to be even at 0, first of all how many how many a how many dopants how many dopants N_D will be not ionized that clearly number is equal to $N_D - N_A$.

That means, even at 0 then; that means, if I a a this number is 10^{16} and this number is 10^{15} is what I added center of 4 fif 15 boron in silicon and to a 16 phosphorous in silicon, then $10^{16} - 10^{15}$ sorry $10^{16} - 10^{15}$, is the only number of donor atoms, which can

participate rest of the rest are already compensated. And hence, such semiconductors are called compensated semiconductors.

That means this is the only $n_D - n_A$ is the concentration n which I can expect which in this case is two, this is the n I can expect here, when I raise the temperature that is the that is the only amount I can get. I hope you can note see this. qualitatively which I will prove a exactly precisely. Similarly by same arguments, you can see here in this case where I have this phosphorous. So, many phosphorous levels e_D and which means n_D of these I have added here.

Now, in this case this is, this number is 10^{15} and this number is 10^{16} what will happen, these electrons will at least these four electrons will come down here even at 0 case, suppose temperature is 0 k. So, this four electron will come down here, so what do we see, we see that in this picture just like in this picture, all n_A were ionized and n_A number of donor same number, of donors have also ionized at 0 k.

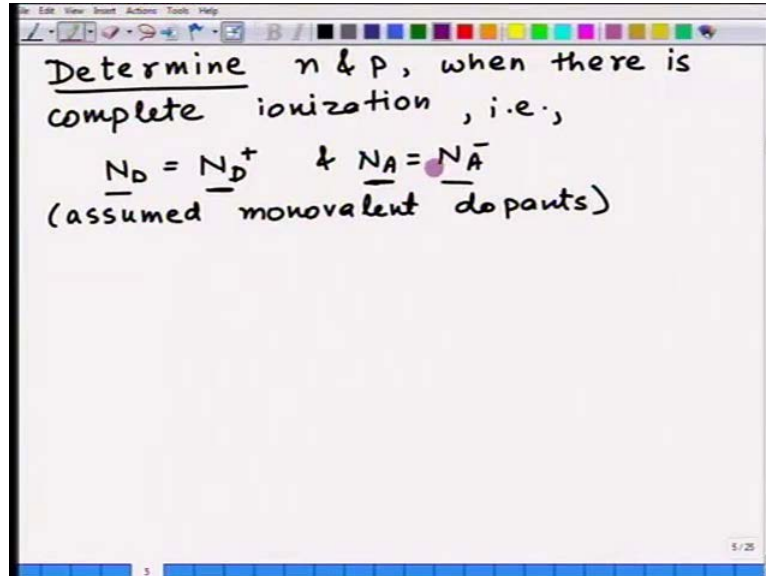
$N_D - N_A$ there for were not ionized, these are the at 0 k when I raise the temperature it is only these which can give there electrons to conduction band there for same argument here, in this case these electrons would have come down here, even at 0 k. So, there for now say N_D smaller than N_A in this case all N_D atoms will be ionized, whatever is the number of N_D that many number of acceptors will also be ionized boron minus.

So, the once which are not ionized are there for $N_A - N_D$, this is the number which is not ionized right here, these are the one which are not ionized they did not have taken the electron, they have not accepted the electron. So, boron is still in 0 charge state, when they accept the electron only then they will go to minus 1 charge state; that means, when I raise the temperatures, temperature the once which will accept their electrons and create hole will be equal to only this.

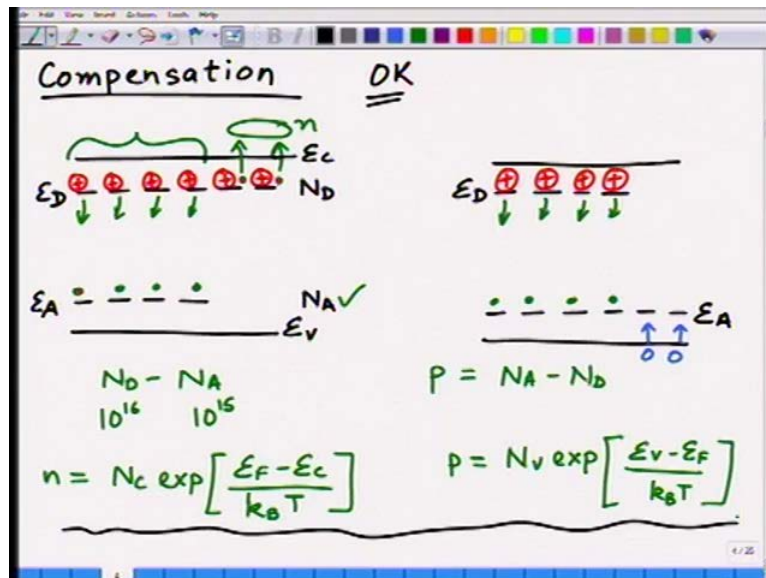
So, this is the number which can then, create the holes when I raise the temperature. So, these are the few things these are few things which qualitatively you can understand, now having given then explain explanation let us look at that then, let us look at what happens let us look at a this compensation idea mathematically also. So, what is our picture, our picture is a is something like this, I know I know my n is equal to n_c times e

to power $e F$ minus $e c$ by $k T$. Similarly, I know my p is equal to (no audio from: 23:32 to 23:42)

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Now, my problem is question is determine, so these are by assumptions, you want to determine n and p . When there is complete ionization, what is that mean; that means, when N_D is equal to; that means, whatever dopants have added, whatever donor I have added, phosphorous atoms I have added all of them have ionized, whatever boron atoms I have added in silicon, all of them have ionized. And we are assuming monovalent,

you are assuming monovalent implies that N_D is equal to N_D plus, plus N_D in 0 charge state.

Similarly, we mean by monovalent do pants as N_A is equal to N_A minus plus N_A in 0 charge state, they only two charge state phosphorous 0 and minus 1 charge state, that is what is implied by monovalent do pant, it gives only one transition and only for one charge state, to another charge state only exactly one transition is what it does. If, so then I am interested in doing what n and p 's, that n and p off-course is given by this expressions, which is just written here these are general expressions.

So, only thing were we know need to know is, where is the Fermi energy if we knew where the Fermi energy is, then we can solve this problem, just like an intrinsic case, in intrinsic case what did we do. In intrinsic case, what we did was that we wrote down the definition of intrinsic condition, that n is equal to p when, we wrote n is equal to p that expression then gave us, where the Fermi energy is we calculated $e I$, since that the Fermi energy level.

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Determine n & p , when there is complete ionization, i.e.,
 $N_D = N_D^+$ & $N_A = N_A^-$
 (assumed monovalent dopants)
charge neutrality
 $n + N_A^- = p + N_D^+$

Band diagram showing energy levels E_C , E_D , E_A , and E_V . A green bracket is drawn between E_D and E_A with a question mark.

Something similar will have to do here, will have to define now what additional we need one more additional condition alright. So, let us write down that additional condition, that additional condition if you look at in intrinsic semiconductor n equal to p was very natural for us to write, that exactly represented charge neutrality condition. That means, that you see when a electron is created, by thermal means in a intrinsic semiconductor,

correspondingly I must have positive charge also, if I did not then charge neutrality would not be maintained.

So, in to maintain the semi in a semiconductor pure silicon, then I there for demanded that n should be equal to p , that we did wrote in a logical way, but if you think about it that really represents charge neutrality. In this semiconductor also, where we have doped the charge neutrality may be should be maintained; that means, in silicon the boron I add the boron atoms are not charged, they come with three electrons; that means, they are in 0 charge state, that is what I have added in silicon I have added phosphorous, they come with five electrons, they are in not charged they are not charged they are not charged.

They are neutral that is what I have added in here, but when they go in there, some of them depending on where the Fermi energy is some of them will get ionize and some will not be ionized, when they ionize they have a charge. If they have a charge, but the net total charge must say will be 0 that is what the requirement on this semiconductor will be. So, let us write down this ex charge neutrality condition.

It is a what are the negative charges, my negative charges are n what else is my negative charge of course n_A minus, the boron atoms I have added and they have become minus charge. Some may be 0 charge state, but since I am assuming complete ionization in this case of course, there for I will assume the immediately I will assume that same thing, all of them are ionized just after this. But, this is general expression similarly, number of positive charge should be equal to positive charge and positive charge is because, of holes plus N_D plus, this by charge neutrality condition.

Now, let us look at the consequence of these assumption, I have made assumption that N_D is equal to N_D plus N_A is equal to N_A minus as a consequence of complete ionization what; that means, is that if this is e_c , this is e_v I do not need the details of where e_A and e_D levels are e_D and e_D as long as Fermi energy is somewhere, in this region here below e_D and above e_A then I have shown to you, that complete ionization will occur. As long as e_A is below e above e_A Fermi energy is above e_A and below e_D , then all the do pants will be ionized.

Let us say $3 k T$ from each of these points, these are a Fermi energy away from that, in that case that all the do pants will be ionized. So, since I have assumed that there is there

is complete ionization, I am assuming that Fermi energy is going to whatever Fermi energy we calculate, is going to come out in between these region.

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Compensation DK

$n = N_D - N_A$
 $10^{16} - 10^{15}$

$p = N_A - N_D$

$n = N_C \exp\left[\frac{E_F - E_C}{k_B T}\right]$

$p = N_V \exp\left[\frac{E_V - E_F}{k_B T}\right]$

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Determine n & p , when there is complete ionization, i.e.,

$N_D = N_D^+$ & $N_A = N_A^-$ ←
 (assumed monovalent dopants)

charge neutrality

$n + N_A^- = p + N_D^+$

$\rightarrow n + N_A = p + N_D$

if $N_D > N_A$

And if it is going to come out in this region, then clearly then, these expressions also for n and p are also valid in this region. In this region as you know $3 k T$ away from $e c$ and $e v$ these expressions for valid and hence, these expressions would be still be valid if Fermi energy is even $3 k T$ away from $e D$ and $e A$ levels. So, there for in that case

complete ionization will also occur and these expressions will be valid which I am about to use there for.

So, first thing I will write in this special case, while this is a general case and the special case there for I will write, in this special case n is equal to N plus N_A equal to p plus N_D . So, I will write it down like this because, N_A minus was equal to N_A and N_D plus was equal to N_D now let us, solve for a case if N_D is greater than N_A let us assume, u called assume N_A is greater than N_D that is find, will find what the result it gives us. So, let us continue on this next page.

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$$n + N_A = p + N_D, \quad N_D > N_A$$

$$n = N_c \exp\left[\frac{E_F - E_c}{k_B T}\right], \quad p = N_v \exp\left[\frac{E_v - E_F}{k_B T}\right]$$

$$np = n_i^2, \quad n_i^2 = \underline{N_c N_v} \exp\left[\frac{-E_g}{k_B T}\right]$$

$$n + N_A = \frac{n_i^2}{n} + N_D$$

$$n^2 - (N_D - N_A)n - n_i^2 = 0$$

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$$

$$p = \frac{n_i^2}{n}$$

So, n plus N_A is equal to p plus N_D alright and we, assuming N_D is greater than N_A you do the exercise, when N_A is greater than N_D just like I am proceeding here. If, so then I have another expression remember since, n is equal to e to N_c times N_c times e to power E_F minus E_c by $k_B T$ and p is equal to N_v times e to power E_v minus E_F by $k_B T$. And remember, product np we have shown is equal to n_i square, that we have that also we know, that must always be true.

So, I am going to write this expression as n plus N_A equal to n_i square by n plus N_D you see now since n_i square I know,, since I already know about n_i square is it is a material proper property recall n_i square is equal to $N_c N_v e$ to power minus E_g which is band gap by $k_B T$ that is what we had calculated. So, there for I know what n_i is

since, I know the band gap whatever temperature you are talking about, this is a constant, this is a material was the material specified is the density of states of that material.

These are numbers, which unknown and the expression for that was also given, if you know the effective mass then you know N_c and N_v alright. So, if that is the case then since I know n_i there for I can substitute in here, now you see is this expression only in n and I can easily calculate, what the value of n will be in this case. So, this is that is half simple it is, so let us do that, so I am going write this as $n^2 - N_D - N_A n + n_i^2 = 0$.

So, I have a quadratic equation here, so what is then equal to n is equal to $n = \frac{N_D - N_A \pm \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$. So, that is what n is equal to clearly you can see, that I must drop this minus sign, I must remove this minus sign from here plus. Because, this quantity will be this, quantity in square root will be greater than this quantity and since n is a positive number.

So, I there is no possibility of having a negative sign, so I am going to remove that now. So, I am going to remove this this sign and a only sign I can carry is plus, so that n is equal to, once I know n I can go back and determine, what e_F is I can substitute in here, what a and of course p is equal to then $n_i^2 = np$. So, you can determine n and p and of course, you can determine by where the Fermi energy is from either of these to expression, will give you same number does not matter, that will be that come out always right. Because, n_i^2 is this quantity anyways it will be used, a these two expression is being used there. Let us, but proceed further little bit.

When much, much greater than n_A greater than equal to $2n_i$ is greater than $2n_i$, in that case what happens I can drop this term this $4n_i^2$ I can drop this term because, $(N_D - N_A)^2$ will be much, much greater than $4n_i^2$ in that case. If, so then what will happen, then I will get basically n is equal to $N_D - N_A$ is a key result. And; that means, also that p is equal to $n_i^2 / (N_D - N_A)$ alright.

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when $(N_D - N_A) \gg 2n_i$

$$n = N_D - N_A$$

$$p = \frac{n_i^2}{N_D - N_A}$$

$N_A \gg N_D$

$$p = N_A - N_D, \quad n = \frac{n_i^2}{N_A - N_D}$$

Si $n_i = 10^{10} \text{ cm}^{-3}$ 300K

$$N_D = 10^{17} \text{ cm}^{-3} \text{ [P]}$$

$$N_A = 10^5 \text{ cm}^{-3} \text{ [B]}$$

If you were to if you were to do the same exercise, which I am asking you a which I have asked you to do that N_A is greater than N_D , then you solve for p in that case, in that case you solve for p and you will find by going through exactly same process process, you will find p is equal to N_A minus N_D . And you will find there for n is equal to n_i square by N_A minus N_D , since N_A is much, much greater than N_D . So, these are the two values will get and you can determine, where the Fermi energy is also based on any of these expression.

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Compensation DK

Energy level diagrams showing donor levels (E_D) and acceptor levels (E_A) relative to the conduction band (E_C) and valence band (E_V). The left diagram shows N_D donor levels and N_A acceptor levels, with n electrons in the conduction band and p holes in the valence band. The right diagram shows the compensated case with $p = N_A - N_D$ holes in the valence band.

$N_D = 10^{16}$ $N_A = 10^{15}$

$$n = N_C \exp\left[\frac{E_F - E_C}{k_B T}\right]$$

$$p = N_V \exp\left[\frac{E_V - E_F}{k_B T}\right]$$

Now, notice this result which we got, this n is equal to N_D minus N_A recall, that this is exactly what the compensate semiconductor was when we said. Since, in this case donors are greater than N_A . You will recall this picture of this picture we have drawn here, where N_D is greater than N_A I have shown six levels here and four levels here, if that happens even at 0 k we had said that at least four of these whatever is the number of N_A would have, would be ionized already and these equal number of donor atoms would also be ionized.

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when $(N_D - N_A) \gg 2n_i$

$$n = N_D - N_A$$

$$p = \frac{n_i^2}{N_D - N_A}$$

$N_A \gg N_D$

$$p = N_A - N_D, \quad n = \frac{n_i^2}{N_A - N_D}$$

Si $n_i = 10^{10} \text{ cm}^{-3}$ 300K

$N_D = 10^{17} \text{ cm}^{-3}$ [P]

$N_A = 10^{15} \text{ cm}^{-3}$ [B]

What is left at 0 k is N_D minus N_A which are not ionized, which eventually will give you the electrons, in the conduction band when you raise the temperature and that can then be in N_D minus N_A , notice, you got an that results now. We, have gotten that results that n is equal N_D minus N_A and of course conversely if N_A was greater than N_D , then precisely similarly, you are going to get the exactly same way you are going to get N_A minus N_D .

May be is worth looking at some numbers, in silicon for example, n_i as you know is about on the order of at room temperature, at 300 k that is I am talking about 3 and k 10 to power 10 per centimeter cube. Now, suppose you have N_D which was equal to, let say this we go to 10 to power 17 per centimeter cube this, is the phosphorous atoms you have added for example, this many and N_A is equal to let us say 10 to power 15 per centimeter cube, this many boron atoms you added.

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$n = 10^{17} - 10^{15} \approx 10^{17} \text{ cm}^{-3}$

$p = \frac{10^{20}}{10^{17}} = 10^3 \text{ cm}^{-3}$

Caution:

$n \rightarrow 10^{10}$

$p \rightarrow 10^{10} ?$

Then what is n equal to n clearly there for is approximately equal to 10 to power 17 minus 10 to power 15, which is approximately equal to 10 to power 17. That's what the value of n will be, then what will be the value of p value of p will be there for equal to 10 to power n i square 20 divided by 10 to power 17 which is equal to 10 to power 3, here is this number let me write down this more cleanly here, 10 to power 3. Now, this requires little bit of discussion.

Now, I am going to put this n red here caution, this is the matter which confuses some students and often I get this question. That calculation we have done shows n is equal to 10 to power 17, this students often I believing to accept because, this you see that I have added 10 to power 7 for 17 phosphorous atoms. So, since they can give the electron to conduction band.

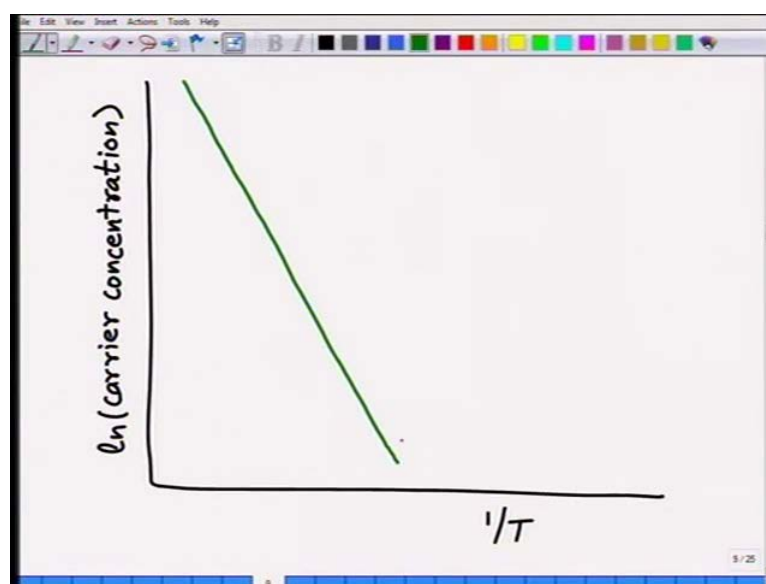
So, one can ace accept the fact that there are 10 to power 17 number electrons, in the conduction band. What call this problem is this concentration of holes, why the argument made is as follows that when you have a intrinsic semiconductor, since we are saying n i is 10 to power 10; that means, at least 10 to power 10 electrons are here, and 10 to power 10 holes are here. That is what it means, intrinsic electron concentration that if the semiconductor were intrinsic it was not doped, in that case I would have gone an 10 to power 10 electrons here and 10 to power 10 holes here, this p and this n would have been this number.

Then I have taken this semiconductor at same temperature 300 k and added now, some phosphorous and boron does not matter boron is there or not because, any way all boron is compensated right, upfront anyways it is like 10^{17} and if you added 10^{15} both 10^{15} boron, then $10^{17} - 10^{15}$ of the phosphorous atoms are anyway, gone to compensate boron. Once they, have compensated then that is out of our calculations, only part, only thing which we going to play part is what is remaining phosphorous phosphorous atoms.

So, anyways, so whether you added only phosphorous atom or you added boron that is not the issue, boron also that is not the issue let, so but, point is that if you added do pant it is a do pant dominated. Then in intrinsic semiconductor itself you have 10^{10} electrons in 10^{10} holes when you added phos do donor atoms then n is increase 10^{17} , that is acceptable that y z that holes have become, only 10^3 even in intrinsic case, they were 10^{10} why did they became less, at least they should have been 10^{10} , that is one problem which I often here from students.

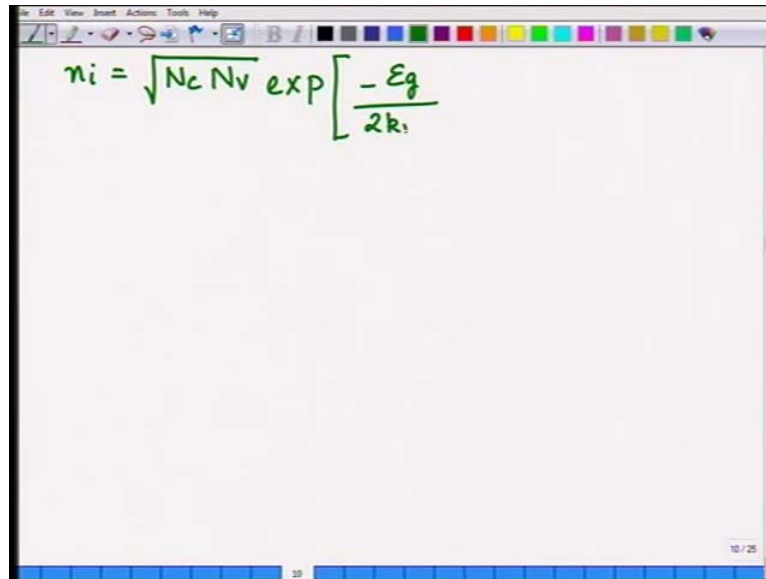
So, I would like to clarify that point to you in a minute. Then us this I will try to clarify to you through a plot, and then hopefully a keep this question I mind, this I am posing as a question which may be in your mind also and I will try to clarify it, but I will do this do, so through an plot, through a plot.

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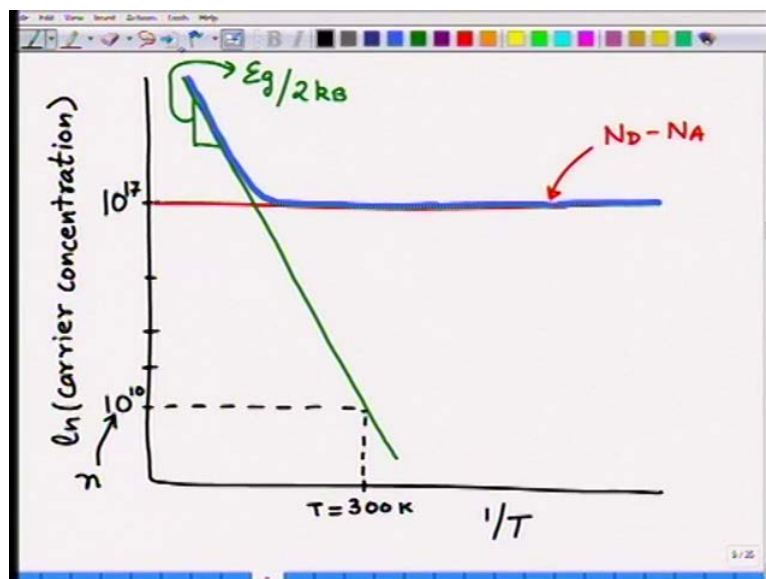
So, let me re plot something which was talked to you earlier, if I plot $\ln n$ versus $1/T$ versus \log of, so is less space here carrier concentration. Whether n or p does not matter n or p whatever, whichever way is more dominant, whichever is more dominant is what we are plotting. You will recall, the way we have plotted was that here is a semiconductor silicon or whatever it is we had said that intrinsic electron concentration.

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$$n_i = \sqrt{N_c N_v} \exp\left[-\frac{E_g}{2kT}\right]$$

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This n_i varies as $N_c N_v$ which has a very weak temperature dependence T^3 to power 2 a type of temp temperature dependence $e^{-E_g / 2k_B T}$ that is

what we had written. And there for we had said that the slope would be equal to $e g$ correspond to $e g$ by $2 k B$ that is what we had said. Alright now let us say $3 \text{ unit } k$, this is silicon if you looking at 300 k let us say, then what happens. At 300 k here is let us write down a number here somewhere, let us mark out the number somewhere here is 10 to power 10 let us say, in case of 10 to power 10 which means I am plotting on log scale, but this is n , this is n equal to this much n_i or n carry concentration is equal to 10 to power 10 percent limited cube.

And if you plog a log of natural log of it, then it is actually natural log of 10 to power 10 that is what this number represents here alright. So, if that is the case then right here and this temperature is 300 k , this temperature is corresponds to T corresponds to equal to this. So, I am plotting 1 by T of course on this scale, but the temperature here corresponds to at this point 300 k . So, this is 10 to power 10 is n_i , in case of silicon which is what we have used here.

But, now notice that what happens to my n , my n is equal to N_D minus N_A and it is independent of temperature, it is independent of temperature it is about 10 to power 17 in this case, in this example which I have constructed here. So, let us plot this n here now also will separate line on the same graph, so if I do, so then let say let us say that here is a somewhere here is 10 to power 17 , let us say and number being repress it is an log scale. So, this is 10 to power 11 10 to power 12 etcetera etcetera and, so on or maybe I take it even higher here. So, here is 10 to power 17 right here.

If, so then let me plot this line here. So, this is line is right here, what is this line representing, this line is representing N_D minus N_A about 10 to power 17 in this case which have been added in our example, which we have taken. Now, what is in this semiconductor carrier concentration as a function of temperature, clearly that profile will look something like this, meaning there by, this is blue line basically if it the semiconductor was intrinsic, then it would have followed green line.

But, when semiconductor is when semiconductor is doped also then the concentration will follow blue line, why clearly now this is where you can clearly see that at 300 k what happened was I had N_D minus N_A as 10 to power 17 .

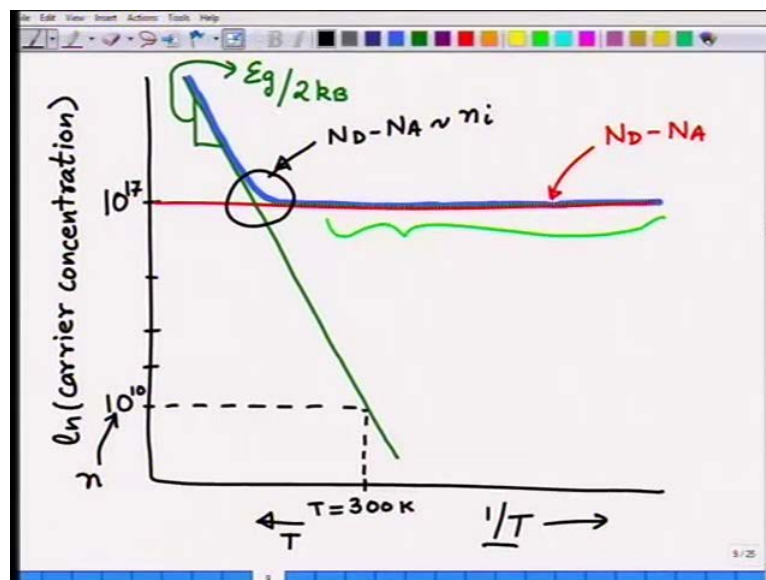
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$$n_i = \sqrt{N_c N_v} \exp\left[\frac{-E_g}{2k_B T}\right]$$

10^{17}

That means, that when I had this semiconductor right here, I had at 300 k 10 to power 17 electrons could go up into the conduction band, intrinsic concentration anyways very, very small it does not dominate.

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But, when we go to temperatures higher and higher this way is temperature is increasing, temperature increasing $1/T$ what I am plotting in this direction. So, temperature is increasing in this direction, so either it go to higher, so I start it somewhere here and I go for higher and higher temperatures, as I go to higher temperatures. Remember around

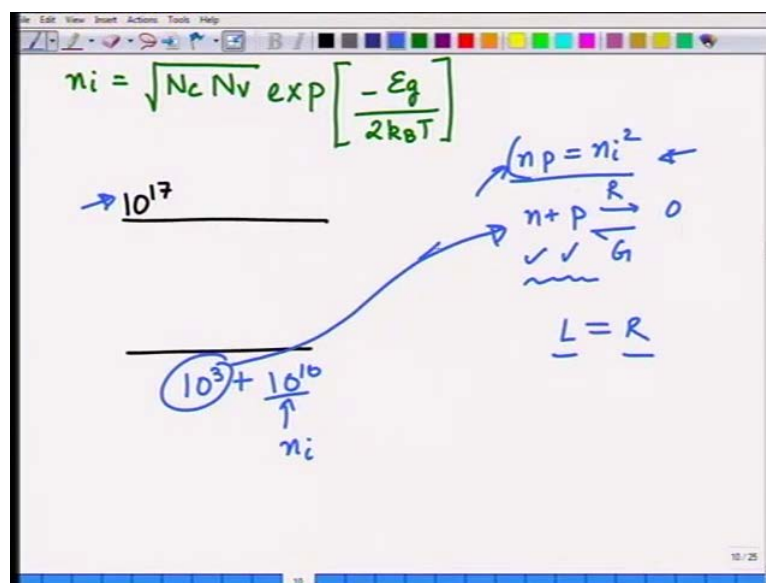
this area, what happens in this region your $N_D - N_A$ is now comparable to n_i . In this point, they equal $N_D - N_A = n_i$.

And what was my assumption here notice $N_D - N_A$ should be much, much greater than $2n_i$ or n_i does not matter, two factor of when we talking about order of a magnitude it matters does not play serious part. So, $N_D - N_A$ should be much, much greater than n_i . So, clearly that is true in this region, in this region it is true. In this region where n_i is this green line and $N_D, N_D - N_A$ is this red line and clearly red line is much, much great higher greater than this blue line remember this is log scale.

So, this much, much greater, but in this circ in circle region this two numbers start becoming equal and what happens, while this number will continue same way. But, now since temperature is large enough, the intrinsic carrier concentration itself is very large and there for carriers are generated through intrinsic means rather than dope dope semiconductor.

That means, a dope semiconductor if you dope it and you, but you continue to raise the temperature, at some point of time it starts behaving like an intrinsic semiconductor. So, that is the point that this is 10^{17} , when 10^{17} and when temperature is large enough, then anyway when intrinsic co carrier concentration becomes 10^{17} or higher, then this number becomes even large and starts following green line.

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Because, of intrinsic region fine, but always remember this that at any given temperature even at 300 K, when $n = 10^{17}$ I do have a law of mass action. It is; that means, that recall that n plus this came from n plus p recombination and generation and the generation process, this equilibrium between these two processes is dictated this, that n product p should be constant. If you; that means, increase this then you must decrease this.

You cannot there for argue that even at 300 K when p was when n became 10^{17} for the reason shown, they cause it is doped semiconductor. So, an n became 10^{17} I did not 300 K, although intrinsic semi, intrinsic means could have given 10^{10} if it semiconductor intrinsic, then p would have been 10^{10} the holes would have been 10^{10} . But, know they cannot be because, the in thermal equilibrium this condition should always be maintain that $n p$ is equal to n_i^2 , square that condition is requirement of thermal equilibrium.

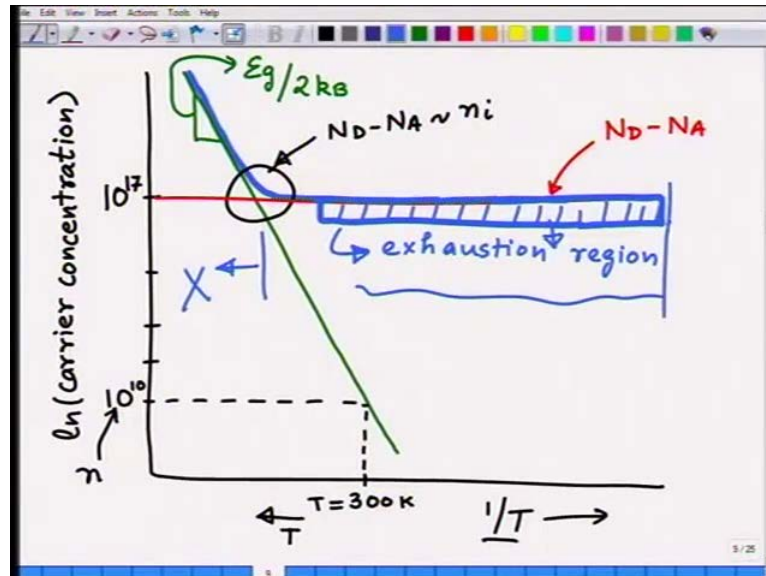
Just in the same way, as you have studied in your chemistry course, where if you take two reactants and if you increase one, if you increase one then other one concentration of other must be decrease, to keep the rate of reaction the same forward in an cu maintain the equilibrium. You to maintain a equilibrium either whatever is the on the left hand side of the equation and right hand side equation, chemical equilibrium either if one quantity on left hand side goes down then other must increase or proportionately on the right hand side everything must increase correspondingly.

Since, in this case it is annihilation of electron and hole that cannot happen, there for only option left is if one increases other must decrease. And that is the reason it requirement of thermal equilibrium that is leading to that thermal equilibrium is leading to that p must become 10^3 , you must never think of concentration in valence band as being equal to this 10^3 plus 10^{10} . Because, that is coming from n_i means, this is a common mistake, which students make that they think that n_i , n_i is because, of this reason it because, of this reason.

And because, of this n_i anyways n_i was 10^{10} at at room temperature, there for add that that number also to it, that will be wrong. This is the hole concentration in valence band because, you are required to maintain this particular equilibrium because,

this particular equilibrium, must be maintain and hence you must have product $n p$ equal to constant and hence this must become 10 to power 3 .

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So, that is the point now, this is what is called exhaustion region. This is where, really the semiconductor semiconductors work. So, this is clearly or or this is or maybe I can go further may be erase this one, draw a little nicely now this is the region this is the region, this is the exhaustion region for a semiconductor. A semiconductor would normally we used in electronics in this region, so I will finish my class here, and I will show eventually you understand the point that whether semiconductor is doped or un doped.

If temperatures increases beyond this point if it goes temperature on this side, then semiconductor will behave like intrinsic semiconductor. You also know devices are made when you have a N junction and a P junction, you make a P N N P junction, but in temperature range is on this side, since semiconductor starts behaving like an intrinsic semiconductor. So, it does not matter that is N type or P type do pants play no role, so where if you try to make a junction and heat it up too much, it go to high temperatures.

Then n characteristic and p characteristic are lost and there for junction is lost all you have is a semiconductor, intrinsic semiconductor. And see hence devise will stop operating so; obviously, you cannot operate in the intrinsic range, semiconductors are made n and p type that you can clearly do, only in the exhaustion region.

So, I will stop my lecture here, next le lecture I will show I will talk you about fees out carrier, fees out characteristic and that will be last lecture on this a equilibrium carrier concentrations. And in that case I will show you, how temp what happens if you go low, so I am talk I have talked about high temperatures what will happen, from this exhaustion region, if you go to still lower temperature what will happen that I will speak about, I will talk about in the next lecture.

Thank you.